

**PROFESSIONAL TRAINING IN MATHEMATICS EDUCATION: A STUDY OF  
PROGRAMMES, PRACTICES AND PROSPECTS**

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PROGRAMMES, PRACTICES AND PROSPECTS**

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**Thesis submitted to the Faculty of Education in fulfilment of the  
requirements for the Degree of Doctor of Education in the  
Department of Mathematics, Science and Technology Education at the  
University of Zululand.**

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**December 2012**

## DECLARATION

I, Khetha Bonginkosi Biyela hereby declare that ***“Professional training in mathematics education: A study of programmes, practices and prospects”*** is my own work both in conception and execution and that all the sources I have used or quoted have been indicated and acknowledged by means of complete references.

Signed by -----

on the ----- day of -----2012

## ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to the people and organisations mentioned below for their invaluable assistance they provided to me during this study. The order in which they are listed does not imply ordinal strength of the assistance they provided to me.

- Dr D.C. Sibaya at the University of Zululand, my supervisor for her constructive criticism in writing.
- Professor P.T. Sibaya, my co-supervisor for his constructive criticism and guidance.
- Ms S. Ntuli for giving her time to typing the manuscript.
- The staff members of the University of Zululand library for their help during my research.
- Mr R. Wilkinson from University of Zululand- Public Relations office for editing my study.
- The university of Zululand Research Committee for giving me financial assistance.

- The gatekeepers of the teacher training institutions in KwaZulu Natal for allowing me to conduct the research in their institutions and their co-operation in the administration of the instruments.
- My wife Zenzile Mamma Biyela (MaMhlongo) who supported and encouraged me throughout my studies. Her sacrifices and motivation contributed a lot to the completion of my study. Also our children Mbali, Nomvelo, Nhlakanipho and Mnqobi for being patient to me during the study.
- Finally, I thank God, Almighty who made it possible for me to accomplish my goals.

## ABSTRACT

The present study investigates the level of mathematics content knowledge acquired by pre-service teachers at the point of exit in their training programmes. The study was conducted on mathematics pre-service teachers. The purpose of study is to determine the level of mathematics attained by the pre-service. It is surmised the level of mathematics knowledge can influence learners' performance in mathematics. The teachers' lack of adequate mathematics content knowledge to teach mathematics proficiently is allegedly the source of poor attainments in mathematics education. On the basis of this perception, the extent to which pre-service teachers are ready to teach must be established. It was therefore compelling to conduct the present study to find answers to the following questions: What is the level of mathematics content knowledge the pre-service teachers possess at the point of exit of their training programmes?. How does the level of mathematics content knowledge possessed by pre-service teachers influence their teaching practices? This assertion forms the basis of the aims of study.

To achieve the aims of the study, a standardised Mathematics Proficiency Test was administered to a sample of final year prospective teachers from two universities in South Africa. Practice teaching assessment and the comparison of high school and teachers education syllabi was also done to achieve the aims of the study. The results revealed that very few pre-service teachers command adequate knowledge of mathematics as they exit their training programmes. The pre-service teachers' knowledge in three sections of mathematics namely,

algebra, trigonometry and geometry is the same. The study also revealed that there is no relationship between the achievements in mathematics content and achievements in teaching practice. Furthermore the study revealed that the teacher training programmes cover most of the themes that are covered by high school syllabus.

The discussion of findings coupled with their implications is highlighted. The avenues for future research are indicated.

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# CHAPTER ONE

## MOTIVATION OF THE STUDY

### 1.1 INTRODUCTION

The current reform efforts in the South African education system have endorsed certain ideals and expectations in mathematics education. While such expectations also apply to other subjects, they are not as clearly pronounced as they are in mathematics education. This is obviously to be expected, because in the South African school system, and in the eyes of the community, mathematics is a high status subject. As part of the reformation in education, educators are expected to implement constructivist and problem-centered approaches to teaching (Onwu & Mogari, 2004; Moll, 2002). In the case of mathematics teaching and learning the reform process is underscored by the desire to shift from a procedural and rule-based (syntactical) view to a conceptual view of mathematics (Benken & Brown, 2008). This transformation seems to challenge educators' content knowledge, pedagogy and the methods of teaching the subject. In the light of this background, it is compelling to investigate mathematics teacher education programmes and practices in the South African situation.

There is a prevalent lack of competence in mathematics that results in poor achievement among South African learners (Howie, 2003; Moloji & Strauss, 2005; van der Walt & Maree, 2007). Outdated teaching practices and lack of basic content knowledge on the part of educators have been reported as some of the causes of poor attainments in mathematics (Mji & Makgato, 2006;



Taylor, 2009; Howie, 2003). The present researcher certainly perceives problems in the teaching of mathematics to be related to educators' lack of subject content knowledge, pedagogical knowledge and the methodology of teaching mathematics.

In this study, the researcher does not confine content knowledge solely to the understanding of the abstractions of mathematics content but includes the ability to connect it to the learners' immediate environment and experiences so that they can make sense of what the teacher intends teaching. Very often in teacher training institutions the emphasis is put on pedagogy and methodology of teaching. In South Africa it is undoubtedly true that most prospective teachers suffer from a weak mathematics content background. The Presidential Education Initiative (PEI) Report points out that South African curriculum initiatives have focused mostly on teaching methodology at the expense of underpinning conceptual knowledge that needs to be taught and learned (Brodie, 2004:66). In the same way the present researcher maintains that pedagogy and methodology should supplement mathematics content knowledge. What this means is that pedagogy and methodology should be built on a strong foundation of mathematics knowledge.

Since the introduction of the outcome-based education (OBE), National Curriculum Statement (NCS) and Curriculum and Assessment Policy Statement (CAPS), a number of workshops and seminars have been held for mathematics educators. These workshops are primarily aimed at equipping educators with methods and strategies of teaching to achieve the stipulated outcomes. The assumption seems to be that educators have adequate content and pedagogical knowledge; however, this is not the case for some educators.

Contrary to this assumption, several studies reveal that despite content, pedagogy and methodology courses taken by mathematics teachers in their training programmes, they often leave these programmes with the same knowledge base as when they first entered into teacher training (Benken & Brown, 2008; Seaman, Szydlik, Szydelik & Beam, 2006). Arising out of this, the implication would be that very little if any effort is made by teacher training institutions to advance students' knowledge of mathematics. Similarly, very few if any studies have been conducted to gauge the level of teachers' mathematics knowledge at the point of exit. It is difficult to measure the extent of change brought about by the exposure of pre-service teachers to training programmes, but nevertheless it seems that some of the programmes are hardly able to equip pre-service teachers with essential mathematics knowledge.

The shortage of studies that evaluate the pre-service teachers' level of mathematics knowledge at the point of exit and the alleged failure of some programmes to equip pre-service teachers with adequate mathematics knowledge necessitated the present study. This study intends finding solutions with regard to the continuous production of mathematics teachers who seem to be inadequately trained. The present researcher strongly believes that there is a need for an intervention in the training of mathematics teachers. A strong collaboration between the Faculty of Education and the Faculty Science are possibly needed to strengthen the desired intervention so as to ensure the acquisition of advanced knowledge of mathematics content and its teaching practices.

In South Africa there may be some criticism of the role played by the current teacher education programmes in reforming mathematics education. However, the insurmountable difficulty in recruiting mathematically competent students into mathematics education should also be noted. Pournara (2005) asserts that the majority of the pre-service mathematics educators would not have been accepted into a science degree programme mainly because their matric mathematics results are extremely low. What this means is that the prospective mathematics educators enter into teacher education programmes with little or meagre conceptual understanding of school mathematics. This, according to the Department of Education (DoE, 2001), continuously produces educators who perpetuate the cycle of mediocrity because some of their matriculants also join the teaching profession after completing their schooling. A revolving door effect is established and maintained by this cycle. Consequently, the system sustains itself as a self-fulfilling prophecy.

Teachers often put emphasis on the manipulative skills and the routine application of algorithmic procedures. This problem is further compounded by the fact that when the prospective teachers enter into mathematics education, their mathematics background is often abstract (Nicol, 2002). Arising out of this, the researcher argues for closer collaboration between schools and teacher training institutions so that informed mathematics programmes can be designed for pre-service educators. In addition to this, it remains the responsibility of the university to change the mediocre mathematics teaching and learning standards by providing the students with a strong mathematical content knowledge (Sam, 2005).

However, a strong content knowledge or knowledge base without pedagogy may be good, but this is not sufficient to enable a teacher to teach learners to construct mathematics knowledge and to use a problem-solving approach. Constructivist and problem-solving approaches are central to the current reform in mathematics education (Moll, 2002). With this in mind, knowing mathematics for teaching transcends the ability to manipulate and apply mathematics in scientific disciplines. Knowing mathematics for teaching involves knowing how learners construct knowledge and meanings in mathematics learning. Furthermore, it includes knowing how learners' intellectual development affects their mathematics learning. As observed by Ball and Bass (Adler & Davis, 2006), the way of knowing and using of mathematics by the teachers differs from the way mathematicians hold and use mathematics.

In mathematics teaching, besides knowing the content, it is very critical for a teacher to know how to impart content knowledge to learners. This demands that a teacher should have knowledge of learners' cognitions, relevant practice activities and teaching strategies in order to teach meaningfully. According to Fennema and Frenke's model, effective teaching and learning of mathematics occurs when the teachers' knowledge of content of mathematics and pedagogy and the students' cognitions, context-specific knowledge and beliefs are integral parts of the teacher preparation programmes. Thus, these five components of Fennema and Frenke's model of teachers' knowledge (Mohr, 2006) need to be advocated when the universities implement teacher training programmes.

In South Africa, the current mathematics teachers, particularly those who have recently emerged from teacher training institutions, often exhibit common practices. These practices are consistent with Crespo (2003), Hristovitch and Mitcheltree's (2004), findings namely that middle school teachers' instructional activities do not encourage achievement of conceptual understanding of mathematical ideas. If the pre-service teachers' mathematics knowledge is procedurally strong and conceptually weak, as Li and Smith (2007) claim, failure to create conceptual understanding is obviously expected.

Very often pre-service teachers, like middle school teachers, fail to arrange mathematical topics coherently and as a result they rely very much on the textbook suggested sequence of topics and subject matter (Hristovitch & Mitcheltree, 2004; Turnuklu & Yesildere, 2007). Presumably, the tendency brought about by such practices would be to spend much time on the topics that the teacher understands the most and to ignore or omit those that are cognitively demanding, thereby not covering a meaningful amount of work.

Ideally a teacher's mathematics knowledge should transcend knowing the content of the textbook. Teachers should be able to contextualize mathematics conceptions in order to facilitate its application even outside the classroom situation. However, in this regard it seems that, like in some other countries, South African teachers' lack of mathematics knowledge is a daunting factor. When teaching, pre-service and novice teachers ask closed and factual questions; mathematical tasks they give to learners are of low cognitive demand focusing on memorization and procedures (Lloyd, 2006; Crespo, 2003). Contrary to the highlighted weaknesses in the current mathematics

teaching practices, the reform in mathematics teaching and learning requires teachers who are committed and competent to encourage learners' ideas (DoE, 2002).

After 1994 the major political changes in South Africa culminated in the transformation of the curriculum which was aimed at addressing the injustices created by apartheid in education (DoE, 2003; CCTELA, 2004). Mathematical literacy was introduced as alternative to mathematics in secondary schools with the addition of new topics to the mathematics curriculum, without prior training of teachers to face the challenges of this reform. For example, transformation geometry was introduced in grades 11 and 12 and this topic might be difficult for some educators to conceptualize. Such changes inspired development in the mathematics curriculum programmes and materials, but did not provide teachers with enacted strategies necessary for successful implementation of the desired reform (Manouchehri & Goodman, 2000). As a result, this exacerbated the already prevalent lack of competence in mathematics.

A number of studies (Manouchehri & Goodman, 2000; Nicol, 2002; Leikin & Levav-Waynberg, 2007; Nicol & Crespo, 2006; Ball & Bass, 2002) suggest that the teachers' background knowledge in the subject matter affects how they present this knowledge to their students. However, Brodie (2004) asserts that in South Africa the teacher preparation programmes tend to focus more on methodology and pedagogy instead of the subject content. Furthermore the transformation process in South Africa is characterized by uncertainty on the part of the teachers, which results in poor exposition of the subject content. On this basis, the present researcher speculates that teacher training

institutions, even in the post-apartheid era, may still produce mathematics teachers with doubtful mathematics competences. This hypothesis is consistent with the view expressed by Nxesi, the former president of South African Democratic Teachers' Union (SADTU), with regard to poor quality of mathematics teaching in South Africa. He says that it is amazing that a national strategy for the professional development of teachers after ten years still does not exist (De Vries, 2005). With this in mind, and noting the poor teaching standards in mathematics, it is compelling to conduct an investigation into the current mathematics teacher preparation programmes. The idea is to determine the level of mathematics content the prospective teachers receive in teacher training institutions.

## **1.2 STATEMENT OF THE PROBLEM**

The legacy of apartheid in the South African education system continuously impacts negatively on the quality of teaching and learning of mathematics. Most of the teachers who currently teach mathematics were trained poorly in under resourced colleges of education, while they had already been exposed to a very low level of mathematics content at school (DoE, 2003). Usiski (Lott & Souhrada, 2000) pointed out that in the old system of education, in general, students were not taught enough mathematics in high school and those who took three years of the teacher college preparatory mathematics programme did not learn the uses of mathematics. In the same way, no remarkable, if any, improvement is observed from transferring the three year teacher preparatory programme from colleges of education to a four year programme at university. In order to address this problem there is a need to re-examine mathematics teacher education in South Africa.

The school mathematics curriculum policies that have been adopted by South Africa focus heavily on learner-centeredness and an activity based approach. Teachers are expected to develop their own learning programmes (DoE, 2002). When teachers develop the learning programmes and materials they have to consider and build on learners' knowledge (Nkhoma, 2002). This demands a detailed knowledge-base from the teacher. For instance, teachers must have a strong knowledge of the content and pedagogy. However, it is not clear how mathematics programmes at teacher training institutions address the issue of lacking content knowledge in mathematics teachers.

There seems to be no in-depth conceptual understanding of mathematics content on the part of the teacher because their teaching is more procedural and rule-based (Nicol & Crespo, 2006). Additionally, the present researcher concurs with Turnuklu and Yesildere (2007), who claim that it is not possible to teach mathematics without having deep mathematics knowledge. With this in mind, teaching based on learner-centeredness and a constructivist approach cannot yield good results unless mathematics teachers are educated with a strong basis in mathematics knowledge as well as pedagogical content knowledge. While the literature regarding teacher preparation (Pournara, 2005; Lott & Souhrada, 2000) indicates that pre-service teachers often do not have adequate mathematics knowledge when they join the teaching profession, it does not indicate how the institutions overcome this problem. The present study will focus on how teacher training institutions address the issue of content knowledge in mathematics teachers.



Most of the weaknesses noted earlier in this thesis are manifested in the way these teachers present their mathematics lessons. For example, Crespo (2003:266) observes that there are disconnections between pre-service teachers' mathematics content knowledge and what they do in practice which eventually hinder effective learning. This creates uncertainty regarding whether it is the pre-service educators who find it hard to put in practice what they have learnt in mathematics education courses or whether it is the syllabus which they teach that does not create connectivity. The teachers' knowledge of mathematics guides them to determine the links between the subject matter they intend teaching and teaching practices (Leikin & Levav-Waynberg, 2007).

One of the aims of the Revised National Curriculum Statements (RNCS) is to promote competence in mathematic knowledge among the teachers (DoE, 2002; DoE, 2005). However, regardless of whether the teacher is doing in-service training or s/he is still doing a teacher preparation course, it is not stated how competence in mathematics knowledge is achieved. The view of mathematics teaching as heavily dependent on the teachers' content knowledge is evident in a comparative study of pre-service mathematics teacher education in Malaysia and in China (Sam, 2005). It is concluded that China produces mathematically competent teachers compared to Malaysia. The reason for this difference is that mathematics education programmes in China focus more on content components while Malaysia's focus more on pedagogy and methodology. Like Malaysia, South Africa's curriculum initiatives are assumed to focus mostly on teaching methodology instead of strengthening the conceptual or content knowledge (Brodie, 2004).

While it is the contention of the researcher that sound mathematics knowledge is essential for good teaching practice, the role of pedagogical knowledge should not be underestimated. This further substantiates the suggestion that effective mathematics teaching results not only from the educator's content knowledge, but also from the interplay between his/her knowledge of learners, their learning and teaching strategies used to enhance learning (Mohr, 2006). When teaching mathematics, it is very critical for a teacher to know his/her learners' diversity in the class. This means that when teachers do not understand how learners differ in their learning approaches, mathematics teaching will not yield the desired outcomes (Downey & Cobbs, 2007). Furthermore Hill and Ball (2004) and Brodie (2004) contend that teachers' mathematics knowledge should transcend common knowledge of content, either conceptual or procedural, to acquire specialized knowledge of content that enables the teacher to determine whether his/her teaching practices lead to meaningful learning. However, the weaknesses in teaching and learning of mathematics outlined in the present thesis indicate that mathematics teachers in South Africa have little or meagre knowledge of mathematics for teaching. This raises questions about the relevance of the current mathematics education programmes to mathematics teacher preparation.

On the basis of poor teaching and attainments in South Africa, it is compelling to conduct a study on mathematics teacher preparation programmes in order to find answers to the research questions and to address some of the weaknesses that are manifested in the teaching practices of pre-service mathematics teachers.

### **1.3 RESEARCH QUESTIONS**

This study attempts to find answers to the following critical questions:

- 1.3.1 How much mathematics content knowledge do teachers possess at the point of exit in their training?
- 1.3.2 What is the impact of pre-service teachers' content knowledge of mathematics on their teaching practices?
- 1.3.3 What is the nature of the content covered in mathematics education teacher preparation programmes and the high school mathematics?

### **1.4 OPERATIONAL DEFINITION OF TERMS**

It is necessary to define the terms that appear in the title or topic of the study. In order to avoid confusion in this study, the terms have been defined operationally as follows:

#### **1.4.1 Professional training**

In this study, professional training shall mean a programme for educating the prospective mathematics teachers for a professional qualification in teaching.

#### **1.4.2 Mathematics education programme**

In this study, mathematics education programme shall mean mathematics content designed for preparing teachers, that is, syllabi for mathematics education and relevant activities. This includes studies of the strategies that enable the teacher to teach mathematics meaningfully to learners.

### **1.4.3 Practices**

In this study, practices shall mean all activities that teachers do regularly during lesson preparation, presentation and a repertoire of related daily routine.

### **1.4.4 Prospects**

In this study, prospects shall mean all that lies ahead for teachers in preparation.

### **1.4.5 Pedagogy/methodology of teaching**

In this study, pedagogy shall mean knowledge of how to teach mathematics. This knowledge involves understanding of the content, the nature of the learners and curriculum, and knowing strategies to teach mathematics effectively.

### **1.4.6 Constructivism and learner-centred approach**

In this study, constructivism and learner-centred approach shall mean a teaching approach that emphasizes learners' construction of knowledge from their own experiences influenced profoundly by cognitions. The teacher only facilitates and gives guidance during the learning the process.

### **1.4.7 Problem solving**

In this study, problem solving shall mean an approach that encourages learners to solve mathematics problems in which solutions are not apparent.

## **1.5 AIMS OF THE STUDY**

The aims of the study are as follows:

- 1.5.1 To determine how much mathematics content teachers possess at the point of exit in their teacher training programme.
- 1.5.2 To investigate the relationship, if any, between achievement in content and achievement in teaching practice.
- 1.5.3 To investigate the relationship, if any, between content covered in mathematics education preparation programmes and high school mathematics.

## **1.6 HYPOTHESES**

The hypotheses of the study are as follows:

- 1.6.1. (a) Pre-service teachers possess adequate mathematics content knowledge when they exit their training programme.
- 1.6.1. (b) Knowledge possessed by pre-service teachers will vary according to three sections of mathematics, namely algebra, trigonometry and geometry.
- 1.6.2. (a) The teachers' knowledge of mathematics content will correlate with his/her teaching practices performance.
- 1.6.2. (b) There will be no difference between pre-service teachers' mean scores in knowledge of mathematics and practicum.

## **1.7 RESEARCH METHODOLOGY**

### **1.7.1 Research Design**

Research design is the way in which a study is laid out. It explicates the plan or strategy by which the answers to research questions are to be obtained. Many studies which explore mathematics teachers' competency in mathematics and pedagogical content knowledge often use either descriptive or field experimental research design.

The descriptive research design provides an accurate description or picture of the status or characteristics of a situation or phenomenon. If the descriptive research design is used, the focus is not on how to ferret out cause-and-effect relationship but rather on describing the variables that exist in a given situation, and sometimes, on how to describe the relationship that exists among those variables. Education sometimes conducts research using descriptive designs to learn about the attitudes, perceptions, opinions, beliefs, behaviours and so forth.

A field experimental design is a research study in a realistic situation in which one or more independent variables are manipulated by the researcher under carefully controlled conditions, as the situation will permit.

As it has been indicated that many studies in mathematics education use descriptive or field experimental design, Li and Smith (2007); Turnuklu and Yesildere (2007); Nicol (2002) used descriptive design to investigate the nature of teacher preparation programmes. In their designs the participants

were pre-service teachers who were at the last stage of study in the programme. These researchers designed survey questionnaires on teachers' knowledge of mathematics and pedagogy. They prepared mathematics tests, in-class problems and group interviews. Mathematics testing focused on mathematics content knowledge and pedagogical content knowledge, while in-class problems focused on pre-service teachers' interpretation of learners' misconceptions in mathematics content. The participants were required to complete the questionnaire before they wrote mathematics test. Both qualitative and quantitative methods were used in the analysis of these studies. The findings describe the nature of teacher preparation programmes, pre-service teachers' perceptions about the programmes and extent of content knowledge they learn in these programmes.

On the other hand, Benken and Brown (2008) used the experimental design to determine the effects of integrating mathematics content and general pedagogy courses in a teacher preparation programme. These courses have previously been disconnected throughout the mathematics education programme. For their study, each content course and general pedagogy course was divided into two sections. In one section (*the experimental section*) of the course, in Benken and Brown's study (2008) the teachers were taught certain sections referred to as experimental, while those in the other section (*control section*) were taught normally by the other full-time faculty. In the same way as other studies of this nature, Benken and Brown's study (2008) used both surveys and content tests to collect data. However, in their study, course artefacts and interviews were also added as instruments. Like in the case of descriptive designs, in their experimental design both qualitative and quantitative methods of data analysis were also used.

For the present study the researcher used descriptive and field study research design. Field studies/descriptive research designs are non-experimental scientific enquiries aimed at discovering the relations and interactions among sociological and educational variables in real social situations. The reason for using descriptive design is that there would be no strict measures of control in the field. The researcher wanted to observe the teaching process in a natural setting and to describe its characteristics.

### **1.7.2 Research Techniques**

Very often studies of this kind use a variety of techniques to collect data. For example, Li and Smith's (2007) study that investigates the extent of knowledge in mathematics and pedagogy that prospective teachers learned during training used survey questionnaires and a mathematics test. Benken and Brown (2008) used survey questionnaires, content examinations, course artefacts and interviews. Their study investigated the effects of integrating mathematics content, general pedagogy and methodology courses. It is a common trend in such studies that the survey is administered first, followed by a content test and interviews. Nicol and Crespo (2006) used participants' observation and document analysis to determine how pre-service teachers interpret and use curriculum materials. They observed pre-service teachers during their practicum teaching, focusing on how teachers use textbooks. They conducted an analysis of students' course work and textbook.

In the present study, an approach similar to the one of Nicol and Crespo (2006) was adopted, although in the present study three techniques of data collection were used. For example, a mathematical proficiency test and



teaching practice evaluation and document analysis were used in order to gauge pre-service teachers' strength in content knowledge after training. The mathematical proficiency test was a standardized Academic Aptitude Test (AAT) (Minnie & Paul, 1982). The mathematics test elicited information about pre-service teachers' content knowledge. The mathematics syllabus for teacher training was compared with the high school mathematics syllabus (content analysis) to determine whether there was any correlation.

### **1.7.3 Research Methods and Procedures**

Three aims were formulated and achieved by using different methods and procedures. The aims were as follows:

- 1.7.3.1 To determine how much mathematics content teachers possess at the point of exit in their teacher training programme.

With regard to aim number 1, a mathematical proficiency test was administered to fourth year student teachers. Several researchers believe that mathematics tests or content examinations are appropriate in measuring the extent of content knowledge one has in mathematics (Li & Smith, 2007; Benken & Brown, 2008).

- 1.7.3.2 To investigate the relationship, if any, between achievements in content and achievement in teaching practice.

Many studies that investigate teachers' knowledge in mathematics and pedagogy often use questionnaires (Li & Smith, 2007; Tirosh, 2000; van der

Walt & Maree, 2007). Depending on the nature of the study, some researchers use different approaches to collect data. For instance, Nicol and Crespo (2006) used interviews and participants' observation to determine how the pre-service teachers interpret and use curriculum materials. However, for the second aim in the present study, participant observation and evaluation were used. A teaching practice evaluation form was used to assess teaching skills of pre-service teachers.

The researcher was assisted by the mathematics education lecturers who evaluated students during their teaching practice. Obtained data for each pre-service teacher's performance in teaching practice evaluation was correlated with the pre-service teachers' results in the Mathematical Proficiency Test to establish whether any impact was made by the teachers' content knowledge on teaching practices.

- 1.7.3.3 To investigate the relationship, if any, between content covered in mathematics education preparation programmes and high school mathematics.

With regard to aim number 3, a copy of the prospective teachers' course syllabus from the first year to the fourth year of their study was requested from the Faculty of Education. A copy of the document that contains the high school mathematics syllabus (Grades 10, 11 and 12) was obtained from the Department of Education. The researcher analysed the content in these documents to determine whether mathematics content covered in the mathematics teacher education programmes adequately linked with high school mathematics syllabi. The rationale for choosing high school grades

10,11 and 12 was that these pre-service teachers were trained to teach the said grades.

#### **1.7.4 Sampling Design**

Most of the studies in this area of investigation select, as participants, students who are in their final year of study and those who already have a degree in mathematics and science (Turnuklu & Yesildere, 2007; Li & Smith, 2007; Nicol & Crespo, 2006).

The present researcher targeted fourth-year pre-service teacher doing Bachelor of Education (B.Ed) degrees. The B.Ed degree is divided into three types, namely the B.Ed (Foundation Phase); B.Ed (Intermediate phase) and B.Ed (Senior and Further Education & Training Phase). The present study targeted only those doing B.Ed at Senior Phase and FET level. The reason for selecting this cohort of pre-service teachers is that they have acquired a sound knowledge of both the content and pedagogy. Secondly, as they were at the completion of their study, the expectation was that they were ready to teach mathematics from grade 8 to 12 the following year. These pre-service teachers were all studying methods of teaching mathematics. This arrangement excludes students who hold BSc degrees. The reason for this is that the researcher wanted to establish whether the teacher preparation programmes in the faculties of education prepare pre-service teachers with the necessary mathematics and pedagogical knowledge as they finish the course.

Based on the nature of the study, sampling design was purposive, that is, only participants who fulfilled the purpose of investigation were included in the

sample (Devers & Frankel, 2000; Gray, 2004). Purposive sampling is designed to enhance understanding of the individual or group's experiences (Devers & Frankel, 2000:264). It means that the groups that are likely to provide more insight into research questions should be selected. According to Merriam (2002: 12), if the researcher seeks understanding and the meaning of the phenomenon from the perspective of the participants, it is important to select a sample from which the most can be learned. To this end, purposive sampling was more appropriate for this study because there was a specific population targeted.

### **1.7.5 Scoring Procedure**

In the mathematical proficiency test, one mark was given for each correct answer. The scoring of the teaching practice evaluation was done by mathematics education lecturers. For content analysis, common themes in the content syllabus were directly recorded and compared.

### **1.7.6 Data Analysis**

Both quantitative and qualitative methods of data analysis were used to analyse the participants' responses. The data from the mathematical proficiency test were analysed quantitatively. The pre-service teachers' total scores were calculated and categorized into levels. A cut-off point indicating students' level of performance was specified. For example, 50% was the cut-off point, which means that any student scoring below this level has insufficient knowledge. The basis for this judgement is that 50% is a cut-off point for a pass. A particular interpretation was assigned to each level. For

instance, any score from 80%-100% is classified in level 1, which implies an excellent performance or good content knowledge. Scores from 50% to 79% fall in level 2, which signifies an average performance, or moderate content knowledge. Any score from the lowest possible score to 49% fall within level 3, which indicates insufficient knowledge.

Chi square ( $X^2$ ) test was used as the statistical test to determine whether there was a significant difference between the students who passed and those who performed poorly. Furthermore F-test was used to compare the students' achievements in the different sections of mathematics, namely in algebra, trigonometry and geometry. The data from the participants' evaluation form were analysed quantitatively. Scores obtained from the evaluation form were compared with the scores from the mathematics proficiency test to determine any correlations. In this regard, linear correlation was used for data analysis. This would determine whether or not teaching practice was dependent on the extent of mathematics knowledge possessed by a teacher.

Data obtained from a comparison of the pre-service teachers' content programme and high school syllabus was analysed qualitatively. Data was coded to determine the common themes. During the data interpretation process, data was classified into categories. Relationship was explored among the categories (Nicol, 2002).

## **1.8 ETHICAL CONSIDERATIONS**

One of the important aspects of this study was the consideration of the ethics of the research. The ethical consideration includes, among other things, taking

into account the rights of the participants, their protection, treating them with respect and viewing them as indispensable and autonomous partners in the study. In ensuring adherence to the ethics of the research, the researcher considered the following ethical issues.

### **1.8.1 Informed Consent**

Participants were required to give their consent to take part in the study. This was “informed consent,” meaning that before the commencement of the field study the researcher had to fully inform the participants about their role in the study and the purpose and objectives of the study. The purpose and objectives of the study were clearly explained and understood by the participants. The participants were informed about why they were regarded as suitable for the study. Details of the researcher were given to the participants, including the name of the institution where the researcher comes from.

### **1.8.2 Capacity**

The researcher preferred to involve students in their final year of study in mathematics education because he was hoping that they would be able to provide information that was desired for the study. Furthermore the documents containing teacher education mathematics syllabus together with the high school syllabus presumably contained information that is relevant to this study. These were issues which were granted ethical clearance from the universities and Department of Education.

### **1.8.3 Deception**

The researcher ensured that the purpose and the objectives of the research were not misrepresented to mislead the participants.

### **1.8.4 Confidentiality, Privacy and Anonymity**

The participants were informed that any information gathered from them would be treated with confidentiality. Anonymity was ensured for every individual who provided information even if the study is publicized.

## **1.9 SIGNIFICANCE OF THE STUDY**

1. The study will do throw light on the shortcomings of mathematics education programmes in the teacher educating institutions.
2. Give suggestions about improving mathematics teacher preparation programmes.
3. Suggest practice activities beneficial for teacher preparation.

## **1.10 PLAN OF THE STUDY**

The study is organized as follows:

### **CHAPTER ONE**

In this chapter the motivation for the study, statement of the problem, aims of the study and the plan for the organization of the whole report of the research is discussed.

### **CHAPTER TWO**

This chapter contains the theoretical framework on which the study is based. The spotlight is on the schools of thought in the teaching and learning theories in general and mathematics in particular.

### **CHAPTER THREE**

Review of previous work in this field is done in this chapter. The focus is on the research on teacher preparation programmes for mathematics education, content knowledge of mathematics, and teaching practice in South Africa and other countries, especially those which perform well in mathematics as measured by PISA.

### **CHAPTER FOUR**

This chapter discusses in detail the research design and methodology of the study. Also described in this chapter, are the procedures for data collection, the selection of the participants, the plan for data organization and analysis and ethics of social research.



## **CHAPTER FIVE**

This chapter focuses on the details of the presentation and the analysis of data. It describes how the fieldwork and the administration of the research instruments were conducted.

## **CHAPTER SIX**

This chapter presents the findings of the study about mathematics education programmes and pre-service teachers' knowledge of mathematics and practices. The discussions of implications of findings are also considered.

## **CHAPTER SEVEN**

This chapter consists of a summary of the study, limitations of the study, recommendations, avenues for future research and the conclusion.

## CHAPTER TWO

### THEORETICAL FRAMEWORK: DIFFERENT SCHOOLS OF THOUGHT AND LEARNING PROCESS

#### 2.1 INTRODUCTION

This chapter consists of the theoretical framework on which the study is based. The focus is on the teachers' conception of mathematics knowledge and the theories related to mathematics teaching and learning. The chapter furthermore examines the impact of the teachers' existing belief and mathematics knowledge on the teaching and learning of mathematics. The assumption is that the way teachers conceive mathematics, their beliefs and the extent of mathematics knowledge they possess influence the teaching and learning of mathematics. However, Staub and Stern (2002:345) concluded that research on teachers' beliefs often focuses mainly on beliefs pertaining to the teachers' role, students' learning and principles that explain interactive behaviour. Little (or inadequate) consideration is given to the subject matter content. This chapter reviews studies and theories in this context.

The lack of consensus in the way in which people describe and explain the human mind and behaviour resulted in the establishment of different schools of thought which are sometimes regarded as the worlds of psychology. For example, *structuralism* emerged as the first school of thought. In reaction to structuralism, *functionalism* emerged as the second school of thought. The first and second schools of thought were in line with the absolutist view of teaching and learning. Philosophies associated with the absolutist view were,

amongst others, associationism and behaviourism. In the middle of last century (20<sup>th</sup> century) a third school of thought, *humanism*, gained popularity when it emerged as a reaction to psychoanalysis and behaviourism, which were underpinned by the ideas of functionalist psychology. Humanism was in line with the fallibilist view of teaching and learning. The philosophies that were associated with fallibilists were, among others, cognitivism and constructivism. In the present study, the schools of thought and philosophies are chronologically discussed taking into account their applicability in education, particularly in the teaching and learning of mathematics.

## **2.2 STRUCTURALISM IN EDUCATION**

Structuralism, as established by Edward Tichener, was the first school of psychology. The main goal of this psychology was to describe the structure of the mind and its processes. The focus was on mental experiences, consciousness and the correlation of mental phenomena with physical events. The present study examines this psychological perspective from a mathematical education point of view. During the 20<sup>th</sup> century scientific discourse, mathematics has been particularly dominated by the absolutist view in which mathematics was seen as a static, absolute, objective and infallible body of knowledge (Ernest, 1991; Staub & Stern, 2002). It is characterized by giving students unrelated routine mathematical tasks which involve the application of learnt procedures, with emphasis on correct answers and criticism of any failure to achieve the right answer.

Influenced by dominant views at the time, scientists also tried to understand how the existence of these structures should be understood in mathematics. The structuralist mathematics perspective holds that mathematical objects are defined by their positions in the structures with no underlying properties or intrinsic nature. For example, the number 4 on a mathematical structure which could be a number line is identified as the successor of 3 and the predecessor of 5 but not intrinsically. Basically, the reason for defining mathematical objects by their positions in a structure rather than intrinsic properties is that mathematical objects do not have an internal composition arranged in structures. The objects in mathematics are structure less points or positions in a structure (Cole, 2010).

Although structuralism seems to have lost popularity, it did have a positive role in mathematics teaching and learning. For example, the research indicates that active teaching methods based on structuralism approach are effective in improving students' performance in meaning usage, solving common problems and in mathematics reasoning (Salehi, 2011). However, it was also indicated that teaching methods based on structuralism had no impact on improving students' knowledge in mathematics.

### **2.3 ASSOCIATIONISM**

Titchener's theory of structuralism was heavily influenced by his teacher (Wundt)'s ideas of association. Associationism, like structuralism falls under the first school of thought (first world of psychology). Associationism is a theory which had a huge impact on mathematics education in the twentieth century (Staub & Stern, 2002). This theory is based on E.L. Thorndike's work

published in the *The psychology of arithmetic* in 1922 (Carrol, 1997). It emphasizes contiguity and reinforcement as central elements of learning. It explains how items are combined in the mind to generate thought and create learning. Learning takes place through the involvement of simple mental structures in which mental bonds or associations connect sets of stimuli and responses (Carroll, 1997). According to Lukas (2002), learning is the formation of associations between unrelated information based on their contiguity.

While associationists construe associations as a linking and contiguity of ideas, Thorndike maintains that associations exist between the situations in which an organism finds itself and the impulses in the organism to action (Wozniak, 1999). In the context of teaching and learning the “situations and impulses” could mean the environment where learning takes place and response to stimuli, respectively. There seems to be commonality between this theory and behaviourist theory, particularly in the subject matter presentation. Teaching and learning based on associationist theory is characterized by a drill and practice approach which may result in little or no comprehension of the learned concepts and procedures (Carrol, 1997).

The principles which define associationist approaches (Carrol, 1997; Lukas, 2002; Holtorf, 2011) are connectedness of ideas and mental elements, contiguity of stimuli and bonds reinforcement by the frequency of use. If bonds are reinforced by frequent use it means that it can be weakened by seldom use.

The role of associationism in mathematics education has been obscured by the growing realization that learning is an active process that involves among other things rational thinking and transforming the significant understanding a learner already has into new ideas or concepts. According to Staub and Stern (2002) appropriate explanation for the construction of arithmetic networks can be accounted for by the underlying knowledge of number facts and the acquisition of computing routines. They argue that teaching based on associationist theory may not account for the acquisition of the conceptual mathematics knowledge needed in complex and new problem situations, or to solve mathematical word problems.

Thorndike recommends that when learning mathematics, students need to perform much drill and practice on correct procedures and facts to strengthen the correct mental bonds. Although Skinner denied this theory of mental bonds, his prescriptions for mathematics teaching were similar to that of Thorndike. He also maintained that drill and practice reinforced by reward for desirable or correct answers and punishment for undesired answers would enhance teaching and learning of mathematics.

The present researcher argues that the application of associationist theory in mathematics teaching may be relevant at elementary level. At this level learners are still memorizing facts, for example, the multiplication tables, square of numbers and shapes of geometric figures. At elementary level most of mathematics revolves around concrete objects which are easy to link with ideas already in the mind. However, for a middle and high school syllabus some mathematical concepts may be abstract and not apparent, thus innate abilities and resemblance may not be effective. The present researcher

supports the criticism by Schoenfeld, (1987) that although some learning occurs from the drill and practice approach, this is superficial and inflexible, and little or no understanding can be associated with the learned procedures. To this end, there is justification for John Locke's position in which he describes the "association of ideas" as detrimental to rational thought (Encyclopaedia of psychology, 2001).

## **2.4 FUNCTIONALISM**

Functionalism is a second school of thought, formed as reaction to structuralism. It is rooted in William James' work and Charles Darwin's evolutionary theory. As opposed to structuralism, which focuses on the elements of consciousness, functionalism studies the functions of consciousness and explains systematically the mental processes focusing on the purpose consciousness and behaviour (Muhammad, 2009). According to Muhammad (2009) the advocators of functionalist principles, like John Dewey emphasize consideration of individual differences in mental abilities, and this reminds us that learners with different mental levels learn at a different pace. On the basis of this, functionalist teachers must use different methods of teaching so that learners can learn in one method and others by means of another method. The emphasis is on activity based teaching supported by meaningful teaching material. Although functionalism gradually lost popularity due to the rise in behaviourism, its lasting legacy in teaching and learning remains significant (Passer & Smith, 2011). The basic tenets of this school are the role of the mind in environmental adaptation and the role of practice, interest and motivation of learners (Muhammad, 2009; Shiraev, 2011).

When mathematics teaching and learning is based on drill and practice, as advocated in structuralism, conceptual understanding is minimized. Mathematics is often applied in other disciplines, at work and in our daily activities. Hence Muhammad (2009) asserts that functionalism does not only provide insight of learning activities, but it also allows the opportunity to practice in real life the task that leads naturally to problem solving, and this is appropriate for mathematics teaching. Functionalism emphasizes individual differences in mental abilities. Certainly, this is supported by Downey and Cobbs (2007) who maintain that when teaching mathematics it is very critical for a teacher to know his/her learners' diversity in class, because learners might not be at the same cognitive level.

## **2.5 BEHAVIOURISM**

In the last century, the teaching of mathematics has been dominated by the behaviourist theory which originated in attempts to refute the aspects of associationism (Carroll, 1997). As Passer and Smith, (2011) and Theron, (2006) put it, the behaviourists are totally opposed to structuralists' focus on consciousness, functionalists focus on heredity and psychoanalysts focus on unconscious mental content .

Behaviourist theory is underpinned by the belief that it is the observable behaviour of the learner that determines how he or she acquires knowledge of the subject. If there is no observable change in the learner's behaviour there can be no assumption that learning has taken place. Behaviourists consider a learner to have an empty mind that needs to be filled with knowledge transmitted directly from a teacher. Rughubar (2003:25) writes "behaviourist



theory portrays the teacher as the all-knowing dispenser of knowledge, and the learners as passive recipients, hence subscribing to a teacher-centred approach to teaching and learning”. For this reason the teachers adopting behaviourism need to study and modify the learners’ behaviour by setting up situations to reinforce the learners’ desired responses.

Behaviourism and the existing beliefs about how learners accommodate new ideas in mathematics teaching and learning are based on certain principles, such as observable behaviour, environmental reinforcement and transfer of learning. It means that if there is no observable change in behaviour, then there was no transfer of knowledge, and thus no learning has occurred.

According to Naikumi (2010), behaviourist theory gave birth to the drill and practice method of teaching which put emphasis on an explain-practice-memorise teaching model. The teacher is at the centre of teaching and learning. He/she breaks the subject matter into units which form a particular sequence starting from known to unknown and from simple mathematics concepts or activities to more complex ones. Moodley, (2009) viewed drill and practice to be essential for mastery of mathematics techniques. However, the present researcher perceives this differently. He supports Golub’s (Cooper, 1993) position that behaviourally based instruction could be useful for clearly delineated content where learners’ responses are categorized as right or wrong.

Such responses are rarely encountered in mathematics learning. Furthermore, in contrast to Moodley’s view of drill and practice in mathematics teaching, Rughubar, (2003) argues that knowledge and skills acquired through

prescriptive methods which are often applied in behaviourism are not necessarily transferable and learners hardly understand mathematics.

Purportedly, Thorndike's view is pertinent to the way many South African teachers taught mathematics, and this necessitated the shift towards constructivist based teaching and learning. As a result the South African government in 1997 announced the inception of a new education system based on Outcome-based education (OBE) principles. This was a move by the Department of Education to a position where it avoided making prescriptions about teaching strategies and allowed them to be a teacher's choice (Moll, 2002). This movement underscored the adoption of "constructivism" to be the basis of new teaching approaches.

Although behaviourist theory is subjected to numerous criticisms, there are nevertheless mathematics sections where it can be applied effectively. As Naikumi (2010) has remarked, teaching methods attached to the behaviourist theory are effective in studying certain concepts, such as learning multiplication tables and squares of numbers. In addition to this notion, Cooper, (1993) suggests that behaviourally based instruction seems most useful for clearly delineated content where branching is constrained and learner responses are categorized as right or wrong. Indeed these perceptions are supposedly true for other subjects, but ideally the unique features of mathematics demand the teacher to focus on both the comprehension of the concepts and processes than on the correctness or wrongness of the answer.

## 2.6 HUMANISM

In psychology, humanism is a third school of thought that was developed in the middle of 20<sup>th</sup> century as reaction to behaviourism and psychoanalysis. Humanists felt that behaviourism and psychoanalysis fail to take into account the role of personal choices (Cherry, 2012). Humanistic psychology takes positive aspects of conscious mental activities to be central to human growth. It entails studying a person in totality, particularly based on the belief that the individual's behaviour is often determined by his/her inner feelings and self-concept (McLeod, 2007). The educational perspective of humanism assumes that when teaching, the students need to be intrinsically motivated to learn. This means that they should not be subjected to any external reinforcement or imposition of knowledge by a teacher. In contrast to the humanistic perspective, behaviourists would expect extrinsic rewards to activate a learner.

Creating a sense of motivation should be the integral part of the lesson being taught. This can be achieved when a teacher is allowing students to take responsibility of their learning, students believing in their potential and the teacher adding value to students' views. Supposedly, if students are given the opportunity to show their creativity and ability to investigate meanings on their own, subjectivity and a positive view of themselves is enhanced. According to McLeod, (2007) objective reality is less important than a person's subjective perception and understanding of the world and this is an essence of mathematics learning. With this in mind, it means that a humanistic teacher strives to develop a learner's self-esteem that would drive him/her towards the achievement of his/her appropriate goals.

Dills & Romiszowski, (1997) assert that humanistic approach in teaching is based on the principles of individualism and self-determination, mutual understanding, affective and experiential learning and teaching approaches. In addition to this, equity, democratic collaboration and use of relevant material should be used during the teaching process. The nature of the subject “mathematics” requires that one who learns it should not just conform to the rules and the teacher’s methods of solving problems. On account of this view, Singer and Moscovici, (2008) say that the student must first build a mental representation of the mathematical problem before taking a decision on how he/she will solve the problem. Building mental representation is an essence of humanism because it advocates the individual’s subjective perception and understanding of the domain. Passer and Smith, (2011) maintain that people’s primary goal is to make sense of the world and to find personal meanings in it, and when this goal is not achieved, the results would be uncertainty and anxiety. Uncertainty and mathematics anxiety is dominant where teaching subconsciously or consciously encourages the learners to view mathematics as abstract, prescriptive, formal and meaningless.

The humanistic approach emphasizes a concrete and pragmatic view of reality. As a result, when applied in mathematics teaching, it helps the students to realize the existence of reality that rules the world of phenomenon and to create an eagerness to acquire endless mathematics knowledge (Woo, 2004). However, it envisages a teacher who is able to connect students with the history in which mathematics discoveries were made. Such connections must be closely related to the students’ cultural heritage so that they can recognise mathematical ideas.

## **2.7 THEORIES ON INSIGHTFUL LEARNING**

Insightful learning theories are influenced largely by the Gestalt philosophy. Unlike behaviourists, Gestaltic psychology holds that the mind cannot be studied by breaking it into elements or associations which combine to form a larger unit. It also rejects explanation of mind functioning in terms of cause and effect. Instead, Gestalt emphasizes the studying of the mind as whole because it interconnects the elements into a pattern that creates meaning which cannot be found in an individual element (Theron, 2006).

In education, insightful learning is sometimes known as Gestaltic learning. This, according to Prakash (2011), means that learning is concerned with the whole individual and arises from the interaction of the individual with his/her situation or environment. Such interactions induce formation of perception, imagination and ideas, which altogether constitute insight. Insightful learning theories emphasize that teaching must be aimed at discovery and meaningful learning. As a result, educational theories which share the common view with Gestaltic psychology are also discussed in terms of their principles and the role they play in the teaching and learning of mathematics.

## **2.8 COGNITIVISM**

Cognitivism is a theory of learning which perceives learning as a process of adapting oneself to the environment using assimilation and adaptation. It is one of the insightful learning theories. Learners organize and interpret concepts and ideas (assimilation) using the existing cognitive structures and thus changing cognitive structures (accommodation) to make sense of the

environment. It emerged as a shift from behaviourism that perceives learners and their behaviours as the product of the environmental stimuli, to seeing learners as a source of plans, intentions and ideas actively used for construction of meanings from stimuli and knowledge from experiences (Woolfolk, 1998). The teacher studies the students in order to establish what is within their experiences and sets up the environment which will induce stimuli to which learners will respond. According to Naikumi (2010), to ascertain the level of intellectual growth and to facilitate the development of insight of a learner the teacher must model the learners' experiences by providing moderately challenging tasks. This is in accord with Piaget and Jerome Bruner who advocated discovery and hands-on learning. As a result Piaget, Chomsky, and Ausubel discarded drilling and the empirical nature of behaviourism, which placed emphasis on thinking skills and thinking processes (Moll, 2002).

Learners actively constructing knowledge in a social context provide an explanation of reality (Piaget, n.d; Conway, 1997). Complexity in learners' actions needs to be matched by a similar level of complexity in the instructor or teacher's actions and teaching methods must actively involve learners and present them with challenges while also taking into account learners' prior knowledge. The aspects of Piaget and Bruner's active construction of knowledge and discovery learning seem to be fundamental in mathematics learning because learners construct mathematics concepts in their own ways without these being imposed on them by a teacher. The assumption is that learners will easily remember what they discovered on their own. By a way of illustration the researcher used Naikumi's example(2010) as follows: In teaching sets, if  $A = \{3,6,2,9,8,0\}$   $B = \{8,6,3,9,2,0\}$  when learners are told to

compare set A with set B, they are able to discover that set A and B have similar elements. The teacher then brings in the concept of ‘equal sets’ and if asked to compare sets C and D where C= [3,1,5,8,7,0] and D= [8,6,3,9,2,0], learners are able to discover that sets C and D are not equal but have some common elements. The teacher then brings in the concept of ‘intersection between the two sets’. The understanding of the stages of cognitive development helps the teacher to realise that learners do not undertake mental exercises that exceed their mental capacity.

## **2.9 CONSTRUCTIVIST THEORY**

Constructivism is the theory of learning that views learning as an active process where learners construct meanings for themselves (Moll, 2002; Bruner, n.d.). It is an insightful learning theory which falls within the fallibilist paradigm. Most constructivists are inspired by Piaget who put the individual at the centre of meaning making, although Vygotsky, on the other hand, purports that communication and social life is central to the meaning making (Lerman, 1996:133). Vygotsky argues for the social aspect of learning, which includes among other things, the interaction with peers and with the educator (Moodley, 2009; McCracken, n.d). Despite these contrasting views of Piaget and Vygotsky, notably they both seem to fight against associationism and the behaviourism view of teaching which is still applied by many teachers in their teaching.

Constructivist theory is reactionary to theories which view learning as a mimetic activity where learners keep on repeating newly presented information. It is based on the very fundamental belief that learners should be

helped to construct knowledge that is meaningful and useful to their own lives (Jacobs, 2004: 46). According to Hobden (2007) constructivist teachers strive to find ways in which individual learners actively create knowledge. This requires a teacher to be a facilitator or a partner in learning rather than being the dispenser of knowledge.

The advocates of a constructivist approach suggest that teachers use a collaborative style where learners learn from their peers, reflective processes where learners talk about what was previously learned, and discussion through which learners construct knowledge and engage in enquiry-based activities (Moodley, 2009). Similarly, the move that was taken by South Africa to reform the education system through the implementation of Outcomes-Based Education (OBE) was informed largely by constructivist theory. Anderson (1996) states that the traditional (absolutist) perspective of seeing mathematics as a fixed body of facts and procedures which are memorized, leads to a teaching approach which emphasizes the acquisition of skills, accuracy and answers rather than processes. Contrary to the absolutists' perspective, she sees constructivism in mathematics to be dynamic in a sense that it leads to a teaching approach which emphasizes active engagement with mathematical tasks. Learners construct their own meanings of the task from their existing knowledge, which is an essence of meaningful learning (Muijs & Reynolds, 2005).

Like other theories, constructivism is defined in terms of its principles such as: Learning is an active process in which learners use sensory input and construct meanings out of it; learning is internally controlled and mediated; learning involves language; learning is contextual and teachers should be



mindful of the fact that learners bring unique prior knowledge, experiences and beliefs to the learning situation (“Constructivist education: Learning principles in constructivism”, 2006; Moll, 2002; Muijs & Reynolds, 2005:63).

Notwithstanding the contrasting ideas of Piaget, Vygotsky and Bruner, the renowned cognitive theorists about individual construction of knowledge, there exists a consensus on the fact that knowledge is actively constructed by a learner through discovery. This is the essence of mathematics teaching and learning. Epistemologies of mathematics and mathematics education view mathematics as a process rather than product, whereby knowledge of mathematics is acquired by doing mathematics (Coben, 2003). As a result there has been an increasing need of problem solving and participative approaches in mathematics education which culminated in a shift from traditional, associationism and behaviourism paradigms towards constructivism.

Although the present researcher agrees with Vygotsky’s notion (Moll, 2002) that a child acquires new knowledge through the articulation of natural (non language-based) and cultural (language-based) lines of development, he believes that this is not always true. For example, in mathematics education the cultural line of development may not always work because some mathematical concepts may not be within the cultural background of a learner and thus not applicable in his or her language.

One of the principles of constructivism states clearly that the language we use influences learning (Lerman, 1996). Certainly this principle endorses the

argument that it is the language of instruction which determines the learner's development in mathematics knowledge and this may be a foreign language to other learners. To this end, the issue of a cultural line of development in mathematics knowledge acquisition may be bias. The researcher supports Piaget's views that knowledge construction is a learners' adaptation to the environment (social context) that leads to the change of schemas and incorporates new ideas in that same level. In the same way he supports Bruner's assertion that a learner relies on cognitive structures to attach meanings to his/her experiences that allow him/her to go beyond the given information (Bruner, n.d.).

## **2.10 THE CONFLICT BETWEEN ABSOLUTIST AND FALLIBILIST PARADIGMS IN MATHEMATICS EDUCATION.**

Staub and Stern, (2002) observed that the legacy of an absolutist paradigm, particularly associationist theories of learning which put contiguity and reinforcement as a central element for learning, continued to shape the contemporary culture of schooling. Absolutists view mathematics as universal, objective and certain with mathematical truth discovered by the mathematicians through their intuition (Ernest, 1996). As a consequence, absolutist teachers see mathematical concepts, theorems and formulae as universal and absolutely true. The resistance to shift from an absolutist and traditional way of teaching to the modern ways of teaching has been very remarkable.

The manifestation of resistance and difficulty in adopting a constructivist view and learner centred teaching has been prominent in South Africa during the

inception of Outcomes-Based Education (OBE). Teachers continued to use traditional, prescriptive approaches at the expense of the participative approaches in teaching mathematics. Teachers taught mathematics in isolation from the everyday experiences while many if not most of them claim to be constructivist in one form or another (Rowlands & Carson n.d.). Implicitly, teachers consciously or subconsciously have an absolutist and fallibilist conception of mathematics although failing to put the fallibilist view into practice. Dengate and Lerman (1995), in their study, found that many teachers were enunciating the implications of constructivist theories for teaching; however they also found that a huge percentage of teachers still believed that mathematics is a set of algorithmic rules that can be transmitted rather easily.

In the light of teachers' difficulty to move away from behaviourist to constructivist theories, Clements (1997) reminds us that constructivism tells us more about learning than about teaching. In addition to this, Clements (1997) makes a claim that after years of being taught in the behaviourist way, it is difficult to suddenly become a facilitator in a constructivist setting. In support of this claim, Goldblatt and Smith, (2004) make a reference to Canadian teachers saying that the way they were taught while they were still students often guides them when teaching. This has some implications to the way pre-service teachers are trained.

As opposed to absolutists, fallibilists view mathematics as incomplete and always in progress to invent new mathematical truths (Ernest, 1996). Ernest (1996) argues that the history of mathematics should be the integral for any philosophical account. By way of illustration, he refers to the historical development of the Euler Relation with regard to number of faces of

mathematical solids. To prove the relationship  $F+V=E+2$  took a hundred years, and during this process different proofs were invented and subjected to criticism and then modified. This shows that in mathematics there is no definition or proof which needs to be considered as the absolute truth. It is for this reason that Nyaumwe and Buzuzi (2007) argue that mathematical claims are not absolutely true unless they prove adequate in the context in which they are created.

On the basis of the fallibilists' view, learners should individually and socially construct mathematical knowledge through experimentation, explanation, justification, proofs and modification of mathematical claims and conjectures. However, in the United States teachers restrict proofs to geometric conjectures where they use axioms, definitions and theorems to prove the validity of the conjecture (Peressin, Borko, Romagnano, Knuth & Willis, 2004). This exacerbates the resistance to shift from the absolutist to the fallibilist beliefs about mathematics teaching.

While we acknowledge that there are teachers who do have a conception of the fallibilist view of mathematics, it nevertheless seems that many teachers have a view of mathematics as a fixed body of facts, skills, concepts and knowledge. In the 1990s, when many educationists, teacher educators and teachers advocated transformation in mathematics education in favour of constructivism, a conflict was created. According to Carrol (1997) curriculum documents represented constructivist interpretations of mathematics as well as absolutist views, and while some teachers had adapted their teaching approaches many continued to teach mathematics in traditional ways.

Besides the created conflict in the interpretation of the documents, the resistance in changing from transmission to a constructivist approach seems to agree with Freudenthal (1991) who maintains that it is easier to teach and learn structured form rather than the unstructured form of content. As a result teachers are tempted to teach mathematics as a set of algorithmic rules. Freudenthal suggested that teachers must conceive mathematics as a complex subject which involves mental activity, common sense, art and a system of algorithms, and most importantly, to consider the extent of the learner's contribution to the learning progress.

The goal of reform efforts to implement constructivist approaches in mathematics education is to create mathematics understanding and also to provide a strong knowledge base for students pursuing post-secondary education or entering the workplace. As a result the focus of many departments of education has been on inculcating constructivist based teaching approaches to pre-service teachers (Andrew, 2006). Notwithstanding the interactive nature of constructivist theory, the teacher remains the critical facilitator of learning who is expected to support and guide learners in their knowledge construction. As Andrew, (2006) put it, a constructivist teacher makes decisions, creates environments and selects assignments which cause learners to be in charge of their own learning. To this end, teachers with deep mathematics knowledge and knowledge of constructivist approaches that will engage students in mathematical thinking are needed.

## 2.11 CONCLUSION

The theoretical perspective of this study highlights the different schools of thought and theories which often inform the teaching approaches in mathematics education. It presents chronologically, the development of theories of education and their role in the teaching and learning of mathematics. It is widely acknowledged that the teacher's beliefs about mathematics teaching and knowledge of mathematics content have influence on the way in which the teacher teaches mathematics (Wilburne & Napoli, 2008; Ernest, 1988). The teachers' beliefs lend themselves to a particular philosophy.

The review of teaching theories provides explanations of the psychological perspectives which underpin the gradual changes and development in the way people view mathematics. Very often teaching is classified into instrumental, relational, procedural and conceptual, without understanding the theory behind it. In light of this review, these classifications are associated with a particular theory or philosophy. When pre-service teachers begin to teach in the teaching profession, they consciously or subconsciously extend the legacy of each of these psychological perspectives.

Mathematics education currently demands the adoption of the constructivism in teaching and learning of mathematics while the legacy of absolutism still prevails. Consequently, teachers often have a conflict of ideas with regard to the acquisition of knowledge during the teaching and learning of mathematics. Although the current mathematics teaching is theoretically constructivist oriented, it is nevertheless clear from the review of theories that

other perspectives still have a role to play in mathematics teaching and learning.

The preoccupation of our teaching with constructivism tends to obscure the existence and the role of other theories in the teaching of mathematics. While the present researcher approves the adoption of constructivist teaching, it is ostensible in the literature that some of these teaching theories have pedagogical implications. For instance, the issue of behaviour and the differences in mental abilities advocated by functionalists (Muhamad, 2009) is crucial for mathematics teaching. Furthermore, McLeod, (2007) in humanism assert that objective reality is less important than subjective perception and understanding of the domain, and this is an essence of mathematics learning. This review of theories is valuable for this study because most mathematics education perspectives have been looked at broadly, which enhances understanding of different behaviours in mathematics teaching and learning.

## **CHAPTER THREE**

### **REVIEW OF RELEVANT STUDIES CONDUCTED IN THIS FIELD**

#### **3.1 INTRODUCTION**

This chapter focuses on the review of literature which is pertinent to the training of prospective mathematics teachers. The chapter examines, among other things, the content base of mathematics and the pedagogical knowledge pre-service teachers often have when they exit preparation programmes. The impact of content knowledge on the teachers' teaching practices is also examined in this chapter. In addition to this, the review includes the establishment of the relationship between the syllabi for teacher preparation and high school mathematics.

#### **3.2 STUDIES ON THE LEVEL OF PRE-SERVICE TEACHERS' MATHEMATICS CONTENT KNOWLEDGE AT THE POINT OF EXIT OF TRAINING PROGRAMMES.**

Research evidence reveals that South African students do not compare favourably with their international counterparts in academic attainments, particularly in mathematics. This has been evident in the Third International Mathematics and Science Study (TIMSS) conducted in 1995, 1999 and 2003, in which South Africa participated with 41 other countries and it was reported that South African mathematics learners came last (Mji & Makgato, 2006). Their achievements in science and mathematics are very often extremely low (Moloi & Strauss, 2005; van der Walt & Maree, 2007; Howie, 2003). While the cause of poor achievements in mathematics is multi-dimensional, many



researchers attribute it to outdated teaching practices, teaching of mathematics by unqualified or under-qualified teachers, and lack of mathematics knowledge among mathematics teachers (Pournara, 2005; van der Sandt & Niewoudt, 2003; Mji & Makgato, 2006; van der Walt & Maree, 2007). This means that, despite the existence of other teaching and learning components, the teacher remains the fundamental determinant of effective learning. For this reason, teachers' knowledge of mathematics content and pedagogy is pivotal in the teaching and learning process of mathematics.

Teacher training programmes are generally expected to produce an individual who is ready to teach. Pre-service teachers training programmes are designed and developed in accordance with the guidelines set out in the policy document (Department of Higher Education and Training (DoHET), 2011) entitled Minimum Requirements for Teacher Education Qualifications (MRTEQ). The teacher training programmes must comply with MRTEQ (DoHET, 2011:49). MRTEQ therefore draws a portrait of the kind of teacher that nations, particularly the South African nation, should have. However, the total quality management approach of pre-service teachers cannot guarantee that the end product is ready to teach. A guaranteed product can be achieved with machine-made products. How can we ascertain that pre-service teachers are ready to 'hit the ground running' at the point of exit of their programmes? In South Africa teacher training is a big industry. The country has seen the sporadic erection of Colleges of Education to promote this industry. In less than 15 years of their existence, Colleges of Education have been closed down. The status quo remains, teachers are highly in demand.

The need to train competent teachers has persisted since the introduction of the Norms and Standards for Educators (NSE) in the year 2001 (DoE, 2000). Sound education and training are the foundations of competent practice in teaching. The NSE did not meet this requirement. Although it was intended to restructure education, the NSE did not stipulate how educators or teachers could be made competent products. The NSE intended to achieve competence through integration of fundamental competence, practical competence and reflexive competence. In addition to these competences, the NSE advocated the establishment of seven educator roles. The framework prescribed by the NSE was a matrix of four competences multiplied by seven roles (DoE, 2000). One study revealed that novice teachers perceived this framework as inadequate for the preparation of teachers (Sibaya & Sibaya, 2008).

Brodie (2004) asserts that the focus of teacher training has been mainly on the teaching methodology and the nature of the school mathematics curriculum at the expense of underpinning content and conceptual knowledge needed for teaching. It is intriguing that the designers of teacher education programmes seem to disregard the fact that, generally, students were not taking enough mathematics in high school and that even those who did take three years of college preparatory mathematics did not learn the use of that mathematics (Lott & Souhrada, 2000:106; Crespo, 2003). Lloyd (2006) goes on to say that pre-service teachers often have weak mathematics knowledge and a narrow view of mathematics.

Revelations of concern about improving the status of mathematics teachers come from TIMSS (Reddy, 2006) and TIMSS (2007) studies. These studies indicate that, internationally, between 43-57 per cent of learners have been

taught by teachers who had participated in professional development workshops. The highest percentages of these workshops were related to content and pedagogy. It is noted in these studies that South African teachers attended a high number of professional development workshops. These workshops focus on, amongst other things, mathematics content, mathematics curriculum, improvement of critical thinking and mathematics assessment. Comparatively, workshops attendance is more pronounced in South Africa than in other countries (Reddy, 2006).

Pre-service and in-service teacher education is an expensive enterprise. It makes us despondent to realize that in South Africa large capital investment in pre-service and in-service teacher education programmes yields diminishing returns. Chapter nine (National Planning Commission (NPC), 2010) indicates capital injection at all levels of education. These measures are taken to improve education, training and innovation. Van der Sandt and Nieuwoudt (2005) have also recorded the same remarks about government expenditure on teacher education for the 2001-2002 financial periods. However, it is surprising that despite these efforts, the status of mathematics education in South Africa has not yet shown conspicuous improvement (TIMSS, 2007; Brodie, 2004).

The manifestation of deficiencies in teachers' knowledge of mathematics is very pronounced among pre-service and novice teachers. Very often novice teachers find it difficult to solve mathematical problems in different ways in the classroom (Leikin & Levav-Waynberg, 2007). The research, which has been in great vogue in the field of mathematics education, focuses mainly on methods of teaching rather than on teachers' mathematics content knowledge

(Onwu & Mogari, 2004; Moll, 2002; Brodie, 2004; Benken & Brown, 2008; Mji & Makgato, 2006; Taylor, 2009). The literature asserts that despite the content and pedagogy courses taken by prospective teachers in their training, they very often leave training programmes with the same knowledge base as when they first entered into teacher training (Benken & Brown, 2008; Seaman, Szydlik & Beam, 2006). This assertion is supported by the findings of Sibaya and Sibaya (2008:94), namely that novice teachers who are the product of NSE did not master the academic content of the subjects they were expected to teach upon completion of their training. As a consequence, these novice teachers felt less competent with regard to their command of the subject matter. This means that teachers do not have adequate content knowledge to teach mathematics proficiently (Even & Ball, 2009; Mulis, Martin, & Foy, Olson, Preuschoff, Erberber, Galia, 2008; Taylor, 2008; Hristovitch & Mitcheltree, 2004; Dorfler, 2003; Crespo, 2003; Kinach, 2002).

Performance of pre-service mathematics teachers depends, amongst other things, on the subject knowledge they possess. Menon (2009) classified this subject knowledge into three categories, namely traditional, pedagogical and reflective. On the other hand, performance in mathematics proficiency tests underpins these broad areas of knowledge. This means that proficiency tests are designed to measure students' competencies in a subject area (Azeem & Gondal, 2011). Mathematical proficiency goes beyond the ability to solve mathematics problems. It involves understanding of mathematics concepts (traditional), application of mathematics knowledge to novel problems (pedagogical) and the ability to reason mathematically as a process of learning mathematics successfully (Adnan, Zakaria & Maat, 2012; Azeem & Gondal, 2011; Whitehurst, 2003). Kilpatrick, Swartford and Findell (2011) go

on to say mathematical proficiency is necessary for the successful learning of mathematics. Sestic and Huttunen (2006) also maintain that the intention of proficiency assessment is not to rank order learners but to diagnose competences, students' strengths and weaknesses, and what students are capable of doing. Mathematics proficiency of pre-service teachers has a direct effect on the quality of mathematics education in their learners (Kayhan & Argun, 2009; Corcoran, 2005).

Pre-service teachers' proficiency also reflects the effectiveness of the teacher education programme that they have gone through (Tatto, Schwille, Senk, Ingvarson, Rowley, Peck, Bankov, Rodriguez & Reckase, 2012). The requirements for pre-service teachers who have trained in mathematics as one of their specialization subjects is the use of the subject knowledge in order to be proficient in teaching mathematics in schools (Benken & Brown 2008; Leikin & Levav-Wynberg, 2007; Toh, Chua, & Yap, 2007; Brodie, 2004; Hill & Ball, 2004).

Kayhan and Argun (2009); Crespo, (2003) reveal in their study that proficiency of pre-service mathematics teachers in their subject matter is acquired through experience. The study conducted by Kayhan and Argun (2009) compared the performance of pre-service teachers on the two assessments conducted during the first and tenth weeks of the students' final year. The scores obtained in the tenth week by pre-service teachers were better than the scores obtained during the first week. The pre-service teachers showed improvement in the performance of subject matter knowledge, teaching process and communication skills. Similarly, the study conducted by Crespo (2003) examined the changes in the problem posing strategies of a

group of 34 elementary pre-service teachers as they posed problems to the learners for 11 weeks.

The pre-service teachers' later problem-posing practices were significantly better than the earlier ones. Instead of posing traditional single step and computational problems, they ventured into posing problems that have multiple approaches and solutions, and are open ended and exploratory, and cognitively more complex. The importance of teaching experience in teaching mathematics is indicated in these studies.

Better performance in mathematics knowledge of pre-service teachers also depends on the quality of the teacher training programme and its coherence (Avalos, Telez & Navarro, 2010). Avalos et al., (2010) examined the knowledge base of first, third and final year mathematics pre-service teachers. The study involved six teacher education institutions in Chile. The result indicated that, for the final year students, 50% responded correctly to mathematics questions, 40% performed well in geometry and algebra and only 32% were able to answer questions on mathematics pedagogy. Avalos et al., (2010) infer that the results indicate that the pre-service teachers might find it difficult to teach mathematics in all the grades they are prepared to teach. The significance of academic knowledge of mathematics is indicated in both the Avalos and the present study although using a different approach. The present study investigates how much knowledge of mathematics pre-service teachers possess as they exit their programme of training. Furthermore the study of Avalos *et al.*, reports group performance (inter-individual analysis) whereas the present study reports the performance of each person (intra-individual analysis).

The study of Avcu and Avcu (2010) shows the performance of pre-service teachers on problem solving. The pre-service teachers were given five problems which required the use of different problem strategies, such as making a drawing, accounting for all possibilities, adapting a different point of view, finding a pattern, organizing data, logical reasoning and working backwards. The results show that pre-service teachers were able to use problem solving strategies, while the use of different strategies was limited. Avcu and Avcu (2010) further conclude that low achievement of pre-service teachers is attributed to lack of knowledge on the effective use of the problem solving strategies. On the other hand, the present study uses a standardized mathematics proficiency test which covers a wide range of problems across the subfields of mathematics.

Peressinet *al.*, (2004) conducted a research exercise that traced the learning trajectories of teachers from two reform-based teacher preparation programmes into their early teaching careers. They provided two examples to illustrate how they understood the process of learning to teach. These were situative perspective on teacher cognition and the domains of professional expertise. From their study they concluded that prospective teachers come to teacher education without the subject knowledge necessary to enact reform-based images of teaching. This conclusion supports the findings of Schoenfeld; Cooney, Ball, Even, Thompson and Thompson (Peressinet *al.*, (2004) that despite many mathematics courses taken by undergraduate mathematics majors and prospective mathematics teachers, they have incomplete rule-based knowledge of mathematical concepts. Thus, according to Peressinet *al.*, (2004) it is the responsibility of the teacher training universities to design teacher preparation and teacher development

programmes that model good mathematics teaching, enable teachers to develop content and pedagogical knowledge.

Based upon the research pertaining to teacher training, the teacher education has been oriented more towards pedagogy and teaching methodology. On the other hand, the issue of teachers' knowledge of mathematics has been a prominent one for several decades. Davis and Simmit (2006:294) and Dorfler (2003:162) say that very little progress has been made towards a consensus on the question of what teachers need to know. Ball and Bass (Davis & Simmit, 2006), who have been a major influence in the project which was intended to better understand teachers' knowledge of mathematics, note with concern that teachers need to be adept at interpreting concepts for learners. According to these authors, competence in interpreting concepts requires knowledge of how mathematical topics are connected, how ideas anticipate others and what constitutes a valid argument.

Teachers with inadequate mathematics knowledge cannot appropriately connect or sequence mathematics topics. This claim is supported by Hristovitch and Mitcheltree's study (2004). Teacher education is an on-going process in which teachers develop even beyond the prescribed formal training period. As a result, while the present study focuses on pre-service teachers, it is important also to draw some information on the studies based on "qualified" teachers' knowledge of mathematics teaching. The study of Hristovitch and Mitcheltree (2004) was part of the project for a professional development programme intended to improve teachers' content knowledge and pedagogical skills. The first phase of the project focused on identifying the areas in which teachers' knowledge needed improvement, and the second



phase focused on the issue of what content should be delivered to teachers in order to improve learners' achievements in mathematics. Three teachers were the source of data obtained during the teaching of the notion of fractions and decimals. It was evident from this study that while teachers try to use innovative approaches like problem solving in their teaching strategies, instructional activities employed do not lead to conceptual understanding and connections of ideas. These teachers failed to organize and sequence mathematical topics in a way that would present mathematical ideas coherently to ensure connectivity.

The consequence of teachers' failure to organize and coherently sequence mathematics topics is to teach topics as isolated bits of information, isolated from other concepts in the same discipline, devoid of real world application which is expected to help learners to conceptualize mathematics (Lott & Souhrada, 2000). Teaching is characterized by very rigid teaching methods which rely on textbooks as the sole source of instruction (Lott & Souhrada, 2000; Turnuklu, & Yesildere, 2007) and thus do not allow learners to explore their ideas. Contrary to this, is the perception that mathematics is the concept and process driven subject that requires learners to authentically connect and apply it in real world context (DoE, 2005). This requirement cannot be accomplished unless the teacher possesses adequate content knowledge.

Notwithstanding failure to organize and sequence mathematics topics, the study of Hristovitch and Mitcheltree (2004) indicates that teachers hardly explain mathematical ideas. For example some teachers have difficulty in relating fractions to division of whole numbers, and to explicitly show transition from common fractions to decimal fraction (Hristovitch &

Mitcheltree, 2004). The problem of pre-service teachers' difficulties in dealing with fractions is evident also in Tirosh's study (2000) which investigated pre-service teachers' knowledge of division of fractions. One of the items in Tirosh's questionnaire was "*Item 1. Following are four division expressions. You are requested to (a) calculate each of these expressions, (b) list common mistakes students in seventh grade may make after finishing their studies of fractions, and (c) describe possible sources for each of these mistakes,  $\frac{1}{4} \div 4$ ;  $\frac{1}{4} \div \frac{3}{5}$ ;  $4 \div \frac{1}{4}$ ;  $320 \div \frac{1}{3}$* " Tirosh (2000:9). In the first part some pre-service teachers gave incorrect quotient of 1 instead of  $\frac{1}{16}$ . They incorrectly used inverse operation as  $\frac{1}{4} \div 4 = \frac{1}{4} \times 4 = 1$ . In the second problem other pre-service teachers gave  $\frac{1}{4} \div \frac{3}{5} = 6^2/5$ , they inverted both dividend and divisor as follows:  $4/1 \times 5/3 = 20/3 = 6^2/5$ . In the last problem some incorrectly argue that  $320 \div \frac{1}{3} = 106.666$ . Tirosh (2000) attributed these errors to intuitive misconception that the quotient must be smaller than the dividend. With regard to part (b) and (c) 27 of the 30 pre-service teachers gave typical incorrect response for each expression. Purportedly, these shortcomings result from the low level of mathematics knowledge possessed by mathematics teachers.

The study of Hristovitch and Mitcheltree (2004) shows that it is expected of professionals in any field of endeavour to practice as they complete their studies or training. The extent to which teachers are ready to teach must be established. It is therefore compelling to conduct the present study to find answers to the questions: What level of mathematics content knowledge do pre-service teachers possess at the point of exit of their training programme? How does mathematics content knowledge of pre-service teachers vary across the mathematics subfields?

### **3.3 THE IMPACT OF PRE-SERVICE TEACHERS' CONTENT KNOWLEDGE OF MATHEMATICS ON TEACHING PRACTICES**

The hypothetical position of the study is that deep knowledge of mathematics content is the prerequisite for effective mathematics teaching. If other components of teaching, such as teaching method, strategies and knowledge of learners, are grounded in deep content knowledge, meaningful teaching of mathematics will be achieved. Furthermore, this conception is underpinned by the belief that the teacher with a deep conceptual understanding of mathematics is able to translate mathematical abstractions, to make connections between mathematics concepts, connect mathematics to other disciplines and also to the real context (Leikin & Levav-Wynberg, 2007). The pre-service teachers' background is a fundamental determinant of conceptual knowledge acquisition. Scholz (Kinach, 2002:53) asserts that if the teacher's mathematical background is procedurally strong, then his/her teaching approaches will not easily transform into conceptually oriented teaching practices. Therefore this study also investigates the effects of the pre-service teachers' mathematics content knowledge to their teaching practices.

Concerns about knowledge of mathematics and quality teaching weigh heavily on the hearts of many mathematics educationists. This is aggravated by the realization of the insurmountable difficulty of recruiting mathematically competent students in mathematics education. This places an undue strain upon teacher training institutions. One of the observed factors that contribute to the poor attainments by prospective teachers is the lack of English proficiency and mathematics proficiency (Du Plessis & Gerber, 2012). Uys, Van der Walt, Botha and Van den Berg (2006) identified the most

important factor in teacher training as English competence. Regardless of the subject taught, teachers must be competent in the medium of instruction. It is through the command of the medium of instruction that the teacher is able to interpret and translate mathematical concepts into mathematical knowledge. Mathematical proficiency is central to effective teaching. Its mastery is imperative at all levels of education (Gierdien, 2012). However, little if any, is documented about the level of academic mathematics content underpinning teaching activities.

Competence in mathematics can be seen to encapsulate four complex elements, namely academic knowledge, pedagogical knowledge, reflexive knowledge and interpersonal relations knowledge. Academic knowledge refers to the possession of requisite knowledge for teaching. Pedagogical knowledge means the necessary skills to apply this knowledge effectively. Reflexive knowledge means the objective judgment to know when to apply those skills. Interpersonal relations knowledge means adherence to the principle of integrity and maintenance of a reasonable standard of interaction in the teaching profession. The responsibility of a teacher suggests that the life and time of novice teachers should be filled with confidence and critical enquiry. To produce teachers who are competent a reasonable number of standards should be postulated for the teaching profession.

Teaching of mathematics is influenced largely by the level of the teachers' content knowledge and pedagogical knowledge (Piccolo, 2008). Many researchers point out that in order to construct mathematical concepts in students' minds, mathematical content knowledge and pedagogical knowledge is needed (Turnuklu & Yesildere, 2007; Mohr, 2006). Hristovitch

and Mitcheltree (2004) concluded that content knowledge and understanding of the methods of enquiry in mathematics is at the core of effective teaching. Hristovitch and Mitcheltree (2004) conducted a study which was part of large project aimed at developing a professional development programme to improve pre-service teachers' content knowledge and pedagogical skills. Their study revealed that although teachers were trying to employ innovative approaches, such as problem solving in their teaching, their instructional activities were not leading to conceptual understanding and connection of mathematical ideas. It was also found that they had difficulty in explaining mathematical ideas. Apparently, this difficulty results from the lack of either content or pedagogical knowledge.

The assertion was made earlier that pre-service teachers complete their training with the same knowledge base of content and pedagogy as when they begin training (Benken & Brown, 2008). Ishler, Edens and Berry (Benken & Brown, 2008) assert that one of the factors that causes prospective teachers to exit with similar knowledge and beliefs as when they commenced their studies is that many teacher education programmes, particularly in mathematics, do not make connections across content, method and general pedagogy throughout the entire programme. As a result Benken and Brown (2008) conducted an experimental study to investigate the effect of integrating mathematics content, general pedagogy and methodology courses during teacher training. The participants were prospective elementary teachers. The findings in their study revealed that when integrated programmes are used, the understanding of pre-service mathematics teachers is enhanced; pre-service teachers are able to translate modelled experiences into ideas about future practices, and this changes their affective attitude

toward mathematics. The study indicated that when content and pedagogy is connected in the programme, pre-service teachers' mathematical conceptions will improve, thus leading to conceptual understanding of mathematics.

Conceptually based teaching is central to effective and meaningful learning. Fennema and Franke (Turnuklu & Yesildere, 2007) argue that if a teacher has a conceptual understanding of mathematics, this influences his/her classroom instruction positively. In addition to this, Hobden (2007) concludes that students with conceptual understanding know more than isolated facts and methods; they understand why a mathematical idea is important as well as the kinds of contexts in which it is useful. If a teacher does not know how to translate mathematics abstractions into a form that enables learners to relate the mathematics to what they already know, learners will not learn with understanding (Turnuklu & Yesildere, 2007).

Research which has been conducted during the last decade reveals that strong mathematics content knowledge for teachers is essential for effective classroom instruction that develops mathematical connections in students' minds (Leikin&Levav- Wynberg, 2007; Nathan & Petrosino, 2002; Li & Smith, 2007; Piccolo, 2008). This implies that pre-service teachers must acquire a deep knowledge of mathematics content during their training. An assumption that often underpins secondary mathematics teacher education, is that prospective secondary teachers already have a mathematical disposition and considerable mathematical competence that need only to be tuned to the needs of the teaching (Adler & Davis, 2006). However, these authors maintain that it is critical that in both pre-service and in-service mathematics teacher education programmes, mathematical know-how and dispositions

need to be in ways that will enable teachers to project strong mathematics identities. Clearly, effective mathematics teaching depends, amongst other things, on robust connection between content knowledge and pedagogical knowledge. This connection is what Shulman (Mohr, 2006:219) perceives as a teacher's specialized knowledge of content, which he calls "pedagogical content knowledge". Shulman defines pedagogical content knowledge as the ability of a teacher to transform the content knowledge into forms which are "pedagogically powerful and yet adaptive to the variations in ability and background presented by the students".

Based on the importance of content and pedagogical knowledge in teaching, there is a need to conduct studies on pre-service mathematics teachers' competency of pedagogical content knowledge in mathematics. Turnuklu and Yesildere (2007) conducted research on pre-service teachers' pedagogical content knowledge in mathematics. The participants were at their final year of study at the university. Data were collected through four mathematical problems which were based on pre-service teachers' interpretations of learners' misconceptions or misunderstanding of mathematical knowledge. The findings of the study indicated that a deep understanding of mathematical knowledge is necessary but not sufficient to teach mathematics. The problem is compounded if teachers do not have sufficient mathematical content knowledge needed for teaching. These authors found that pre-service teachers have difficulty in explaining the difference and similar features of  $(+5) - (+3)$  and  $(+5) + (-3)$ . This is attributed to both lack of understanding the relationship between addition and subtraction and also the ability to use teaching strategies that will enable learners to see a distinction between these two operations. This means that content and pedagogical knowledge are

mutually constitutive. This conclusion is supported by Piccolo (2008) who maintains that the ability to teach mathematics content meaningfully is influenced by general pedagogy, pedagogical content knowledge and mathematical content knowledge.

The studies conducted by Crespo (2003) and Kayhan and Argun (2009), which examined the problem posing strategies of pre-service teachers and proficiency in mathematics respectively, indicated that improvement in these aspects goes with pre-service teachers' experience. However, an intriguing result particularly in Crespo's study, was apparent disconnections between the pre-service teachers' mathematical content knowledge and their instructional practices. In addition to this, the study indicated that regardless of the pre-service teachers being mathematically competent and confident, their beginning problem posing practices were limited and limiting. Generally, mathematical tasks posed by these teachers are based on memorization and procedures or tasks of low cognitive demands. In addition to this, Hengstein and Stein (Crespo, 2003:244) report that, apart from the fact that the task of high cognitive demand is exceptionally rare, it also tends to be reformulated into a less demanding task during instruction. This substantiates that the connection between knowledge of content and pedagogy is pivotal for effective mathematics teaching.

Nathan and Petrosino (2002) assert that content knowledge is a prerequisite for a well-developed pedagogical content knowledge. Pedagogical competencies and understandings are based upon having a deep and thorough understanding of mathematics content (Piccolo, 2008; Hill & Ball, 2004; Nathan & Petrosino, 2002). If teachers do not acquire adequate mathematics



content knowledge in their training it will be difficult for them to acquire the necessary content at work. However, Tirosh (2000) and Sam (2005) argue that it is pedagogical knowledge that can be achieved in the field. To this end, more mathematics content should be covered at an advanced standpoint during the teacher training and acquisition of content knowledge must be a priority in mathematics teacher education programmes.

Mathematics is a dynamic subject that is best conceptualized through application. However pre-service teachers are often teacher dependent, passive learners who rely on memorization of facts and procedures instead of applying their own independent thought (Andrew, 2006). It may be misleading to assume that when the learner is able to follow the procedure that leads to correct answers, then learning has occurred. There must be meanings attached to the procedures and answers. When no meanings are attached to the processes, then teaching is likely to be devoid of application.

For example, if pre-service teachers are encouraged to learn algebraic equations and graphs before any meaning is attached to it, they may be able to draw graphs without knowing that there is a task ahead of them to teach learners that the graph is a mathematical entity which represents a realistic scenario. Meaning oriented teaching consists of constructive activities that allow the learner to integrate new information within his/her own conceptual framework (Buelens, Clement & Clarebout, 2002). The National curriculum statement (DoE, 2005) emphasizes that teaching and learning of mathematics should not be presented in a vacuum; teachers need to contextualize the subject they teach. As a result teacher education must inculcate this notion to the pre-service teachers while they are still in the training process. This means

that the content and the processes involved in teaching have to be entrenched in an appropriate context that will enhance the application of mathematics.

Kahan, Cooper and Bethea (2003:223) argue that content knowledge in the subject area does not suffice for good teaching. Nicol (2002) conducted a study on the impact of connecting subject matter with workplace contexts. She indicated that in her study, teachers had strong academic understandings of content but not in ways which allowed them to connect content to context or locate the content in the context effectively. For example, all participants had already earned a Bachelor of Science degree in their respective areas. However, what is intriguing is that there were indications that some of the participants had difficulty in seeing the world outside the classroom from a mathematical perspective. This confirms the interdependent nature of mathematics knowledge and pedagogy in teaching.

According to Viadero (Li & Smith, 2007) teaching of mathematics requires a special set of skills, and due to this reason, teacher educators must be able to identify the kind of mathematics and pedagogical knowledge the prospective teachers need to acquire in their programme study. However, there are critical factors that might affect the way in which teacher educators teach in training programmes. For instance, some of the teacher educators (lecturers) were previously primary and secondary school teachers themselves and they were trained to enact an old and untransformed education system where learners were passive recipients of knowledge. In addition to this, the professors and some teacher educators in universities usually have advanced degrees in mathematics which they got through mathematics classes which only prepared them for graduate studies (Wu, 1999). Secondly, there is no

legislative requirement for university lecturers to have a teaching professional qualification and they are not assessed during their appointment to determine their teaching skills (Vithal, 2010). Based on these, it is not surprising to find that the majority of mathematics teachers are not competent in both content and pedagogy.

It is widely accepted that expertise in the subject taught is critical for effective teaching (Nathan & Petrosino, 2002). The daunting factor is the tendency that extremely high expertise in the domain often obscures the expert's knowledge of how trainees develop intellectually in that domain. Nathan and Petrosino (2002) conducted a study to investigate the effects of high level of expertise in mathematics content. The findings of the study showed that subject matter experts can be blind to the developmental needs of the pre-service teachers. This notion is supported by Krisman-Scott, Kershbaumer and Thomson (Nathan & Petrosino, 2002) who observed that in medical training, expert nurse clinicians filled faculty positions and demonstrated a notable lack of teaching skills. Very often the way teachers present their lessons is influenced largely by the way they were taught in schools, colleges and universities (Philip, 2007, Thomas & Pederson, 2003; Robinson, 1999). Good teaching skills and strategies must be demonstrated by teacher educators during the pre-service teachers' training. Secondly, teaching of mathematics is influenced largely by general pedagogy, pedagogical content knowledge and mathematical knowledge (Piccolo, 2008; Mohr, 2006). With this in mind, teacher educators must have strong content and pedagogical background which they pass to their student teachers.

In many countries, including South Africa, low levels of mathematics attainment is an issue which raises concern to researchers and educationists. Teachers are perceived as the source of poor achievements in mathematics education (Pournara, 2005; Van der Sandt & Niewoudt, 2003; Mji & Makgato, 2006; Van der Walt & Maree, 2007). The common claim is that teachers are poorly trained, and thus not equipped with necessary mathematics and pedagogical knowledge. However, despite many studies conducted in mathematics education, no agreement has been reached on the issue of what teachers need to know, and the extent of content knowledge they are supposed to have in order to teach well (Dorfler, 2003; Davis & Simmt, 2006). Some researchers believe that the two models of teacher training, for example, the integrated and separated models, have an influence on the pre-service teachers' acquisition of mathematics content knowledge.

The involvement of faculties other than education faculties in teacher education has been debated internationally in recent times (Hoadley, 2009). However, it seems that no agreement has been reached with regard to this issue. It is evident in the lack of uniformity in the manner in which teacher education is conducted in different universities. For example, some universities encourage collaboration between the Faculty of Science and Mathematics and the Faculty of Education during training of mathematics teachers. On the other hand, some universities integrate content and methodology courses in the Faculty of Education.

Collaboration between the Faculty of Science and Faculty of Education is supported by Dorfler (2003), who proposes the consideration of mathematics as an enterprise where mathematicians and mathematics educators engage

together in growing numbers. In addition to this, the study conducted by Grassl and Mingus, (2007) at the University of Northern Colorado explored such collaboration, where the abstract algebra, which is one of the core courses for secondary pre-service teachers, was co-taught by a mathematician and mathematics educator. The assistance from the Science Faculty helped mathematics education to change the nature of pre-service teacher examination to include more challenging problems with higher expectations.

Grassl and Mingus's study (2007) supports the study of Hill and Ball (2004), which evaluated California's Mathematics Professional Development Institutes (MPDI) using novel measures of knowledge for teaching mathematics. MPDI involved both mathematicians and mathematics educators and was designed to provide the subject matter teacher professional development in mathematics. The study indicated that teachers who participated in MPDI improved their performance on the set measures. This accounts for involving lecturers from the faculty of science and mathematics in the training of pre-service mathematics teachers.

As indicated in Hoadley's study (2009), the teaching of mathematics content and educational courses in different faculties is not without criticisms. Garegae and Chakalisa (2005) give an outline of how the University of Botswana structured its teacher preparation programme in which a separated approach in teacher training is used. Bachelor of Education (B Ed) students study their mathematics content courses with their BSc counterparts in the Faculty of Science and education courses from the Faculty of Education. Their study was based on pre-service mathematics teacher preparation programmes and early years of teaching in Botswana. Separated model has

been criticized on the basis that it offers student teachers with mathematics courses which are part of the BSc menu and this has limitations with regard to the way the content is taught (Garegae & Chakalisa, 2005). When a separated model is used, the style of teaching is at variance with the advocated methods and strategies of teaching school mathematics; pre-service teachers are left without ample support to internalized teaching models suggested in education courses, and pre-service teachers do not get sufficient content tailored towards the school curriculum (Hoadley, 2009; Garegae & Chakalisa, 2005).

Benken and Brown (2008) found the integrated teacher education model in which the academic content and method is taught by faculty of education fostering an increased willingness to teach mathematics. It also helps prospective teachers to focus their lessons to an understanding of the content. Like in the case of the separated model, the integrated model also has some criticisms. For example, the integrated model is criticized for teaching only to high school level, thus limiting the teachers' mathematical advancement (Hoadley, 2009).

These criticisms support Sibaya and Sibaya's study (2008), which was intended to ascertain how novice teachers perceive the new teacher education programme based on the policy of the Norms and Standards for Educators (NSE). The NSE model advocated integrated teaching in which both the academic content and method are taught by the Faculty of Education lecturers. Sibaya and Sibaya (2008) argue that the integrated model of teacher education has failed the test of time. They found that novice teachers perceived the integrated teacher education programme as inadequate. Furthermore novice teachers felt that the programme does not prepare

prospective teachers to teach effectively. Based on the findings of this study the novice teachers who were exposed to that teacher preparation programme did not master the academic content of the subjects they were expected to teach upon the completion of their studies.

Ball and Bass (Davis & Simmit, 2006) argue that mathematics content knowledge needed for teaching is not a watered down version of formal mathematics but a serious and demanding area of mathematical work. This means that irrespective of whether the content course is offered in the Faculty of Science and Mathematics or in the Faculty of Education, it is imperative that pre-service teachers acquire a strong knowledge of the mathematics that they are expected to teach in schools when they complete their studies. This gives justification for the universities of the countries like Malaysia and China to offer a Bachelor Degree of Science with Education to secondary school mathematics teachers (Sam, 2005). Sam (2005) supports the idea of ensuring that prospective teachers get a strong content knowledge at training stage. The assertion by Kinach (2002) and Peressin *et al.*, (2004) that the subject matter understanding which the prospective teachers bring to teacher education coursework is not the sort of conceptual understanding that they will need to develop in their future students to accentuate the need for thorough training in content knowledge acquisition.

While the researchers argue for a teacher education approach which involves collaboration between the Faculty of Science and Faculty of Education with the aim of providing strong mathematics content knowledge to prospective teachers, it must be noted that strong content knowledge in the subject area does not suffice for good teaching (Kahan, Cooper & Bethea, 2003). This has

been evident in the Italian system of teacher education, which is characterized by secondary school teachers who have university education which encompasses courses in disciplines related to those they will teach in school but with no courses in education nor training in classrooms (Furinghetti, 2000). The mathematical education for these teachers was theoretically comparable with that of a beginning researcher in pure mathematics. Their policy was underpinned by the idea supported by most mathematicians that “whoever knows mathematics, knows how to teach mathematics”. According to Furinghetti (2000), the perception that one who knows mathematics knows how to teach it, proved itself more and more inadequate to the expectations of the new role of the school due to the lack of didactic aspects and appropriate methodologies.

### **3.4 STUDIES ON THE RELATIONSHIP BETWEEN CONTENT COVERED IN MATHEMATICS EDUCATION PREPARATION PROGRAMMES AND HIGH SCHOOL MATHEMATICS**

For several decades, in mathematics teaching and learning a spotlight has been on teachers’ content knowledge and teaching strategies. This has been accentuated by the low level of attainments in mathematics by the students. In this context, teachers’ mathematics content knowledge is often measured by learners’ achievements. For this reason, there is a need to investigate the relationship between the content covered by teacher preparation programmes and the content covered by the school syllabus.

The issue which often brings controversy is whether the mathematics for teacher preparation courses should be within the outskirts of high school



topics with little emphasis on advanced aspects or whether it should go beyond and incorporate advanced tertiary mathematics.

According to Wu (1999) there is a need to carve out a solid middle ground between the two extremes of teaching, namely what is seen by the academics as good mathematics and what the designers of the high school mathematics syllabus consider to be instantly useful to learners. This would suggest that pre-service teacher preparation programmes should consolidate mathematically those topics which do not stray far from the high school mathematics curriculum.

Mathematicians and engineers often do highly theoretical and abstract mathematics which may sometimes be outside learners' imaginations. Brodie (2004:71) observes that mathematics as practiced by mathematicians is different from mathematics as practiced in schools. The reason for this is that, for schools, mathematics is re-contextualized and crucially implicates theories of learning and schooling. However, there is a need for a good transition from high school to tertiary mathematics because some of the learners taught, will pursue studies in mathematics-oriented fields where a strong mathematics background is needed.

High school mathematics is a demanding learning area which requires a teacher to have some conceptual sophistication. For this reason pre-service teachers must acquire knowledge and modes of reasoning similar to those of experts (Garcia *et al.*, 2006). The researcher sees the expert as a mathematician in the faculty of science or a teacher educator in the faculty of education. In addition to this, the Conference Board of the Mathematical

Sciences (CBMS) (2001) points out that the high school pre-service teachers should complete the equivalent of an undergraduate major in mathematics which also connects university or college mathematics with high school mathematics. Purportedly, this notion supports the University of Botswana when it prepares teachers at degree level through the departments of Mathematics and the faculty of Science, in which B. Ed mathematics pre-service teachers take the same content programme for mathematics as BSc students.

Klein (Furinghetti, 2000:44) pointed out a problem of discontinuity in the university preparation of mathematics teachers. The advanced mathematics done at university must remind the prospective teacher of what he/she learned at school and what he/she will be teaching upon the completion of the training. The lack of connectivity between the university and school mathematics content syllabi compels the pre-service teacher to go back to school experience and teach the way in which he/she was taught in secondary school in order to meet the teaching expectations (Furinghetti, 2000). Based on this, CBMS, (2001) and Shoaf (2000) note with concern that there is a need to design the core mathematics major courses in such a way that pre-service teachers make insightful connections between the advanced mathematics that they learn and the high school mathematics that they will teach.

The integration of advanced mathematics with high school mathematics seems to be appropriate to the South African context, where the mathematics curriculum always changes. For example, some advanced mathematics topics from college and university are brought down to the high school curriculum. By way of illustration, in 1989 topics such as mathematical induction were

phased out from the high school mathematics syllabus; calculus and analytical geometry were then introduced. Obviously, teachers who never studied calculus in their training programmes had problems when teaching these topics. Recently topics such as transformation geometry and mathematical modelling, for example, were introduced in the high school mathematics curriculum. Like in all other mathematical topics, there are high expectations with regard to the teaching of these new topics. Teaching becomes very difficult when teachers have never encountered these topics during their teacher training programme.

Cohen and Ball (Ellis, Contreras & Martinez-Cruz, 2009: 2) have this to say: “how can teachers teach mathematics that they never learned, in ways that they never experienced?” This means that although there are workshops and in-service training programmes offered by Department of Education, mathematics teachers may not reach the desired competence as expected from pre-service teachers. One argument is that one cannot expect to change observed ill teaching of many years in one week or one semester. This has some implications to the teacher preparation programmes regarding the content and pedagogy courses.

It is possible that pre-service teachers did most of the teacher education mathematics topics when they were still in high school. However, at teacher training level, there is a need to tackle those topics from an advanced standpoint and enliven them with historical background, interconnections among ideas, analogies and proofs (Wu, 1999; Davis & Simmt, 2006; Furinghetti, 2000).

This implies that the designers of teacher preparation programmes must entrench the links between the teacher preparation mathematics and high school mathematics.

The absence of a commonly agreed upon core body of knowledge in mathematics education brings about lack of uniformity in teacher preparation programmes. For example, Pournara (2005) suggests the following content courses for teacher preparation: *college algebra, geometry and trigonometry; introduction to calculus, vectors and matrix algebra; mathematical modelling, statistics and probability; financial mathematics and mathematical connections between different domains of mathematics*. Sam (2005) indicated that four universities who participated in his study which compared pre-service teacher education between Malaysia and China included six major areas of mathematics in their mathematics teacher preparation programme. These were *calculus, algebra, probability, statistics, differential equations and complex equations*. Shoaf (2000) suggests that future mathematics teacher courses must include *calculus, linear algebra, mathematical statistics, geometry, abstract algebra and computer science* at advanced level.

Wu (1999) believes that for the high school prospective teachers' programme, a history of mathematics must be included. For instance, they need to know something about *Euclid's elements, Archimedes, Descartes, Newton, Euler, Gauss and Riemann*. History links university and secondary mathematics teaching, particularly for future teachers (Furinghetti, 2000:49). In order to illustrate this point, Furinghetti considered a university mathematics student who confessed that he could not understand algebraic concepts without

knowing why they had been introduced and how they developed to their present form. It means that the history of mathematics explains the originality of the concepts, and hence, the understanding of the mathematical concepts will be improved.

Ostler (2000) asserts that mathematical modelling is an area where teacher preparation programmes can help pre-service mathematics teachers refocus on instructional techniques that embrace the less traditional classroom methods of problem solving and modelling. Mathematics modelling is one of the richest forms of representation in mathematics because it encourages learners to work with and apply a variety of mathematical concepts, processes and relationships (Dossey *et al.*, 2002). Doerr (2007) observes that one of the reasons for the limited use of application and modelling at primary and high school level is lack of knowledge by the teachers to teach through applications and modelling. Mathematical modelling does not only deepen students' understanding of mathematics but it also expands their repertoire of mathematical tools for solving real-world problems (DoE, 2005).

One may argue that mathematics modelling is not a mathematical concept by itself; it is an application of mathematics in real-life situations or problems. However, Pournara (2005) and Shoaf (2000) felt that inclusion of mathematics modelling in their proposed pre-service teacher content curriculum would be appropriate. According to the study of Sam (2005) which compares pre-service mathematics teacher education between Malaysia and China, universities in China offer mathematical modelling and geometry as compulsory courses for their pre-service teachers, while in Malaysia these are not offered at all. In addition to this, China gives much

time for all its mathematics content courses when compared to Malaysia. As a result Sam (2005) found that Chinese mathematics teachers are more competent than teachers from Malaysia and other countries. Teacher training institutions need to focus more on geometry because it seems that students forget geometric content easily when they have left school. This perception is supported by Van der Sandt and Nieuwoudt's (2005) investigation of geometry knowledge among pre-service teachers. They found that as the number of years of training increases, a decline in content knowledge is observed. The following table is adapted from Sam (2005) in order to illustrate the differences between the two countries with regard to mathematics curriculum for pre-service mathematics teacher education programmes.

**TABLE 3.1 List of research studies for literature control**

<b>AUTHOR AND YEAR</b>	<b>TITLE OF ARTICLE</b>	<b>PARTICIPANTS</b>	<b>SOURCE</b>	<b>RELEVANCE</b>
Adler, J. & Davis, Z. 2006.	Researching mathematics for teaching in mathematics teacher education.	Education institutions	Journal for research in mathematics education,	Gives suggestions about ideal teacher education courses
Adnan, M., Zakaria, E., & Maat, S.M. (2012).	Relationship between mathematics beliefs, conceptual knowledge and mathematical experience among pre-service teachers.	Pre-service teachers	Procedia – Social and Behavioural Sciences	Provides understanding about beliefs and conceptual knowledge
Andrew, L. 2006	Pre-service teachers' reaction to their final constructivist mathematics class	Final year pre-service teachers	The journal, [Online],	The place of constructivist approach in teaching of mathematics

Avalos, B., Tellez, F., & Navarro, S. 2010.	Learning about the effectiveness of teacher education: A Chilean study	Pre-service teachers in the first, third and fourth year of study.	Perspectives in Education	Provides information about problems faced by teacher education in general
Avcu, S. & Avcu, R. 2010.	Pre-service elementary mathematics teachers' use of Strategies in mathematical problem solving	93 Pre-service elementary mathematics teachers	Procedia Social and Behavioural Sciences	Enhancement of problem solving strategies
Azeem, M. & Gondal, M.B. 2011.	Mathematics Proficiency Assessment Based Upon Item Response Theory	2680 students from 143 schools	The international Journal of interdisciplinary Social sciences	Like the current study, it used mathematics proficiency test to collect data.
Benken, B.M. & Brown, N. 2008	Integrating teacher candidates' conceptions of mathematics teaching and learning: A cross University collaboration	Pre-service mathematics teachers	<b>The journal, [Online],</b> <a href="http://www.k-12prep.math.edu/journal/contentk">http://www.k-12prep.math.edu/journal/contentk</a> .	Highlights the importance of pedagogical content knowledge.
Brodie, K. 2004.	Re-thinking teachers' mathematical knowledge	Theory	Perspectives in Education.	Teachers' mathematical knowledge.
Buelens, H.; Clement, M. & Clarebout, G. 2002.	University assistants' conceptions of knowledge, learning and instruction	Desktop research	Research in education	How knowledge, learning and instruction is conceived.
Corcoran, D. 2005.	Mathematics subject knowledge of Irish primary pre-service teachers	Primary school pre-service teachers	Paper presented at The European Conference on Educational Research, University College Dublin.	Informs us about the kind of mathematics knowledge needed in teaching of mathematics.
Crespo, S. 2003.	Learning to pose mathematical problems: Exploring changes in pre-service teachers' practices.	34 Pre-service teachers who completed 3 years of course work	Educational studies in mathematics.	Pre-service teachers Practices
Department of education 2005.	Learning programme guidelines.	Report	National Curriculum statement Grades 10-12.	School learning Programmes

Davis, B. & Simmit, B. 2006.	Mathematics for teaching: An on going investigation of the mathematics that teachers (need to) know.	No participants	Educational studies in mathematics.	Investigate mathematics teacher need to know for teaching
Department of Education, 2000	Norms and standards for educators	None	Norms and standards for educators. Government Gazette	Educators guide
Department of Higher Education and Training (DoHET) 2011.	The minimum requirements for teacher education qualifications.	Report	Government Gazette 34467. Pretoria: Department of Education.	Teacher qualifications
Doerr, H.M. 2007	What knowledge do teachers need for teaching mathematics through applications and modelling?	Prospective teachers	The 14 <sup>th</sup> ICMI Study: Modelling applications in Mathematics education.	The role mathematical modelling in learners learning
Dorfler, W. 2003.	Mathematics and mathematics education: Content and people, relation and difference.	Theory	Educational studies in mathematics.	Mathematics and mathematic education
Dossey, J.A., Mc Crone, S., Giordano, F.R. & Weir, M.D. 2002.	Mathematics methods and modelling for today's mathematics classroom.	Teachers	A contemporary approach to teaching grades.	Give knowledge about the teaching methods
Du Plessis, L., & Gerber, D. 2012.	Academic preparedness of students_ an exploratory study.	First year students at the university	The journal for Trans-disciplinary Research in South Africa	Throw light to how first year students should be placed in appropriate programmes
Ellis, M.W.; Contreras, J. & Martinez-Cruz, 2009.	The mathematical preparation of prospective elementary teachers.	One mathematics teacher	The journal, 2. <a href="http://www.k-12prep.math.ttu.edu">www.k-12prep.math.ttu.edu</a> .	Problem solving
Furinghetti, F. 2000.	The history of mathematics as a coupling link between secondary and university teaching.	Theory	International journal of mathematical education in science and technology	Knowledge of origins of mathematics concepts
Garcia, M.; Sanchez, V. & Escudero, I. 2006.	Learning through reflection in mathematics teacher education.	Theory, perspectives	Educational studies in mathematics	Relationship between theory and practice



Garegae, K.G. & Chakalisa, P.A. 2005.	Pre-service mathematics teacher preparation programmes and early years of teaching in Botswana.	Report about the curriculum	A proposal to be presented at the 15 <sup>th</sup> ICMI study conference on the Professional Education and Development of Teachers of Mathematics. Brazil 2005	Structuring teacher education curriculum
Gierdien, F. 2012.	Pre-service teachers' views about their mathematics teacher education modules	Intermediate and senior phase pre-service teachers	Pythagorus	Provides knowledge about how pre-service teachers view their education modules
Grassl, R. & Mingus, T.T.Y. 2007.	Team teaching and cooperative groups in abstract algebra: nurturing a new generation of confident mathematics teachers.	A mathematician and mathematics educator	International journal of mathematical education in science and Technology.	Collaboration between mathematicians and mathematics education
Conference Board of the Mathematical Sciences 2001.	Issues in mathematics education.	Desktop research	The mathematical education of teachers: American mathematical society, 11.	Ideals for mathematical education of teachers
Hill, H.C. & Ball, D.L. 2004.	Learning mathematics for teaching.	Mathematics teachers	Journal for research in mathematics education.	Developing teachers' mathematics knowledge for teaching
Hoadley, U. 2009.	A distant reality: aligning the B.Ed curriculum at North West University.	Report	Human Science Research Council	Aligning B. Ed curriculum
Kahan, J., Cooper, D. & Bethea, K. 2003.	The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers.	Student teachers	Journal of mathematics teacher education, 6:223-252.	Explains the importance of content knowledge in the teaching of mathematics
Hobden, S.D. 2007.	Towards successful mathematical literacy learning.	Pre-service teachers	Unpublished dissertation, Durban: University of KwaZulu Natal.	Mathematical literacy

Hristovitch, S. & Mitcheltree, M. 2004.	Exploring middle school teachers' pedagogical content knowledge of fractions and decimals.	Three Grade six teachers	Paper presented at the annual meeting of the North American chapter of the international group for the psychology of mathematics education, October 2004, Canada	Teachers' knowledge of fractions
Kilpatrick, J., Swartford, J., & Findell, B. 2011.		Desktop research	Adding it up: Help children learn mathematics. Washington D.C.	Suggests ways in which mathematics can be taught to learners
Kinach, B.M., 2002.	A cognitive strategy for developing pedagogical content knowledge in the secondary mathematics methods course: Towards a model of effective practice.	21 pre-service teachers	Teaching and teacher education 18, 51-71.	Development of pedagogical content knowledge
Leikin, R. & Levav-Waynberg, A. 2007.	Exploring mathematics teacher knowledge to explain the gap between theory-based recommendations and school practice in the use of connecting tasks...	Secondary school mathematics teachers.	Educational studies in mathematics, 66:349-371.	Connecting tasks in teaching mathematics.
Howie, S.J. 2003.	Language and other background factors affecting secondary pupils' performance in mathematics.	8000 pupils from 200 schools.	A journal of research in SMT education, 7:1-20.	Factors causing Learners' poor performance.
Li, Y. & Smith, D. 2007.	Prospective middle school teachers' knowledge in mathematics and pedagogy for teaching.	Mathematics pre-service teachers.	Proceedings of the 31 <sup>st</sup> conference of the international group for the psychology of mathematics education.	Content knowledge and pedagogy.
Lloyd, G.M. 2006.	Pre-service teachers' stories of mathematics classroom.	Pre-service teachers	Exploration of practices through fictional accounts. Educational studies in mathematics, 63:57-87.	Teachers perceptions

Lott, J.W. & Souhrada, T.A. 2000.	As the century unfolds: A perspective on secondary school mathematics content.	Theory, perspective	Learning mathematics for a new century.	Secondary school mathematics content
Loucks-Horsley, S. 1998.	The role of teaching and learning in systemic reform.	Theory	Science educator, 7(1):1-6	Curriculum reform
Menon, R. 2009.	Pre-service teachers' subject knowledge of mathematics.	Pre-service teachers	International journal for mathematics teaching and learning.	Contribute to the knowledge about pre-service teachers' knowledge of mathematics
Mji, A. & Makgato, M. 2006.	Factors associated with high school learners' poor performance: A spotlight on mathematics and physical science.	10grade11 learners in 3 districts of Tswane North	South African journal of education.	Highlighting causes of learners poor performance in mathematics
Mohr, M. 2006.	Mathematics knowledge for teaching.	Theory	School science and mathematics, 106: 219-243.	Mathematics knowledge for teaching
Martyn, S. 2008.	Descriptive research design.	Theory	Available at: <a href="http://www.experiment-resources.com/descriptive-research-design.html">http://www.experiment-resources.com/descriptive-research-design.html</a>	Defines and explains when to use descriptive research design
Moll, I. 2002.	Clarifying constructivism in a context of curriculum change.	Theory	Journal of education, 27: 5-31.	Constructivism
Moloi, M. & Straus, J. 2005.	The SACMEQ II Project in South Africa	Report, no participants	A study of the conditions of schooling and the quality of education. Harare: SACMEQ.	Curriculum and learners' performance
Nathan, M & Petrosino, A. 2002.	Expert blind spot among pre-service mathematics and science teachers.	Pre-service high school teachers	Expert blind spot.	Expertise in teaching
National Planning Commission (NPC) 2010.	National Development Plan 2030.	Report	Our future makes it work. Government Gazette <a href="http://www.info.gov.za/issues/national-development-plan/index.html">http://www.info.gov.za/issues/national-development-plan/index.html</a> .	Planning for educational activities

Nicol, C. 2002.	Where's the math? : Prospective teachers visit the workplace.	22 pre-service teachers in integrated maths, science and technology class	Educational studies in mathematics.	Linking mathematics to work place (practical)
Onwu, G.O.M. & Mogari, D. 2004.	Professional development for out-comes based education curriculum implementation.	Foundation phase teachers	Journal of education for teaching,	Professional development and curriculum implementation
Ostler, E. (2000).	Mathematical modelling: Some ideas and suggestions for pre-service teacher preparation.	Theory	Mathematics Department University of Nesbraka. Omaha	Importance of mathematical modelling
Peressini, D., Borko, H., Romagnano, L., Knuth, E. & Willis, C. 2004.	A conceptual framework for learning to teach secondary mathematics: A situative perspective.	Pre-service teachers	Educational studies in mathematics.	Perspective on teacher cognition
Philip, R.A. 2007.	Mathematics teachers' beliefs and affects.	Theory	Second handbook of research on mathematics teaching and learning.	The effects of beliefs in teaching
Piccolo, D. 2008.	School science and mathematics.	Theory	Official journal of The school Science and mathematics association.	Views of content and pedagogical knowledge for teaching
Pournara, C. 2005.	The glass is half-full: Rising to the challenge of pre- service mathematics teacher education in South Africa.	Theory	15 <sup>th</sup> ICMI study conference in the professional education and development of teachers of mathematics. South Africa.	Courses for content and methodology
Reddy, V. 2006.	Mathematics and science achievement at South African schools in TIMSS 2003.	TIMSS Report	Human science research council.	Mathematics curriculum
Robison, M. 1999.	Initial teacher education in changing South Africa: experiences, reflections and challenges.	Theory	Journal of education for teaching.	Teacher education in South Africa.

Sam, C.L. 2005.	A comparison of pre-service mathematics teacher education between Malaysia and China.	Four universities, two from China and two from Malaysia	School of educational. Universiti Sains.	Teacher preparation programme
Seaman, C.E., Szydlik, S.D., Szydlik, J.E. & Beam, J.E. 2006.	A comparison of pre-service elementary teachers' beliefs about mathematics and teaching mathematics.	Pre-service teachers	School science and Mathematics.	Beliefs about mathematics and its teaching.
Sestic, L. & Huttunen, I. 2006.	A guide: Implementing the revised Core Curriculum for Modern languages in Bosnia and Herzegovina. DGIV/LANG Council of Europe.	Desktop research.	DGIV/LANG Council of Europe.	Informs the researcher about the implementation of the revised curriculum.
Shoaf, M.M. 2000.	A capstone course for pre-service secondary mathematics teachers.	Theory	International journal of mathematical education in science and technology.	Use of technology in the teaching of mathematics.
Sibaya, P.T. & Sibaya, D.C. 2008.	Novice educators' perceptions of the teacher education programme proposed by the Norms and Standards for Educators.	Novice teachers	Perspectives in education, 26 (4), 86-100.	Evaluates the teacher preparation programmes.
Tatto, M.T., Schwille, J., Senk, S.L., Ingvarson, L., Rowley, G., Peck, R., Bankoy, K., Rodriguez, M. & Reckase, M. 2012.	Policy, practice and readiness to teach primary and secondary mathematics in 17 countries.	Primary and secondary mathematics teachers	TEDS_ International Report	Gives information about policies and practices of mathematics teachers in different countries.
Taylor, N. 2009.	South African school mathematics fails to prepare students for challenges of university.	Theory	City Press, 17 May. p.5.	State of mathematics in South African schools
Thomas, J.A. & Pedersen, J.E. 2003.	Reforming elementary science teacher preparation: What about extant teaching beliefs?	Theory	School science and mathematics.	Teacher preparation

Tirosh, D. 2000.	Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions.	Pre-service teachers	Journal for research in mathematics education.	Role of teachers' knowledge of learners' conceptions
Toh, T.L., Chua, B.L. & Yap, S.F. 2007.	School mathematics mastery test and pre-service mathematics teachers' mathematics content knowledge.	In-service teachers	The mathematics educator.	Upgrading teachers content knowledge.
Turnuklu, E.B. & Yesildere, S. 2007.	The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey.	45 pre-service Teachers	The journal.	Pedagogical content knowledge.
Uys, M., Van der Walt, J.L., Botha, S.U., & Van der Berg, R. 2006.	An integrated course for English medium of instruction teacher trainees in South Africa.	Teacher trainees	Journal for language teaching.	Informs the current study about effects of English medium of instruction in training of teachers.
Van der Sandt, S., & Nieuwoudt, H.D. 2005.	Geometry content knowledge: pre-service training making a difference?	Prospective teachers.	African journal of research in SMT Education, 9,109-120.	Gives contrition on how to help prospective teachers acquire geometry content knowledge.
van der Walt, M. & Maree, K. 2007.	Do mathematics facilitators implement meta-cognitive strategies?	Twelve mathematics facilitators in six schools.	South African Journal of Education.	Meta-cognitive strategies in teaching mathematics.
Vithal, R. 2010.	Moves underway to train university lecturers to teach.	Theory	UKZNTOUCH, A University of KwaZulu-Natal Alumni Magazine: University of KwaZulu-Natal, Durban.	The level of teacher educators' expertise.
Whitehurst, R. 2003.	IES Director Grover.	Desktop research	<a href="http://www.ed.gov/print/rschstat/research/progs/mathscience/Whitehurst.html">http://www.ed.gov/print/rschstat/research/progs/mathscience/Whitehurst.html</a> .	It explains why mathematics is challenging.
Wu, H. 1999.	Pre-service professional development of mathematics teachers.	Pre-service teachers	University of California, Berkeley, USA.	Improvement of teacher preparation programs.

### 3.5 CONCLUSION

The previous work done in this field indicates that in many countries mathematics education has been a subject of great concern particularly to researchers and mathematics educators. In the light of the previous studies in mathematics education, most mathematics teachers begin their teaching career with inadequate content knowledge and the sophistication needed in mathematics teaching. Clearly, teachers' mathematics content knowledge is paramount for the meaningful teaching and learning of mathematics (Brodie, 2004; Furinghetti, 2000; Natan & Petrosino, 2002; Benken & Brown, 2008; Leikin & Levav-Waynberg, 2007; Loucks-Horsley, 1998). It is the teacher's possession of deep mathematics content knowledge that enables him/her to acquire a well developed pedagogical content knowledge.

The present investigation is intended to determine whether the pre-service teachers possess adequate mathematics content knowledge, as recommended by many researchers, as a primary source of effective teaching. There have been many studies and intervention recommendations pertaining to the upgrading of the teaching standards in mathematics, yet no conspicuous improvement has been observed. There is a need to determine the level of content knowledge possessed by pre-service teachers as well as the relevance of the current programmes in providing pre-service teachers with appropriate and adequate mathematics knowledge.

## **CHAPTER FOUR**

### **RESEARCH METHODOLOGY**

#### **4.1 INTRODUCTION**

In this chapter the detailed research design and methods of the study are discussed. The research methodology includes the selection of the study sample, the research instruments, procedures for data collection and the plan for data organization and analysis.

#### **4.2 RESEARCH DESIGN**

Researchers sometimes conduct research using descriptive and field study designs to learn about attitudes, perceptions, opinions, beliefs, behaviours and relationships, among other variables. In order to address the research questions and to achieve the aims of the present study, a similar approach or design is used.

Field studies/descriptive research designs are non-experimental scientific enquiries aimed at discovering the relations between and the interactions among sociological, psychological and educational variables in real social situations. Any research project that pursues relations and/or tests hypotheses in real-life situations as found in communities, schools, factories, organizations and institutions can be considered to be field studies or descriptive designs (Kerlinger, 1992:312; Turney & Robb, 1971:62).



One of the advantages of using descriptive design is that the participant is observed in a completely natural and unchanged environment (Martyn, 2008). The design is not intended to ferret out cause and effect relationships, but to describe relations that already exist among variables (Grimes & Schulz, 2002; Shuttleworth, 2008; Sibaya & Sibaya, 2008; Key, 1997). Descriptive design is ideal in the present study because the aim is to achieve an in-depth understanding about the existing pre-service teachers' level of mathematics content knowledge. Descriptive design is commonly used with field experiment methods in which data are collected to describe persons, organizations or phenomena in their natural settings (Sibaya & Sibaya, 2008; ECS & McRel, 2004).

#### **4.3 SAMPLE DESIGN**

The first stage of the sampling design procedure involved cluster sampling. A table of random numbers (Behr, 1988) was used to select one province, that is, KwaZulu-Natal out of the nine provinces of South Africa. In this province very few higher education institutions offer the Bachelor of Education (B.Ed) degree as a teacher preparation programme. The B.Ed degree is divided into three types, namely the B.Ed (Foundation phase); B.Ed (Intermediate Phase) and B.Ed (Senior, Further Education & Training Phase) teaching (Government Gazette, 2011:21).

The second stage involves all elements in the chosen cluster. These elements were the fourth year pre-service teachers of the B.Ed (Senior Phase & Further education & Training Phase). Upon completion of their programme these pre-service teachers are expected to teach mathematics at grades 8 to 12

inclusive. These pre-service teachers were all studying the subject called Method of Teaching Mathematics. Fourth year students were selected based on the assumption that they have acquired both content and pedagogical knowledge and in view of the expectation that the following year they would begin their teaching career. As a result, the final stage of sampling design used in this study was purposive, that is, only participants who fulfilled the purpose of investigation were included in the sample (Devers & Frankel, 2000; Gray, 2004).

Sam (2005) also used the same sample design in his study, which compares pre-service mathematics teacher education between two countries, namely Malaysia and China. Only two universities were selected in each country, which makes up a total of four universities that participated in the study. Based on the study of the syllabi documents, the comparison focused on the structure and mathematical content programmes offered by two universities in each country. Although the sample was too small to generalize the situation or to represent all of the universities in Malaysia and China, it nevertheless gave a glimpse of how the universities' programmes differ from each other, or how they are similar in terms of content and structure. In a similar way, in this study only two universities were selected from one province, out of the many universities to be found in the nine provinces of this country.

#### **4.4 RESEARCH INSTRUMENTS**

Studies of this nature often make use of a variety of data collection techniques. Li and Smith (2007) used survey questionnaires and mathematics tests to investigate prospective teachers' knowledge of mathematics and

pedagogy acquired during their training period. The study conducted by Benken and Brown (2008) used survey questionnaires, content examination and interviews to investigate the effects of integrating mathematics content, general pedagogy and methodology courses.

The present study used three instruments, namely the mathematical proficiency test, practice teaching forms and syllabi documents. There seems to be commonality in measuring the level of pre-service content knowledge. Most studies, including the present study, use tests which are based on the content knowledge. However, instead of using survey questionnaires and interviews, the present study uses practice teaching assessment forms. The reason for such a different approach is that the present researcher wanted to get data based on what pre-service teachers practically do to illuminate their existing knowledge of content and pedagogy.

The approach of the present study was to allow pre-service teachers to solve mathematical problem that are similar to what they would be teaching. This approach would elicit information on the level of mathematics content knowledge that pre-service teachers have acquired by the time that they exit their training. With regard to study aim number one, the researcher used the mathematical proficiency subtest of Minnie & Paul(1982). For aim number 2 of the study, the student teachers' practice teaching assessment form was used to get information regarding their pedagogical content knowledge and teaching practices.

The teacher assessment forms were obtained from the universities in which the practice teaching assessment was done. For aim number 3 of the study,

documents containing high school and university mathematics syllabi were used to get data regarding the relation between high school and teacher education mathematics syllabi. The documents containing the high school syllabus were requested from the Department of Education offices and university syllabi were obtained from three universities.

#### **4.4.1 Description of the mathematical proficiency test**

The mathematical proficiency test is the subtest of the Academic Aptitude Test (AAT) (Minnie & Paul, 1982). The mathematical proficiency test is based on the high school mathematics content. It contains a variety of mathematical problems for algebra, trigonometry and Euclidean geometry topics of high school mathematics. It was surmised that this subtest would elicit pre-service teachers' knowledge of mathematics content needed for high school mathematics teaching. The test has 30 items. Each question has five possible answers from which the respondent is expected to select one correct answer. One mark is allocated for each correct answer, which means that a perfect total score would be 30. The participants responded by shading the correct answer in the answer sheet. Test material was collected after administration. Nothing was left with the candidate. The time limit for the test was 70 minutes. The following details had to be filled in by the student on the front page of the instrument: Student number, Degree/Diploma (e.g. PGCE). This form of identification enabled the researcher to correlate marks obtained in the mathematics proficiency test with the marks obtained in practice teaching.

The present researcher considers the subscale to be suitable for this study because it seems to share almost a common aim with the present study. For example, the test came about as an initiative of the Department of Education and Training to design a comprehensive test to be used to evaluate standard 10 learners' level of mathematical knowledge for the purpose of guiding them in choosing courses in respect of further training. It was also used by the universities to obtain information that could be used at first year level in conjunction with the matriculation examination results in order to guide and select students for particular courses or subjects. The subscale precisely measured the academic potential or ability of a student to cope with mathematics at university level. It gave an indication of a learner's mathematical ability and level as well as spatial ability, which is an essential part of the present study, although at teacher education level. For the purpose of this research, it would indicate the pre-service teachers' level of mathematics content knowledge. Notably, during the standardization process the test was applied to all South African ethnic and racial groups consisting of males and females. Similarly, the present study sample, namely pre-service teachers, is also multiracial, multicultural and includes both males and females. The AAT is a battery of tests. These different tests are called subscales. The mathematical proficiency test is one of these subscales. It was standardised on African Blacks. As a result, the present researcher strongly believes that this subscale is appropriate for the present research project.

#### **4.4.2 Validity and reliability of the mathematical proficiency test**

The Academic Aptitude Test (AAT) was standardized on a representative standardization sample of 1463 matriculants in September 1972. The

standardization sample consisted of male and female learners from all African languages. For example, there were 893 males and 570 females. Initially, three equivalent forms or set of questions were formulated and administered to the standardization sample. The scrutiny of items led to the establishment of two parallel forms. Forms A and B were administered to matriculants and to university students, respectively, for item analysis. To establish the norms, the final form of the test battery for matriculants was applied to a representative sample of 1298 learners in 1973. The AAT has a manual that contains norms, procedures for administration, scoring and interpretations. The whole administration process was done under the supervision of a psychologist registered with the Health Professions Council of South Africa (HPCSA).

According to Huysamen (Soman, 2006), validity and reliability are the two psychometric properties that are of critical importance in any measuring instrument in social science research. Validity of a test is the extent to which a test measures what it supposed to measure, while reliability is the degree of accuracy and consistency with which a test measures the same trait with repeated measurements (Minnie & Paul, 1982). In order to determine the validity of the AAT battery of tests, correlations between examination marks obtained in various subjects and scores of the battery were computerized. This was done by the test designers, namely Minnie and Paul (1982). Table 6.3 in the test manual (Minnie & Paul, 1982:37) shows that only the subtest on spatial perception did not correlate with school subjects. The validity correlation coefficients between mathematics proficiency and school subjects are shown in table 4.1. This is the internal consistency method for establishing validity.

**TABLE 4.1      Validity correlation coefficients between mathematics proficiency test and school subjects**

Subject List	School Subjects	Correlation Coefficients	Strength	R <sup>2</sup> %
A	Ethnic Languages	0.15	Very slight	2.3
B	Afrikaans	0.25	Slight	6.3
C	English	0.34	Slight	11.6
D	Biology	0.32	Slight	10.2
E	Physical Science	0.42	Moderate	17.6
F	Mathematics	0.76	High	57.8
G	Geography	0.23	Slight	5.3
H	History	0.36	Slight	13.0

The correlation coefficients between mathematics proficiency and school subjects range from 0.15 (very slight) through 0.42 (moderate) to 0.76 (high). These correlation coefficients were tested for statistical significance at 1% and 5% levels. The correlations between the Mathematical Proficiency Test and subjects B to F were significant at 1% level. There were no significant correlation coefficients between the Mathematics Proficiency Test and two subjects, namely Ethnic Languages and Geography.

In the present study, the researcher attempted to determine the effect size of the significant correlation coefficients. Table 4.1 contains these values in the last column. A measure of the effect size should follow statistically significant results to be on the edge of statistical sophistication. The effect size informs us about the magnitude of change among variables. Here we attempt to find an

answer to the question of how much change in one variable is accounted for in another variable. Put differently, how big is the association between mathematics proficiency and subjects? How strong is the correlation? Table 4.1 shows that a correlation coefficient of 0.15 has an effect size of 2.3%, 0.42 has an effect size of 17.6%, and 0.76 has an effect size 57.8%.

The implication of these percentages is that a change in one variable will account for an equal change in another. If we know these values for one subject score, we can predict an equal condition that obtains in another subject. Therefore the effect size percentages apprise us about the magnitude of change between two variables, that is, explained variation. For the correlation coefficients of 0.15, 0.42 and 0.76 the unexplained variance is 97.7%, 82.4% and 42.2% respectively. The correlation coefficients of the test indicate the validity of all the mathematics proficiency tests.

The reliability of the AAT was established for the battery as a whole. The Kuder-Richardson (KR) formula 20 was applied to get accurate values of the reliability. For all the subtests the reliability coefficients range from 0.69 to 0.90. Their values can be regarded as very high coefficients of the reliability. The subtest on the Mathematics Proficiency Test is reported to have a KR 20 reliability of 0.75 (Minnie & Paul, 1982:31-47).

The present study makes use of a standardized instrument, that is, the norms of the instrument have been established. The instrument is both valid and reliable. It is used for research purposes and not for psychological assessment. The standardization sample of the test was up of male and female matric learners consisting of males and females. In other words, the test was



designed to test learners' proficiency in high school mathematics. The present study intends to determine the proficiency of pre-service teachers in high school mathematics as they exit their training. The pre-service teachers are expected to teach effectively the same mathematics content which was applied to the standardisation sample when they completed their training. To this end, the mathematics proficiency test was deemed suitable for this study sample.

#### **4.4.3 Description of practice teaching form**

The practice teaching assessment forms consisted of items which focused on the teachers' knowledge of the subject and pedagogy. The very common aspects contained in the teacher assessment form for these universities were as follows: The introduction of the lesson or activities, evidence of a good plan, motivation of learners, relevance of teaching method used, involvement of learners, evidence of extensive reading around the topic/theme, interpretation of learning programmes, ability to assess and share assessment with learners and the conclusion of the lesson (Appendices B and C). In each of these aspects the pre-service teachers were awarded marks by their individual supervisors. The present study was conducted during the second semester of 2011 and mark schedules were readily available.

The items in the practice teaching assessment form have the potential to elicit information that will inform the researcher about the current status of the pre-service teachers' pedagogical knowledge. It is for this reason that the form was chosen as one of the instruments for the present study.

#### **4.4.4 Description of syllabi comparison procedure**

The study also explored the relationship between the high school and teacher education mathematics syllabi. To compare the content coverage of these two levels of education, it was deemed appropriate to use the syllabi documents. The syllabi documents contain all of the mathematics topics covered in teacher preparation and at high school (Appendices D and E).

The researcher developed a structural framework consisting of four columns. The first column was for the university content, the second for the high school content, the third for the similar or common themes count and the fourth column was for the different themes count. The researcher identified and coded common and different themes in the syllabi for high school and teacher preparation programmes. Instead of normal coding, the researcher used italic writing to indicate common themes and bold for different themes. Common and different themes were quantified to facilitate comparison.

Apart from wanting to find the relationship between high school and university syllabi, the researcher also wanted to find out how the content knowledge possessed by pre-service teachers contributes to the way in which they teach mathematics. The data collection technique is determined by the research question and the source, which has the potential of yielding the best information to answer the question (Merriam, 2002). As a result, the techniques used in this study are presumed suitable to produce information that is necessary to address the research questions of the study.

#### **4.5 PROCEDURES FOR EMPIRICAL STUDY AND ETHICAL CONSIDERATIONS**

The researcher first wrote letters to the relevant institutions to request permission to conduct research (Annexure A). The institutions required the researcher to obtain ethical clearance from his university of registration to confirm his registration. Having obtained ethical clearance from university of registration, the researcher proceeded to apply for ethical clearance certificates from the universities from which external samples were drawn. One of the important aspects of this study was to show respect to the participants and to the institution in which the study would be conducted. The researcher visited two institutions. The purpose of the study was explained to the gatekeepers of each institution. The gatekeepers required the submission of the following documents before the researcher was granted an ethical clearance certificate: a letter of informed consent, copies of instruments to be administered and a copy of the proposal of the study.

An ethical clearance letter/document was required by the gatekeepers at the institutions where the instruments would be administered. It served as confirmation that the researcher was a registered university student and to certify that the researcher had obtained permission from his/her university to conduct researcher in another institution.

Having submitted all the documents that were required by the gatekeepers of each institution, the researcher was issued with an ethical clearance certificate (Annexure B). The ethical clearance certificate confirms that the researcher had been granted permission to conduct research in that institution.

The researcher was required to explain in writing the procedures for the informed consent to the participants. The participants were assured of anonymity and that they would be allowed to discontinue the process should they feel uncomfortable. As Henning et al., (Powell, 2010) put it, when doing research, the participants need to know that their privacy and sensitivity will be protected and they also need to know what will happen with the information thereafter. The participants were required to sign the informed consent letter (Annexure C). This served as confirmation that each participant participates in this research on a voluntary basis. The subject teacher accompanied the researcher during the administration of the instrument.

A copy of the mathematical proficiency test was submitted to the authorities at each of the universities where the research would be conducted. The authorities of the universities wanted to ascertain the nature and the appropriateness of the test to be administered to the intended study sample groups before they granted permission. The researcher complied with these requests. This copy was fetched after the administration of the mathematics proficiency test. Standardized psychological tests are regulated and controlled. Their distribution is subject to strict control by the Health Professions Council of South Africa (HPCSA).

One of the requirements for ethical clearance was to submit the proposal or the summary of the study in which all the details of the study are outlined. Included in the proposal is, among other things, the topic, aims, research questions, participants, methodology and procedures of the study.

## 4.6 ADMINISTRATION OF RESEARCH INSTRUMENTS

The aims of the study were re-iterated as stated in chapter one. For the measurement of each aim, the instrument used, scoring procedure for each instrument and the interpretation of results were discussed, aim by aim, as follows:

### 4.6.1 Aim number one

**To determine how much mathematics content do teachers possess at the point of exit of their teacher training programmes. Put differently, to determine to what extent teachers have mastered the prescribed mathematics content at the point of exit of their teacher training programmes.**

The mathematics proficiency test was used to measure this aim. The mathematics proficiency test contained mathematical problems that covered most areas of the school mathematics content syllabus. The same procedure for the administration of the research instrument was followed in both institutions. Before students entered into the classroom, the researcher ensured that a question paper was placed on every desk. On entering the classroom, students were advised to read the instructions carefully. Students were also requested to write their registration numbers on the answer sheets. The question and answer sheets were arranged sequentially according to the numbering. This was done to make it easier to see whether there were any missing question and answer sheets.

As one of the imperatives of the instrument, the researcher together with the students solved the first three problems, which served as examples in the mathematical proficiency test to show the students how they are expected to answer the questions. Thereafter, the students were allowed to write the test on their own. They wrote up to the last minute of the allocated time. When they had finished writing, the answer sheets were collected together with the test booklets. No answer sheets or test booklets were left behind.

The mathematical proficiency test has thirty questions. One mark was allocated to each correct answer. This means that the highest possible score was 30 and the lowest possible score zero. Scores were categorized into levels of performance. Scores from 24-30 (80% -100%) were considered as level 1, which represented excellent performance or adequate mathematics knowledge for high school teaching. Hypothetically, teachers at this level were likely to have adequate knowledge to teach high school mathematics effectively if they were equipped with proper pedagogical knowledge during training. Scores from 15- 23 (50% - 79%) were considered as level 2 (mediocre) performance or moderate knowledge. While teachers at this level can teach mathematics, one may nevertheless expect them to have shortcomings in their teaching. Scores below 15 (50%) were considered as level 3 (inadequate) knowledge. Presumably, these teachers have inadequate mathematics content knowledge necessary for teaching high school mathematics.

The continuous scale was dichotomized and a cut-off point was 50%. Any students who scored below 50% would be considered as having an inadequate knowledge of mathematics or an inadequate potential of teaching

mathematics effectively. Any student who scored 50% and above was considered to possess adequate mathematics knowledge needed for mathematics teaching.

#### **4.6.2 Aim number two**

**To establish the relationship, if any, between achievement in content and achievement in teaching practice**

At both universities, the researcher requested access to the students' practice teaching assessment scores, which indicate their performance in practicum. Each pre-service teacher's score in the mathematical proficiency test was compared with his/her score in the teaching practice assessment in order to find out whether there is any correlation between the pre-service teachers' knowledge of mathematics and the way in which they teach mathematics.

The teacher's performance score during teacher evaluation was taken as if it had been awarded by the assessing lecturers according to the university's specifications. These forms contain students' registration numbers and marks in percentage. The registration numbers were crucial for identification of data for X variables and Y variables in the working out of correlation between two sets of data. High correlation will indicate an association between two sets of scores. The opposite would indicate that there is little or no association between the achievements in mathematics knowledge and the teaching practice result.

### 4.6.3 Aim number three

**To establish the relationship, if any, between content covered in the mathematics teacher preparation programmes and high school mathematics**

In order to measure this aim, content analysis was used. According to Palmquist (1967), content analysis is used to determine the presence of concepts and themes within texts or sets of texts. As a result the documents that contained high school mathematics and teacher education mathematics were used to compare their syllabi content.

The comparison of the syllabi would determine whether there is any relationship between the teacher preparation mathematics and school mathematics. The assumption was that a similarity between what is taught in teacher education and what they would be teaching at high school is essential for effective teacher preparation. In order to compare the syllabi, the researcher tabulated the categories, themes/main topics and subtopics/subthemes covered by both syllabi. Categories consisted of the mathematics sections (e.g. algebra, trigonometry and geometry). Themes were the main topics in the category. For example, in algebra the main topic is exponents. Subtopics are the parts of the main topic. For example, exponential laws, exponential equations, exponential functions and graphs are parts of main topic, the exponent. The researcher highlighted the common and different themes or topics. The total number of common themes in relation to the total number of different themes was calculated and compared. In the teacher education syllabus, themes and topics which were beyond high school



level were identified in order to ascertain the extent of advanced mathematics covered by pre-service teachers in their training.

Content analysis is done in order to summarize the comparison of high school and teacher education syllabi for the three sections of mathematics content, namely: algebra; trigonometry and geometry. The summary indicates the frequency of the themes and the intensity and direction of the content themes in each section.

This process went through the following stages, as suggested by Jacob (2006):

1. The establishment of the unit of analysis.

This entails working out the field of the study of mathematics, that is, algebra, trigonometry and geometry.

2. Specifying variables and categories.

In the present study we worked out the categories in terms of mathematics sections. We created objective data for scientific analysis. These are in the form of sub-themes. These sub-themes lend themselves to classification.

3. Construction of frequency distribution table.

We simply counted the number of sub-themes or topics that are done exclusively at high school level. We did the same for university level and this formed the second column of frequency distribution. In the third column we placed entries which are done in both institutions. We summed up the entries row-wise in order to obtain the entries for the fourth column.

## **4.7 CONCLUSION**

This chapter dealt with the design and the methodology of the study. In each aim of the study, an appropriate instrument is used. For example, the mathematical proficiency test, practice teaching form and the syllabi documents were used to measure aim number one, two and three respectively. The nature of the study requires the use of field/descriptive research design. The presentation and analysis of data is covered in chapter five.

## CHAPTER FIVE

### PRESENTATION AND ANALYSIS OF DATA

#### 5.1 INTRODUCTION

This chapter is concerned with the presentation, analysis and interpretation of data. The hypotheses that were formulated in chapter one are also tested in this chapter.

#### 5.2 RE-ITERATION OF HYPOTHESES

The hypotheses to be tested are as follows:

Hypothesis 1a:

*Pre-service teachers possess adequate mathematics content knowledge when they exit their training programme.*

Teacher training programmes are expected to prepare candidates for a teaching career and proficiency in their method subjects.

Hypothesis 1b:

*Knowledge possessed by pre-service teachers will vary according to the three sections of mathematics, namely algebra, trigonometry and geometry.*

Teachers' mathematical knowledge of algebra, trigonometry and geometry vary. The individual pre-service teacher is likely to achieve differently in each of these three sections of mathematics.

Hypothesis 2a:

*The teacher's knowledge of mathematics content will correlate with his/her teaching practice performance.*

This hypothesis is based on the assumption that one cannot teach what one does not know or understand. Pre-service teachers who are mathematically competent at the exit of their training programmes are expected to do better in teaching practices.

Hypothesis 2b:

*There will be no difference between pre-service teachers' mean scores in knowledge of mathematics and practicum.*

Pre-service teachers are expected to get almost the same scores in mathematics content and teaching practice.

## **5.3 RESULTS**

### **5.3.1 Hypothesis 1 (a)**

*Pre-service teachers possess adequate mathematics content knowledge when they exit their training programme.*

In order to determine the level of mathematics content knowledge possessed by pre-service teachers when they exit their training, a mathematics proficiency test was administered. Table 5.1 reflects grouped frequency distribution of percentages obtained by the participants in a mathematics proficiency test. Looking at the cumulative frequency column, we conclude that only 37 participants passed by scores ranging from 51% - 100%. This

category possesses adequate knowledge of mathematics at the point of exit. This claim is made because the pass mark is 50%. This leaves us with 128 participants who failed the test.

**TABLE 5.1 Grouped frequency distribution of scores on mathematics proficiency test (N= 165)**

<b>Class interval</b>	<b>Frequency</b>	<b>Cumulative Frequency</b>
91 – 100	1	1
81 - 90	1	2
71 – 80	4	6
61-70	10	16
51- 60	21	37
41 -50	36	73
31- 40	43	116
21- 30	34	150
11- 20	13	163
1- 10	2	165

The marks obtained from the mathematics proficiency test were categorized into three levels of performance (Table 5.2). In level 1 we enter the frequency of students who obtained scores from 80% to 100%, in level 2 the entries range from 50% to 79% and level 3 entries are 49% and below.

**TABLE 5.2      Distribution of participants according to categories of mathematics knowledge (N=165)**

<b>Distribution</b>	<b>Levels</b>		
	<b>Level 1</b>	<b>Level 2</b>	<b>Level 3</b>
<b>Categories</b>			
<b>Percentages</b>	<b>80-100</b>	<b>50-79</b>	<b>49 and below</b>
<b>Frequencies</b>	<b>2</b>	<b>43</b>	<b>120</b>

We dichotomised this continuous scale into a cut-off point of 50%. Any score below the cut-off point was regarded as a failure, while a score above the cut-off point is regarded as a pass. Out of 165 participants only 27.3% passed and 72.7% failed. A chi-square ( $\chi^2$ ) statistical test was used to test whether the difference between those who passed and those who failed was significant. The chi-square test was preferred due to its advantage as a nonparametric test for categorical data. There are very few assumptions that need to be met prior to the application of the test (Grimm, 1993). For example, the assumptions that must be met prior to application of chi-square test are listed below:

- ❖ The data consists of two or more variables.
- ❖ The data is in the form of a frequency count that is tabulated and placed in appropriate cells (Grimm, 1993:451; Elifson, Runyon & Haber, 1990: 410)
- ❖ Each observation or frequency is independent of all other observations (Elifson, Runyon & Haber, 1990:413).

The chi-square test is usually applied when variables are discrete.

**TABLE 5.3**      **Distribution of students' performance in mathematics test (N=165)**

Distribution	Classification	
	Passed	Failed
Frequencies	45	120

$$\chi^2 = 34.09, \quad df = 1, \quad p < .05 \qquad \Phi = .455$$

At  $df = k-1 = 1$  and .05 Alpha level, the critical value for the chi-square test read from the table is 3.84. The calculated value of chi-square is 34.09, which is greater than the tabled critical value. This means that a chi-square of 34.09 can occur by chance in less than 20 times in a hundred, which is highly significant at .05level. As a result, the null hypothesis is rejected. Since chi-square = 34.09 at  $df = 1$  it would be significant even at the .01 level. This indicates that very few students command a sound knowledge of mathematics as they exit their training programme. This observed/apparent difference is most probably not due to chance factors but is statistically significant.

The research hypothesis, namely that pre-service teachers possess an adequate mathematics content knowledge when they exit training programmes, is not tenable. Table 5.3 clearly indicates that out of 165 pre-service teachers only 45 teachers have a good command of mathematics content. On the basis of the claims made by Turnuklu and Yesildere (2007) that it is not possible to teach mathematics without having deep mathematics

knowledge, this means that only 45 teachers can be expected to teach mathematics effectively. However, table 5.2 shows that out of 45 teachers with a good command of mathematics knowledge only two teachers are expected to be extremely effective in their mathematics teaching. The significant difference between the pre-service teachers who passed and those who failed give us the confidence to conclude that when pre-service teachers exit training programmes they do not possess adequate mathematics knowledge needed for teaching at further education and training (FET) level (grades 10,11 and 12).

The statistical test only tells us whether the experiment has an effect or not, it does not inform us about the magnitude of the effect. As a result, if the experimental result is significant it is vital to calculate the effect size: Effect size  $\Phi = \sqrt{34.09/165}$   
 $= .455$

This value indicates that the difference between the pre-service teachers who passed and those who failed is moderate.

### 5.3.2 Hypothesis 1b

*Knowledge possessed by pre-service teachers will vary according to the three sections of mathematics, namely algebra, trigonometry and geometry.*

The researcher felt that knowledge of mathematics could vary with the field of study in mathematics. This notion influenced researchers to explore pre-service teachers' performance in ***algebra, trigonometry and geometry***. As a result, it is compelling for the researcher to know the pre-service teachers' performance in the different sections of mathematics that they will



be teaching in schools. These would be algebra, trigonometry and geometry. To this end, the F-test was considered to be a suitable statistical tool because we compared three mean scores obtained in algebra, trigonometry and geometry. A total score for each of the three areas was summated from the marks obtained in the Mathematics Proficiency test.

It is important to ensure that the following conditions are satisfied prior to the application of the F-test.

- ❖ The treatment populations should be normally distributed.
- ❖ Treatment populations should have equal variances.
- ❖ Individual observation is independent of any other observations (Keppel & Zedeck, 1989:108).

The F-test was preferred to the t-test because it can provide information about a significant difference between more than two means of the study. Since the present study deals with a large sample size, it is more appropriate to use the F-test because if the sample size increases, the critical F value decreases, which increasingly makes it easier to find a significant difference among the means (Grimm, 1993:280; Keppel & Zedeck, 1989).

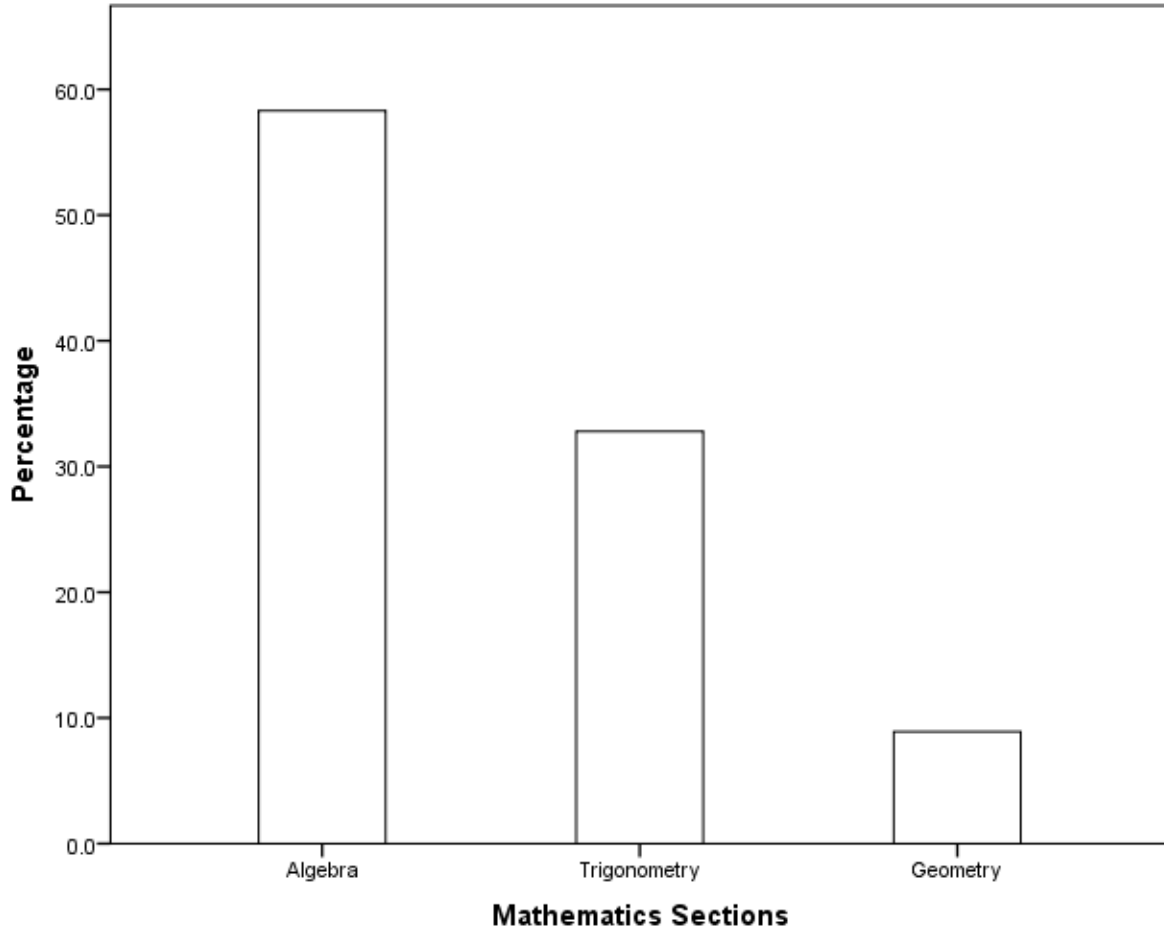
**TABLE 5.4 Comparison of pre-service teachers' achievements in three sections of mathematics(N=165)**

Mathematics Sections	Mean	N	Std Deviation	Variance	F ratio	P. value
					6.60	p <.05
Algebra	7.13	165	2.551	6.510		
Trigonometry	4.01	165	1.962	3.848		
Geometry	1.09	165	.993	.986		
Total	4.07	495	3.137	9.843		
F = 6.60		p < .05		$\eta^2 = .62$		

The critical value of F read from the table (Grimm, 1993) was 3.02 at  $df_{BG} = 2$  (k-1) and  $df_w = 492$  (N-k). The obtained F value of 6.60 is larger than the tabled critical value at .05 level (3.02) and at .01 level (4.66). This shows that the calculated value of F is significant. As a result the null hypothesis is rejected. In other words F is significant at both levels of significance. We can conclude that pre-service teachers' differ with regard to their command of knowledge of the three sections of mathematics. The effect size  $\eta^2 = 0.62$ .

According to Durrheim (2002), this means that 62% of the total variance between scores is accounted by difference in achievement among the three sections of mathematics. The sections of mathematics are algebra, trigonometry and geometry.

The mean scores in Table 5.4 confirm the formulated hypothesis that pre-service teachers' mathematics knowledge varies according to three sections of mathematics, namely algebra, trigonometry and geometry. Although the ANOVA test indicates significant differences among the three mean scores, the pictorial presentation shows a vivid variation (Figure 1) in achievement. The performance of pre-service teachers was best in algebra, second-best in trigonometry and the lowest in geometry. The pictorial presentation provides a vivid image of differential attainment in three sections of mathematics. These results support Avalos *et al.*, (2010).



**Figure 1** : Distribution of percentage across three sections of mathematics

The results obtained from the ANOVA test reaffirms hypothesis one, namely that pre-service teachers do not possess adequate mathematics knowledge when they exit their training. These results suggest that generally, the teachers who are currently produced by the teacher training institutions are not adequately prepared to teach the three sections of mathematics at high school level. They cannot hit the ground running.

### 5.3.3 Hypothesis 2a

*“The pre-service teachers’ knowledge of mathematics content will correlate with his/her teaching practices performance.”*

Based on the literature (Benken & Brown, 2008; Brodie, 2004; Fennema & Franke, 1992; Hristovitch & Mitcheltree, 2004; Leikin & Levav-Waynberg, 2007; Wu, 1999), the researcher believes that teachers’ knowledge of content will correlate with the teaching practices. In other words, pre-service teachers with deep mathematics knowledge will employ various and appropriate teaching strategies to enhance learners’ learning. This would be manifested by a strong relationship between the individual pre-service teachers’ achievements in Mathematics Proficiency tests and in practice teaching scores. Pre-service teachers’ marks obtained in the Mathematics Proficiency test were compared with their scores obtained in teaching practice assessment.

A Pearson  $r$  statistic was used to test this hypothesis. The Pearson  $r$  was deemed appropriate because it is the statistical test that measures the correlation between the variables (Keppel & Zedeck, 1989:55). In order to use this statistical test, the following assumptions must be met:

- ❖ The relationship between the variables should be linear, meaning that an increase in one variable causes an increase in the other.
- ❖ There should be an adequate sample size in order to meet the assumptions for carrying out a correlation coefficient.
- ❖ Both variables should be interval or ratio which ensure that meaningful mean is determined (Baker, 1994:385).

As the data for the present study reasonably meet these three assumptions, Pearson  $r$  is indicated as the best option in this regard.

**TABLE 5.5 The relationship between content knowledge and teaching practice marks**

Test marks	N	Correlation between	Significance
Pearson $r$	165	.074	$P > .05$

At  $df = N-2$ , the obtained correlation coefficient of 0.074 is less than the tabled critical value of  $r = 0.1946$ . Since  $p > 0.05$  the null hypothesis cannot be rejected. This means that there is an insignificant correlation at a 0.05 level of significance. This indicates that there is no linear association between the achievements in mathematics content knowledge and the teaching practice.

The formulated hypothesis, namely that pre-service teachers' knowledge of mathematics content will correlate with his/her teaching practices marks, was untenable. The null hypothesis with regard to the correlation between content knowledge and practicum is therefore upheld. The results of the present study therefore show that teaching is not a correlate of content knowledge of the subject. This implies that a teacher can teach regardless of the extent of the content knowledge that he/she commands in the subject. The result of the present study with regard to hypothesis 2a is in contrast to the assertion by many researchers (Manouchehri & Goodman, 2000; Nicol, 2002; Nicol & Crespo, 2006; Ball & Bass, 2002; Piccolo, 2008), namely that teachers' background knowledge in the subject matter affects how they present this knowledge to their learners.

### 5.3.4 Hypothesis 2b

*“There will be no difference between pre-service teachers’ mean scores in knowledge of mathematics and practicum.”*

Turnuklu and Yesildere (2007) argue that it is not possible to teach mathematics without having deep mathematics knowledge. However, the obtained correlation results are not consistent with these claims. As a result the researcher decided to use a t-test to test whether the pre-service teachers’ mean scores for mathematics proficiency test and the practicum are significantly different. In order to use the t-test the following assumptions must be met.

- ❖ There should be one random sample of interval or ratio scores.
- ❖ The scores of the population from which the sample is taken should be normally distributed and have equal variances.
- ❖ Each score within the sample should be independent of all other scores (Grimm, 1993:161).

The selection of the t-test was based on the fact that the data reasonably meet the assumptions and it is a parametric measure. The parametric tests are often powerful because they produce minimal errors (Grimm, 1993).

**TABLE 5.6 Student teachers' mean scores for content and teaching practice**

	Mean	N	SD	SEM	t	df	Significance
Maths content	40.79	165	14.877	1.158	-23.117	164	p < .000
Teaching practice	70.08	165	7.779	.606			

The calculated t-value of -23.117 at  $df = 164$  is greater than the tabled t-value of 1.960 and this leads us to reject the null hypothesis and retain the alternative hypothesis. The obtained t-value (23.117) indicates that it would be significant even at 0.01 level. Thus, we conclude that the content mean score of 40.79 is significantly different from the practical mean score of 70.08. This indicates that, generally, pre-service teachers obtain much higher marks in teaching practice than in mathematics content. Therefore,  $d = 40.79 - 70.08 / 11.835 = -2.475$ . The value of Cohen's d indicates that the difference between the pre-service teachers' achievements in content and the teaching practice is very large.

The significant mean scores for content and teaching practice when t-test is used highlights the inconsistency between what the pre-service teachers achieve in content and what they achieve in teaching practice. This reaffirms the results obtained for hypothesis 2a. Furthermore, it indicates that generally pre-service teachers are awarded extremely high marks in their teaching practice when compared with what they get in mathematics content tests. Crespo (2003:266) states that there are disconnections between pre-service teachers' mathematics knowledge and what they do in practice and this



hinders effective teaching. On this basis, it looks as though the practice teaching scores do not truly reflect the pre-service teachers' performance. Secondly, if pre-service teachers get extremely high marks in teaching practice and very low marks in content, it means that a pre-service teacher can teach what he/she does not know.

#### **5.4 RELATIONSHIP BETWEEN HIGH SCHOOL AND UNIVERSITY MATHEMATICS SYLLABI**

The data for aim number three were analysed qualitatively. The aim was re-iterated as follows:

*“To establish the relationship, if any, between content covered in mathematics education preparation programmes and high school mathematics.”*

In order to ascertain whether there is link between the mathematics content covered in teacher preparation programmes and high school mathematics the data were categorized in a tabular form and content analysis was conducted. Content analysis assumes that it is possible to identify content in terms of units. Once you have identified which units of data belong to which categories, it is then possible to count them in order to determine how much of each type can be found in the content (David & Sutton, 2004). Content analysis is done on the basis of explicit rules of coding to ensure that the same results can be obtained, even by different researchers (Nachmias & Nachmias, 1992: 311). For example, David and Sutton (2004) say that there should be clear definitions of the meanings of the categories prior to coding or during

the process of coding. In addition to this, Weber (1990:12) has this to say: “to make valid inferences from the text, it is important that the classification procedure be reliable in the sense of being consistent: Different people should code the same text in the same way.”

According to Terre Blanche, Durrheim and Kelly (2006) there are different ways of highlighting pieces of text: some like to use coloured marker pens and others use index cards, labels and words (descriptive codes) to reduce data into manageable form. In the same vein, for data at hand, instead of using these mentioned ways of coding, the researcher used **bold** and *italic* writing to highlight common/similar and different themes. The common themes (themes that appear in both high school and university syllabi) were written in *italic* while the different themes were written in **bold** type in the table to facilitate distinctions. In each topic of the syllabi the common and diverse themes were counted to compare the number of themes done at university and those done at high school.

The first reason for using bold type and italic writing is based on the fact that there are only two categories, which made it convenient to use these two forms of writing to identify common and diverse themes. **Coloured marker pens, labels, index cards** and **words** are suitable for coding data with very diverse and multiple categories. The second reason was that if coloured pens are used, they do not show up if the text is photocopied; all colours turn up as black. It was for these reasons that the present researcher preferred to use bold and italic writing for coding the data.

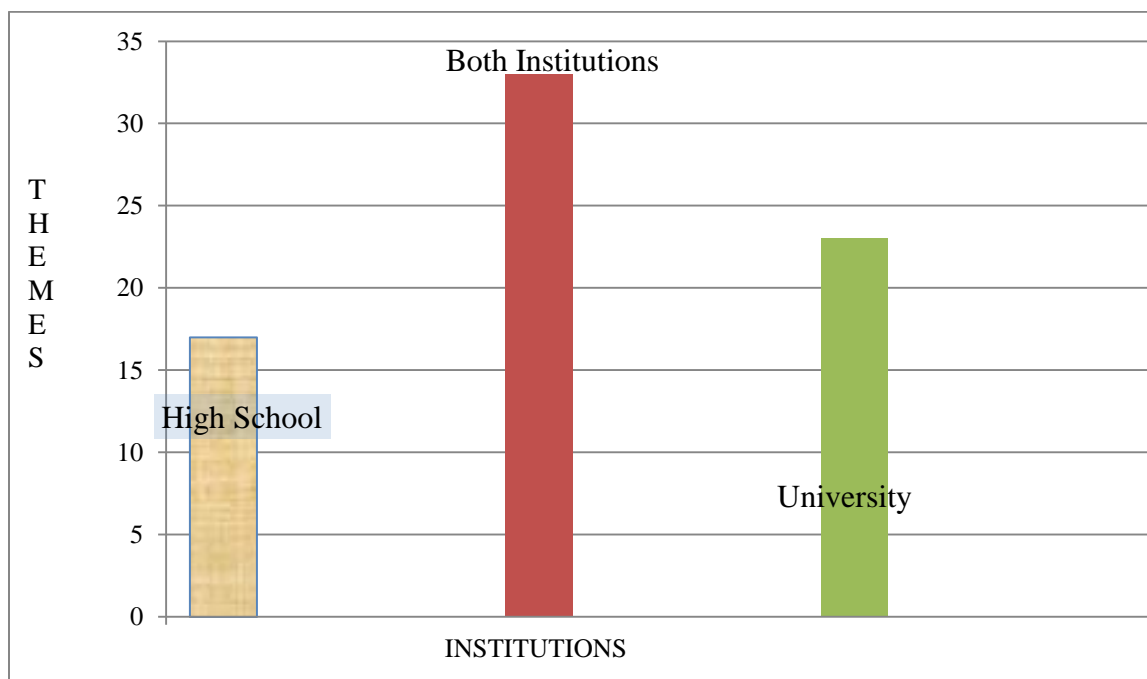
**TABLE 5.7 COMPARISON OF UNIVERSITY TEACHER EDUCATION  
SYLLABUS WITH HIGH SCHOOL MATHEMATICS  
SYLLABUS: Algebra Content**

<b>University Algebra Content</b>	<b>High School Algebra Content</b>	<b>Word Count Similar themes</b>	<b>Word Count Different themes</b>
<b>EXPONENTS:</b> <i>Exponential laws, exponential equations, exponential functions, graphs, inverse functions</i>	<b>EXPONENTS:</b> <i>Exponential laws, surds, exponential equations, Exponential functions, graphs and inverse functions</i>	Five themes Common	One theme done at high school only (surds)
<b>NUMBER SYSTEM:</b> <b>Complex numbers, conjugates, polar form, De Moivres theorem, complete a square</b>	<b>NUMBER SYSTEM:</b> <b>Rational numbers, recognition of non-real numbers, complete a square</b>	Only one theme common.	Four themes done at university only. Two themes done in high school only.
<b>LOGARITHMS:</b> <i>Logarithmic laws, including ln, equations</i>	<b>LOGARITHMS:</b> <b>Definitions, logarithmic laws, equations</b>	Two themes common	One theme done at university (ln). One theme (definition) done at high school only.
<b>SEQUENCE ANDSERIES:</b> <i>Sum of sequences, generalizations,</i>	<b>NUMBER PATTERNS:</b> <i>Sum of sequences, generalizations, prove conjectures, recursive formulae interpretations,</i>	Two themes common	Two themes done at high school only.
<b>CALCULUS:</b> <i>Limits, continuity, derivative of exponential and logarithmic functions, derive using rules, integration, differential equations</i>	<b>CALCULUS:</b> <i>Limits, continuity, rate of change and average gradient, derive from the first principle, using rules cubic graphs</i>	Three themes common	Three themes done at university only. Three themes done at high school only.

<b>FINANCIAL MATHEMATICS:</b> <i>Simple interest,</i> <i>Compound interest,</i> <i>Nominal &amp; effective interest rates, <b>Changing interest, annuities,</b></i>	<b>FINANCIAL MATHEMATICS:</b> <i>Simple and compound interest, <b>foreign exchange, Effective &amp; nominal interest rates ,annuities, bonds, sinking fund, investments and loan options</b></i>	Four themes common.	One theme at university only. Four themes done at high school only.
<b>STATISTICS AND PROBABILITY:</b> <i>Stem and leaf diagrams, mode, mean, median and standard deviation <b>range, inter quartile range, variance., Relative frequency, cumulative frequency, relative cumulative frequency, outliers, skewedness of distribution, independent probability, tree diagram, central tendency and dispersion ,regression</b></i>	<b>STATISTICS AND PROBABILITY:</b> <i>Sample size linked to mean, mode, median and standard deviation, stem and leaf diagram, normal distribution, compare relative frequency to probability, <b>dependant and independent events, tree diagrams, symmetric vs skewed data, central tendency and dispersion. calculate correlation coefficient, regression function,</b></i>	Twelve themes common	Six themes done at university only. Three themes done at high school only
<i>Mathematical Modelling</i>	<i>Mathematical Modelling</i>	Common theme	
<i>Linear Programming</i>	<i>Linear Programming</i>	Common theme	
<b>Matrix algebra and polynomial theory, Piecewise functions, Partial functions, Transcendental functions, Countability</b>	None	none	All six themes done at university

There are 31 common themes, while 21 themes are exclusively done at university and 16 themes are done in high school exclusively. This means that the total number of themes done by pre-service teachers at university is 52,

while the total number of themes done at school is 47.



**Figure 2:** Relationship between high school and university algebra syllabi

The commonality between the themes/topics covered in both syllabi indicates that there is a relationship between algebra covered in teacher preparation programmes and in high school. The teacher preparation programmes allow pre-service teachers to acquire basic algebra knowledge needed for high school teaching. It further indicates that although most of the algebra content themes covered by teacher preparation programmes focus on high school mathematics, it nevertheless enriches the pre-service teachers with mathematics knowledge that is beyond high school level.

It appears in Table 5.6.1 that 31/52 themes, that is 60% of the algebra content done by pre-service teachers, is a repetition of high school work. We would therefore expect pre-service teachers to achieve very high scores in the

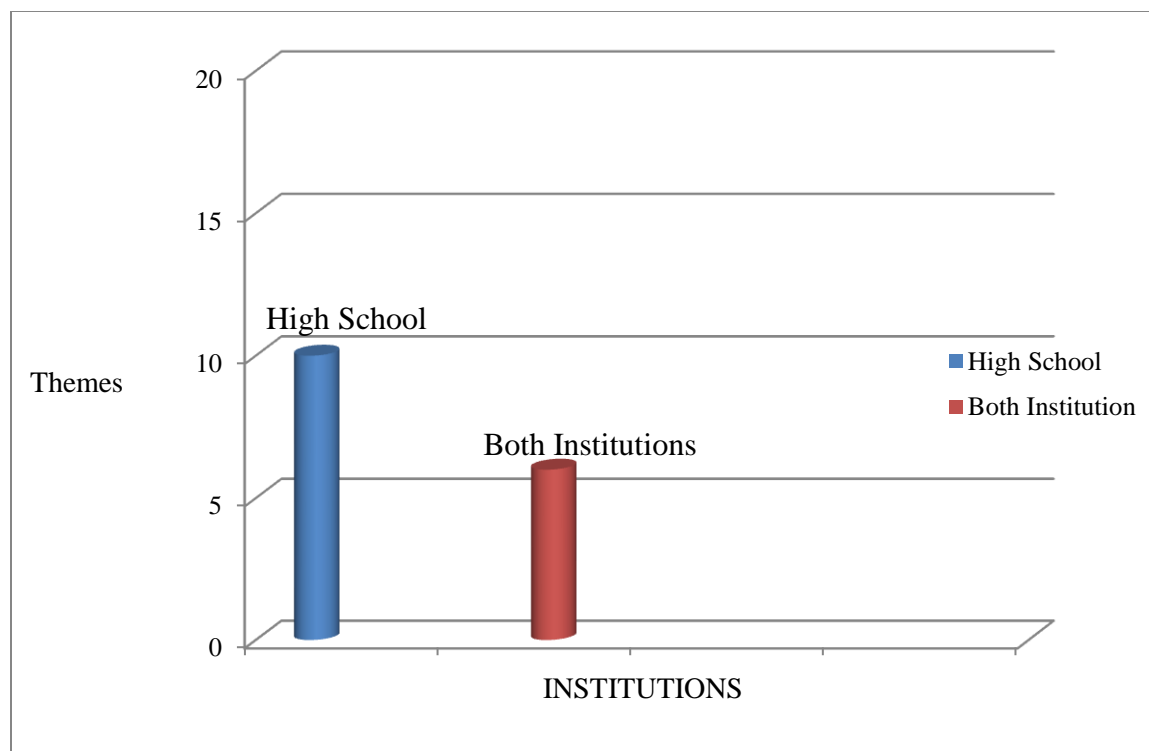
algebra section of the mathematics proficiency test. Secondly, they are expected to teach this section of mathematics effectively. However, pre-service teachers performed poorly in the test, which means that we would not expect them to excel in their teaching practice.

There is a belief that teacher education syllabi should go beyond high school level to include advanced mathematics. This has apparently been considered because 21/52 themes, which is 40% of the pre-service teachers' algebra content themes, are more advanced than the high school mathematics syllabi. Part of this 40% is based on exposing pre-service teachers to topics which are completely new to them, while another part is an extension of high school themes treated at an advanced level.

Since 31 out of 47 themes which are covered by the high school syllabus are common for both high school and university, it implies that 16 themes are exclusively done at high school. Presumably, pre-service teachers did these 16 themes when they were still at school. We can therefore conclude that, based on the teacher education syllabus, pre-service teachers ought to be well prepared to teach high school mathematics.

**TABLE 5.8 COMPARISON OF UNIVERSITY TEACHER EDUCATION  
SYLLABUS WITH HIGH SCHOOL MATHEMATICS  
SYLLABUS: Trigonometry content**

<b>University Trigonometry Content</b>	<b>High School Trigonometry Content</b>	<b>Word count Similar themes</b>	<b>Word count Different themes</b>
<i>Trigonometric functions and inverse functions</i>	<b>Definition using similar <math>\Delta_s</math>, trigonometric functions and inverse functions, <b>graphs, special angles, 2-D problems</b></b>	Two themes common	Four themes different
<i>Reduction formulae</i>	<i>Reduction formulae <b>including negative angles</b></i>	One theme common	One theme different
<i>Trigonometric identities</i>	<b>Derive</b> and use <i>Identities</i>	One theme common	One theme not common (derivation)
<i>Trigonometric equations General solution of trigonometric equations</i>	<i>Trigonometric equations, general solutions of trigonometric equations. <b>Sine, cosine and Area rules, 3-D problems</b></i>	Two themes common	Four themes different



**Figure 3 : Relationship between high school and university trigonometry syllabi**

There are only 6 themes that are common to both university and high school syllabi. Ten themes are covered exclusively in high school. There are no trigonometry themes that are done exclusively at the university. This means that the total number of trigonometry themes that are covered in high school is 16, while teacher education selects only 6 trigonometry themes for teacher preparation.

Although the teacher preparation programmes cover most of the themes that are covered by the high school syllabus, the basic concepts such as definitions of trigonometric terms, derivation of the identities, sine rule, cosine rule and area rule are done only at high school level. Presumably, at university level pre-service teachers are expected to apply these concepts in solving



trigonometric problems. This commonality in the programmes indicates that there is a strong relationship between trigonometric content covered in teacher preparation programmes and in high school. To this end, one can conclude that pre-service teachers are allowed to strengthen their knowledge in trigonometry, which is an essence of teacher training.

Table 5.7 shows that the teacher education syllabus for mathematics repeats all themes that pre-service teachers covered when they were still at high school level. On the basis of this repetition in the syllabus, we would expect pre-service teachers to attain high marks in the trigonometry section of the test and also to be able to teach trigonometry meaningfully. However, the results indicate that pre-service teachers perform poorly in this section. We can therefore expect them to have difficulty when teaching trigonometry.

**TABLE 5.9 COMPARISON OF UNIVERSITY TEACHER EDUCATION SYLLABUS WITH HIGH SCHOOL MATHEMATICS SYLLABUS: Geometry content**

<b>University Geometry Content</b>	<b>High School Geometry Content</b>	<b>Word count Similar themes</b>	<b>Word count Different themes</b>
<b>EUCLIDEAN GEOMETRY:</b> Figures and shape geometry. <b>Spheres</b>	<b>EUCLIDEAN GEOMETRY :</b> <i>Polygons, Theorems and proofs on quadrilaterals, circles,</i> <b>Pythagoras theorem, Proportionality and similarity</b>	Only two themes common	Three themes done at high school only and one theme done at university only (spheres)

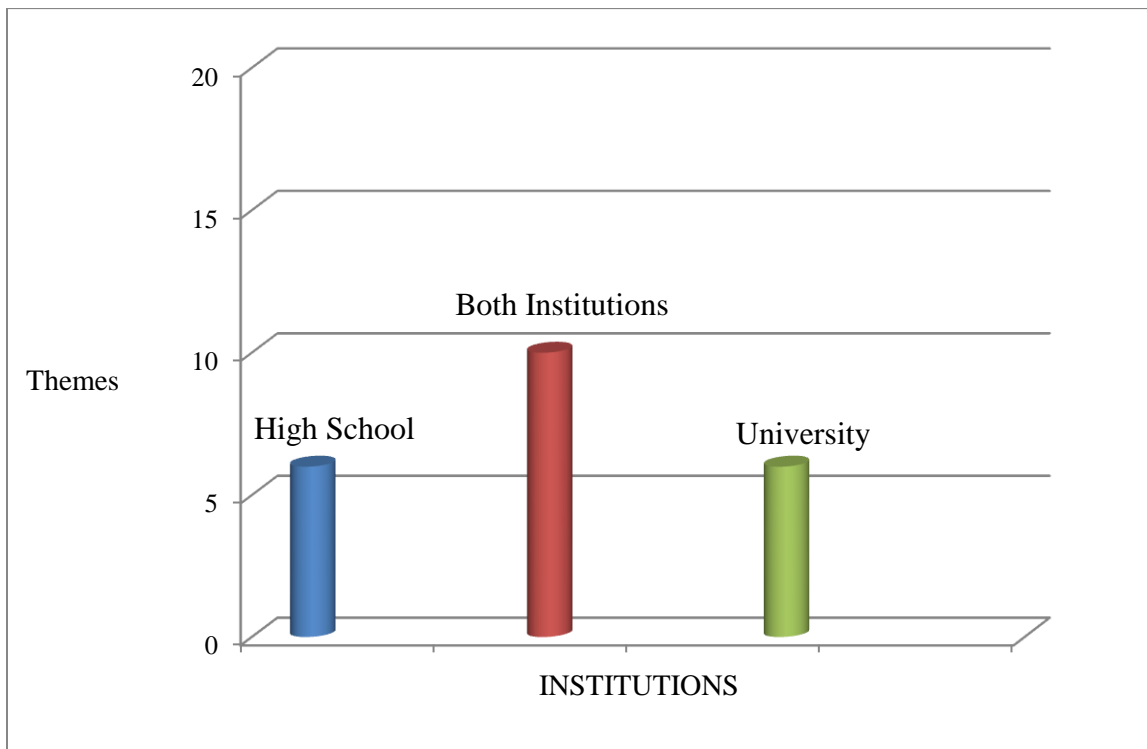
<b>COORDINATE GEOMETRY:</b> <i>Consolidation of high school topics on coordinate geometry, Conic sections</i>	<b>COORDINATE GEOMETRY:</b> <i>Equation of line, circle and tangent to circle. Inclination of a line</i>	Four themes common	One theme, Conic sections done at university only
<b>TRANSFORMATIONAL GEOMETRY:</b> <i>Consolidation of high school topics on transformations</i>	<b>TRANSFORMATIONAL GEOMETRY:</b> <i>Rotations around origin, enlargements, Rigid transformations and enlargements</i>	Four themes are common	

In geometry there are 9 themes that were found to be common to both the university and high school syllabi, and 3 themes that are exclusively done at high school, while 2 themes are exclusively done at university. This means that the total number of themes done at university is 11, while the total number of themes done at school is 12. This precisely indicates that the teacher preparation programmes cover most of the geometric content covered in high school.

It was remarkable that the teacher education programmes do not clearly state the subtopics covered in geometry. In one teacher training institution the programme only gives the main topics of geometry, for example “*figures and shape geometry*,” while in another institution geometry does not appear at all in the programmes. The teacher preparation programmes offer sections which are beyond the high school syllabus. For example, geometry on sphere and conic sections is only studied at university level. Notably, in one teacher training institution *where figures and shape geometry* is done, concepts

which are very basic in geometry are only done at high school level and students are expected to apply these concepts in solving geometric problems at university level.

Nine out of 11 (9/11), that is 82% of themes covered by the teacher education mathematics syllabus, is taken from the high school geometry content. The remaining 2/11 (18%) themes are at university level. This exposes pre-service teachers to an advanced level of geometry. As a result we would expect pre-service teachers to attain very high marks in the geometry section of the test. However, the test results indicate that geometry is the most problematic section for pre-service teachers to pass. With this in mind, we cannot expect pre-service teachers to teach geometry effectively.



**Figure 4: Relationship between high school and university syllabi**

## **5.5 CONTENT ANALYSIS**

For content analysis, common themes between school syllabus and teacher training programmes were coded and grouped to form categories. The total number of common themes and different themes were calculated and compared so that the total number of these themes could be determined. If the number of common themes is greater than different themes, this would indicate that there is an overlap between teacher education mathematics and high school mathematics. If a greater number of the themes are found solely in teacher education mathematics or at tertiary level, it would indicate that most of what is taught to pre-service teachers cover high school level and beyond. If a greater number of themes are found in the high school syllabus but not in teacher education mathematics, one may assume that there is small amount of high school work covered by pre-service teacher preparation programmes.

The frequency table displays frequencies for all units of analysis (TABLE 5.10)

**TABLE 5.10 Frequency, intensity and direction of mathematics themes**

Mathematics Sections	Exclusively done at		Themes common to both institutions	Total
	High School	University		
Algebra	16	21	31	68*
	<b>23</b>	<b>31</b>	<b>46</b>	<b>100</b>
Trigonometry	10	0	06	16*
	<b>63</b>	<b>0</b>	<b>37</b>	<b>100</b>
Geometry	03	02	09	14*
	<b>21</b>	<b>14</b>	<b>65</b>	<b>100</b>

**Bold type indicates percentage**

**Asterisk indicates intensity and direction**

If we compare column 2 and column 3 in the table it appears that there is more than a 50% overlap between high school and university mathematics education syllabi. This observation applies to all three field of mathematics, that is, algebra, trigonometry and geometry. The observation supports the finding that there is a reasonable relationship between high school and teacher preparation mathematics syllabi. The numbers with an asterisk in the last column indicate the intensity of the content that ideally the pre-service teachers are supposed to have learnt if all high school mathematics topics were included in the teacher education mathematics syllabus.

## **5.6 THE COURSE FOR THE METHODS OF TEACHING MATHEMATICS**

The aim of the study was to analyse the structure of the methods course. For example, hypothesis number two “*The pre-service teachers’ knowledge of mathematics content will correlate with his/her teaching practices*” requires comparison between achievement in content between the test and the teaching practice mark. Practicum is about putting into practice what has been dealt with in a method class. One of the results from the analysis of the present study is that other institutions specify in their programmes the topics that they do in methods classes while others are very generic in their programmes. Below is the mathematics methods course taken from one of the universities where the method course outlines specific topics to be covered. It clearly states that a student will qualify to do method 1 course provided he/she passed mathematics at level 5 or obtained 60% in the first year mathematics content course. In some universities, topics which do not appear in the content course are covered in the methods course. For example, in the case of Euclidean geometry, it was not mentioned in the content syllabus for teacher education in one institution, but it appeared in a method course.

**TABLE 5.11 Mathematics teaching methods syllabus**

<b>LEVEL</b>	<b>ALGEBRA</b>	<b>TRIGONOMETRY</b>	<b>GEOMETRY</b>
<b>1</b>	Teaching and learning of algebra, Statistics related to the current documents and international research		
<b>2</b>			Issues related to the teaching of geometry, Some interesting geometric theorems not in school curriculum, Creation of appropriate classroom material for the teaching of geometry.
<b>3</b>	Getting familiar with the FET curriculum, Issues related to the teaching of functions, calculus at FET level	Issues related to the teaching of trigonometry at FET level	
Use of technology in the FET classroom covers all the three sections of mathematics; technology is also used to create appropriate classroom materials.			

It is evident from the analysis of the present study that much of the work done in the methods course is based on issues related to the teaching of specific topics such as **statistics, functions, calculus, trigonometry and Euclidean geometry**. The rationale behind the selection of these specific topics for the

method course is not indicated. Presumably, the university believes that the pre-service teachers often have problems in these topics, which then necessitates teaching methods to focus more on such topics. This notion is evident in pre-service teachers' low attainments in the geometry and trigonometry sections of the mathematics proficiency test. On the other hand, the universities in which there are no specifications pertaining to the syllabus for the method course means that there is no indication of the topics that will inform the syllabus for the method course. This indicates that teacher educators have to use their own discretion.

## **5.7 CONCLUSION**

This chapter has dealt with the presentation and analysis of data. The results of data analysis in this chapter have revealed the following:

Very few pre-service teachers command a sound knowledge of mathematics as they exit training programmes. It was also evident from the calculated mean scores that students' achievements in three sections of mathematics differed significantly. In this study, there was no association found between the achievements in mathematics and teaching practice. The documented information indicated that the syllabus for teacher preparation programmes covers most of the high school mathematics topics. However, there is no certainty about the extent to which these topics are taught to the pre-service teachers. Chapter six discusses the findings and the implications of the study



**CHAPTER SIX**  
**DISCUSSION OF FINDINGS, IMPLICATIONS, PROSPECTS**  
**AND RECOMMENDATIONS**

**6.1 INTRODUCTION**

The amount of time that the teachers spend in their training is presumably of adequate duration to prepare and equip prospective teachers with the skills and knowledge needed for teaching. This assumption is based on the fact that the teacher education syllabus covers most of the high school mathematics content and goes beyond to enrich pre-service teachers with university mathematics within four years of B.Ed. However, it seems that although the duration of training is presumed adequate, nonetheless the training very often does not adequately equip pre-service teachers with the necessary skills and knowledge for teaching. The adequacy of teacher training and preparedness to teach a subject is often judged by a teacher's performance in that subject. It is clear from Sam's study (2005) that it is the focus of the training that determines pre-service teachers' achievements in the training. For example, China focuses more on mathematics content, while Malaysia puts more stress on educational courses (Sam, 2005). As a result, China's teachers are more competent than Malaysian teachers. In South Africa, the low student achievement in mathematics has raised questions and speculations about the system of education and efficacy of teacher training programmes.

By implication, one may be tempted to assume that it is possible that much of the work during teacher training focuses on the philosophies and theories of teaching at the expense of the acquisition of the desired subject matter

knowledge. The notion of subject knowledge being the fundamental component of teaching is based on the belief that other components serve as the vehicle to convey content knowledge to the learners. To this end, knowledge of theories and teaching strategies needs to be grounded on a strong subject knowledge base.

The speculations about the apparent poor teaching of mathematics by mathematics teachers, which results in a lack of literacy in mathematics among the South African learners, often culminates in a number of questions. In this study, such questions were as follows:

- (1) How much mathematics content knowledge do teachers possess at the point of exit in their training?
- (2) What is the impact of pre-service teachers' content knowledge of mathematics on their teaching practices?
- (3) What is the relationship, if any, between the content covered in mathematics education teacher preparation programmes and high school mathematics?

In accordance with these questions, the hypotheses of the study were formulated as follows:

1(a) Pre-service teachers possess inadequate mathematics content knowledge when they exit their training programme.

In accordance with this hypothesis, the analysis was based on the pre-service teachers' scores obtained in the achievement test. The rationale was that content/achievement was ideal to measure the extent of the content

knowledge that pre-service teachers have as they begin their teaching careers.

1(b) Knowledge possessed by pre-service teachers will vary according to three sections of mathematics, namely: algebra, trigonometry and geometry. The analysis was based on the pre-service teachers' attainments in the three sections of mathematics.

2(a) The pre-service teachers' knowledge of mathematics content will correlate with his/her teaching practice performance.

For this hypothesis, the analysis was done in terms of the pre-service teachers' attainments in the mathematics proficiency test and in teaching practice. The objective is to check whether there is any causality in these two variables.

2(b) There will be no difference between pre-service teachers' mean scores in knowledge of mathematics and practicum.

The analysis was based on the calculated mean scores for pre-service teachers' attainments in content test and practicum.

The obtained data based on each aim of the study and research instrument used were analysed to test the formulated hypothesis. To this end, the findings of this study were discussed in relation to each aim.

## 6.2 FINDINGS IN RELATION TO AIM NUMBER ONE

The first aim of the study is *to determine how much mathematics content knowledge teachers possess at the point of exit of their teacher training programme.*

The findings of the present study indicate that pre-service teachers generally lack academic content knowledge in mathematics. Pre-service teachers do not possess adequate knowledge of mathematics at the point of exit of their training programmes (Even & Ball, 2009; Mullis et al., 2008). These findings support the existing theory and previous findings that pre-service teachers' mathematics understanding of mathematics courses that they have gained from university and college, is inadequate for the teaching of elementary and high school mathematics (Crespo, 2003); Mji and Makgato (2006). Whether these teachers will be able to teach effectively is the question. In the same vein, previous research has found that newly qualified teachers' level of understanding of mathematics is inadequate for the delivery of mathematics in elementary and high school classrooms (Crespo, 2003; Mji & Makgato, 2006).

The duration of teacher training was extended from three years to four years at university. Based on the findings of the present study, the duration of teacher training is by analogy, like putting a duck in the water and taking it out dry. This means that at the point of completion of training pre-service teachers still have the same knowledge base as they had at the entry point (Benken and Brown, 2008; Seaman, Szydelik & Beam, 2006; Tailor, 2008; Hristovitch & Mitcheltree, 2004; Dorfler, 2003). The aforementioned authors found that

pre-service teachers possess insufficient and inadequate knowledge of mathematics when they exit training programmes. With this in mind, we can conclude that while there are numerous factors that lead to a consistent lack of competence and poor attainment in mathematics by South African learners, this situation is also strongly associated with a lack of basic content knowledge on the part of the teachers.

Another finding of the present study is that pre-service teachers demonstrate the same performance or achievement in different sections of mathematics. These subfields of mathematics are algebra, trigonometry and geometry. Pre-service teachers do not manifest individual variation in abilities in these areas. This is the new, unique, original finding of the present study. Studies reviewed in this research did not report anything about the subfields of mathematics or about pre-service teachers' attainments. Even a study by Avalos et al., (2010) discusses only two subfields of mathematics, namely algebra and geometry. Taking into account the research findings by Li and Smith (2007) that teachers' knowing mathematics for teaching is essential to effective classroom instruction, yet teachers lack mathematics knowledge there is the ostensible reason for the continuous high failure rate in grade 12 mathematics, particularly in geometry.

### **6.3 FINDINGS IN RELATION TO AIM NUMBER TWO**

The second aim of the study is *to establish the relationship, if any, between achievement in content and achievement in teaching practice.*

The findings of the present study reveals that there is no relationship between

the achievements in mathematics content and the achievements in teaching practice as measured by the mathematics proficiency test and teaching practice evaluation forms respectively. It was found that pre-service teachers are awarded very high marks in their teaching practice evaluation. As a consequence, the finding is apparently at odds with the findings of a number of studies (Manouchehri & Goodman, 2000; Leikin & Levav-Waynberg, 2007; Nicol & Crespo, 2006; Ball & Bass, 2002), namely that teachers' background knowledge in the subject matter affects the manner of classroom presentation of the subject matter. The better the command of subject matter, the better is the mode of presentation.

The inconsistency between pre-service teachers' performance in content and practice teaching seems to support Crespo's findings (2003) that there are disconnections between pre-service teachers' mathematics knowledge and what they do in practice. However, Crespo's finding does not invalidate the assertion by Ball and Bass (2002); Nicol (2002); Nicol and Crespo (2006) and Manoucheri and Goodman (2000) that teachers' background knowledge in the subject matter is fundamental in the teaching of the subject.

Furthermore, the achievement of very high scores for practice teaching while the mathematics content achievement test scores are very low seems to defy our explanations, as Turnuklu and Yesildere (2007) claim that it is not possible to teach mathematics effectively without having a deep knowledge of mathematics. Their study indicates that if the teachers do not have adequate mathematics content knowledge they do not even see the relationship between the simple concepts such as addition and subtraction. To them,  $5-3$  means "+5 plus -3". Based on the findings of Turnuklu and Yesildere's study (2007) one

would expect a strong positive relationship between students' performance in mathematics content and practice teaching results.

The obtained correlation results are inconsistent with many researchers' claims that teachers' knowledge of mathematics is associated with their performance in teaching practice. For example Fennema and Franke (1992) argue that a conceptual understanding of mathematics influences classroom instruction positively. According to these researchers' claim, teachers who performed well in a content test should also get high scores in teaching practice and those who performed poorly in a content test should get low scores in teaching practice. However, the present study's results do not support this expectation. Generally, all pre-service teachers get higher scores in teaching practice than in mathematics content tests and some dismally failed the content test (Annexure D). As a result the present researcher was tempted to conclude that generally, pre-service teachers in teaching practice are awarded marks which are incredibly high

#### **6.4 FINDINGS IN RELATION TO AIM NUMBER THREE**

The third aim of the study was *to establish the relationship, if any, between content covered in mathematics education preparation programmes and high school mathematics.*

The findings of the present study revealed that the teacher training programmes cover most of the themes covered by the high school syllabus. However, it appears that although some of the themes or concepts which are regarded as basic, such as the definition of terms, rules and derivation of

formulae, are done in high school and at university, they are only applied when solving problems. Presumably, the assumption is that students learnt these while they were still at school. It was also found that in the same main topic there are subtopics which are done exclusively at university while others are only done at high school. For example, in algebra, logarithms and calculus are main topics done in both levels, that is, in high school and university, but the concept of *ln*, *integration* and *derivative of logarithmic functions* are only done at university level. The findings of the present study indicate that there is link between high school mathematics and teacher preparation mathematics, particularly in algebra. This link supports CBMS's suggestion (2001) that the high school pre-service teachers' mathematics course must connect university or college mathematics with high school mathematics.

The finding of the present study also revealed that mathematics for teacher preparation programmes transcends high school mathematics to cover some advanced topics. For example, matrix algebra, polynomial theory, piecewise functions, transcendental functions and others are only done at teacher preparation level. This indicates that teacher preparation programmes take pre-service teachers to advanced level mathematics. The findings support Shoaf (2000) and CBMS (2001) beliefs that the core mathematics major courses for pre-service teachers must make connections between the advanced mathematics that they learn and the high school mathematics they will teach.

In trigonometry it was found that teacher preparation programmes cover all the themes in the high school syllabus with the exception of *sine, cosine and area rules*. As noted, in the case of algebra, these trigonometry formulae in



teacher preparation programmes are only applied in solving trigonometric problems. In trigonometry, the teacher preparation programmes seem to contain more repetition of what was done in high school mathematics. Ideally, this repetition ought to have enhanced pre-service teachers' understanding of trigonometry concepts, and thus it should help in teaching such concepts effectively. These findings indicate that despite the existing commonality in trigonometric themes covered by teacher preparation programmes and the high school syllabus, pre-service teachers still find it hard to form concepts in trigonometry.

The findings of the present study show that there is no consistency in teacher training particularly with regard to geometry. We found that some teacher preparation programmes do not specify the extent of Euclidean geometry expected to be covered by pre-service teachers. In one institution Euclidean geometry does not appear in the programme. In this regard the researcher assumed that obviously it is not done in their teacher training. Contrary to this, most of teacher preparation programmes suggested by different researchers, for example, Pournara (2005) and Shoaf (2000), suggest geometry as one of the priorities for future teachers. The findings of the study, however, revealed that the teacher preparation programmes include some topics which are not covered in the high school syllabus. For example, spheres and conic sections are only covered in teacher preparation programmes. This is in accordance with Garcia (2006), who believes that pre-service teachers' development must be based on knowledge and modes of reasoning similar to those of experts. However, if there is a gap between high school and teacher training programmes, this leads to discontinuity, as pointed out by Furinghetti (2000). Based on the findings of the study, the extremely low achievement in

geometry by pre-service teachers is attributed to this gap and the already poor achievement they suffered in school.

The findings of the study revealed that universities have different programmes (syllabi) for teaching methods. Some university syllabi have structured methods programmes that target issues related to the teaching of specific topics such as **statistics, functions, calculus, trigonometry and geometry**. Presumably, the selection of these topics for a method course is based on the notion that they are problematic to the students. In contrast, some universities have an unstructured and generic programme for methods courses. This would suggest that the individual teacher educators in this case are allowed to use their discretion to select topics for method course, and it is likely that teacher educators are biased in their selection. The poor attainments by pre-service teachers in the trigonometry and geometry sections of the mathematics content test give justification for its inclusion in the method course. Notwithstanding the disparities in method courses for these different universities, the way pre-service teachers are evaluated in the practice teaching situation is the same. It was found that in two universities who participated in this study, pre-service teachers were awarded very high marks for teaching practice.

## **6.5 IMPLICATIONS**

The main focus of the study is on the professional training of mathematics pre-service teachers. There are three components which are perceived as cornerstones for teacher preparation. These are teacher preparation programmes, the kind of teacher produced from the programmes and the

general teaching practices shown by these teachers. For the present study, the findings with regard to these components have certain implications.

There is an urgent need to revise the syllabus of the content and method of mathematics in the teacher training programmes. These programmes must produce teachers who command knowledge of the discipline of mathematics. The revision of the syllabus will ensure that teachers complete training with adequate knowledge of the subject. Upon leaving the institutions, these teachers will hit the ground running when they enter the classroom for the first time.

The study reveals that we must strengthen the academic content of mathematics. Poor performance indicates that the knowledge base is lacking. Teachers who are inadequately prepared in mathematics will produce learners who are incompetent in the subject. This is a revolving door effect phenomenon.

The failure rate at grade twelve level in mathematics can be reduced if we have competent teachers in mathematics. The training programme for teachers must put more emphasis on content rather than on principles of education and methods of teaching over four years. A mechanism should be established to enable government to develop policy document backed by scientific findings from research.

The present study found that during teaching practices, students are awarded very high marks which do not correlate with their performance in the mathematics content test. This may imply that supervisors for pre-service

teachers do not critically evaluate students during their practice teaching. Certainly, the credibility of the evaluation is questionable.

In algebra there seems to be a link between teacher preparation and high school in terms of mathematics content. The advanced mathematics taken by pre-service teachers is not far removed from high school mathematics. This supports Wu's suggestion (1999) that teacher preparation programmes should carve out a solid middle ground between advanced mathematics and high school mathematics.

Most of the trigonometry topics in teacher preparation programmes are covered in high school syllabi. Pre-service teachers are given enough time to consolidate and get more understanding of the trigonometric problems related to what they did in school mathematics. This is supposed to increase the chances of pre-service teachers to do well in trigonometry, but in reality it is not the case. This means that pre-service teachers are still experiencing difficulty in conceptualizing trigonometry.

Geometry has always been a problematic area in high school mathematics. This would suggest that current pre-service teachers also had the same experience when they were still at high school. With this in mind, geometry should be one of the priority mathematics sections for pre-service teachers as one would expect them to compensate for the gaps left by their high school experience. Whereas pre-service teachers showed tremendously low achievement in Euclidean geometry, the teacher preparation programmes gave very little information with regard to the extent of geometric content that they covered. The teacher preparation programmes should clearly indicate the

kind of geometry topics and subtopics expected to be learnt by pre-service teachers.

## **6.6 RECOMMENDATIONS**

The results of the present study show that there is a need for intervention with regard to the training of pre-service teachers in order to improve their knowledge of mathematics content and pedagogy. There are speculative views regarding the prevalent lack of mathematics knowledge in teachers. For example, Pournara (2005), Lott and Souhrada (2000) assert that the majority of pre-service teachers join teacher education programmes with extremely low matric mathematics results. On the other hand, the Presidential Education Initiative (PEI) report pointed out that South African curriculum initiatives focus mostly on teaching methodology at the expense of conceptual knowledge, which needs to be taught and learnt (Brodie, 2004:66). The present researcher purports that while there is a link between mathematics for teacher education and high school, very little, if any, effort is made by teacher education to advance the pre-service teachers' existing mathematics knowledge. As Sam (2005) puts it, it is the responsibility of the university to change the mediocre mathematics teaching and learning. To this end, the following recommendations were made:

- (a) The content and the depth or the intensity of mathematics courses for high school pre-service teachers should be at the level of students who do mathematics as a major course. If possible the department of mathematics education must collaborate with the department of science and mathematics within the university. In addition to this, the programme should encourage the students to revisit the concepts that

are regarded as basic in high school mathematics rather than to assume that students know and understand those concepts.

- (b) Mathematics lecturers, in their work programmes, should clearly indicate topics and subtopics that they intend teaching, and the pre-service teacher should be aware of these topics. This will reduce speculations about the coverage of the pre-service teachers' mathematics content.
- c) The findings of the study reveal that very often pre-service teachers are given very high marks in teaching practice when compared with what they get in the mathematics content test. It is the responsibility for all universities to improve teaching practice assessment. Assessment scores during teaching practice should realistically and honestly represent a student's performance.
- (d) During the data analysis it appeared that universities have different mathematics programmes for teacher training. For example, some seem to offer very little in Euclidean geometry while others do not specify the topics to be covered; meanwhile, students suffer as result of a weak knowledge base in this section. In order to achieve equality in teacher training and to ensure that teachers with a Bachelor of Education degree have the same pedagogical content knowledge regardless of the university in which the student has studied, the content programme should be standardized.
- (e) The universities must use similar and standardized instruments to evaluate teaching practice for pre-service teachers. Standardization of the teaching practice instrument will ensure that pre-service teachers are subjected to the same assessment procedures regardless of the university at which they were trained.

- (f) There must be a balance between high school and university syllabi. In other words the teacher training programme must cover the mathematics content that the pre-service teachers will teach at high school while also tackling some mathematics topics at university level.
- (g) There must be a balance between mathematics content covered in teacher preparation programmes and pedagogy. For example, the training of pre-service teachers should not focus mainly on pedagogy and methodology at the expense of mathematics content needed for teaching. Furthermore, it should be borne in mind that while content knowledge is necessary but not sufficient for teaching, there is a need for deep pedagogical knowledge.
- (h) Advance mathematics must be introduced gradually from year 1 to year 4 in teacher training. This means that there must be a smooth transition (continuity) from high school to university level mathematics.

## **6.7 THE IDEAL INTERVENTION MODEL**

The findings of the present study show that pre-service teachers' knowledge of mathematics content is inadequate for teaching. This supports the assertion of Pournara (2005), Lott and Souhrada (2000) and PEI's report (Brodie, 2004) that the majority of pre-service teachers join teacher education programmes with extremely low high school mathematics knowledge. It was obvious from the pre-service teachers' achievements in the mathematical proficiency test that their level of high school mathematics is very low. The role of the teacher's pedagogical knowledge in effective teaching cannot be ignored. This means that pedagogical knowledge is indispensable for ideal classroom practices. However, the present study indicates that there is no relationship

between pre-service teachers' mathematics knowledge and their teaching practices.

Certainly, these findings account for South African learners' low achievements in science and mathematics when compared with their international counterparts (Moloi & Strauss, 2005; van der Walt & Maree, 2007; Howie, 2003). The teacher-training institutions must play a role in creating a strong bond between high school and teacher education mathematics content. Such a bond would be aimed at consolidating and integrating high school and university mathematics. This would close the gaps that were experienced by pre-service teachers in mathematics when they were still at school and also advance their knowledge of mathematics for teaching. The training must enhance the relationship between what pre-service teachers learn in content and method courses with what they do in teaching practice. The current status of the teacher training compels the researcher to think about an intervention model that will address the shortcomings of the existing teacher preparation programmes.

## **6.8 THE PILLARS UNDERLYING THE PROPOSED MODEL**

These pillars are based on the findings of the study and the literature review. The study revealed that teachers that are produced by teacher-training institutions possess an extremely low level of mathematics content knowledge. This lack of basic mathematics knowledge in teachers results in the poor attainment of learners in the subject (Mji & Makgato, 2006; Taylor, 2009; Howie, 2003). There is a need to overcome this problem by improving the pre-service teachers' conceptual understanding of mathematics. Then and



only then can we expect them to teach effectively. Wu (1999) has the following to say in this regard: to overcome the pedagogical problem requires a deep knowledge of the subject on the part of the prospective teachers and their instructors alike.

The study found that there is lack of relationship between pre-service teachers' knowledge and what they do in practice. This is evident in the low marks that pre-service teachers obtain in content tests and the very high marks that they attain in teaching practice. Brodie (2004) argues that the relationship between knowledge and practice in mathematics is important. She asserts that the kind of practices that a teacher engages in embraces mathematical knowledge.

Adequate subject knowledge on the part of the teacher enables him/her to select appropriate teaching strategies while, on the other hand, knowledge of the methods cannot substitute for content knowledge. Teacher preparation programmes that focus more on the subject content can substantially improve the prospective teachers' level of content knowledge.

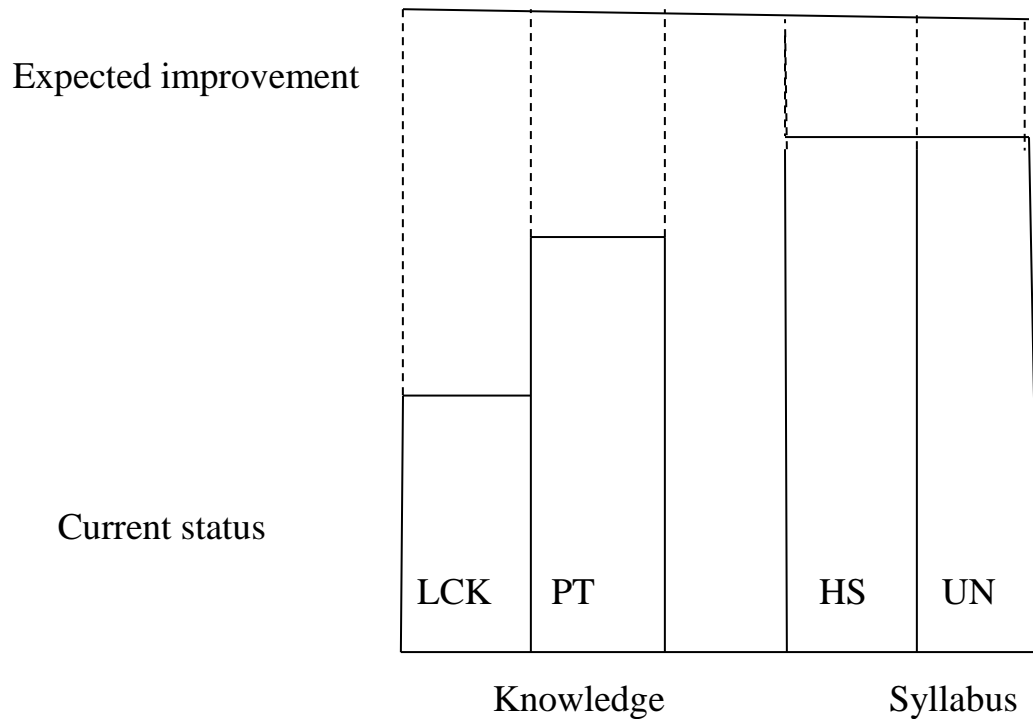
The literature review on teachers' level of content knowledge (Mji & Makgato, 2006) shows that a lack of basic content knowledge results in poor teaching standards. This has an adverse effect on the teachers' expected role as mediators, designers of learning programmes and materials, and as subject specialists (DoE, 2000). In order to fulfil this role, the teacher education syllabus must contain basic high school mathematics in addition to advanced topics for enrichment benefits.

Teachers effectively teach what they know the way they understand it. This is evident in Tirosh's study (2000) in which pre-service teachers were required to find answers to calculations that involve fractions, to list common mistakes that are often made by the students in those calculations and to identify the sources of the mistakes. The study revealed that five of the thirty pre-service teachers gave incorrect answers and with regard to the listing of students' common mistakes and their sources, 27 of the 30 pre-service teachers gave one typical incorrect response to each expression. Clearly, if the teacher does not understand the problem, he/she cannot realise the mistakes and their sources. Furthermore, it is indicated in the literature (Turnuklu & Yesildere, 2007) that some pre-service teachers do not understand the connection between addition and subtraction. For example, pre-service teachers could not explain the different and similar features of  $(+5) - (+3)$  and  $(+5) + (-3)$ . If the teachers struggle to get correct solutions and to identify learners' mistakes and their sources, then it is absolutely clear that their teaching will not be effective. The level of content knowledge determines the potential of a teacher to teach the subject. The strong relationship between what the prospective teachers have learnt in mathematics and what they do in practice is essential. Several writers (Brodie, 2004) argue for closer links between the mathematics that teachers learn and the core activities of their practices. Chances are very slim for teachers to teach effectively if they lack mathematics content.

Universities prepare teachers to teach mathematics at either primary or high school. As a result there is a need to maintain a balance between the high school and university mathematics during the training process. Furinghetti (2000:44) argues that when prospective teachers arrive at university as

students, they often deal with the problem of advanced mathematics. When these teachers begin teaching they are expected to teach school mathematics and sometimes the advanced mathematics they learnt at university becomes a distant memory which does not help them at all. As a result it is imperative that school mathematics content should constitute a larger part of the B.Ed mathematics course in order to enhance teachers' understanding of what they will teach.

Based on the pillars discussed above, this study proposes a model mathematics programme for teacher training. The proposed programme is intended to improve high school pre-service teachers' knowledge of mathematics content. Consequently, the results of the present study lead us to the model of pre-service teachers' pedagogical content knowledge improvement. It comprises of basic high school mathematics and advanced level, or university mathematics, which is above the level of the high school mathematics. Table 6.1 represents the model which illustrates the current status and the scope of improvement of the pre-service teachers' level of mathematics knowledge. Model B illustrates the proposed B.Ed mathematics content and method syllabus.



**Figure 5 :** Module A: The relationship between current knowledge and expected improvement in level of content knowledge, practice teaching and syllabus  
 LCK = Level of Content Knowledge    PT = Practice Teaching    HS = High School  
 UN = Univeristy

= Curernt status     
 = Scope of improvement

The model indicates the level of mathematics knowledge that the pre-service teachers generally possess when they complete their teacher training (LCK) and the scope of improvement needed. It indicates the relationship between the pre-service teachers' mathematics knowledge and their teaching practices (PT) and the expected improvement. The model also relates high school mathematics (HS) to the advanced level mathematics they learn at the university (UN). The scope of improvement in the syllabi for university and high school is of a lesser range. This notion is, however, wider for content knowledge. There is a need to find a way to improve the current situation. In

figure 5 the height of the bar (LCK) with solid line indicates the prospective teachers' current level of mathematics. The top section of the bar, consisting of dotted lines, indicates the expected improvement in order to achieve the ideal level of pre-service teachers' mathematics knowledge. This improvement will be achieved by making our mathematics programmes focus more on the basic high school mathematics. More mathematics content topics, including advanced topics, must be covered in the syllabus. This means that much time will be allocated to content courses.

The height of the bars (LCK and PT) indicates that currently the pre-service teachers' level of content knowledge is lower than their knowledge of teaching methods. With the dotted lines the model shows the extent of improvement needed in order to strengthen the relationship between content and practice. The model emphasizes changing of the focus of the teacher training from being methodology-orientated to content-orientated. The literature (Sam, 2005) shows that countries such as China where pre-service teachers' mathematics programmes tend to focus more on academic components, the teachers' competency in mathematics content is better than their counterparts who focus more on educational components, such as Malaysia. The teacher preparation programme needs to entrench the links between teacher education mathematics and high school mathematics.

The model supports the point of view of CBMS (2001) and Shoaf (2000), namely that pre-service teachers' mathematics curriculum design should make insightful connections between the advanced mathematics that they learn and the school mathematics that they will teach. The model shows that very little need to be done with regard to improving the relationship between

high school mathematics acquired and the advanced mathematics that they learn in their training.

Table 6.1 is the model B for mathematics content and method themes that will constitute the syllabus for mathematics teacher education (B. Ed). It indicates the ideal level of mathematics and the amount of mathematics coverage in each year of study. Similarly, with the methodology course, the amount of work to be covered in each level is indicated. The high school mathematics content forms the basis for teacher preparation mathematics content. As a result, it must take the bigger part of the teacher preparation mathematics content. On the other hand, an advanced knowledge of mathematics is essential and therefore advanced mathematics is additional.

The model B proposes a B. Ed mathematics syllabus which consists of the basic high school mathematics and advanced level mathematics for university level. The programme focuses more on the content. That is why it covers 80% content and 20% of educational and practical components. The first column indicates high school mathematics consisting of grade 10 and grade 11 mathematics. The clarity of the syllabus and the bigger fraction of teacher preparation mathematics being the school mathematics content will reduce the tendency of being biased towards certain topics and neglecting others. The university mathematics is done for enrichment and to extend pre-service teachers' knowledge to advanced level. When teaching the pre-service teachers, the lecturers have to link the university topics to the high school topics in such a way that there is continuity. For example, when they do quadratic equations in the high school content, they will do complex numbers in advanced mathematics. This alignment of topics will advance pre-service

teachers' knowledge of real solution of quadratic equations to non-real (imaginary) solutions which introduce a new system called complex numbers.

**TABLE 6.1: Proposed model B for B.Ed method mathematics syllabus**

YR	HIGH SCHOOL COMPONENT Maths Content 50%	UNIVERSITY COMPONENT Maths Content 30%	EDUCATION COMPONENT Teaching Methods 10%	PRACTICAL COMPONENT Teaching Practice 10%	ENRICHMENT Ratio of Content and Activities
1	<b>Grade 10 &amp; 11 Level</b> Algebra expressions, Real & non-real numbers, Probability: dependent & independent events, Symmetric and skewed data, correlation coefficient , special angles, 2-D problems, Exponents, Number patterns , Equations & inequalities, <b>Surds</b> , Trigonometric functions, Euclidean geometry, Analytical geometry, Finance & growth, Measurement, Statistics	Complex numbers, De Moivre's theorem, Synthetic division, Partial fractions, Natural logarithms, Mathematical modelling through applications	Principles of education Methods of teaching	Microteaching	50:30:10:10
2	<b>Grade 11 &amp; 12</b> Equations, Analytical geometry, Functions & inverse functions, Sequences & series, <b>Trigonometry: reduction formulae including negative angles</b> , graphs, equations, Compound angles, Sine, Area & Cosine rule, 3-D problems Logarithms, Financial growth & decay, <b>Foreign exchange</b> , bonds, <b>sinking funds</b> , investment & loan options, Euclidean geometry, Statistics, Limits, Differential calculus	Matrix algebra Determinants, Binomial theorem (Pascal triangle) Mathematical induction, Piecewise functions, Ellipses, Transformations, Conic sections	<b>Method of teaching mathematics</b>	<b>Observational Learning</b>	50:30:10:10

	<b>HIGH SCHOOL COMPONENT</b> <b>Maths Content 30%</b>	<b>UNIVERSITY COMPONENT</b> <b>Maths Content 20%</b>	<b>EDUCATION COMPONENT</b> <b>Teaching Methods 10%</b>	<b>PRACTICAL COMPONENT</b> <b>Teaching Practice 40%</b>	<b>ENRICHMENT</b> <b>Ratio of Content and Activities</b>
3	<b>Grade 12</b> Exponential & logarithmic functions, Linear programming, Trigonometry 2-D & 3-D, Polynomial functions, Differential calculus, cubic graphs, Geometry: Pythagoras theorem, Proportionality & similarity. Statistics: Regression & correlation, Counting & probability	Limits involving trig ratios, Differentiation rules: Chain rule, Product rule, Quotient rule. Differentiation formulas involving logarithms and trig ratios. Integration: indefinite integral, definite integral, Calculating the area of irregular figure.	<b>Method of teaching mathematics</b>  <b>Theories of learning.</b> <b>Methods of teaching mathematics.</b>	<b>PRACTICUM</b>	30:20:10:40
4		Geometry on plane and Spheres, Fractals, Transcendental functions, Differential equations Modelling parametric equations	<b>Methods of teaching of mathematics and theories of learning</b>	<b>PRACTICUM</b>	0:10:40:50

The proposed content and method teacher education programme intends focusing on basic mathematics topics at the first year of study. For example, grade 10 and 11 topics are integrated and supplemented by few university mathematics topics. Students have a tendency of forgetting easily what they have learned some months or years back. As in the case of mathematics, some students at grade 12 level often do not remember some of the mathematical concepts that they encountered in previous grades. Similarly, prospective teachers may have a problem of recalling some of the mathematical concepts that they learnt when they were still at school, particularly in grade 8, grade 9,



grade 10 and grade 11. At the second year we expect these pre-service teachers to go out for teaching practice, and if they do not have a good command of basic high school mathematics, failure to teach at these levels is expected. The ratio of content and pedagogical activities shows that in the first and second year the programme focuses more on the high school content than on university and method courses. It is intended to prepare pre-service teachers thoroughly with content so that they can face the learners with confidence during teaching practice.

Very often when pre-service teachers go out for teaching practice, they are given junior classes and this compels them to recall what they learnt many years ago. To this end, there is a need to revisit mathematics topics which they had learnt in junior classes when they were still at school. The method courses are gradually integrated at a small scale to their education until they are in the final year of study. Much work done at this level is based on educational and practical components. The reason for this is that we expect these pre-service teachers to begin their teaching career the following year. Secondly, in the final year prospective teachers are in a position to transform content knowledge into pedagogical knowledge because they have dealt extensively with the mathematics content. As a result, during the final year of study pre-service teachers are given enough time to gain teaching experience.

## **6.9 PROSPECTS**

Undoubtedly, teachers are the most significant factors that affect students' learning of mathematics. One of the primary responsibilities of mathematics teachers is to mould and develop the learners' mathematical reasoning. Some

of the fundamental determinants of the teacher's ability to successfully perform his/her responsibilities is possession of specialized mathematics content knowledge, knowing his/her students as learners and knowledge of pedagogy. The present study indicates that there is a gap between content and pedagogical knowledge.

The adoption of the proposed intervention models can benefit the prospective teachers in many ways. For example, one of the prospects of the study is that the teacher educators will become aware of the imbalances that exist between content and pedagogy when they teach pre-service teachers. The models focus more on content in the first two years in order to accommodate the lack of content knowledge on the part of pre-service teachers. The early focus of teacher education on mathematics content will make teachers to be competent and confident with regard to subject knowledge. This means that pedagogical knowledge will be grounded on a strong content knowledge base. The pre-service teachers will be able to critically evaluate the content of the syllabus, coherently arrange the topics of the content to ensure continuity and to suit the current situation without following the textbook sequence. If this happens, teachers will be able to use textbooks appropriately, that is, they will select tasks from the textbook which have a potential to engage learners' intellect instead of those tasks that require application of well-known procedures to reach expected solutions.

If the teacher is not competent and confident, particularly in content, he/she cannot manoeuvre the sequence of the topics that appear in the textbook to suit the arising need of the learners. Such teachers frequently rely on teaching algorithms rather than cognitive thought processes. He/she avoids learners'

questions and other sections or topics in the syllabus, thus creating gaps which ultimately threaten the learners' future in mathematics. Since the proposed programme focuses more on content courses, the prospective teachers will be fluent in the subject matter. They will have a wide scope of thinking, even beyond the call of duty, about the students they will be teaching and the concepts of the subject. The manner in which the university topics are aligned to high school topics in the proposed model will enhance understanding and continuity. Pre-service teachers will be able to connect school mathematics to university mathematics and eventually possess both the basic and advanced mathematics.

Secondly, the fact that the models suggest that much high school content work should be done during the beginning of the teacher education programme, means that by the time that the pre-service teachers go out for teaching practice they would have revisited mathematics topics that they had covered when they were still at school. To this end, the pre-service teachers will reach almost the same level of competence in content and pedagogical knowledge.

## **6.10 CONCLUSION**

Most of the pre-service teachers are one page ahead of high school learners in terms of mathematics knowledge. With this in mind, one cannot expect conspicuous improvement in South African learners' results for mathematics. The comparison between achievements in content and teaching practice may have contradictory interpretations. For instance, one could say although pre-service teachers do not have adequate mathematics content knowledge, they nevertheless teach well, and this seems to be confusing. On the basis of

the shortcomings of the teacher preparation programmes and the lack of relationship between teachers' content knowledge and teaching practices the researcher proposes a restructuring of the content and method course. Much work and time is given to the content course in the first two years of study while in the last two years the content and method courses are equally considered. Presumably, the existing gap between content knowledge and pedagogy will be addressed.

**CHAPTER SEVEN**  
**SUMMARY, GENERALISATION, LIMITATIONS AND AVENUES**  
**FOR FURTHER RESEARCH.**

**7.1 SUMMARY**

**7.1.1 The problem**

The study was designed to investigate the nature of training that the pre-service mathematics teachers get from the universities. The study focused mainly on the pedagogical content knowledge possessed by the pre-service teachers when they completed their training.

**7.1.2 The aims of the study were as follows:**

- (a) To determine how much mathematics content do teachers possess at the point of exit of their teacher training programme. By mathematics content we mean all mathematical themes, concepts and rules that a teacher is expected to know in order for him/her to be considered as having knowledge of mathematics. Presumably, knowledge of mathematics content would enable the teacher to teach mathematics effectively.
- (b) To establish the relationship, if any, between achievements in content and achievements in teaching practice. By achievements in content we refer to the marks obtained by pre-service teachers in a mathematics proficiency test. Achievements in teaching practice refer to the marks obtained by the pre-service teacher during teaching practice assessment.

- (c) To establish the relationship, if any, between content covered in mathematics education preparation programmes and high school mathematics.
- (d) Content in mathematics education refers to the topics, themes and concepts that are taught to pre-service teachers during teacher training. High school mathematics means all mathematics topics and themes that are taught to high school learners.

### **7.1.3 The following hypotheses were formulated**

#### ***Hypothesis in relation to aim number one***

- (a) Pre-service teachers possess adequate mathematics content knowledge when they exit their training programme.
- (b) Knowledge possessed by pre-service teachers will vary according to three sections of mathematics, namely algebra, trigonometry and geometry.

#### ***Hypothesis in relation to aim number two***

- (a) The teachers' knowledge of mathematics content will correlate with his/her teaching practices performance.
- (b) There will be no difference between pre-service teacher mean scores in knowledge of mathematics and practicum.

#### **7.1.4 Methodology**

The structure of the present work comprises of seven chapters. Chapter one is concerned with the statement of the problem and motivation for an evaluation of the current teacher training programmes at different universities. Chapter two deals with the different schools of thought and the theories of the teaching and learning of mathematics. Chapter three consists of the review of the previous work done pertaining to teacher training and teaching and learning of mathematics. Chapter four gives the detailed methodology of the study. The instruments used are as follows: Academic Aptitude Test (AAT), teaching practice inventory and syllabi documents. Chapter four also indicates the procedures followed with regard to administration of the instruments. Chapter five consists of presentation and analysis of data. Chapter six contains the discussion of findings and implications of findings of the study. Chapter seven provides the summary of the study, its limitations and recommendations.

#### **7.1.5 Findings**

The present study revealed the following: *Pre-service teachers generally have very little mathematics content knowledge, which is necessary for teaching when they exit their training programmes.*

The majority of pre-service teachers fell far below the cut-off point of 50% which in this study demarcates the point between those considered as having adequate knowledge and those with inadequate knowledge of mathematics. In terms of performance levels, based on this study, seventy-three percent of

pre-service teachers are in level 1, which indicates scores from 49% and below. Twenty-six percent of the teachers have a moderate knowledge of mathematics. Only one percent of pre-service teachers were found to have a strong content knowledge of mathematics. This means that only 27% of the pre-service teachers who participated in this study would be ready to teach mathematics upon completion of their studies.

There is no significant difference between the pre-service teachers' knowledge of the three sections of mathematics, namely algebra, trigonometry and geometry. The difference in pre-service teachers' attainments in three sections of mathematics is statistically insignificant. However, despite this insignificant difference it appears that they perform better in algebra than in trigonometry and geometry.

There is no correlation between pre-service teachers' knowledge of content and their performance in teaching practice. The remarkable trend is that, in a content test, pre-service teachers obtained low marks while in the practice teaching they scored extremely high marks. This implies that, generally, in teaching practice assessment pre-service teachers are awarded very high marks. As a consequence, there seem to be no association between pre-service teachers' content knowledge and their teaching practice results.

There is a significant difference between pre-service teachers' mean scores for mathematics content and practicum. The mean scores for content and practicum were significantly different. The mean score for practical was far above the mean score for content. This emphasizes the point that there is no correlation between content knowledge and teaching practice.



There is a relationship between mathematics for teacher education and high school mathematics. Most of the high school mathematics topics and themes are covered in the teacher education mathematics syllabus. However, it seems that geometry is not given enough time, because it does not appear in the work programmes of some of the universities. Basic aspects of the content, for example definition of terms and concepts, do not appear in the syllabus for the teacher education. This means that prospective teachers are assumed to know the definition of mathematic terms and concepts. There is evidence of a remarkable effort to advance pre-service teachers' content knowledge because the teacher education mathematics syllabus goes beyond high school mathematics and includes advanced level mathematics. Although there is a relationship between high school and mathematics education syllabi there is no certainty about whether these topics, including advanced topics, are thoroughly taught to the prospective teachers.

## **7.2 GENERALISATION**

The primary aim of the study has been to ascertain the level of pre-service teachers' pedagogical content knowledge at the point of exit of their training process. For this purpose, it is important to know whether the findings of the study are applicable to other pre-service teachers elsewhere. The sample of the study consisted of pre-service teachers from two universities in one province of South Africa. Using cluster sampling design we exhausted all institutions which offer teacher training in the province (A) or county (A). In a cluster, all student teachers in the fourth year B. Ed degree group were included as participants in the study. Based on the cluster sampling and the

fact that the teacher training system in South African universities is not significantly different, we can generalize the findings with certainty.

### **7.3 LIMITATIONS**

The study was able to achieve the aims that were formulated at its beginning. However, the study has limitations with regard to sampling. For example, the study sample was taken from only two universities out of many universities in South Africa. One may argue that the universities differ in terms of the teacher training. For example, in some universities pre-service teachers take their content course in the faculty of science while others take their content course in the faculty of education. This may produce different results with regard to content knowledge acquisition. The participation was voluntary, and as a result some students opted not to participate, which eventually reduced the number of participants.

### **7.4 AVENUES FOR FUTURE RESEARCH**

The study on professional training in mathematics education has opened the following avenues for future research.

- (a) There is a need to investigate why pre-service teachers, after having been in the university for such a long period, yet still obtain tremendously low marks in high school mathematics. For example, are the universities doing justice when teaching mathematics content to the pre-service teachers or do they assume that pre-service teachers know the mathematics they will be teaching to learners as they finish the training.

- (b) There is a need to investigate how pre-service teachers are assessed during the teaching practice. According to the findings of the present study there seems to be a loophole in the evaluation process. It appeared in the study that while most of the students fail the mathematics content test, they get very high marks for their practice teaching. One may argue that there should be proportionality between the content and practice teaching marks.
- (c) There is a need to investigate why pre-service teachers perform comparatively poorer in trigonometry and geometry than in algebra. In the same way, the question arises why pre-service teachers perform more poorly in geometry than in trigonometry.
- (d) The findings of the study indicated that universities have different method courses. Some are formally structured while others do not have visible structures. It is important to investigate the benefits of having a structured method programme as opposed to an unstructured method programme.

## **7.5 CONCLUSION**

The study informs us that although there have been many changes in education, both in schools and teacher training, there is still difficulty in improving a conceptual understanding of mathematics. Based on the findings of the study, it is clear that pre-service teachers for mathematics begin their teaching careers with inadequate content knowledge needed for teaching the subject. Most of the pre-service teachers seem to be at the same level as high school learners in terms of mathematics knowledge. With this in mind, one cannot expect conspicuous improvement in South African learners' results for mathematics. The study established that, generally, the marks awarded to

pre-service teachers in practice teaching are incomparably higher than what they achieve in content tests. Such a finding implies that honesty should be the focus of the assessment during teaching practice so as to avoid deceptive information about pre-service teachers' levels of pedagogical content knowledge.

## REFERENCES

- Adler, J. & Davis, Z. 2006. Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education*. 37(4): 270-296.
- Adnan, M., Zakaria, E. & Maat, S.M. 2012. Relationship between mathematics beliefs, conceptual knowledge and mathematical experience among pre-service teachers. *Procedia – Social and Behavioural Sciences*. 46(9): 1714 – 1719.
- AECT (Association for educational communications and technologies) 2001. Descriptive research methodologies. *The handbook of research for educational communications and technology*. Available: [www.aect.org/edtech/ed1/41/41-01.html](http://www.aect.org/edtech/ed1/41/41-01.html). [2011, June 5].
- Anderson, J. 1996. Some teachers' beliefs and perceptions of problem solving. In *Technology in Mathematics Education*. P.C. Clarkson, Ed. Melbourne: Education research group of Australia. 30-33.
- Andrew, L. 2006. Pre-service teachers' reaction to their final constructivist mathematics class: A case study. *IUMPST: The journal*. Available: <http://www.k-12prep.math.ttu.edu>. [2010, April 16].
- Constructivist education: Learning principles in constructivism. 2006. Available: <http://www.adraptics.com./topoi.pdf>. [2011, July 17].

Avalos, B., Tellez, F. & Navarro, S. 2010. Learning about the effectiveness of teacher education: A Chilean study. *Perspectives in Education*. 28(4): 11-21.

Avcu, S. & Avcu, R. 2010. Pre-service elementary mathematics teachers' use of strategies in mathematical problem solving. *Procedia Social and Behavioural Sciences*. 9(2):1282-1286.

Azeem, M. & Gondal, M.B. 2011. Math Proficiency Assessment Based Upon Item Response Theory. *The international Journal of Interdisciplinary Social Sciences*.6(1):105-122. Available: <http://www.SocialSciences-Journal.com>. [2012, July 12].

Baker, T.L. 1994. Elementary statistics for social research. *Doing social research*. New York, USA: McGraw-Hill, Inc.

Ball, D.L. & Bass, H. 2002. Towards a practice-based theory of mathematical knowledge for teaching. In *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group*. B. Davis & E. Summt, Eds. Edmonton: CMESG /GCEDM.

Behr, A.H. 1988. *Empirical research methods for the human sciences*. Durban, South Africa: Butterworth.

Benken, B.M. & Brown, N. 2008. Integrating teacher candidates' conceptions of mathematics teaching and learning: A cross University collaboration. *The journal*. Available: <http://www.k-12prep.math.edu/journal/contentk>. [2010, January 28].

Brodie, K. 2004. Re-thinking teachers' mathematical knowledge: A focus on thinking practices. *Perspectives in Education*. 22(1): 65-79.

Bruner, J. n.d. Cognitive theory. Available:

<http://tip.psychology.org/bruner.html>. [2011, February 18].

Buelens, H.; Clement, M. & Clarebout, G. 2002. University assistants' conceptions of knowledge, learning and instruction. *Research in Education*. 67(2): 44-57.

Canadian Council of Teachers of English Language Arts (CCTELA). 2004. Gifting democracy: English language education in South Africa. *English Quarterly*. 36(2):1193-9966.

Carl, A.E. 2009. Effective curriculum design for dynamic curriculum development. *The teacher empowerment through curriculum development*. 3<sup>rd</sup> ed. Cape Town: Juta & Company.

Carrol, J.E. 1997. Primary school teachers views of mathematics teaching and learning. Ph. D. Thesis. Monash University.

Cherry, K. 2012. Humanistic psychology. Available: <http://www.About.com/psychology>. [2012, April 12].

Clements, D. H. 1997. Constructing constructivism. *Teaching Children Mathematics*. 4(4): 198-200.

- Coben, D. 2003. Adult numeracy: Review of research and related literature. *National research and development centre for adult literacy and numeracy*. London : NRDC.
- Cole, J.C. 2010. *Mathematical structuralism today*. Buffalo, New York: Buffalo state college.
- Conference Board of the Mathematical Sciences 2001. *The mathematical education of teachers*. Available: <http://www.cbmsweb.org/METDocument/index.htm> [2009, September 20]
- Conway, J. 1997. Educational technology's effect on models of instruction. Available at: <http://udel.edu/~jconway/EDST666.htm#dirinst> [2010, June 12].
- Cooper, P.A. 1993. Paradigm shift in designed instruction: From behaviourism to cognitivism to constructivism. *Educational Technology*. 33(5): 12-19.
- Corcoran, D. 2005. Mathematics subject knowledge of Irish primary pre-service teachers. *Paper presented at the European Conference on Educational Research*. Dublin, 7-10 September 2005. Dublin: University College Dublin.
- Crespo, S. 2003. Learning to pose mathematical problems: Exploring changes in pre-service teachers' practices. *Educational Studies in Mathematics*. 52(1): 243-270.
- David, M. & Sutton, C.D. 2004. Coding qualitative data: Qualitative content analysis. *Social Research*. London, UK: Sage.



Davis, B. & Simmit, B. 2006. Mathematics for teaching: An on-going investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics*. 61(1):293-319.

Dengate, B. & Lerman, S. 1995. Learning theory in mathematics education: using the wide angle lens and not just the microscope. *Mathematics Education Research Journal*.7 (1): 26-36.

Department of education. 2000. *Why some “disadvantage” schools succeed in mathematics and science: a study of “feeder” schools*. Pretoria: Government Printer.

Department of education. 2000. Norms and standards of educators. *Government Gazette* 415 (20844). Pretoria, Republic of South Africa: Department of Education. Available: <http://education.pwv.gov.za/content/documents/170.pdf>. [2009, August 20].

Department of education. 2001. *National strategy for mathematics, science and technology education in general and further education and training*. Pretoria: Government Printer.

Department of education. 2002. *Revised national curriculum statement Grade R-9*. Pretoria: Government Printer.

Department of education. 2003. Mathematical literacy. *National curriculum Statement Grades 10-12*. Pretoria: Government Printer.

Department of education. 2004. *National strategy for mathematics, science and technology education: Creating tomorrow's stars today. Implementation plan 2005-2009*. Pretoria: Government Printer.

Department of education 2005. Learning programme guidelines. *National Curriculum statement Grades 10-12*. Pretoria: Government Printer.

Department of Higher Education and Training (DoHET). 2011. The minimum requirements for teacher education qualifications. *Government Gazette 34467*. Pretoria: Department of Education.

Devers, K.J. & Fraenkel, R.M. 2000. Study design in qualitative research-2: Sampling and data collection strategies. *Education for Health*.13: 263-271.

De Vries, A. 2005. Pandor vra passie vir onderwys. (Pandor asks passion for teaching.) *Net- Rapport, 4 June 2005*. Available:  
[http://www.news24.com/Rapport/nuus/0,752-795\\_1716353,00.html](http://www.news24.com/Rapport/nuus/0,752-795_1716353,00.html).  
[2005, June 5].

Dills, C.R. & Romiszowski, A. J. 1997. *Institutional development paradigms*. Eaglewood cliffs, New Jersey: Educational Technology publications.

Doerr, H.M. 2007. What knowledge do teachers need for teaching mathematics through applications and modelling? In *The 14<sup>th</sup> ICMI Study: Modelling applications in mathematics education*. W. Blum, P.L. Galbraith, H.W. Henn & M. Niss, Eds. New York: Springer. 69- 78.

- Dorfler, W. 2003. Mathematics and mathematics education: Content and people, relation and difference. *Educational Studies in Mathematics*. 54(1): 315-316.
- Dossey, J.A., Mc Crone, S., Giordano, F.R. & Weir, M.D. 2002. Mathematics methods and modelling for today's mathematics classroom. *A contemporary approach to teaching grades 7-12*, California: Brooks/ Cole.
- Downey, J.A. & Cobbs, G.A. 2007. "I actually learned a lot from this": A field assignment to prepare future pre-service math teachers for culturally diverse classrooms. *School Science and Mathematics*. 107(1): 391-394.
- Du Plessis, L.& Gerber, D. 2012. Academic preparedness of students\_ an exploratory study. *The journal for Trans-disciplinary Research in South Africa*. 18(1): 81-94.
- ECS & McREL, 2004.A policymakers' primer on education research.  
Available: [www.ecs.org/html/educationIssues/Research/primer/index.asp](http://www.ecs.org/html/educationIssues/Research/primer/index.asp).  
[2011 May 26].
- Elifson, K.W., Runyon, R.P. & Haber, A. 1990. Statistical inference with categorical variables: Chi Square and related measures. *Fundamentals of social statistics*. New York: McGraw-Hill, Inc.

Ellis, M.W., Contreras, J. & Martinez-Cruz, 2009. The mathematical preparation of prospective elementary teachers: Reflections from solving an interesting problem. *IUMPST: The journal*, 2. Available: [www.k-12prep.math.ttu.edu](http://www.k-12prep.math.ttu.edu). [2009, April 20].

Encyclopedia of psychology, 2001. Associationism. *Gale encyclopaedia of psychology*. Gale group 2001. Available: <http://findarticles.com/p/articles/mi> [2011, June 06].

Ernest, P. 1988. The impact of beliefs on the teaching of mathematics. 6<sup>th</sup> *International congress of mathematical education*. Budapest, August 1988. Available: <http://webdoc.sub.gwdg.de/edoc/e/pome/impact.htm> [2008, July 01].

Ernest, P. 1991. *The philosophy of mathematics education*. London, UK: Falmer Press.

Ernest, P. 1996. New angles on old rules. *Times higher educational supplement*. Exeter, UK: University of Exeter.

Even, R. & Ball, D. L. 2009. The professional education and development of teachers of mathematics: *The 15<sup>th</sup> ICMI Study*. New York, USA: Springer.

Fennema, E. & Franke, M. 1992. Teachers' knowledge and its impact. In *Handbook of research on mathematics teaching and learning*. D.A. Grouws, Ed. New York, USA: Macmillan.

- Freudenthal, H. 1991. *Revisiting mathematics education: China lectures*.  
Dordrecht, Netherlands: Kluwer Academic Publishers.
- Furinghetti, F. 2000. The history of mathematics as a coupling link between secondary and university teaching. *International Journal of Mathematical Education in Science and Technology*. 31(1): 43-51.
- Garcia, M., Sanchez, V. & Escudero, I. 2006. Learning through reflection in mathematics teacher education. *Educational Studies in Mathematics*. 64(1):1-17.
- Garegae, K.G. & Chakalisa, P.A. 2005. Pre-service mathematics teacher preparation programmes and early years of teaching in Botswana. *A proposal to be presented at the 15<sup>th</sup> ICMI study conference on the Professional Education and Development of Teachers of Mathematics*. Gaborone, Botswana: University of Botswana.
- Gierdien, F. 2012. Pre-service teachers' views about their mathematics teacher education modules. *Pythagorus*, 33(1):134.
- Goldblatt, P.F. & Smith, D. 2004. Illuminating and facilitating professional knowledge through case work. *European Journal of Teacher Education*. 27(3): 335-354.
- Gray, D. 2004. *Doing research in the real world*. London, UK: Sage publications.

- Grassl, R. & Mingus, T.T.Y. 2007. Team teaching and cooperative groups in abstract algebra: nurturing a new generation of confident mathematics teachers. *International Journal of Mathematical Education in Science and Technology*. 38(5): 581-597.
- Grimes, D.A. & Schulz, K.F. 2002. Descriptive studies: What they can and cannot do. *The Lancet*, 359.
- Grimm, L.G. 1993. Testing the significance of a single mean: The single sample z and t tests. *Statistical applications for the behavioural sciences*. New York, USA: John Wiley & Sons, Inc.
- Heffner, C.L. 2004. Research methods: Experimental design. *AllPsych and Heffner media group. Inc.* Available: [allpsych.com/researchmethods/index.html](http://allpsych.com/researchmethods/index.html). [2011, June 5].
- Hill, H.C. & Ball, D.L. 2004. Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*. 35(5): 330-351.
- Hill, H.C., Rowan, B., & Ball, D.L. 2005. Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*. 42(2):371-406.

Hoadley, U. 2009. A distant reality: aligning the B.Ed curriculum at North West University. In *Opportunities and challenges for teacher education curriculum in South Africa*. G. Kruss, Ed. Potchefstroom, South Africa: Human Science Research Council.

Hobden, S.D. 2007. Towards successful mathematical literacy learning: A study of a pre-service teachers education module. Ph. D. Thesis. University of KwaZulu Natal.

Holtorf, C. 2011. *Associationism*. Available:  
<https://tspace.library.utoronto.ca/citd/holtorf/3.7.html> [2011, February 18].

Howie, S.J. 2003. Language and other background factors affecting secondary pupils' performance in mathematics in South Africa. *A Journal of Research in SMT Education*. 7(1):1-20.

Hristovitch, S. & Mitcheltree, M. 2004. Exploring middle school teachers' pedagogical content knowledge of fractions and decimals. *Paper presented at the annual meeting of the North American chapter of the international group for the psychology of mathematics education*. 21 October 2004. Available:  
[http://www.allacademic.com/meta/p117700\\_index.html](http://www.allacademic.com/meta/p117700_index.html) [2008, October 10].

Hunt, O. 2007. *A mixed method design*. Available:  
[http://www.articleallery.com/article\\_185975\\_22.html](http://www.articleallery.com/article_185975_22.html). [2009, July 13].

- Jacob, B. 2006. *Content analysis as a research tool*. Available:  
<http://en.articlesgratuits.com/content-analysis-as-a-research-tool-id998.php>  
[2009, September 08].
- Jacobs, M. 2004. Curriculum design. In *Teaching-learning dynamics: A participative approach for OBE*. M. Jacobs, N.C.G. Vakalisa & N. Gawe, Eds. Sandton: Heinemann Publishers.
- JeongHo W. 2004. School mathematics as a major subject for humanistic Education. *International conference of mathematics education*. September 2004. Korea: National University.
- Kahan, J., Cooper, D. & Bethea, K. 2003. The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education*. 6(1):223-252.
- Kayhan, H.C. & Argun, Z. 2009. Evaluation of pre-service mathematics teachers in aspect of teacher proficiency. *Procedia Social and Behavioral Sciences*.1(9): 2632-2336.
- Keppel, G. & Zedeck, S. 1989. Significance and hypothesis testing. *Data analysis for research designs*. New York, USA: W.H. Freeman and Company.
- Kerling, F.N. 1992. *Foundations of behavioural research*. New York, USA: Harcourt Brace College.



- Key, J.P. 1997. *Research design in occupational education*. Oklahoma: Oklahoma State University.
- Khan, F. 2000. Contextual factors associated with mathematics anxiety: Perceptions of teachers and students in secondary school in KwaZulu Natal. M. Ed. Dissertation. University of Natal.
- Kilpatrick, J., Swartford, J., & Findell, B. 2011. *Adding it up: Help children learn mathematics*. Washington D.C, USA: National Academy Press.
- Kinach, B.M. 2002. A cognitive strategy for developing pedagogical content knowledge in the secondary mathematics methods course: Towards a model of effective practice. *Teaching and Teacher Education*. 18(1): 51-71.
- Leikin, R. & Levav-Waynberg, A. 2007. Exploring mathematics teacher knowledge to explain the gap between theory-based recommendations and school practice in the use of connecting tasks. *Educational Studies in Mathematics*. 66(1):349-371.
- Lerman, S. 1996. Inter-subjectivity in mathematics learning: A challenge to the radical constructivist paradigm? *Journal for Research in Mathematics Education*. 27(2):133-15.

- Li, Y. & Smith, D. 2007. Prospective middle school teachers' knowledge in mathematics and pedagogy for teaching: The case of fraction division. In *Proceedings of the 31<sup>st</sup> conference of the international group for the psychology of mathematics education*. J.H. Woo, H.C. Lew, K.S. Park & D.Y. Seo, Eds. USA: Seoul. 185-192.
- Lloyd, G.M. 2006. Pre-service teachers' stories of mathematics classroom: Exploration of practices through fictional accounts. *Educational Studies in Mathematics*. 63(1):57-87.
- Lott, J.W. & Souhrada, T.A. 2000. As the century unfolds: A perspective on secondary school mathematics content. In *Learning mathematics for a new century*. M.J. Burke & F.R. Curcio, Eds. Reston, Virginia: NCTM.
- Loucks-Horsley, S. 1998. The role of teaching and learning in systemic reform: A focus on professional development. *Science Educator*.7(1):1-6.
- Lukas, S. 2002. Principles of instructional technology: Theories of learning. Available: <http://www.edexcellence.net/library/epciv.htm>. *Principles of instructional technology.htm* [2011, February 18].
- Manouchehri, A. & Goodman, T. 2000. Implementing mathematics reform: The challenge within. *Educational Studies in Mathematics*. 42(1): 1-34.
- Martyn, S. 2008. *Descriptive research design*. Available: <http://www.experiment-resources.com/descriptive-research-design.html>. [2011, May 23].

- McCracken, C. n.d. Reconstructing mathematics: Constructivism, art, integration, and coming to know. *Daily Journal of the Teaching Experience*. Available: [http://www.dpi.state.nc.us/curriculum/ArtsEd/AE\\_SCS.html](http://www.dpi.state.nc.us/curriculum/ArtsEd/AE_SCS.html). [2012, April 16].
- McLeod, S.A. 2007. *Humanism*. Available: <http://www.simplypsychology.org/humanistic.html>. [2012, April 24].
- McNeil, N.M. & Alibali, M.W. 2000. Learning mathematics from procedural instruction: Externally imposed goals influence what is learned. *Journal of Education Psychology*. 92(4): 734-744.
- Menon, R. 2009. Pre-service teachers' subject knowledge of mathematics. *International Journal for Mathematics Teaching and Learning*. Available at: <http://www.limit.plymouth.ac.uk/journal/menon>. [2011, May 5].
- Merriam, S.B. 2002. Introduction to qualitative research. *Qualitative research in practice*. San Francisco: Jossey-Bass.
- Minnie, R. & Paul, V.H. 1982. *Manual for the academic aptitude test (Standard 10) (AAT)*. Human Science Research Council. Pretoria, South Africa: HSRC.
- Mji, A. & Makgato, M. 2006. Factors associated with high school learners' poor performance: A spotlight on mathematics and physical science. *South African Journal of Education*. 26(2): 253-266.

- Mohr, M. 2006. Mathematics knowledge for teaching. *School Science and Mathematics*. 106(1): 219-243.
- Moll, I. 2002. Clarifying constructivism in a context of curriculum change. *Journal of Education*. 27(1): 5-31.
- Moloi, M. & Straus, J. 2005. *The SACMEQ II Project in South Africa: A study of the conditions of schooling and the quality of education*. Harare, Zimbabwe: SACMEQ.
- Moodley, U. 2009. A study of learners' conceptual development in mathematics in a grade eight class using concept mapping. M. Ed Dissertation. University of KwaZulu Natal.
- Muhammad, S. 2009. Theory of functionalism in educational psychology. *Education: Education awareness research*. Available: <http://www.research-education-edu.blogspot.com> [2012, April 20].
- Muijs, D. & Reynolds, D., 2005. Constructivist teaching. *Effective teaching*. London, UK: SAGE.
- Mullis, I.V.S., Martin, M.O. & Foy, P., Olson, J.F., Preuschoff, C., Erberber, E. & Garlia, J. 2008. *TIMSS 2007 International mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eight grades*. Chestnut Hill, MA: Boston College. Available: <http://timss.bc.edu/TIMSS2007/mathreport.html> [2011, April 23].

- Nachmias, C. & Nachmias, D. 1992. Secondary data analysis. *Research methods in the social sciences*. St Martin's Press, Inc.
- Naikumi, M. 2010. Linking behaviourism, Cognitivism and Constructivism approaches to the teaching of mathematics. *Education Africa*. Available: <http://www.blog.education-africa.com/archives/685> [2011, January 17].
- Nathan, M & Petrosino, A. 2002. Expert blind spot among pre-service mathematics and science teachers. *Expert blind spot*. Colorado: University of Colorado-Boulder.
- National Planning Commission (NPC) 2010. National Development Plan 2030: Our future makes it work. Pretoria, Republic of South Africa. Available: <http://www.inf.gov.za/issues/national-development-plan/index.html>. [2012, May 8].
- Nicol, C. 2002. Where's the math? : Prospective teachers visit the workplace. *Educational Studies in Mathematics*. 50(1): 289-309.
- Nicol, C.C. & Crespo, S.M. 2006. Learning to teach with mathematics textbook: How pre-service teachers interpret and use curriculum materials. *Educational Studies in Mathematics*. 62(1): 331-355.
- Nkhoma, P.M. 2002. What successful black South African students consider as factors of their success. *Educational Studies in Mathematics*. 50(1): 105-107.

Norms and Standards for Educators. 2000. 415(20844), Pretoria: Government gazette.

Nyaumwe, L.J. & Buzuzi, G. 2007. Teachers' attitudes towards proof of mathematical results in the secondary school curriculum: The case of Zimbabwe. *Mathematics Education research Journal*. 19(3): 21-32.

Nyawuza, P.R. 2004. An investigation into development planning by primary schools management. M. Ed Dissertation. University of Zululand.

O'Connor, T. 2010. *Experimental and quasi-experimental research design*. Available: <http://www.drtoconnor.com/rest> of URL [2011, April 5].

Onwu, G.O.M. & Mogari, D. 2004. Professional development for out-comes based education curriculum implementation: The case of UNIVEMALASH, South Africa. *Journal of Education for Teaching*. 30(2):161-167.

Ostler, E. 2000. *Mathematical modelling: Some ideas and suggestions for pre-service teacher preparation*. Nesbraka, Omaha: University of Nesbraka.

Palmquist, M. 1967. Content analysis. *Research methods and theory*. Available at: <http://www.colostate.edu/Depts/WritingCenter/references/research/content/page2.htm>. [2011, May 28].

Passer, M.W. & Smith, R.E. 2011. The phenomenological-humanistic perspective. *Psychology: The science of mind and behaviour*. New York, USA: McGraw-Hill.

- Peressini, D., Borko, H., Romagnano, L., Knuth, E. & Willis, C. 2004. A conceptual framework for learning to teach secondary mathematics: A situative perspective. *Educational Studies in Mathematics*.56(1): 67-96.
- Philip, R.A. 2007. Mathematics teachers' beliefs and affects. In *Second handbook of research on mathematics teaching and learning*. F. Lester Ed. Charlotte, NC: Information age publishing 106.
- Piaget, J. n.d. Genetic epistemology. Available: [piaget.cognitive.theory.htm](http://piaget.cognitive.theory.htm) [2011, January 11].
- Piccolo, D. 2008. School science and mathematics. *Official Journal of the School Science and Mathematics Association* 108(2): 1-3. Available: <http://www.ssmj.tamu.edu/rib> [2010, February 5].
- Pillay, S. 2005. The relationship between educators' perceptions on change management and educators' attitudes towards change: A case study of IQMS implementation at primary school. M.Ed. Dissertation. University of KwaZulu Natal.
- Pournara, C. 2005. The glass is half-full: Rising to the challenge of pre- service mathematics teacher education in South Africa. *15<sup>th</sup> ICMI study conference in the professional education and development of teachers of mathematics*. Johannesburg, South Africa: University of Witwatersrand.

- Powell, P. 2010. *A critical investigation into curriculum development discourses of academic staff at a South African university of technology*. Ph.D. Thesis. Durban: University of KwaZulu Natal.
- Prakash, J. 2011. *Educational implications of insight theory of learning*. Available: <http://www.preserveArticle.com>. [2012, April 18].
- Reddy, V. 2006. Mathematics and science achievement at South African schools in TIMSS 2003. *Human science research council*. South Africa: HSRC Press.
- Robison, M. 1999. Initial teacher education in changing South Africa: experiences, reflections and challenges. *Journal of Education for Teaching*. 25(3):191-201.
- Rowlands, S. & Carson, n.d. *The contradictions in the constructivist discourse*. Montana :State university.
- Rughubar, S. 2003. The mathematics education of youth: Nellie and Wiseman. M. Ed Dissertation. University of Durban- Westville.
- Rule, A.C. & Harrell, M.H. 2006. Symbolic drawings reveal changes in pre-service teacher mathematics attitudes after a mathematics methods course. *School Science and Mathematics*. 106(1): 241-243.
- Salehi, M. 2011. Creative teaching methods: Impacts based on structuralism approaches in improving mathematics performance. *Australian Journal of Basic and Applied Sciences*. 5(9): 2290-2294.



- Sam, C.L. 2005. *A comparison of pre-service mathematics teacher education between Malaysia and China*. Universiti Sains.
- Schoenfeld, A.H. 1987. Cognitive science and mathematics education: An overview. In *Cognitive science and mathematics education*. A.H. Schoenfeld, Ed. New Jersey, USA: Lawrence Erlbaum Associates.
- Seaman, C.E., Szydlik, S.D., Szydlik, J.E. & Beam, J.E. 2006. A comparison of pre-service elementary teachers' beliefs about mathematics and teaching mathematics: 1968 and 1998. *School Science and Mathematics*. 105(4):197-210.
- Serrao, A. 2008. New roadmap compiled at education indaba. *The star*. 14 November:1
- Sestic, L. & Huttenen, I. 2006. *A guide: Implementing the revised Core Curriculum for Modern languages in Bosnia and Herzegovina*. DGIV/LANG Council of Europe.
- Shiraev, E. 2011. Psychology and the mass society at the beginning of the 20<sup>th</sup> century. *A history of psychology: A global perspective*. London, UK: Sage Publications, Inc.
- Shoaf, M.M. 2000. A capstone course for pre-service secondary mathematics teachers. *International Journal of Mathematical Education in Science and Technology*. 31:151-1602.

- Sibaya, P.T. & Sibaya, D.C. 2008. Novice educators' perceptions of the teacher education programme proposed by the Norms and Standards for Educators. *Perspectives in Education*. 26 (4): 86-100.
- Singer, F.M & Miscovici, H. 2008. Teaching and learning cycles in a constructivist approach to instruction. *Teacher and Teacher Education*. 24(1): 1613-1634. Available: <http://www.science-direct.com>. [2010 November 16]
- Skemp, R.R. 1978. Relational understanding and instrumental understanding. *The Arithmetic Teacher*. 26(3): 9-15.
- Soman, R. B. 2006. Educators' perceptions of quality assurance in education. Ph.D. Thesis. University of Zululand.
- Staub, F.C. & Stern, E. 2002. The nature of teachers' pedagogical content beliefs matters for students' achievement gains: Quasi-experimental evidence from elementary mathematics. *Journal of Educational Psychology*. 94(2): 344-355.
- Tatto, M.T., Schwille, J., Senk, S.L., Ingvarson, L., Rowley, G., Peck, R., Bankoy, K., Rodriguez, M. & Reckase, M. 2012. *Policy, practice and readiness to teach primary and secondary mathematics in 17 countries. Findings from the IEA Teacher Education and Development Study in Mathematics*. Amsterdam: IEA.
- Taylor, N. 2008. What's wrong with our schools and how can we fix them? *Paper presented at the CSR in Education Conference, TSIBA Education*. 21 November 2008. Cape Town.

- Taylor, N. 2009. South African school mathematics fails to prepare students for challenges of university. *City Press*. (Cape Town).17 May: 5.
- Terre Blanche, M., Durrheim, K., Kelly, K. 2006. First step in qualitative data analysis. In M. Terre Blanche, Durrheim, K. & Painter, D. Eds. *Research in practice*. Cape Town, South Africa: UCT Press.
- Theron, A. 2006. Perspectives on general and work behaviour. In *Psychology in the work context*. Z. Bergh & A. Theron, Ed. Oxford: Oxford University Press 45-76.
- Thomas, J.A. & Pedersen, J.E. 2003. Reforming elementary science teacher preparation: What about extant teaching beliefs? *School Science and Mathematics*.103(1): 319-332.
- Tirosh, D. 2000. Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education*. 31(10): 5-25.
- Toh, T.L., Chua, B.L. & Yap, S.F. 2007. School mathematics mastery test and pre-service mathematics teachers' mathematics content knowledge. *The Mathematics Educator*.10(2): 85-102.
- Tredoux, C & Durrheim, K. Ed. 2002.*Numbers, hypotheses and conclusions: a course in statistics for the social sciences*. Cape Town, South Africa: UCT Press.

- Turney, B.L. & Robb, G.P. 1971. *Research in education: an introduction*. Illinois, USA: The Dryden Press.
- Turnuklu, E.B. & Yesildere, S. 2007. The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. IUMPST: *The journal*, 1. Available: <http://www.k-12prep.math.ttu.edu> [2009, June 13]
- Uys, M., Van der Walt, J.L., Botha, S.U., & Van der Berg, R. 2006. An integrated course for English medium of instruction teacher trainees in South Africa. *Journal for Language Teaching*. 40(1): 69-86.
- Van der Sandt, S., & Nieuwoudt, H.D. 2005. Geometry content knowledge: Is pre-service training making a difference? *African Journal of Research in SMT Education*. 9:109-120.
- van der Walt, M. & Maree, K. 2007. Do mathematics facilitators implement meta-cognitive strategies? *South African Journal of Education*. 27(1): 223-241.
- Vithal, R. 2010. Moves underway to train university lecturers to teach. *UKZNTOUCH, A University of KwaZulu-Natal Alumni Publication*. Issue 1. 2010. University of KwaZulu-Natal.
- Weber, R.P. 1990. *Basic content analysis*. 2<sup>nd</sup> ed. Newberry Park, CA: Sage.

Whitehurst, R. 2003. *IES Director Grover*. Available:

<http://www.ed.gov/print/rschstat/research/progs/mathscience/Whitehurst.html>

[2012, March 6].

Wilburne, J.M. & Napoli, M., 2008. Connecting mathematics and literature: An analysis of pre-service elementary school teachers' changing belief and knowledge. *IUMPST: The journal* 2. Available:

<http://www.k-12prep.math.ttu.edu> [2011, November 12].

Woolfolk, A. E. 1998. *Educational psychology*. 7<sup>th</sup> ed. Boston: Allyn and Bacon.

Wozniak, R.H. 1999. Introduction to animal intelligence. *Classics in psychology, 1855-1914: Historical Essays*. Bristol, UK: Thoemmes Press.

Wu, H. 1999. *Pre-service professional development of mathematics teachers*. Berkeley, USA: University of California.

Zbiek, R. & Conner, A. 2006. Beyond motivation: Exploring mathematical modelling as a context for deepening students' understanding of curricular mathematics. *Educational Studies in Mathematics*. 63(1):89-112.

## **ANNEXURE A**

**A letter to request permission to do research**

P.O. Box 24018  
KwaDlangezwa  
3886  
12 June 2011

The Head of Department  
University of KwaZulu Natal  
School of Science, Mathematics and Technology Education  
Edgewood Campus

Dear Sir/Madam

**APPLICATION FOR ETHICAL CLEARANCE AND PERMISSION TO CONDUCT RESEARCH**

My name is Khetha Bonginkosi Biyela. I am a Doctorate degree student at University of Zululand in the Department of Mathematics, Science and Technology education. I hereby apply for a permission to conduct a research in your institution. The field work of the study is intended to take place in July 2011. The topic of my study is: **PROFESSIONAL TRAINING IN MATHEMATICS EDUCATION: A STUDY OF PROGRAMMES, PRACTICES AND PROSPECTS**

The study attempts to address the following research questions:

1. How much mathematics content knowledge do teachers possess at the point of exit of their training?
2. What is the relationship, if any, between the content covered in mathematics education teacher preparation programmes and the high school mathematics?
3. What is the impact of pre-service teachers' content knowledge of mathematics on their teaching practices?

**Instrument and Participants**

The mathematics achievement test will be administered to fourth or final year Bachelor of Education (B.Ed) students at the university in order to examine the level of mathematics knowledge they possess when they exit their training programmes. It is intended to be administered when **university re-opens from winter holidays**. As part of the project the teaching practice assessment results for the participants will be correlated with the achievement test results. This will be done in order to establish if the extent of content knowledge has an effect on teaching practices. For this reason a permission to have **an access also to the students' teaching practice results is requested**. Mathematics content covered by teacher preparation programme will be compared with the high school mathematics syllabi. To this end, the researcher would also request the access to **the mathematics content themes or topics covered by the prospective teachers' syllabus from first year to the fourth year**.

On the basis of the nature of this project, the researcher requests the permission to:

- **Administer the achievement (proficiency) test to all mathematics fourth year students.**
- **Access the students' practice teaching marks as required by the project.**
- **Access the documents containing mathematics content covered by the participants from the first to the fourth year of their training**

I will appreciate your cooperation in this regard.

Yours Faithful  
K.B. BIYELA (Mr)

Cell: 0833962066  
Email: biyelak1967@gmail.com

My name is Kibret Mengistaw Biyela. I am a Doctoral Degree Student at University of Addis Ababa in the Department of Mathematics, Science and Technology Education. I hereby apply for a permission to conduct a research in your institution. The field work of the study is intended to take place in July 2011. The topic of my study is: **PROFESSIONAL TRAINING IN MATHEMATICS EDUCATION: A STUDY OF PROGRAMMES, PRACTICES AND PROSPECTS.**

The study attempts to address the following research questions:

1. How much mathematics content knowledge do teachers possess at the point of exit of their training?
2. What is the relationship, if any, between the content covered in mathematics education teacher preparation programmes and the high school mathematics?
3. What is the impact of pre-service teachers' content knowledge of mathematics on their teaching practices?

#### Instruments and Participants

The mathematics achievement test will be administered to fourth or final year Bachelor of Education (B.Ed.) students at the university in order to examine the level of mathematics knowledge they possess when they exit their training programme. It is intended to be administered when all entry re-open from winter holidays. As part of the project the teaching practice assessment results for the participants will be compared with the achievement test results. This will be done in order to establish if the extent of content knowledge has an effect on teaching practices. For this reason a permission to have an access also to the students' teaching practice results is requested. Mathematics content covered by teacher preparation programmes will be compared with the high school mathematics syllabi. To the end, the researcher would also request the access to the mathematics content themes or topics covered by the prospective teachers' syllabi from 5<sup>th</sup> year to the fourth year.

On the basis of the nature of this project, the researcher requests the permission to:



## **ANNEXURE B**

**Ethical certificate granting permission to conduct research**



Research Office (Govan Mbeki Centre)  
Private Bag x54001  
DURBAN, 4000  
Tel No: +27 31 260 3587  
Fax No: +27 31 260 4609  
Ximbap@ukzn.ac.za

2 August 2011

Mr. KB Biyela (206000924)  
School of Mathematics, Science and Technology Education

Dear Mr. Biyela

PROTOCOL REFERENCE NUMBER: HSS/0659/011D  
PROJECT TITLE: PROFESSIONAL TRAINING IN MATHEMATICS EDUCATION: A STUDY OF PROGRAMMES,  
PRACTICES AND PROSPECTS

**EXPEDITED APPROVAL**

I wish to inform you that your application has been granted Full Approval through an expedited review process:

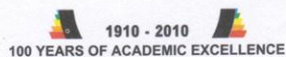
Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

Professor Steven Collings (Chair)  
HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE

cc. Supervisor – Dr. DC Sibaya  
cc. Prof. PT Sibaya  
cc. Ms. T Mnisi



Founding Campuses: ■ Edgewood ■ Howard College ■ Medical School ■ Pietermaritzburg ■ Westville

## **ANNEXURE C**

**A letter to request students to participate in the study  
(informed consent)**

P.O. Box 24018  
Kwa-Dlangezwa  
3886  
21 July 2011

University of KwaZulu Natal  
Edgewood Campus  
Department of mathematics

Dear Student

**REQUEST FOR STUDENT PARTICIPATION IN RESEARCH PROJECT**

I am a Doctorate student at the University of Zululand in the department of Mathematics, Science and Technology. One of the fundamental requirements of this degree is to conduct research and write a thesis. The topic of my research is: **PROFESSIONAL TRAINING IN MATHEMATICS EDUCATION: A STUDY OF PROGRAMMES, PRACTICES AND PROSPECTS.**

To complete my research project I need to get information from fourth year pre-service teachers at university. The research project intends to find out the extent of mathematics content knowledge possessed by pre-service teachers when they exit their training. Another important aspect of research is to determine the impact of teachers' mathematics knowledge on their teaching practices. I therefore request you to participate in this project by:

- **Writing an achievement test which is based on mathematics content**
- **Allowing me to have an access to your practice teaching marks**

It is important to understand that the researcher adheres to the ethics of the research. Therefore the information gathered in this regard will be treated with confidentiality and anonymity. Furthermore note that your participation is voluntary, which means that no student will be forced to write the test.

I will appreciate your co-operation in this regard.

Yours Faithful  
K.B. Biyela (Researcher)

**STUDENT'S CONSENT**

If you agree to participate please indicate by filling in your details below.



STUDENT NO:

-----  
-----  
-----  
-----

SIGNATURE:

-----  
-----  
-----  
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## **ANNEXURE D**

### **Comparison of students' attainments in content and in practice teaching**

RESP NO	CONTENT	PRACTICAL	AVERAGE
1	33	70	52
2	33	77	55
3	57	75	66
4	47	59	53
5	23	72	48
6	30	30	30
7	23	70	47
8	53	78	66
9	47	70	59
10	37	72	55
11	50	65	58
12	40	68	54
13	10	76	43
14	37	70	54
15	33	74	54
16	43	68	56
17	30	78	54
18	23	70	47
19	37	76	57
20	40	58	49
21	40	70	55
22	37	71	54
23	30	73	52
24	27	74	51
25	30	72	51
26	43	72	58
27	53	63	58
28	43	60	52
29	40	69	55
30	30	76	53
31	30	58	44
32	33	70	52
33	57	75	66
34	30	67	49
35	30	76	53
36	37	67	52
37	27	67	47

38	30	73	<b>52</b>
39	37	69	<b>53</b>
40	37	68	<b>53</b>
41	20	75	<b>48</b>
42	3	70	<b>37</b>
43	20	50	<b>35</b>
44	47	69	<b>58</b>
45	40	52	<b>46</b>
46	43	69	<b>56</b>
47	30	76	<b>53</b>
48	30	63	<b>47</b>
49	40	70	<b>55</b>
50	20	71	<b>46</b>
51	37	90	<b>64</b>
52	67	58	<b>63</b>
53	23	63	<b>43</b>
54	20	60	<b>40</b>
55	43	75	<b>59</b>
56	53	74	<b>64</b>
57	37	64	<b>51</b>
58	33	64	<b>49</b>
59	30	66	<b>48</b>
60	33	81	<b>57</b>
61	40	72	<b>56</b>
62	67	60	<b>64</b>
63	43	75	<b>59</b>
64	30	68	<b>49</b>
65	17	70	<b>44</b>
66	43	70	<b>57</b>
67	40	57	<b>49</b>
68	27	60	<b>44</b>
69	57	78	<b>68</b>
70	33	82	<b>58</b>
71	47	71	<b>59</b>
72	73	72	<b>73</b>
73	67	75	<b>71</b>
74	40	50	<b>45</b>
75	20	67	<b>44</b>
76	30	71	<b>51</b>
77	20	65	<b>43</b>
78	83	70	<b>77</b>

79	47	69	<b>58</b>
80	43	79	<b>61</b>
81	63	70	<b>67</b>
82	63	70	<b>67</b>
83	20	64	<b>42</b>
84	40	72	<b>56</b>
85	67	70	<b>69</b>
86	73	65	<b>69</b>
87	50	70	<b>60</b>
88	43	68	<b>56</b>
89	27	66	<b>47</b>
90	37	75	<b>56</b>
91	37	75	<b>56</b>
92	43	72	<b>58</b>
93	27	70	<b>49</b>
94	33	66	<b>50</b>
95	33	75	<b>54</b>
96	43	65	<b>54</b>
97	60	62	<b>61</b>
98	57	67	<b>62</b>
99	50	78	<b>64</b>
100	60	72	<b>66</b>
101	53	65	<b>59</b>
102	33	50	<b>42</b>
103	37	63	<b>50</b>
104	43	71	<b>57</b>
105	30	70	<b>50</b>
106	77	68	<b>73</b>
107	63	75	<b>69</b>
108	43	81	<b>62</b>
109	23	71	<b>47</b>
110	43	64	<b>54</b>
111	50	60	<b>55</b>
112	43	62	<b>53</b>
113	13	78	<b>46</b>
114	27	57	<b>42</b>
115	40	62	<b>51</b>
116	57	75	<b>66</b>
117	60	73	<b>67</b>
118	50	70	<b>60</b>
119	57	58	<b>58</b>



120	60	70	<b>65</b>
121	47	62	<b>55</b>
122	40	76	<b>58</b>
123	20	65	<b>43</b>
124	23	54	<b>39</b>
125	27	83	<b>55</b>
126	40	61	<b>51</b>
127	27	68	<b>48</b>
128	50	70	<b>60</b>
129	40	80	<b>60</b>
130	20	70	<b>45</b>
131	53	70	<b>62</b>
132	43	70	<b>57</b>
133	27	70	<b>49</b>
134	63	75	<b>69</b>
135	47	70	<b>59</b>
136	43	80	<b>62</b>
137	37	80	<b>59</b>
138	53	75	<b>64</b>
139	53	75	<b>64</b>
140	37	70	<b>54</b>
141	40	85	<b>63</b>
142	23	70	<b>47</b>
143	73	65	<b>69</b>
144	63	75	<b>69</b>
145	50	75	<b>63</b>
146	57	80	<b>69</b>
147	50	60	<b>55</b>
148	37	55	<b>46</b>
149	27	50	<b>39</b>
150	27	70	<b>49</b>
151	30	75	<b>53</b>
152	63	85	<b>74</b>
153	53	75	<b>64</b>
154	43	95	<b>69</b>
155	93	70	<b>82</b>
156	47	70	<b>59</b>
157	57	85	<b>71</b>
158	37	70	<b>54</b>
159	47	75	<b>61</b>
160	20	85	<b>53</b>

161	40	70	<b>55</b>
162	40	70	<b>55</b>
163	53	95	<b>74</b>
164	17	70	<b>44</b>
165	43	70	<b>57</b>
	44.23469	70.540816	<b>57.387755</b>

## **APPENDIX A**

### **COMPARISON OF THREE SECTIONS OF MATHEMATICS**

<b>RESP NO</b>	<b>ALGEBRA</b>	<b>TRIGONOMETRY</b>	<b>GEOMETRY</b>	<b>TOTAL 30</b>
1	6	3	1	<b>10</b>
2	6	4	0	<b>10</b>
3	10	5	2	<b>17</b>
4	8	4	2	<b>14</b>
5	5	1	1	<b>9</b>
6	5	2	2	<b>9</b>
7	3	4	0	<b>7</b>
8	8	6	2	<b>16</b>
9	10	4	0	<b>14</b>
10	6	4	1	<b>11</b>
11	9	6	0	<b>15</b>
12	7	4	1	<b>12</b>
13	1	2	0	<b>3</b>
14	8	2	1	<b>11</b>
15	7	2	1	<b>10</b>
16	7	5	1	<b>13</b>
17	6	3	0	<b>9</b>
18	3	3	1	<b>7</b>
19	8	2	1	<b>11</b>
20	6	4	2	<b>12</b>
21	9	3	0	<b>12</b>
22	9	2	0	<b>11</b>
23	7	2	0	<b>9</b>
24	5	3	0	<b>8</b>
25	7	2	0	<b>9</b>
26	8	5	0	<b>13</b>
27	9	6	1	<b>16</b>
28	5	5	3	<b>13</b>
29	7	5	0	<b>12</b>
30	6	2	1	<b>9</b>
31	8	1	0	<b>9</b>
32	7	1	2	<b>10</b>
33	10	5	2	<b>17</b>
34	5	3	1	<b>9</b>
35	6	3	0	<b>9</b>
36	6	5	0	<b>11</b>
37	7	0	1	<b>8</b>
38	4	5	0	<b>9</b>

39	8	2	1	11
40	8	2	1	11
41	5	0	1	6
42	1	0	0	1
43	4	2	0	6
44	8	5	1	14
45	8	2	2	12
46	8	3	2	13
47	6	3	0	9
48	7	2	0	9
49	7	5	0	12
50	2	3	1	6
51	7	4	0	11
52	10	7	3	20
53	5	2	0	7
54	4	1	1	6
55	11	1	1	13
56	10	5	1	16
57	7	4	0	11
58	5	4	1	10
59	5	4	0	9
60	5	3	2	10
61	8	3	1	12
62	12	6	2	20
63	4	7	2	13
64	5	4	0	9
65	3	2	0	5
66	8	4	1	13
67	7	5	0	12
68	6	1	1	8
69	10	6	1	17
70	5	5	0	10
71	8	4	2	14
72	11	8	3	22
73	10	7	3	20
74	6	5	1	12
75	4	2	0	6
76	8	0	1	9
77	2	3	1	6
78	14	8	3	25
79	9	4	1	14

80	7	5	1	13
81	12	6	1	19
82	10	7	2	19
83	6	0	0	6
84	7	4	1	12
85	13	5	2	20
86	12	7	3	22
87	9	4	2	15
89	9	3	1	13
90	5	3	0	8
91	6	4	1	11
92	7	2	2	11
93	7	5	1	13
94	6	2	0	8
95	6	2	2	10
96	6	3	1	10
97	9	4	0	13
98	9	7	2	18
99	9	6	2	17
100	8	6	1	15
101	11	6	1	18
102	9	6	1	16
103	4	4	2	10
104	6	4	1	11
105	4	7	2	13
106	6	3	0	9
107	12	8	3	23
108	11	6	2	19
109	6	4	3	13
110	5	2	0	7
111	8	4	1	13
112	7	6	2	15
113	9	4	0	13
114	2	2	0	4
115	3	4	1	8
116	8	3	1	12
117	10	6	1	17
118	10	7	1	18
119	9	6	0	15
120	11	6	0	17
121	12	6	0	18

122	10	4	0	14
123	8	3	1	12
124	5	1	0	6
125	4	2	1	7
126	4	3	1	8
127	5	6	1	12
128	5	3	0	8
129	10	4	1	15
130	6	6	0	12
131	5	1	0	6
132	10	4	2	16
133	8	3	2	13
134	3	2	3	8
135	9	7	2	19
136	6	7	1	14
137	5	7	1	13
138	5	5	1	11
139	9	4	3	16
140	8	7	1	16
141	8	3	0	11
142	7	3	2	12
143	5	2	0	7
144	11	6	4	22
145	10	5	4	19
146	6	6	3	15
147	10	6	1	17
148	7	6	2	15
149	7	3	1	11
150	4	3	1	8
151	5	3	0	8
152	5	2	1	9
153	10	7	2	19
154	8	6	2	16
155	4	6	3	13
156	14	10	4	28
157	6	6	2	14
158	10	5	2	17
159	7	4	0	11
160	8	5	1	14
161	4	1	1	6
162	6	6	0	12

163	8	3	1	<b>12</b>
164	8	6	2	<b>16</b>
165	3	2	0	<b>5</b>
166	9	2	2	<b>13</b>



## **APPENDIX B**

### **Teaching Practice Evaluation Form for University A**

**TEACHING PRACTICE FORMATIVE ASSESSMENT FORM**

B.ED & PGCE- University Tutors and School Mentor Teachers

Name & Surname: \_\_\_\_\_ Student No: \_\_\_\_\_

School: \_\_\_\_\_ Cluster: \_\_\_\_\_ Grade: \_\_\_\_\_

Learning Area: \_\_\_\_\_ Topic: \_\_\_\_\_

CATEGORIES	RATING	JUSTIFY RATING
	4 = Excellent 3 = Good 2 = Needs Improvement 1 = Inadequate	
Planning of lessons		
Teaching Methods		
Subject Content		
Classroom Management		
Assessment Strategy		

**General Comments**


Signature : \_\_\_\_\_

Date : \_\_\_\_\_

## **APPENDIX C**

### **Teaching Practice Evaluation Form for University B**

**Faculty of Education  
CLASSROOM OBSERVATION AND ASSESSMENT FORM**

Student : \_\_\_\_\_ Student No.: \_\_\_\_\_ School : \_\_\_\_\_ Mark: \_\_\_\_\_  
 Subject : \_\_\_\_\_ Topic/Theme: \_\_\_\_\_ Grade: \_\_\_\_\_

Lecturer : \_\_\_\_\_ Signature : \_\_\_\_\_ Date: \_\_\_\_\_ Time: \_\_\_\_\_

Introduction	Specific Comments
Is the educator able to introduce the lesson topic/theme in a manner that arouses the learners' interests and curiosity? Has the educator made learners aware of what the lesson seeks to achieve? i.e. Learning Outcomes and/or Lesson Outcomes.	
<b>Learning Mediator and specialist</b> Is the educator audible enough? Is (s)he able to explain and describe concepts in his/her subject in a manner that shows that (s)he understands his/her subject well? Is (s)he able to identify learners who are struggling with understanding the lesson and assist them appropriately? How are the methods of teaching the educator is using? i.e. do they cater for the level of development of learners and their ability to understand the language used?	
<b>Leader, administrator and classroom manager</b> Is the educator in control of his/her classroom learners do as they please? Has the educator used group work or individual work? Has (s)he used these or one of these appropriately? i.e. appropriate for the lesson topic/theme and Learning Outcomes or Lesson Outcomes Do the educator's teaching and learning strategies motivate learners, arouse and maintain their interest in the lesson? Would you say this educator's classroom environment was conducive to learning?	
<b>Scholar, researcher and lifelong learner</b> Does the educator show some evidence of having read extensively around this lesson's topic/theme? Is there any evidence of being a scholar in this educator?	
<b>Assessor of learning</b> Are the assessment activities given to learners assessing achievement of Lesson outcomes? Does the manner in which this educator assess reflect good understanding of his/her role as an assessor of learning?	
<b>Interpreter and designer of Learning Programmes</b> Was the lesson delivered in a logical order? Was it learner centered? What is your comment on the teaching and learning materials used? What about the pace of the lesson?	
<b>Conclusion</b> How did the educator conclude the lesson? E.g. any lesson summary, preview of next lesson, homework etc. In your view were the Learning Outcomes and Lesson Outcomes achieved? How was achievement of these done? Any comment about the time it took for the lesson vs lesson duration as per plan?	

**General**

Comments:

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## **APPENDIX D**

# **Teacher Education Mathematics Content Syllabus for University A**

## **MATHEMATICS CONTENT COVERED FROM 1<sup>ST</sup> YEAR TO 4<sup>TH</sup> YEAR**

YEAR 1: Algebraic equations and inequalities; Quadratic theory; Algebraic functions, their graphs and interpretations; Logarithms; Sequences and Series; Trigonometry

YEAR 2: Matrix Algebra; Polynomial Theory; Partial Functions; Trigonometry; Complex Numbers; Statistics and Probability; Coordinate Geometry and Conic Sections

YEAR 2: Piecewise functions; Exponential functions; Inverse functions and Logarithms; Inverse trigonometric functions; Limits and Continuity; the Derivative; Applications of derivatives; Transformations

YEAR 3: Further applications of derivatives; Derivatives of exponential and Logarithmic functions; Integration; Simple integration techniques; Some application of integrals

YEAR 3: Further application of integrals; Transcendental functions and Differential equations; Further integration techniques; Ordinary differential equations; Modelling of differential equations; Numerical method

YEAR 4: The Real number System; Series; Sequences; Continuity; Countability; Cluster Points; Financial mathematics

YEAR 4: Exploration in Mathematics using Technology

## **APPENDIX E**

### **Teacher Education Mathematics Content Syllabus for University B**

## Year 1. Module

### A. Semester 1(Algebra)

**Description:** What are functions? From an introduction of the basic concepts of functions to more advanced functions met in mathematics, science and technology education, these topics provide an excellent foundation for undergraduate study.

#### Topics:

- Introduction to functions
- Linear functions
- Polynomial functions
- Exponential and logarithm functions
- Hyperbolic functions
- Composition of functions
- Inverse functions
- Matrices
- Systems of equation
- Properties of straight line segments
- The gradient of a straight line segment
- Equations of straight lines and circle
- Complex numbers
- **Pre-calculus:**

### B. Semester 2(Trigonometric)

**Description:** Pythagoras' Theorem and pizzas? The theorem is meticulously described and extension material that brings the concept to real life (yes, using pizzas!). Among various other sections covered are sine, cosine, tangent and measurements in radians.

#### Topics:

- Trigonometric functions
- Pythagoras' Theorem
- Trig ratios in a right-angled triangle
- Trig ratios of an angle of any size
- Radian measure
- Trig equations
- Trig identities
- Triangle formulae
- Cosec, sec and cot
- The addition formulae
- The double-angle formulae
- $a \cos x + b \sin x = R \cos(x - \alpha)$
- De Moivre's theorem



- geometry of the circle
- Ration and similarity
- pre-calculus

## Year 2. Modules

### A. Semester 1(linear algebra and geometry)

**Description:** Essential revision or learning for a wealth of disciplines like physics, Technology education. Understanding the qualities of circles, lines and cones together with a thorough explanation of vectors, provide an excellent foundation to further study.

#### Topics:

- Sigma notation
- Arithmetic and geometric progressions
- Limits of sequences
- The sum of an infinite series
- Conic sections
- Polar co-ordinates
- Vectors
- Cartesian components of vectors
- The scalar product
- The vector product
- financial mathematics

### A. Semester 1(Calculus)

**Description:** How do you find a rate of change, in any context, and express it mathematically? You use differentiation. Differentiating logs and exponentials, sines and cosines, and 3 key rules explained, providing excellent reference material for undergraduate study.

- Differentiating powers of x
- Differentiating logs and exponentials
- The quotient rule
- The product rule
- The chain rule
- Differentiation by taking logarithms
- Implicit differentiation
- Maxima and minima

## **A. Semester 2(Statistics and probability )**

**Description:**

**Topics:**

- Univariate data
- Bivariate data
- Single event
- Tree diagrams

## **A. Semester 2(Calculus II)**

**Description:** Integration is often introduced as the reverse process to differentiation, and has wide applications, for example in finding areas under curves and volumes of solids. This section explains what is meant by integration and provides many standard integration techniques.

**Topics:**

- Integration as summation
- Integration as the reverse of differentiation
- Integration using a table of anti-derivatives
- Integration by parts
- Integration by substitution
- Integrating algebraic fractions
- Integrating algebraic fractions
- Integration using trigonometric formulae
- Finding areas by integration

## **Year 4. Modules**

### **A. Semester1(Calculus III)**

**Description:** Essential revision or learning for a wealth of disciplines like physics, Technology education. Understanding the qualities of circles, lines and cones together with a thorough explanation of vectors, provide an excellent foundation to further study.

Topics:

- integration :
- Integration of logarithmic, exponential and hyperbolic
- double and triple integral
- Volumes of solids of revolution
- Differential Equation
- Parametric equation

**A. Semester 1(Calculus III)**

**Description:** Essential revision or learning for a wealth of disciplines like physics, Technology education. Understanding the qualities of circles, lines and cones together with a thorough explanation of vectors, provide an excellent foundation to further study.

**Topics:**

**A. Semester 2(ESMT412: Calculus III)**

**Description:** Essential revision or learning for a wealth of disciplines like physics, Technology education. Understanding the qualities of circles, lines and cones together with a thorough explanation of vectors, provide an excellent foundation to further study.

**Topics:**

**A. Semester 1(ESMT422: Calculus III)**

**Description:** Essential revision or learning for a wealth of disciplines like physics, Technology education. Understanding the qualities of circles, lines and cones together with a thorough explanation of vectors, provide an excellent foundation to further study.

**Topics:**