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ANALYSIS OF MATLAB INSTRUCTION ON RURAL-BASED PRE-SERVICE TEACHERS' SPATIAL-VISUALISATION SKILLS AND PROBLEM SOLVING IN VECTOR CALCULUS.

By

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DECLARATION

I hereby declare that I am the sole author of this dissertation and I have not submitted it to any other university for a qualification. This is a true copy of the dissertation, including the required final revisions, as recommended by my supervisors and examiners. All sources I have used or quoted have been indicated and acknowledged as complete references. I understand that my dissertation may be made electronically available to the public.

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GODFRED AMEVOR   **Date: 10 DECEMBER 2018**

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PROF. A BAYAGA   **Date:……………………………..**

DEDICATION

To my late cousin, Ebenezer Yaw Amevor

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ABSTRACT

Background

Studies from interdisciplinary have noted positive correlation between spatial-visualization skills and mathematical problem solving. However, majority of these studies that interrogated this shared link between spatial-visualization and problem solving were carried in the urban settings only few interrogated rural settings. Also, studies have identified family social economic status (SES) which mainly described one's geographical settlement to be one of the major effects on cognitive development.

Thus, research finding from cognitive discipline revealed that students from poor SES background are less advantagous to cognitive activities (e.g., problem solving) compare to their counterpart.

However, one of research achievements is providing evidence-based that cognitive skills can be enhanced through computer technology and spatial activities hence, the integration of several graphical tools such as: MATLAB, GeoGebra, and many other computer environments in mathematics education. These graphical tools are believed to enhance students' conceptual and procedural knowledge in problem solving in mathematics areas such as: Euclidean geometry, multivariate calculus, and trigonometry which require more spatial skills in their problem solving. However, little has been researched on vector calculus even though vector calculus by its definition is accompanied by spatial reasoning. Students find it easy to evaluate a given vector integral using analytical techniques for integrations but struggle to visualize and transform it from one coordinate system to another.

Objectives

Based on the background, the current research employed the theoretical frameworks of Duval semiotic representation and the visual-analyser (VA) proposed by Zazkis et al., to analyse MATLAB instruction on rural-based pre-service teachers' spatial-visualisation skills and problem solving in vector calculus. The examination was guided by the analysis of the dynamic software MATLAB instruction on Spatial-Visualization, problem solving, and achievement in Vector Calculus. The three objectives were to 1) Analyse how rural-based

pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus. 2) To investigate the degree to which rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement and 3) To assess how a dynamic software environment such as MATLAB influences rural-based pre-service teachers' spatial-visualisation skills.

Method

A sample size of 100 students, guided by a mixed method, was applied in the current study to examine MATLAB instruction on rural-based pre-service teachers' spatial-visualisation and problem-solving skills in vector calculus. A pre-test, post-test, and Purdue spatial-visualization test/rotations (PSVT/R) were used to collect quantitative data. Qualitative data: content analysis was also collected through analysing participants conceptual, factual, and procedural knowledge in problem solving in vector calculus. In addition, the dynamic visual tool MATLAB was employed to examine its effect on enhancing rural-based pre-service teachers' visualization skills.

Findings

Based on the three objectives the main conclusions reached are 1) the dynamic visual tool MATLAB enhanced rural-based pre-service teachers' problem solving in vector calculus, 2) rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement, and 3) dynamic software environment MATLAB influenced rural-based pre-service teachers' spatial-visualisation skills. The recommendations are that; 1) the dynamic software should be considered into every educational curriculum as a pedagogical tool for instruction and 2) the usage of the dynamic software should be extended to rural-based educational institutions.

Keywords: spatial-visualisation skills, problem-solving, achievement, vector calculus, MATLAB dynamic visual tool, and rural-base pre-service teachers.

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Abreviation

2D-Two dimensional

3D-Three dimensional

CG- Control group

CT- cathode ray tube

EEG- electroencephalographic

EG- Experimental group

ET-Eye tracking

fMRI- functional -magnetic resonance imaging

GSP- Geometer's Sketchpad

GUI-Graphic User Interface

MRI- magnetic resonance imaging

MRT-mental rotation test

MSTE- Department of Mathematics, Science, and Technology Education

NCTM-National Council of Teachers of Mathematics

PSVT/R- Perdue Spatial-visualisation Test/Rotation

STEM- Science, Technology, Engineering, And Mathematics

SV- Spatial-Visualisation

TRSR- theory of register of semiotic representation

UNIZULU-University of Zululand

VA- Visual- analyzer

CHAPTER ONE

INTRODUCTION AND BACKGROUND

1.1 Overview of the study

The present study analysed MATLAB instruction on rural-based pre-service teachers' spatial-visualization skills and problem-solving skills in vector calculus, with application at the University of Zululand (UNIZULU). This is based on the fact that studies such as: Stieff and Uttal (2015), Baltaci and Yildiz (2015), Cheng and Mix (2014) and Uttal et al. (2013) which interrogated the relationship between spatial-visualisation skills and mathematics problem solving, were mostly directed towards urban-based universities. Only a few such as AvRuskin (2000), Cumbee (2017) and Okoye (1991) addressed rural-based universities. Furthermore, it is evidenced that family social economic (SES) background has a direct link with spatial-visualisation skills, mathematics problem-solving skills, and achievement (Farmer et al., 2013). To large extend, family SES described person's geographical settlement, hence it is anticipated that students who grew up in the rural setting are less advantageous to this shared relationship between spatial-visualization skills, mathematics problem-solving skills, and achievement.

Research findings on spatial-visualization skills and problem-solving skills revealed that, analysing the effect of MATLAB instruction on pre-service teachers' spatial-visualisation skills and problem-solving skills in vector calculus, attention to the conceptualization of vector calculus is of great importance (Gire & Price, 2012; Newcombe, 2010; Newcombe & Shipley, 2015).

Hence, in order to effectively analyse MATLAB instruction on rural-based pre-service teachers' spatial-visualisation skills and problem-solving skills in vector calculus, various leading studies on spatial-visualization and problem solving have been considered on the basis of the research objectives. Some of these include: Mix and Cheng (2014), Stieff and Uttal (2015), Miller and Halpern (2014), Jasen et al. (2013), Kozhevnikov, Yu and

Blazhenkova (2013), Barniol and Zavala (2012), Fleisch (2011), Fyhn (2010), Sorby (2009) and Wang and Sayre (2010).

The outline of this study is as follows: the background addresses spatial skills' application in different disciplines and theoretical framework which was used to interrogate the research questions. Specific attention was given to: how rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus, to what degree rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement as well as how a dynamic software environment such as MATLAB influences rural-based pre-service teachers' spatial-visualisation skills. Other sections include the methodology which addresses data collection methods and analysis. The last three chapters discuss the data analysis and the summary of the findings of the results, limitations, recommendations, and conclusions.

1.2 Background

Spatial skills have been considered to form the basis of several learning disciplines. For instance, mathematics psychology researchers argued that the ability to rotate, compose, and decompose 2 dimensional and 3 dimensional (2D or 3D) objects in space mainly describes spatial skills (Zhang, Koponen, Räsänen, Aunola, Lerkkanen, & Nurmi, 2014; zazkis, Dubinsky, & Dautermann, 1996; Wai, Lubinski, & Benbow, 2009; Wade, Joshi, Gutman, & Thompson, 2017; Vézard, Legrand, Chavent, Faïta-Aïnseba, & Trujillo, 2015; Sorby, Casey, Veurink, & Dulaney, 2013; Stieff & Uttal, 2015; Šipuš & Cizmešija, 2012; Price, Mazzocco, & Ansari, 2013; Nguyen & Rebello, 2011). There is also consensus that visualisation skills are a measure of an individual's spatial reasoning (Sorby, Casey, Veurink, & Dulaney, 2013). Researchers are also of the view that visual interactions: for example the mental rotation of an object, is a powerful tool that could be used to explore mathematical problem solving. This is because of the ability to draw or the mental representation of the problem aids in finding the problem's solution.

However, research is not firm on which mechanisms underline spatial and cognitive reasoning in terms of problem solving (Jansen et al., 2013; Shepard & Metzler, 1971).

Additionally, spatial-visualisation skill is only one facet of a person's overall intelligence yet, research suggests that spatial reasoning is an important predictor of achievement for example in science, technology, engineering and mathematics disciplines (STEM) (Wai, Lubinski, & Benbow, 2009; Uttal, Meadow, Tipton, Hand, Alden, Warren, & Newcombe, 2013). Furthermore, Stieff and Uttal (2015) argued that STEM to a large extent relies on spatial skills. This phenomenon may offer researchers a reason for the extensive study of spatial reasoning.

For instance, Baltaci and Yildiz (2015) explored the potential use of GeoGebra for teaching analytic geometry concepts. Ibids evidence justified associations of spatial reasoning, mental rotations, and achievement. However, other studies such as: Sipus and Cizmesija (2012) and Clements-Stephens, Rimrodt and Cutting (2009) mapping these associations between spatial reasoning, mental rotation, and mathematics achievement found some inconsistencies in gender and age. Thus, males outperformed females in mental rotation tasks. Though there are other factors, this leads to the question, whether mental task performance is dependent on gender and age (Sipus & Cizmesija, 2012; Clements-Stephens, Rimrodt, & Cutting 2009; Jasen et al., 2013).

1.3 Research on tools, approaches and mathematics cognition processes

Current and ongoing research in the field of cognitive science suggests that one of the factors, the ability to mentally rotate geomrtric objects in space, is singled out as a central metric of spatial reasoning and a predictor of mathematics success (Jansen et al., 2013; Shepard & Metzler, 1971). This ability to mentally rotate gemetric objects in space is considered as a "cognitive tool" and is attainable at an early age. On the other hand, Farmer and colleagues (2013) hypothesized that, family social economic status has a positive effect on one's spatial-visualisation skills and as such, the predictor of his/her mathematics achievement. Additionally, students from rurban family backgrounds will tend to perform better than their counterparts in rural-based environments. Farmer and colleagues (2013) further claimed that an early introduction to computer games and spatial activities develop one's cognitive reasoning. This may stem from the fact that elementary mathematics mainly deal with geometric objects and it is assumed to

stimulate cognitive reasoning. On the other hand, computer-aided programs and geometric tasks are aimed at enhancing cognitive and spatial reasoning, but exactly which mechanism explains this association cannot be accounted for, which is part of the foundation of the current research.

Thus far the research draws from cognitive neuroscience, which is a research field that studies how the brain performs cognitive tasks (Bechtel, 2001; Gazzaniga, 2000). It also employs mathematical prototypes, hypothetical analysis and concepts of the brain to apprehend the principles that govern the development, structure, information-processing, physiology, and cognitive abilities of the nervous system (O'Reilly & Munakata, 2000). Cognitive neuroscience includes experimental tradition and the use of computer and theoretical tools, which therefore overlaps with computer neuroscience. Like any other discipline such as mathematics education which employs technology in learning, and in addition, neuroscientists also process three dimensional images from different medical imaging modalities that capture structural (e.g., magnetic resonance imaging- MRI and cathode ray tube - CT) and functional (e.g., functional -magnetic resonance imaging fMRI) information about the human brain (Wade, Joshi, Gutman, & Thompson, 2017).

On the other hand, some tools and approaches have been specifically tailored to grasp the complexity of brain electrical activity through the analysis of electroencephalographic (EEG) signals (Vézard, Legrand, Chavent, Faita-Ainseba, & Trujillo, 2015). This also employs a technological learning tool which enhances mathematics problem solving and achievement. Zazkis et al. (1996) proposed a model which accounts for the coordination between spatial visualization and analytical reasoning during problem-solving situations, even though these are two distinct forms of reasoning. This may have increased the curiosity of researchers to investigate how the brain processes mathematics tasks.

Contemporary neuroscientists have also found reason to propose that a section of the human brain lights up when it handles arithmetic tasks (Clements-Stephens, Rimrodt, & Cutting, 2009; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Kozhevnikov, 2007; Wai, Lubinski & Benbow, 2009). Additionally, brain imaging research revealed that two sets of brain regions have been associated with number processing (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). These regions, the bilateral intraparietal and prefrontal

systematically activated during number analytical reasonig (Dehaene et al., 1999). Moreover, there is evidence that fMRI revealed bilateral parietal lobe activation increases in mental rotation object depending on the rotational angle (Gogos, Gavrilescu, Davison, Searle, Adams, Rossell, Bell, Davis, & Egan, 2010). The idea is that the difficulty of the mental rotation task correlates with the degree of angle at which it was rotated. For instance, when the isometric block (see figure 2.1) is rotated 30^0 it will be easier to predict the orientation compared to when rotated 120^0. In their studies, Clements-Stephens et al. (2009) matched mathematics performance and found gender differences for the activation of the brain network related to visuospatial processing existed, but noted that mental rotation is a highly distributed task. What can be drawn from Clements-Stephens and colleagues' (2009) claim is that mental rotation shares a link with mathematics aptitude and spatial-visualisation skills

Studies have also proven that spatial-visualization skills are trainable through visual interaction (e.g. mental rotation tasks) and dynamic software and other computer environments (Cheng & Mix, 2014; Uttal et al., 2013; Höffler & Leutner, 2011). The integration of dynamic software (e.g., GeoGebra, Mapel, MATLAB, CAS and other computer environments) in learning and teaching have been highly welcomed by the mathematics community and stakeholders but predominately in urban settings. These computer aided tools have been employed in learning and teaching some sections in mathematics such as geometry, calculus, and probability and their effects have been widely acknowledged.

For example, Hodanbosi (2001) used Geometer's Sketchpad (GSP) a dynamic geometry software and found that students in the GSP group had higher results in the geometry performance test than students in the traditional teaching group. However, this relationship between spatial-visualization skills and problem solving is far less explored in vector calculus (Nguyen & Rebello, 2011; Ferrer, 2016; Fleisch, 2011; Gire & Price, 2012; Robertson, 2013). Hence, this contestation forms the basis of the current research and thus the problem statement.

1.4 Problem Statement

While spatial-visualization skills, mathematics problem solving and achievement has been previously investigated, we do not yet sufficiently understand rural-based pre-service teachers' spatial-visualization skills in problem solving in vector calculus. Also, in the melieu of research evidence about family SES and early exposure to spatial activities (e.g., computer games) enhance SV skills it is presumed that students who grew up in the rural settings are less avantgatage. Hence, it is also anticipated that there may be some inconsistencies in the association between spatial-visualization skills, problem solving and achievement in a rural context. The researcher Nguyen (2011), argued that many traditional mathematics and physics courses focus inadequately on graphical approaches. Mahir (2009) furthered the argument by claiming that students have difficulty in visualizing and sketching space figures and that the transitions between graphical and algebraic representation in 3D is often the most difficult part of finding the solution to vector problems. Similarly, students find it easy to evaluate a given integral using analytical techniques for integration but struggle to visualize and transform it from one coordinate system to another system (e.g., in R^2 transferring a double integral from $\iint dxdy$ to $\iint dydx$ or transforming it to a polar cordinate). The assumptions made by some researchers on the positive role of spatial-visualization skills on mathematics problem solving have led widely to the use of symbolic packages such as MATHEMATICA, MATLAB, GeoGebra and many more computer environments in the mathematics classroom. However, this shared relationship has been largely investigated in urban-based university students and less has been done on rural-based pre-service teachers. This indicates a gap in the study of spatial-visualization skills and vector calculus.

This investigation considers the following research questions: 1) how do rural-based pre-service teachers apply their visualisation skills in problem solving in vector calculus? 2) To what degree do rural-based pre-service teachers' spatial-visualization skills correlate with their vector calculus achievement? and 3) How do dynamic software environments such as MATLAB influence students' spatial-visualisation skills? Answers to these

questions will provide insight into students' understanding that could be used to inform and improve instruction in mathematics cognition and specifically vector calculus.

1.5 Objectives

The objectives of this study are to:

- Analyse how rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus.
- Investigate the degree inwhich rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement.
- Assess how dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills.

1.6 Research Questions

1. How do rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus?

2. To what degree do rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement?

3. How do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills?

1.7 The Purpose of the Study

The current study provides analyses of rural-based pre-service teachers' spatial-visualization skills in problem solving in vector calculus; investigates the degree of correlation between rural-based pre-service teachers' spatial-visualization skills and vector calculus achievement; and assess how dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualization skills. The researcher believes the research will bridge the gap by addressing the paucity of research

on spatial-visualization skills and vector calculus. The findings could be used by mathematics instructors and students in dealing with problem solving in vector calculus.

1.8 DATA COLLECTION METHOD

1.8.1 Participants

A simple random sampling technique was used in choosing a sample size of 100 from a total population of 137 second year rural-based pre-service teachers who registered for vector calculus at UNIZULU for the academic year 2018/2019. This is because each rural-based pre-service teacher has the same characteristics traits and equally likely to be chosen as part of the sample. The sample size was divided into two equal groups; the experimental group and the control group (refer to table 1). This enabled the researcher to account for how the manipulation of one variable of interest, MATLAB instruction, on the variables of interest: problem-solving, achievement in vector calculus, and spatial-visualization skills. This experimental design identified a comparison group (i.e., the control group) which was as similar as possible to the experimental group in terms of baseline characteristics.

1.8.2 Data collection Instruments

The study employed both quantitative and qualitative methods in data collection. A pre-test, post-test, and Purdue spatial visualization t-test/rotations (PSVT/R) was used for the quantitative data collection. Qualitative data was collected by analysing the prior knowledge, conceptual understanding, factual knowledge, and procedure skills of rural-based pre-service teachers by drawing from the sample of the pre-test and the post-test administered (content analysis).

1.8.2.1 Pre-test

A pre-test covering vector calculus topics: vector operations (e.g., addition), vector fields, cross and dot operation of vectors, and vector integration was administered to both groups (refer to appendix 4). This was done to ensure both groups had similar prior knowledge on vector calculus topics under consideration and this serves as a baseline for the study.

1.8.2.2 Achievement test (Post-test)

An achievement test the same as the pre-test (refer to Appendix 4) was examined on both groups. This was done after the experimental group was given treatment by employing the dynamic visual tool, MATLAB, as a pedagogical teaching tool in teaching and learning vector calculus. The teaching session was accompanied by laboratory activities such as; using MATLAB in plotting vector fields, and generating and viewing geometrical shapes from different angles (refer to figures 3.3, 3.4 and 3.5). The session of teaching and learning lasted for three weeks for 2 hours each day. The control group had the same time frame of teaching and learning by the same researcher however, vector diagrams and 3D shapes associated with vector calculus were drawn on the chalkboard and explained. Students were asked questions and given examples to solve (i.e., using a traditional approach).

1.8.2.3. Purdue Spatial-Visualization Test/Visualization of Rotations (PSVT/R)

Section A (see appendix 5) of Purdue spatial-visualization test/rotations test (PSVT/R) consisting of 10 multiple questions on rotation was adapted and used as an assessment tool to analyse the influence of MATLAB instruction on rural-based pre-service teachers' spatial-visualization skills.

1.8.2.4 Content analysis

Qualitative data was collected by a random selection of scripts on pre-test and post-test written from both the experimental and control groups and closely scrutinised empolying Duval's (1996) conceptual framework of semiotic representation and Zazkis et al's. (1996) V-A model. From which rural-based pre-service teachers' conceptual understanding, factual knowledge, and procedural skills required in problem-solving situations were analysed.

1.8.3 Research design

A research design is viewed as functional design that integrates research methods or procedures to achieve reliable results. A triangulation mixed method approach was used in this regard. The study employed both quantitative and qualitative approaches in data

collection. Both methods are different but complement each other when used on the same topic.

An experimental research approach suited this research. The experimental design identifies a comparison group (i.e., the control group) which was as similar as possible to the experimental group in terms of baseline characteristics, such as prior knowledge and the learning environment in which the study was conducted. Furthermore, experimental design allows participants to be randomly assigned to a group and this control the effects of intrinsic and extrinsic variables that can negatively affect the internal validity of the results (Campbell & Stanly, 2015). Hence, a simple random sampling technique was used in choosing a sample size of 100 from a total population of 137 rural-based pre-service teachers doing second year vector calculus at UNIZULU. The sample size was divided into two equal groups; the experimental group and the control group (refer to table 3.1). A pre-test, post-test, and Purdue spatial-visualisation test/visualisation of rotations (PSVT/R) control group experimental design was used as quantitative design.

1.8.3.1 Research approach

A pre-test (refer to Appendix 4) was administered to the groups. This was to ensure similarity in prior knowledge on the vector calculus topics; vector operations, sketching of vector fields, and double and triple integral of 3D shapes. Data was collected and recorded. Thereafter, the researcher taught both groups differently on the vector calculus topics: vector operations, double and triple integration for two hours each day for three weeks. The experimental group was given a treatment by employing the dynamic visual tool, MATLAB, in teaching and learning of vector calculus. While learning vector calculus, they went through some laboratory activities such as: using MATLAB in plotting vector diagrams, viewing planes in space, generating 3D shapes and rotating and viewing them from different angles (refer to figure 3.3, 3.4 and 3.5). For the control group teaching and learning was solely by a traditional approach (the use of chalk and chalkboard). Thus: drawing/sketching vector fields and 3D geometric shapes associated with vector calculus and they were given explanations followed by worked examples. After, a post-test the same as pre-test (refer to appendix 4) was administered from which data was collected. Qualitative data was then collected by means of content analysis. This was achieved

through a random selection of three scripts from the groups before and after the teaching experiment and the solutions were scrutinised for: conceptual understanding, factual knowledge, and procedural skills.

A separate test, Purdue spatial-visualisation test/visualisation of rotations test (refer to Appendix 5) was then used as an assessment tool for diagnosing the effect of the visual tool, MATLAB, on the experimental group's spatial visualisation skills.

1.9 Data presentation and analysis

Paired sample t-tests are usually used in cases where the experimental subjects are divided into two independent groups, with one group having treatment (experimental group) and the other group not having any treatment (control group). Hence, two types of results were collected for each group (i.e., prior to treatment and after the treatment): pre-test and post-test and analysed using the independent sample t-test.

Table 1: Methods of data analysis

Research Questions	Approach	Source of Data	Methods of Analysis
1. How do rural-based pre-service teachers apply spatial-visualisation skills in problem solving in vector calculus?	Qualitative	Pre-test/Post-test.	Content analysis
2. To what degree do rural-based pre-service teachers' spatial-visual skills correlate with their vector calculus achievement?	Quantitative	Pre-test/Post-test	Paired sample t-test
3. How do Dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visual skills?	Quantitative	SPVR-T	Paired sample t-test

1.9.1 Content analysis and descriptive analysis

Content analysis in this study is the type of analysis, which is defined as "a research method for the subjective interpretation of the content of the text data through the systematic classification process of coding and identifying themes and patterns" (Hsieh & Shannon, 2005, p.1278). In addition, it is the systematic reading for making replicable and valid, interference from text or other symbolic matter (Krippendorff, 2018).

In this study content analysis was employed to analyse rural-based pre-service teachers' problem-solving skills in vector calculus before and after the teaching experiment (treatment). Three scripts were randomly selected from each group and guided by Duval's (1999) semiotic representation and Zazkis et al's (1996) conceptual framework. The solutions were closely scrutinised in terms of: rural-based pre-service teachers' conceptual understanding, factual knowledge, and procedural skills required in problem-solving situations.

1.10 Ethical Considerations

Ethical guidelines and policies were observed regarding plagiarism, protection of the rights and the welbeing of participants in the study, in accordance with the University of Zululand's research guide dated January 2016. First, participants were taken through the purpose of the research and made to understand the following: voluntary participation; the right to opt out anytime they wanted without any consequences; and that there is no reward involved in the participation of the study. However, instead they were encouraged to participate by emphasizing how the research designed intrument is relevant to their syllabus. Second, a healthy environment was chosen to conduct all elements of the study that suited comfort, privacy and confidentiality of the participants. The researcher assured the participants that pseudonyms will be used to respect privacy and data collected were used for the purposes of research only.

The participants were given written consent forms (see Appendix 3) to sign before partaking in the research. Finally, the researcher obtained ethical approval by following and submitting the documentation required:

- Research ethics protocol/application form.

- A fully motivated research proposal.

- Participant informed consent form.

- Data collection instruments.

- Letter of request to conduct the research.

1.11 Conclusion

The study set out to analyse the instruction of MATLAB on rural-based pre-service teachers' spatial-visualisation skills and problem solving in vector calculus. In this regard, the study sought to investigate the following research questions:

1. How do rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus?

2. To what degree do rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement?

3. How do dynamic software environments, such as MATLAB, influence rural-based pre-service teachers' spatial-visualisation skills?

The next chapter presents a review of relevant literature interrogating the analysis of MATLAB instruction on rural-based pre-service teachers' SV skills and problem solving and the theoretical frameworks guided in unpacking the research questions.

CHAPTER TWO

LITERATURE REVIEW AND THEORITICAL FRAMEWORK

2.1 Overview

Numerous studies have emerged from different disciplines which explore the shared links between spatial-visualization skills and problem solving (Nguyen & Rebello, 2011; Gire & Price, 2012; Robertson, 2013; Wai et al., 2009; Uttal et al., 2013). In mathematics education, psychologist researchers describe spatial skills as the ability to rotate, compose, and decompose 2D or 3D objects in space and as such this correlate to an individual's spatial and analytical reasoning (Uttal et al., 2016; Jasen et al., 2013). Hence, they: Wai et al. (2009), Jasen et al. (2013) and Uttal et al. (2013) hypothesised that mental rotation is a powerful tool that can be used to promote problem-solving skills and achievement. This idea is based on the notion that the use of representations (such as graphs and sketches) aids in problem-solving situations even though the underlying mechanism explaining this association is not yet fully known.

Spatial skills are only one aspect of the overall intelligence of a person, yet research shows that spatial thinking is an important predictor of achievement in science, technology, engineering and mathematics disciplines (Wai et al., 2009; Uttal et al., 2013). Furthermore, Stieff and Uttal (2015) advance the notion that sciences, technology, engineering, and mathematics (STEM) mainly depend on spatial skills. The ability to assemble, compose or rotate 3D cubes at different angles which are mirror images of each other is associated with both spatial skills and achievement (refer to figure 2.2). Several longitudinal studies have confirmed this association spatial-visualization skills, mental rotation and achievement (Wai et al., 2009; Uttal et al., 2013). However, other studies mapping the relationships between spatial reasoning, mental rotation, and mathematics achievement found some trace of inconsistencies in gender (Sipus & Cizmesija, 2012). Thus: claiming males outperform females in mental rotation tasks (MRT).

In the field of cognitive science, the idea to mentally rotate an object in space and its association with problem solving has been acknowledged however, there is a further claim that this ability correlates with an individual's mathematics aptitude (Jansen et al., 2013; Shepard & Metzler, 1971). They: Jansen et al. (2013) and Shepard and Metzler, (1971) further claimed that the ability to rotate 3D object in space generally enhances cognitive development and it is assumed to be attainable at an early age. Early stage mathematics mainly deals with identifying, matching, and assembling of 2D and 3D shapes have made it convinceable to support the claim by Jansen et al. (2013) and Shepard and Metzler (1971).

Also, this shared relationship between spatial-visualization skills, mental rotation tasks, and mathematics aptitude has led to the development of computer-aided programs. Infact, other varieties of computer games are tailored to achieve this aim (Uttal et al., 2013; Stieff & Uttal, 2015). Interestingly, Zazkis et al. (1996) have advanced the notion by going to the extent of theorising a model which accounts for the transformation process between spatial-visualization skills and analytical reasoning during problem solving. However, the underlying mechanism linking this association between spatial skills and cognitive reasoning is lacking, and since these are two distinct forms of reasoning, it is thus important to research this phenomenon. Surprisingly, contemporary neuroscientists such as biologists have recently gained an interest in investigating the cognitive processes that take place when the brain perceives and processes mathematics information. For instance: the same region of the brain lights up when it handles mental rotation tasks and number lines (Dehaene et al., 199); parietal lode bilateral activation increases as a function of rotational angle in mental rotation of objects (Gogos et al., 2010); and gender differences exist for the activation of the brain network related to visuospatial processing (Rimrodt & Cutting, 2009) are all evidence of the shared link between SV skills and mathematics aptitude.

In addition, evidence on spatial-visualization skills is trainable through visual interaction (e.g. mental rotation tasks) and dynamic software (Cheng & Mix, 2014; Uttal et al., 2013; Höffler & Leutner, 2011) which gives some hope of closing the "spatial gap" in terms of gender. In their longitudinal study, Höffler and Leutner (2011) use technological visual

tools in learning and teaching mathematics and they found that the dynamic software did not only improve students' achievement but also their spatial-visualization skills. Hence, we expect this association between spatial and analytical reasoning to be extended to vector calculus since vector calculus by its definition is accompanied by spatial reasoning.

2.2 Conceptualisation of Vector calculus (Vector calculus cognition)

Vector calculus is a branch of mathematics which deals with differentiation, integration and vector fields, mainly in 3-dimensional Euclidean space R^3 (Robertson, 2013; Galbis & Maestre, 2012). Vector operations; vector addition, vector subtraction, vector multiplication, dot and cross products, gradient, divergence, and curl form the basis of vector calculus. Vector is defined as a mathematical representation of physical quantity that is characterised by size (or magnitude) and direction and it is commonly depicted graphically as a directed arrow (Fleisch, 2011). Force, electric field, momentum and magnetic field are just a few examples that can be represented by vector diagrams. This may offer us the reason why vector calculus is often regarded as a spatial visual topic and hence, demands students' high spatial skills in problem-solving situations. The aforementioned gives the overview (refer to figure 2.1) of the complexity of spatial-visualization skills and its significant role in learning in many, if not all disciplines.

Figure 2.1: A flow chat of the applications of Spatial skills in transdisciplines.
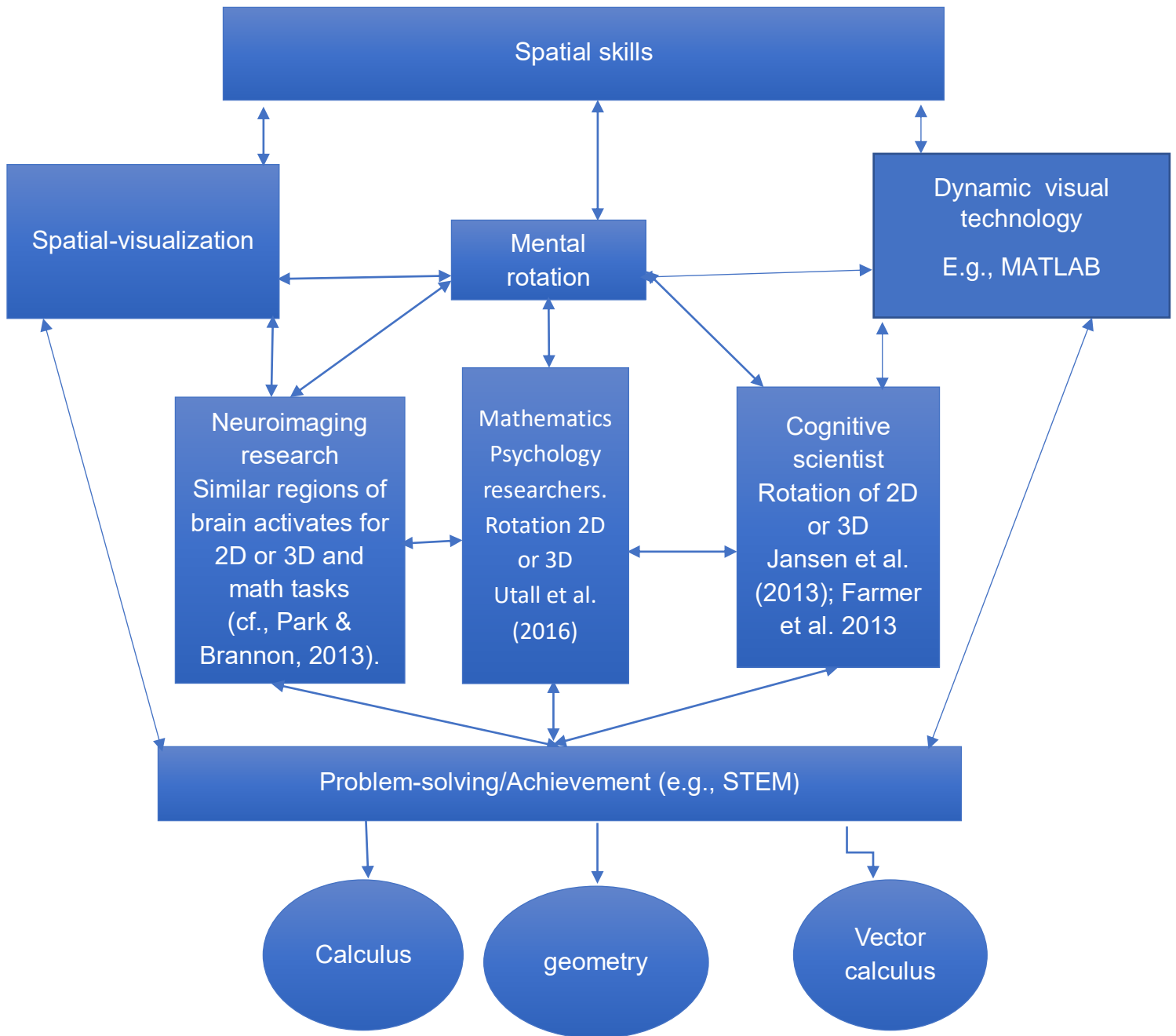
2.3 Spatial Skills

Spatial skills were not thought of as a reflection or measurement of intelligence, problem solving, and achievement until recently when it was well proven in many disciplines such as STEM (Wai et al., 2009; Uttal et al., 2013; and Jansen et al., 2013, Park & Brannon, 2013; Farmer et al., 2013). Perhaps owing to its complexity or the pursuit in assessing its significant role in learning in several disciplines (refer to Figure 2.1) may have led to the distinct classifications from different researchers and scholars such as: Linn and Petersen (1985), Lohman (1993), Maier (1996) and Smith (2009) from interdisciplinary areas of research. Among these include: spatial sense, spatial orientation, spatial perception, spatial-visualisation, mental rotation and spatial reasoning; but there is little consensus on the definitions of these (Linn & Petersen, 1985; Lohman, 1993; Smith, 2009). Accordingly, spatial skills vary in definition according to different researchers and scholars. Linn and Petersen (1985) for instance, define spatial skills as the ability to visualise, transform, and manipulate nonverbal information into 2D and 3D objects. For Lohman (1993), spatial skills are the ability spatial to generate and transform well-structured visual images. Smith (2009) hypothesised spatial skills is the ability to view, conceive, and manipulate objects or ideas within the mind's eye. Having mentioned this disparity in the definitions of spatial skills, researchers have also failed to provide one common classification of its components.

One common classification suggested by Linn and Petersen (1985) examined spatial ability according to three main factors (i.e., spatial perception, mental rotation and spatial visualisation) based on cognitive load demand. Maier (1996), went as far as to classify spatial ability into five distinct components (i.e., spatial perception, spatial visualisation, mental rotation, spatial relation, spatial configuration, and spatial orientation). The most widely used distinction between spatial skills, is the one by McGee (1979), namely spatial-visualisation and spatial orientation/mental rotation. Regardless as to the distinct components and definitions, it can be concluded that all forms of spatial skills require some sort of manipulation or rotation.

However, for the purposes of the main objectives of this research, the researcher adopts the classification that is all-inclusive by the National Research Council. Thus, spatial skills

involve three components: *"concept of space, tools of representation, and processes of reasoning."*

 Concept of space: *It is the extent to which 3-D objects have relative orientation. This is considered to be fundamental in understanding the physical universe. It involves understanding within and between spatial structures.*

Tools of representation: *These are computer aided programs developed through spatial visualisation and aimed at enhancing spatial reasoning and cognitive skills.*

Processes of reasoning: *It involves relating information or often describing the process of drawing conclusions to inform how people solve problems and make decisions. (NRC, 2006, p. 3)*

Studies have found evidence to suggest spatial skills play a significant role in mathematics abilities (Wai et al., 2009; Uttal et al., 2013; and Jansen et al., 2013). However, exactly how these two distinct forms of reasoning connect is unknown. According to Mix and Cheng (2012), "the relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related" (p.206). Apparently, this relationship does not seem to be narrowed to only mathematics. For instance, in their longitudinal study, Mix and Cheng (2012) claim their findings revealed that, spatial reasoning predicts students' achievement in science, technology, engineering, and mathematics (STEM). Largely, among the spatial skills studied researchers have identified spatial-visualisation skills and mental rotation skills to be linked with numeric proficiency. Certainly, spatial-visualization skills are similar to the descriptions of mental rotation skills.

2.3.1 Spatial-visualisation skills and problem solving

Spatial visualisation is defined as "the ability to generate, retain, retrieve and transform well-structured visual images" (Maeda & Yoon, 2013, p. 69). Thus, spatial-visualization skills are mainly limited to the ability to manipulate 2D or 3D objects by means of translation, rotation, composing, or decomposing either physically or mentally. The drawback nevertheless, as asserted by Smith and Strong (2001, p. 2), is that "spatial visualisation is the ability to manipulate an object in an imaginary 3-D space and create

a new representation of the object from a new viewpoint." What could then be drawn from Smith and Strong (2001), is that spatial visualisation could be seen as a cognitive tool for mathematical problem solving. In their views, Denis (1991) and Piaget and Inhelder (1966) argued that spatial-visualisation skills are a powerful tool for mathematics problem solving. In support of this idea: the ability to sketch a graph to depict a problem, or with visual imagery, aids in problem-solving situations.

Just as studies have identified spatial-visualisation skills as a highly cognitive process, so is problem solving. The ability to draw a simple figure to represent a mathematical problem, to interpret the figure with understanding, and to use the figure in problem solving is believed to be fundamental in spatial visualisation.

In mathematics education, problem-solving is considered to be a cognitively challenging task involving a mathematical problem/situation for which a solution is not readily known. Problem solving is commonly accepted as a core component of doing mathematics. According to NCTM (2000, p. 50), "problem-solving is not only a goal of learning mathematics but also a major means of doing so". Literature reviews on problem solving identify the following factors: lack of conceptual knowledge, procedural skills, lack of motivation, lack of self-efficacy, anxiety, and lack of problem-solving skills which hinder students in problem-solving skills (Stylianides & Stylianides, 2014).

For Van Garderen and Montague (2003), students' difficulties in problem solving are as a result of their inability to visualise mathematical concepts and manipulate geometrical shape and space meaningfully. In view of this, recent research revelations on spatial-visualisation skills indicate that dynamic tools motivate and enhance students' self-efficacy, reduce anxiety, and above all conceptual and procedural skills are gained through interaction between the students and in the dynamic environment (Ke, 2008; Kebritchi et al., 2010). Research has also made strong connections against spatial reasoning as a predictor of mathematics achievement (Wai et al., 2009; Mix & Cheng, 2012). This relationship is generally assumed to be more associated with mental rotation skills.

2.3.2 Mental rotation skills and problem solving/achievement

Mental rotation skills are the ability to mentally rotate 2D or 3D object in a space. This ability measures individuals' spatial reasoning through cognitive tasks. For example, mental activities such as: mental rotation task (Shepard & Metzler, 1971), mental paper folding (Shephard & Feng, 1972) and Purdue visualization rotation (Guay, 1976) are arguably considered to have better spatial reasoning. An illustrative example in figure 2.1 gives an overview of spatial measurement. That is, the ability to accurately predict or imagine orientation of the cubes (refer to figure 2.2 below) measures an individual's spatial skills thus; by accurately providing answers to questions such as: are these two shapes different or are they identical and merely oriented? The dependent time coupled with providing an accurate answer to this question is believed to measure an individual's spatial abilities.

Illustrated by Jennifer L. Oeste
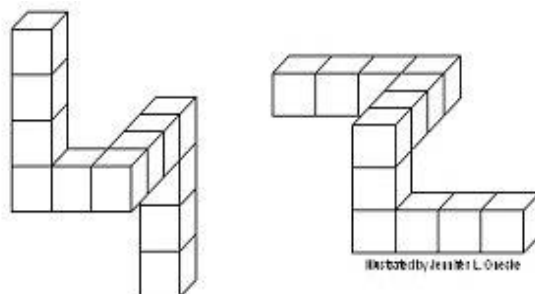
Figure 2.2: Isometric block

Researchers have also found evidence to suggest that an individual's performance in mental rotation tasks correlate with his/her achievement in STEM disciplines (Mix & Cheng, 2014). Thus, giving much promise to the claim that students who attain a high score in these disciplines tend to exhibit high spatial abilities (Ho & Lowrie, 2014; Hoffkamp, 2011; Höffler, 2010; Jansen et al., 2013; Koch, 2006; Mahir, 2009; Wai et al., 2009; Utall et al., 2013).

Despite these studies, there is still evidence from the work of Mix and Cheng (2014) that suggests a relationship between mental rotation skills and STEM achievement. Others such as Wai, Lubinski, and Benbow (2009) are of the notion that, mental rotation ability is a predictor of STEM achievement and even determinant of an individuals' future career.

Höffler and Leutner (2011) advance that, spatial ability could be enhanced through dynamic software and mental rotation tasks (rotation of 2D or 3D). In spite of the studies conducted by Mix and Cheng (2014), Wai, Lubinski and Benbow (2009), and Höffler and Leutner (2011) a contrary view is offered by Sipus and Cizmesija (2012) who argue that mental skills and achievement is inconsistent with gender differences.

A few studies which investigated this disparity between MRT achievement and gender equate this to hormonal difference. For example, as early as in the 90s, Alington, Leaf and Monaghan (1992) hypothesised mental rotation task achievement and gender disparity may be as a result of hormonal differences. Others such as Puts et al. (2007) and Pintzka et al. (2015) extend the notion attempting to claim that, mental rotation skills are linked with the amount of testosterone a foetus encounters in the womb. However, more research is needed to substantiate this claim. Interestingly, whether or not gender difference in mental rotation skills is based on hormones, there is convincing evidence showing that spatial abilities can be enhanced with practice (Newcomb et al., 2010; Cheng & Mix, 2014).

Research also identified the connection between mental rotation skills and early age. Thus, development of mathematics aptitude and spatial reasoning are closely tied, and early spatial intelligence predicts a child's achievement in mathematics (Newcombe et al., 2015; Verdine et al., 2014). Thus, young children with high visualising skills tend to perform better in mathematics achievement. Cognitive scientists such as Farmer et al. (2013) demonstrated that family social economic status has an effect on both a child's spatial reasoning and academic achievement compared with their peers. The drawback from this statement indicates that children from high social economic background who had early introduction to computer games or spatial activities such as; building of blocks, playing chess, matching of designs, spatial puzzles, and many more tend to outperform in academic achievement than their peers who are less advantaged such as those in rural settings. Ibid advanced the notion attempting to claim that spatial skills play important role in predicting overall mathematics success even greater predictive power than general mathematics skills. This is quite convincing from the milieu of mathematics as in learning

in the elementary mainly involves identification, orientation, and assembling of 2D and 3D geometric shapes.

Numerous experimental studies on children's spatial reasoning and mathematics achievement have justified this claim. For instance, in an experimental study, Newcomb et al. (2010), divided 28 children (between the ages of 6-8 years) into two groups. Half the children participated in structured block play for 30-minutes. The other children spent the same time playing the game "scrabble". The researchers scanned the brains of the children (using fMRI technology) before and after the training, while the children solved mental rotation tasks. They found that, unlike children in the "scrabble" control group, children who had participated in the structured block play sessions showed improvements in reaction time and accuracy. Another experimental study found that short training in mental rotation task can increase the performance of mathematics (Cheng & Mix, 2014). After a single, 20-minute session of practice with mental rotation puzzles, children (ages 8-6) in the experimental group obtained higher score in mathematics test compared with control-group peers. Mix and Cheng (2012) went ahead to propose that, an early emphasis on mental rotation skills provides learners with a 'cognitive tool' that can be wielded throughout school mathematics. Drawing from the claims of Mix and Cheng (2012), Farmer et al. (2013) and Newcombe et al. (2015) it is quiet convicing that students who grew up in the rural setting will less perform in cognitive skills (such as problem solving) compare to their counterpart who grew up in the urban setting.

The relationship between spatial-visualization skills and cognitive skills may depend on shared underlying processes. Hence, this may arouse the curiosity of mathematics psychology teachers and neuroscientists to get to the basis of the underlying mechanism associating these two factors.

2.3.3 Cognitive strategy in mental rotation task (MRT)

Evidence based on mental rotation task and cognitive process was first discovered in the classical experimental rotation task by Shepard and Metzler (1971) in which participants made similarity judgment which includes comparing the orientation of pairs of rotated abstract blocks (refer to figure 2.2). The consistently replicated results found that similarity judgment answer time for the same figures increased with increasing the linear function-

often referred to as the angular disparity effect. This observation provides strong ground for mental rotation, a cognitive process thought to involve rotation of analogous mental images until aligned to match. This mental rotation process is thought to reflect the limitations of physical rotation; objects with larger angular differences take longer to physically rotate to match (assuming a constant rotation speed).

One of the experimental factors identified with mental rotation task performance is cognitive strategy. Kozhevinkov (2007) described cognitive strategy as an individual's behaviour or manner of solving a mental rotation task. Certainly, the type of strategy chosen depends on the demands of the task. Research has placed distinct cognitive strategy into three categories namely: mental rotation, mental perception, and spatial visualisation (Gluck & Fitting, 2003). From these distinctions, it can be deduced that cognitive processes differ in these categories. For a mental rotation task, an individual is identified with either *holistic* or *analytic strategies* of solving mental rotation tasks.

Holistic strategy; this involves an individual encoding the spatial information of the accessible image as a whole unit (i.e., mentally rotating the object as a whole).

Analytical strategy (piecemeal strategy): this involves an individual encoding spatial information of the accessible image in pieces. Thus, an individual may mentally rotate one piece in analogy with the comparison figure and then apply the same rotation to the other parts of the figure to see if they match (Khooshabeh et al., 2013).

Research have evidenced that, individuals identify with *holistic strategy* tend to perform better than their counterparts with an *analytic strategy* of solving mental rotation tasks (Khooshabeh & Hegarty, 2010; Khooshabeh et al., 2013). Henceforth, it can be assumed that females and males choose different cognitive strategies in solving mental tasks, but it is still somehow not clear if the same can be said for a female who has a similar spatial ability as their male counterpart which is perhaps an area for future research. Studies on spatial intelligence however, may be drawing completely different picture. Thus, individuals' performance in mental rotation tasks (MRT) directly link with his/her spatial intelligence and this also affects their mathematics learning as a whole.

From this claim, it is worth questioning if an individuals' spatial reasoning is dependent on his/her cognitive strategy. The ability to accurately compare direction and orientation of pair abstract blocks has been cogently proven by research in different disciplines to evoke cognitive reasoning and enhance spatial reasoning. In addition, the continuous trace in inconsistencies on MRT achievement based on gender (Shepard & Metzler, 1971) and activation of different regions of the brain for females and males (Sipus & Cizmesija, 2012) have given some reason to believe that, individuals employ different cognitive processes in solving mental rotation task problems. In addition, neuroimaging reaserch confirms this contention; increasing task difficulty in relationship with increasing activation (Milivojevic, Hamm, & Corballis, 2009), providing further evidence that different cognitive strategies are employed during problem solving situations.

2.3.4 Spatial reasoning and cognitive reasoning - Mental Rotation Task processing -MRT

Neuroscience studies confirm that similar regions of the human brain light up when an individual process both spatial and number tasks (Hubbard, Piazza, Pinel, & Dehaene, 2005, Umilta`, Priftis & Zorzi, 2009; Park & Brannon, 2013). This affirms the claim by cognitive scientists (e.g., Farmer et al., 2013) about the association between spatial and cognitive reasoning. Shum et al. (2013) argued that the brain is made up of 'distributed networks', and when we handle information, different regions of the brain lights up and interconnect with each other.

Functional magnetic resonance imaging (fMRI) research on brain activation during mental rotation indicated that an increase in activation depends on the time taken for completion, error rates, and the difficulty of the task (Gogos et al., 2010). Ibid further indicated that: an increase in brain activation, time, and increased error rates show that task difficulty is relative to the angle of rotation. This corresponds with Shepard and Metzler's (1971) classical experiment on using MRT, comparing and orientation using cubes boxes. Outcomes from the classical experiment revealed that the time it takes to decide if an object was the same (just rotated) or different (a mirror reflection) depends on the angle of rotation (refer to figure 2.2). In other words, it will take a longer period to decide if the 30-degree rotated version compare to the 15-degree rotated version of the

cube boxes depending on an individuals' spatial reasoning. This provides further evidence of the association between MRT and mathematics learning.

The analogy of physical rotation of 2D or 3D shapes and MRT has made it conceivable to use varying degrees of rotation, precision and the duration of finishing as measures of individual differences in mental rotation ability. However, these assumptions are not quite clear on whether or not there is similarity between mental rotation and physical rotation or better still if there are differences in reaction times relevant to gender and age to support earlier claims that males outwit females in MRTs, mathematics problem-solving and achievement in STEM which may be another key section for future research.

Neuroimaging studies using MRTs have shown that individuals with low spatial skills activate different brain regions while performing the same tasks (Logie, Pernet, Buonocore, & Della Sala, 2011) again, providing further evidence to support the claim that different cognitive strategies are used to solve the same spatial tasks. In their study, Clements-Stephens and colleagues (2009) found that, even when matched for achievement, gender differences existed for brain network activation related to visuospatial processing, given reason that MRTs is a highly distributed processing task. This affords to mention that, spatial reasoning forms the basis of science discoveries, and many other learning disciplines. Hence, it can be concluded that spatial skills' applications have contributed to bridging the gaps between interdisciplinary research of which neuroscience and mathematics education disciplines are not an exception.

2.4 Computational Neuroscience and Mathematics Education

Recently, neuroscientist perspectives of employing theoretical analysis and concepts of the brain to understand mathematics processing have drawn increasing interest among education researchers. That is to say, many challenges remain in designing innovative learning environments and examining their impacts on learning for students, developing learning theories, such as understanding how learners perceive and process mathematical information. Also, another dimension of productive research is the use of technology and evidence-based improvement of classroom activities drawn from research based in enhancing learning. Computer-aided representations such as graphs,

games, and simulation are designed to include concrete cognitive representations, or improved spatial visualization skills, as means of enhancing learning (Chang & Linn, 2013). Having mentioned that, some researchers have proposed that it would be useful to integrate different dicisplines in neuroscience, cognitive science, and education to improve learning (Anderson & Contino, 2013; Duncan & Rivet, 2013; Lawson, 2004; Longo, Anderson, & Wicht, 2002). This proposed idea seems to materialize thus: neuroscience methodologies have recently been linked to research in the educational technology field. For instance, little research exists in mathematics education exploring some possible inferences of neuroscience for mathematics education (Campbell, 2011).

This level of integration of neuroscience and mathematics education have taken into the dimension of using mathematical models, theoretical analysis and brain concepts to understand the principles governing nervous system development, structure, information processing, and cognitive reasoning (O'Reilly & Munakata, 2000). This branch is known as computational neuroscience (i.e., theoretical or mathematical neuroscience). It is interdisciplinary science that links the diverse fields of neuroscience, cognitive science, psychology, mathematics, computer science and many others. According to Stern and Travis (2006), the term 'computational neuroscience' consist of two different enterprises. One is the use of computer models and simulations to study brain functions and the other regards the brain as a model system itself and it computes. Neuroscience in the first discipline is not very different from other disciplines in which computer simulation and mathematical models are used to study.

The experimental tradition as well as the use of computational and theoretical tools have seen increase in recent years. Like any other discipline that employs technology in learning, such as mathematics education, neuroscientists also process/analyse three dimensional images from different technological imaging modalities for example eye-tracking (ET), electroencephalography (EEG), and fMRI and so on. This is seen as a new means for understanding individual differences in student brain functions while performing some typical cognitive functions in mathematics learning, such as problem solving, self-directed learning, and interaction with digital-based learning environments. Although, the combination of neuroscience and mathematics education may face some setbacks,

nevertheless this combination tends to validate, refine or refute, and justify learning and teaching theories in mathematics education (Campbell, 2011; Obersteiner et al., 2010).This understanding of neuroscientific methodologies only confirms what has already been proposed and developed by mathematics education researchers however, "this is still valuable information because convergent findings from different research methodologies provide a more solid empirical ground for a given hypothesis, model or theory, than findings obtained by only one research method" (Smedt et al., 2011, p. 234).

In this regard, it is important to view neuroscience as a new methodology which complements and extends available knowledge in mathematics education. Just as in mathematics education, dynamic visual tools are used as a source of spatial visualisation for enhancing cognitive learning also, in the field of neuroscience, methodologies such as eye-tracking (ET), electroencephalography (EEG), and functional magnetic resonance imaging (fMRI) also implicate spatial visualization skills in understanding cognitive activity such as learning. These tools (i.e., ET, EEG and fMRI) are typically adopted to examine how visual information is processed and the cognitive processes characterised in problem-solving situations. This technological tool such as ET has been employed to study human-computer interactions and perhaps this may help uncover the mechanism leading to the claim that students' interactions with dynamic environments enhance their cognitive reasoning.

Several experimental studies from neuroscience discipline have elucidate, validate, and provide answers to existing learning and teaching strategies and theories in mathematics education (Hyönä, Lorch, & Kaakinen, 2002; Susac et al., 2014; Lindström et al., 2013). For example, in their experimental research Susac et al. (2014) investigate students' strategies in simple equation solving using ET technology. Susac and colleagues related the eye movement measures to participants' efficiency and the use of different strategies during solving equations. They also investigated whether there is a different pattern of eye movements between experts and non-experts. Their findings revealed that the eye-tracking methodology provides insights into otherwise unavailable cognitive processes and may be used for exploring problem difficulty, student expertise, and metacognitive processes.

2.4.1 The Role of Technology in Mathematics Education

Mathematics Education with digital Technology explores ways in which widely available digital technologies can be used to benefit the teaching and learning of mathematics. In the context of research evidence that technology enhances learning (Hodanbosi, 2001; Höffler & Leutner 2011; Baltaci & Yildiz, 2015) offered the insights to find the value of digital technology for mathematical learning.

Key pedagogical uses of digital technologies are evaluated in relation to the effective learning of mathematics and the critical analysis of practical ideas for teaching and learning mathematics with digital technology. The majority such as: Kazimovich and Guvercin (2012), Ogunkunle and Charles-Ogan (2013) and Baltaci and Yildiz (2015) concluded by looking at imminent developments and by considering the ways in which digital technology could be used as a catalyst to achieve greater academic achievement. The importance of technology in mathematics education has been underscored by the National Council of Mathematics Teachers (NCTM); it influences both teaching mathematics and improves the learning of students (NCTM, 2000).

The computerized nature of the world has increased the use of computers in teaching many of the subjects in mathematics (Niess, 2006). Research has shown that computer-assisted instrumental materials have a major influence on teaching and learning mathematics (Ogunkunle & Charles-Ogan, 2013).

Among these are:

- Stimulates students' interest in mathematics
- Promotes student and teacher interactions.
- Students build their own prior knowledge and reasoning abilities.
- Introduces mathematics instructional activities and occupies students in activities for a long period of time (Chaamwe, 2010).
- It offers opportunities for students to interact deeply and sustainably with key mathematical ideas (Kazimovich & Guvercin, 2012).
- Student-centred learning become more defined (i.e. students work on their own and are able to develop their own understanding).

- Students' visualize mathematical concepts and explore mathematics in multimedia environments which can foster their conceptual understanding (Andreatos & Zagorianos, 2009).

The noteworthy use of technology in mathematics education has led to the invention of computer aid tools such as MATLAB, GeoGebra, and other computer environments. These computer aided programs empower students to outdo the limitations of their mind in thinking, learning and problem-solving skills and facilitate the visualisation of three-dimensional objects (Andreatos & Zagorianos, 2009; Kazimovich & Guvercin, 2012; Ogunkunle & Charles-Ogan, 2013). These software programs contain well-designed activities which engage students intuitively and allow various geometric interactions between surfaces, space curves, vectors, vector fields, and other calculus related shapes to be visually explored and manipulated (Andreatos & Zagorianos, 2009).Thus, interaction between surfaces (space curve) and rotation to gain 3D perspective is easily visualised. Furthermore, research reveals that computer aided tools enhance students' spatial visualisation which is believed to be one of the defining characteristics of intelligence. According to Wai, Lubinski, and Benbow (2009), spatial visualisation is a measurement of the ability to understand, reason, and predict the spatial relations among objects in 3D. Research findings on spatial visualisation processing of mathematical ideas may explain the many research studies which indicate that teachers who emphasize visual mathematics and who use well-chosen manipulatives encourage higher achievement for students, not only in elementary school but middle school, high school and university (Mix & Cheng, 2012). These skills are key elements in solving mathematical and physical problems. Those who excel in the fields of science, technology, engineering, mathematics-STEM typically show great aptitude in spatial skills (Wai, Lubinski, & Benbow, 2009). It has been shown that spatial skills are helpful in understanding and solving problems in nearly every mathematics field. Children who are more adept at spatial reasoning are better able to master basic math concepts (magnitude and counting, for example) in preschool as well as more complex concepts (word problems, algebra, calculus, and vectors) in middle school and beyond (Hawes, Moss, Caswell, Naqvi, & MacKinnon, 2017). This may offer considerable evidence for the need for enhancing students' spatial skills.

2.5 The MATLAB as a pedagogical tool for learning

Dynamic software and its computer environments are gradually becoming available to enhance and promote mathematical understanding (Ogunkunle & Charles-Ogan, 2013). Also, it has been reported that the integration of advanced computing algebra systems in classroom teaching had positive influence on students' mathematics achievement (Tokpah, 2008). To add to this, engineering mathematics students are requested to have access to appropriate technology due to the perpetually increasing complexities of engineering programs. This dynamic software and its computer environments have found a way into the learning process across all levels of education (de Sousa, Richter, & Nel, 2017) and they are accessible for teaching and learning. MATLAB for example is one of the most recent and dynamic pieces of software which has its own in-built documentation and is designed to reference and manipulate vectors extremely efficiently. The accroym MATLAB is derived from Matrix LABoratory, excels at matrix operation and graphics. MATLAB can be used to produce a wide variety of plots and curves, including 2D plots, 3D plots and 3D surface plots.

Beside these features, the followings also amount to the reasons for the choice of MATLAB over other learning tools:

- It is easily accessible and interactive.
- It has built-in functions to perform many operations (e.g. for signal processing, graphing, generating geometric shapes and many more).
- It has many integrated tools for problem solving and graphic illustration development.

MATLAB is not only exploited in computations but also in the process of teaching and learning. In the MATLAB environment, there are applications that can be created to improve learning (Andreatos & Zagorianos, 2009). The contests to the use of MATLAB in teaching and learning of mathematics in schools are theorized on the fact that it is seen as the most recent cognitive technology that creates new opportunities for mathematics educators. Niess (2006) argued that to be prepared to teach mathematics in this 21st century, the instructor needs integrate teaching and learning (the pedagogy) with

technology for in-depth understanding. Studies have adequately proven that dynamic software, like MATLAB and its computer environments is positively linked to students' spatial visualization, mathematics problem-solving, and achievement. For instance, in their various studies Majid, Huneiti, Balachandran and Balarabe (2013), Kazimovich and Guvercin (2012) and Chaamwe (2010) claimed that the use of the dynamic visual tool, MATLAB as a pedagogical tool is an immediate solution of enhancing students' problem-solving skills.

Furthermore, Majid and colleagues (2013) extended the notion that computer technologies such as MATLAB is a cognitive tool for the development of students' mathematical reasoning, logical reasoning and problem solving. The interactive Graphical User Interface (GUI), animation and graphics visualisation capabilities of the MATLAB are factors that considered to associate with students' cognitive development. Also, a follow up on Kazimovich and Guvercin (2012) and Chaamwe (2010) studies shown that the use of the MATLAB as a pedagogical tool enhance students' motivation to learning and promotes teamwork.

For example, Colgan (2000) verified the effect of MATLAB on teaching engineering mathematics students at the University of South Austrial. The graphing capability of the MATLAB was demonstrated to the students illutstrating graphs functions and multiple commands for plotting functions and many more. Thus, the MATLAB was employed to enhance students' understanding of the mathematical concepts as limits, continuity, the Mean Value Theorem, Rieman Sums, volume of revolution ecetera. The conclusion was the course was considered to be the most successful.

In addition, Puhak, Hall and Street (2011), Chaamwe (2010), Brake (2009) in their variuos studies examined the effectiveness of the MATLAB in learning mathematics. Suprisignly all their findings revealed that the software fosters positive attitude towards learning maths, self confidence, and understanding. Hence, from the afforementioned we anticipate that the ues of the MATLAB as a pedagogical tool for teaching and learning of vector calculus in a traditional classroom will enhance students' spatial-visualisation skills, problem-solving skills and achievement.

2.6 Vector Calculus – related research

Vector calculus is an integration of vectors and calculus and its applications are extensively studied in both the fields of mathematics and physics (Gire & Price, 2012; Wagner, Manogue, & Thompson, 2012; Bollen, van Kampen, & De Cock, 2015; Ferrer, 2016; Fleisch, 2011; Robertson, 2013). Vector calculus employs integration theorems in the calculation of the areas and volumes of 3D shapes (e.g., cones, square, cylinder, cube, spheres etcetera) in different coordinate systems. Hence, what can be deduced from this description is that high spatial skills are required in learning vector calculus.

Accordingly, several studies have noted that students encounter various challenges ranging from inability to sketch vector diagrams correctly, graphical addition of vector components, interpretation of vector diagrams, and switching between different coordinate systems (Rebello, Engelhardt, & Singh, 2012; Dray & Manogue, 2003; Hinrichs, Singh, Sabella, & Rebello, 2010). For instance, Törnkvist, Pettersson and Tranströmer (1993) examined second year engineering university students and they found that students have difficulties in assigning the correct characteristics to field lines. Ibid contended that majority of the students did not notice that field lines never cross. Others incorrectly stated that field lines cannot form loops or claimed that field lines should always be closed. Also, in their interviews with the engineering students, Törnkvist et al (1993) found out that one of the major difficulties of the students is switching between vector fields and field lines which require high spatial reasoning.

In addition, Nguyen and Meltzer (2003) investigated students' understanding of vector addition, magnitude and direction in the first year of undergraduate physics for problems presented in graphic form. In all introductory general physics courses, they administered a seven-piece quiz, including free reaction problems. The results showed that most students could not add two-dimensional vectors after completing a physics course.

In conclusion, Gire and Price (2014, pg. 27) in their study of "an analysis of the material features of electric field vector arrows" found out that, many errors are made by students when sketching graphical representations of vector fields, and they suggested that instructors should be aware that some representational features may have two potential

meanings (e.g., length, meaning both distances between points and strength of a field) or do not match what is being represented (e.g., closer spacing in field line diagrams corresponds to a greater magnitude). This may result in much emphasis on the traditional approach of instruction focusing insufficiently on graphical representations. The conclusion drawn from the works of Törnkvist et al. (1993), Nguyen and Meltzer (2003) and Gire and Price (2014) is that students need to grasp vector concepts and apply their spatial reasoning in vector calculus problem-solving situation.

Integral functions (i.e., integration techniques and theorems) are employed in vector calculus for the calculation of area bounded by curves, the volume of solid revolution, the centroid, the moment of inertia, fluid pressure, work, et cetera (Ferrer, 2016). These calculations are grounded on three theorems namely: Green's theorem which relates to line integral, Stokes' theorem, relates to surface integral and Guass' theorem, relates to flux through a given surface volume integrals involving field's divergence.

However, integration of area bounded by curves and volume of solid revolution of geometrical figures requires high spatial reasoning ability for instance: the ability to draw a sketch or visualized 3D mathematical object in space for a given algebraic expression, interpret it with understanding, and use the 3D object as an aid to write out the integral function requires students' spatial visualization and analytical skills (Zazkis, Dubinsky, & Dautermann, 1996). Furthermore, it is widely accepted, students find it easy to evaluate a given vector integral using analytical techniques for integrations but struggle to visualize and transform it from one coordinate system to another which highly require in problem-solving situation. Research has found evidence to propose that imagery, spatial skills, and multiple representations aid in problem-solving situations (Cunningham & Zimmerman, 1991; Dray & Manogue, 2003; Hinrichs, 2010; Rebello, Engelhardt, & Singh, 2012; Casey, Pezaris, Fineman, Pollock, Demers, & Dearing, 2015).

Despite these studies, there is still evidence from the work of McLeay (2006) that suggests that a relationship exists between imagery, spatial skills and problem solving. Others such as Newcombe (2010) advance the notion that illustrations or pictures can enhance spatial thinking. Newcombe and Shipley (2015) have even attempted to advance the notion by evaluating *visual and spatial reasoning for design creativity* with mixed

results. In spite of the studies conducted by Newcombe (2010), McLeay (2006) and Newcombe and Shipley (2015) a contrary view obtained from Nguyen and Rebello (2011) argues that learners encounter challenges when confronted with multiple representations with particular reference to image movements of 3-D figures, alternatively manipulation in the mind.

2.6.1 Role of Spatial-Visualising Skills in Problem Solving in Vector Calculus

Mathematical formulas and theorems in vector calculus have a direct link with visual representations. Integral, differential, vector operators (e.g., vector operators, vector theorem, and curl) are among the constituents of vector calculus which have a direct link with visual representation. For example: divergence and curl, are typically expressed using the nabla ∇ symbol to denote the del operator. Thus, the divergence of a vector field ($\nabla \cdot A$) is a scalar quantity that measures the magnitude of a source or sink of the field at a given point. Also, curl of a vector field ($\nabla \times A$) describes the infinitesimal rotation at any point in the field. This deepens the argument why problem solving in vector calculus requires students' high spatial-visualization skills hence requires extensive use of illustrations in problem solving. However, research conducted in physics education indicated that students' difficulties in problem solving in vector calculus are as a result of: their inability to apply mathematical knowledge in a physical context thus, they focus on equations and calculations rather than on the physical meaning behind the symbols (Redish, 2006).

Multiple representation is another major challenge identified. Students have difficulties in combining information: words, equations, graphs, and vector diagrams, into correct problem illustration (Kohl & Finkelstein, 2005; Wagner et al., 2012). However, this claim is not quite clear, raising the question of how does multiple representation tends to affect students' problem solving in vector calculus since vector calculus tasks demand high spatial-visualisation skills. Multiple representation as a challenge to students in problem solving gives us further reason to support Kozhevinkov's (2007) claim that, the cognitive factors responsible for the selection of correct representation(s) for a particular task hence, students rather struggle in the selection of correct representation. In their study, Doughty, McLoughlin and van Kampen (2014) investigated Durban City University

students' idea about integrals employing an approach based on the idea of the concept image. They found that all the mental processes were activated when students encounter certain concepts (e.g., an integral of a vector operator).

Although most of, that is, if not all, concepts and problems of vector calculus involve multiple representations which inform that different representations are used in teaching, however, few tasks are designed to develop students' ability to solve problems graphically. In a study on "visual thinking in calculus", Zimmermann (1991) discussed the role of visualisation in calculus: "the role of visual thinking is so fundamental to the understanding of calculus that it is difficult to imagine a successful calculus course which does not emphasise the visual elements of the subject" (p. 136). In addition, Ibid mentioned that it is almost impossible to understand a majority of problems (especially, finding the areas of regions that lie under the graphs of functions and the volume of a solid or to calculate triple integrals) in calculus without adequate visual representation. The ability to sketch 3D solid figures such as a cone, cylinder, cube, pyramid and transacted shapes which are involved in vector integration problems is cognitively demanding and as such requires students' spatial reasoning skills. In a traditional vector calculus course, the teaching of vector calculus focused on more algebraic representations, and the importance of visualisation was neglected. Also, in a traditional vector calculus course, static drawing of a solid region on two-dimensional paper may be incomplete, thus causing erroneous perceptions. Several studies such as: Kösa (2016), Bako (2003), Ertekin (2014) and Delice and Ergene (2015) have noted that, static drawings of a solid region on two-dimensional paper contributes to students' difficulties in interpreting static diagrams representing three-dimensional geometric objects.

However, the incorporation of technology in mathematics education has expanded the dimension where these 3D figures can easily be displayed and rotated in different viewpoints. Since numerous freely assessable dynamic tools (e.g., GeoGebra, Derive, Mathematica, MATLAB, any more) and other computer environment have paved their ways into mathematics classrooms, they have brought about a significant change in the way of learning and teaching vector calculus concepts. The dynamic technology provides a visual and analytic environment where graphical, numerical, and analytical functions

can be combined. It has an interactive environment which allows students to perform visual tasks such as; rotating curves in space, translation from one coordinate system to another and observing 3D solid objects from different viewpoints. They are seen as new cognitive technology.

A fair amount of prior studies has confirmed that instruction using computer-based visualisations can help students in developing spatial visualisation skills (Baki et al., 2011; Karakuş & Peker, 2015; Höffler & Leutner, 2011; Tokpah, 2008; Utall et al., 2013). Exactly the mechanism underlying this connection is not yet known. For instance, visual interaction between the students and computer capabilities in the processes of: drawing curves in triple integrals; manipulating projection on the plane; and rotating or imagining views from different perspectives may improve their spatial skills. Studies such as Ke (2008), Kebritchi et al. (2010), Subrahmanyam, Greenfield, Kraut and Gross (2001) and Karakuş and Peker (2015) have employed varieties of these technological tools such as MATLAB, GeoGebra and many more in teaching and learning mathematics areas (such as; geometry and calculus) and they play a significant role such as enhancing students' problem-solving abilities, spatial visualisation skills, and achievement. For instance, Subrahmanyam et al. (2001) conducted a survey on the impact of computed aided technology in learning and teaching circle geometry. Their findings revealed that computer aided technology supports the development of visual mental rotation, spatial visualisation, the ability to deal with two- and three-dimensional space, and the ability to read pictures and diagrams, which are all essential for conceptual understanding of Circle Geometry.

If this is the case, we expect this shared relationship between spatial visualisation and problem solving in geometry and calculus could be extended to problem solving in vector calculus. Vector calculus tasks can be classified as visual problems, due to their context, or presentation, or because the visual solution is more powerful (Presmeg, 2014).

2.6.1.1 Visualizing vector field conceptualisation

Vector field is the project of vector in each point in a subset of space. It can be visualised as the collections of arrows with given magnitude and directions. Vector operations such as gradient of a vector, curl, and divergence by their definitions require some kind of spatial-visualisation skills and analytical reasoning in performing related tasks. For instance, the vector operator curl of the vector $\vec{F}$ represented as $\nabla \times \vec{F}$ measures how much a vector field circulates around a given a point as given in the illustrative example 1.

Illustrative example 1

With the aid of sketch evaluate the curl of the vector function $\vec{F} = -y_i + x_j$. We first of all calculate the determinant to represent the curl.

$$Cur\vec{F} = \nabla \times \vec{F} = \begin{bmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ -y & x & 0 \end{bmatrix} = 2k$$

Hence, the $Cur\vec{F} = \nabla \times \vec{F} = 2k$ can be represented as shown in figure 2.3 and 2.4.
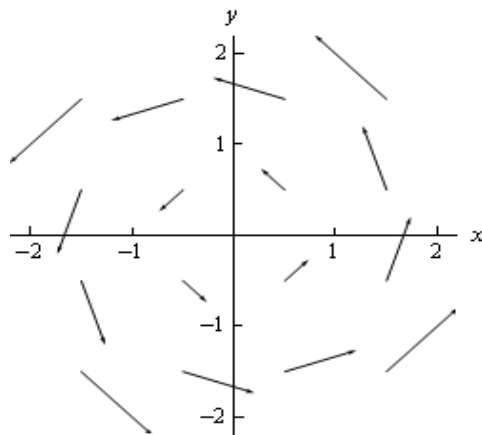
Figure 2.3 shows the curl of the vector function $\vec{F} = -y_i + x_j$

Computer aided graphing such as the GeoGebra and MATLAB give significantly many points plotted. Field plot in Figure 2.3 can be sketched with many more vectors generated with MATLAB as shown in Figure 2.4 below.
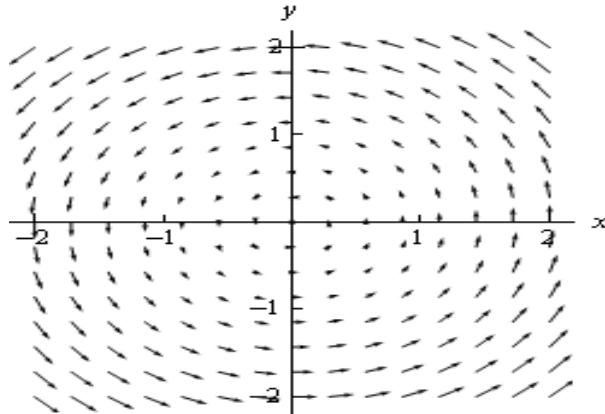
Figure 2.4: Vector function $\vec{F} = -y_i + x_j$ generated using the MATLAB

A close scrutiny of the solutions of the illustrative example 1 clearly shows a constant transition between analysis (calculating of the curl) and visualisation (sketching the vector field) of the vector function. Hence, the obvious reason why modern-day mathematics and physics textbooks have a lot of examples accompanied with their representations. A follow up on Gire and Price (2014) revealed that students' failure to simply draw a sketch to represent vector field may be as a result of insufficient focus on graphs. Subsequently, many studies postulated that graphical representations foster the conceptualisation of abstract mathematical structures in vector calculus (Gire & Price, 2012; Manogue & Dray, 1999; Newcombe, 2010; Newcombe & Shipley, 2015). It is often too cognitively demanding to visualise, sketch or manipulate 3D shapes from different angles. However, the development of computer aided programs provides a learning environment where; students access 3D shapes in different angles and also the visual interactions environment is believed to enhance their cognitive skills such as problem solving.

2.6.1.2 Multiple Integral Calculus Conceptualisation

Multiple integral is a definite integral of a function of more than one variable. The function of two variables $f(x, y)$ over a region R^2 called the double integral, and the integral function of the three variables $f(x, y, z)$ over a region R^3 also known as triple integral are both integral techniques employed in the calculation of areas and volume of regions in the plane and in space. Hence, from this description, the integral function $f(x, y)$ over the region R^2 can be represented as in figure 2.5.

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2.6.1.2.1 Visualizing double integral
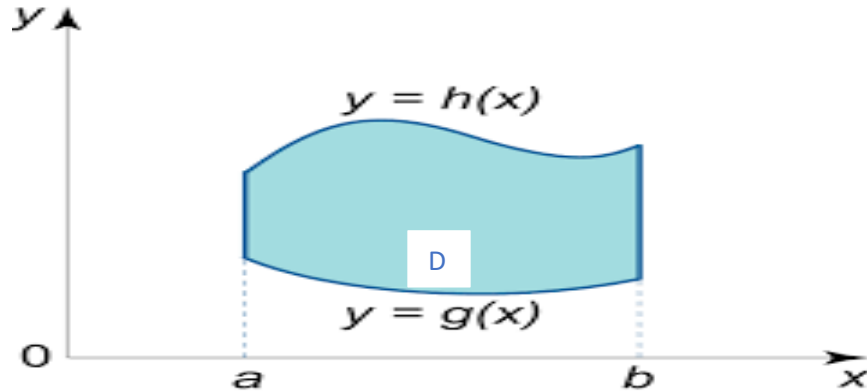
Figure 2.5: A graph showing area under double integral

We let $z = f(x, y)$ be a function of two variables. If D is the region defined on the continuous intervals $a \le x \le b$ and $g(x) \le y \le f(x)$, then we can write the integral formula as;

$$\iint_D f(x,y)dydx = \int_a^b \left[\int_{g(x)}^{h(x)} f(x,y)dy \right] dx$$

The integral formula is used to evaluate the surface area of D (refer to figure 2.5) and the region bounded by two functions $f(x, y)$ as demonstrated in the illustrative example 2.

Illustrative Example 2

Evaluate $\iint_D f(x + 2y)dA$ where D is the region bounded by the parabola $y = 2x^2$ and $y = 1 + x^2$. We first sketch the region D as shown in figure 2.6.
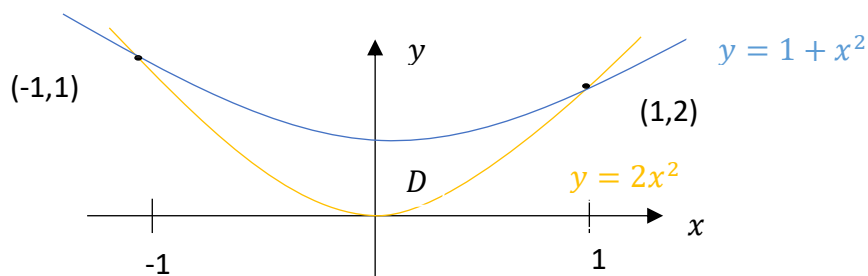
Figure 2.6: A graph of $y = 2x^2$ and $y = 1 + x^2$

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From the graph (refer to figure 2.6), we clearly see that the boundaries of x is given as $-1 \le x \le 1$ and also $y = 2x^2$ and $y = 1 + x^2$ are lower and upper boundaries respectively. The integral equation written as $\iint_D f(x + 2y)dA = \int_{-1}^{1} \int_{2x^2}^{1+x^2} (x + 2y)dydx$

$$= \int_{-1}^{1} [(x + 2y)]_{2x^2}^{1+x^2} dx = \int_{-1}^{1} (-3x^4 - x^3 + 2x^2 + x + 1)dx$$

$$= -\frac{x^5}{5} - \frac{x^4}{4} + 2\frac{x^3}{3} + \frac{x^2}{2} + x]_{-1}^{1} = \frac{32}{15}$$

A close scrutiny of the solution of the illustrative example 2 indicates that there was a constant transition between two registers: geometric register (i.e., graph) and algebraic register (i.e., performing the integration). This means there was coordination between visual and analysis in the problem-solving situation. Reflection on both Duval's (2003) theory of semiotic representation and Zazkis et al's, (1996) V-A model, visual and analysis are tools in every problem-solving situation. For Duval (2003), every mathematical concept is grounded in representations and also, Zazkis et al. (1996) V-A model theorized along the same line but further explains that every problem-solving situation starts with visualization. Hence, the idea is that an accurate sketch, graph or diagram to depict a problem facilitates finding the problem's solution as demonstrated in the illustrative example 2.

2.6.1.2.2 Visualizing triple integral

Triple integral is used to integrate 3-dimensional region. According to Fubini's theorem for triple integral, if $K = f(x, y, z)$ are three variable real-valued function such that f is continuous on the rectangular box $B = [a, b] \times [c, d]$ and $[r, s]$. Figure 2.7 gives the visual representation of the triple integral.
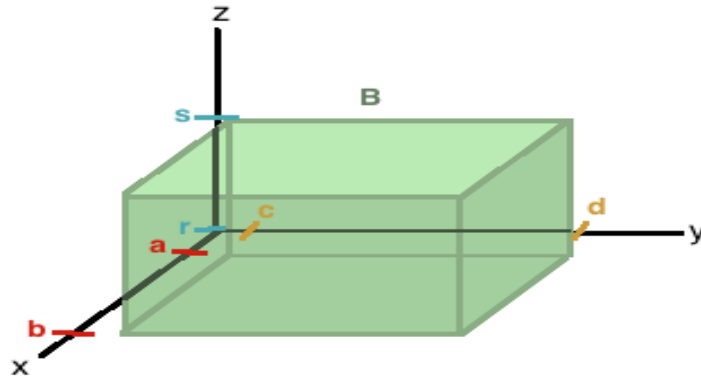
Figure 2.7: Triple integral over box adapted from mathonline.wikidot.com

Hence, from the definition the analytical formula of the triple integral can be written as; $\iiint_R f(x, y, z)dV = \int_r^s \int_c^d \int_a^b f(x, y, z)$. The triple integral equation is employed in the calculation of volume of solid figures (e.g., cone, rectangular, cylindrical surface) as given in the illustrative example 3.

Illustrative example 3

Evaluate $\iiint_E 3 - 4x dV$ where E is the region by $z = 4 - xy$ and above in the $xy-$plane defined by $0 \leq x \leq 2, 0 \leq y \leq 1$.

Solution.

We apply Zazkis et al (1996) V-A model, hence we begin with the sketch of the region E (refer to figure 2.8).
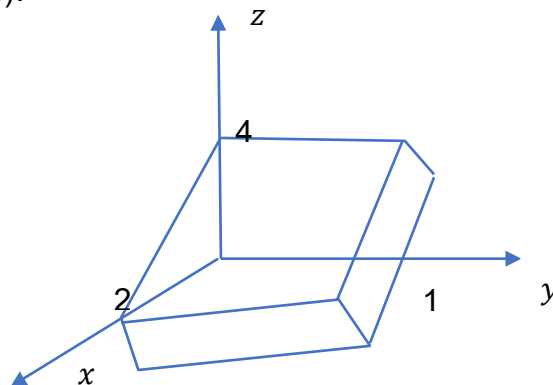
Figure 2.8: A graph showing the region where E is the region by $z = 4 - xy$ and above in the

$xy-$plane defined by $0 \leq x \leq 2, 0 \leq y \leq 1$.

From the graph (refer to figure 2.8), we can see that the limit for z is $0 \leq z \leq 4 - xy$ and also, we can clearly see that the intervals for xy $-$plane are $0 \leq x \leq 2$ and $0 \leq y \leq 1$. Hence, we write the integral equation for the region $E = \iiint_C 3 - 4x dV = \int_0^2 \int_0^1 \int_0^{4-3x} 3 - 4x dz dy dx$

$$= \int_0^2 \int_0^1 3 - 4x |_0^{4-3x} = \int_0^2 \int_0^1 (3 - 4x)(4 - 3xy) dx dy$$

$$= \int_0^2 \int_0^1 4x^2 y - 3xy - 16x + 12 dy dx = \int_0^2 2x^2 y^2 - \frac{3}{2} xy^2 - 16xy + 12y \Big|_0^1$$

$$= \int_0^2 12 - \frac{35}{2} x + 2x^2 dx$$

$$= \left(12x - \frac{35}{4} x^2 + 2x^3 \right) \Big|_0^2 = -\frac{17}{3} \text{ units}$$

In Duval's (1996) theory, representation generates mathematics concept and as such, this exist between registers. From a close scrutiny of the solution in the illustrative example 3, we can make the following conjectures thus: the graph promotes conceptual understanding (writing the integral equation), factual knowledge (writing the limit of $x, y,$ and z) , and procedural skills (substituting in the triple integral equation and calculating the volume of the rectangle).

2.7 Theoretical Framework

This section discusses the theoretical frameworks underpinning the role of spatial-visualisation skills in problem solving, with emphasis on vector calculus. The study was based on the works of Duval's theory of register of semiotic representation (TRSR) and Zazkis et al's, visual-analytical model (VA). The section continues to describe how these theories were used as important components of the analysis of MATLAB instruction on rural-based pre-service teachers' spatial-visualisation and problem-solving skills in vector calculus.

2.7.1 Duval's Theory of Register of Semiotic Representation (TRSR)

The theory of register of semiotic representation: in the context of cognitive science, the notion of representation plays a significant role regarding the procurement and treatment of an individual's cognitive reasoning. As Duval (1995) theorized: "there's no knowledge that can be mobilised by an individual without a representation activity" (p. 15). The idea drawn from this is that every analytical reasoning starts with representation and this exists between at least two registers. In principle of the framework, every cognitive skill (e.g., problem-solving) starts with geometric register to algebraic register (e.g., graphic register to analytical register).

Accordingly, Ernest (2006) in a similar view claimed semiotic systems are characterised by elementary signs and underlying meaning of structure developing from the relationship between the signs within the system. Hence, the comprehension of the theory requires the consideration of these three characteristics:

- Different multiple semiotic representations of the same mathematical object.
- Each different semiotic representation of the same mathematical object does not clearly indicate the same properties of the represented object.
- Semiotic representation content should never be confused with the mathematical objects they represent.

One of the distinctions of the semiotic representations is dependence on an organised system of signs such as linguistic, algebraic, symbolic writing and cartesian graphs. This means that all semiotic representations must be taken into account, mainly on the basis of the register in which they were formed; then, on the basis of what they clearly represent and what they cannot represent. Founded on this assessment, knowledge is analysed based on the properties of the object.

Another important distinction of the semiotic representations is the cognitive operation of conversion of the representations from one system into another, in other words, there is transformation of semiotic representations into other semiotic representations. Duval (2006) argued that the idea that "the notion of semiotic representation presupposes the consideration of different semiotic systems and a cognitive operation of conversion of the

representations form one system into another" (p.17). This supports the view of the coordination between representation and cognitive reasoning such as problem solving.

Duval (2006) further theorises that, the part played by representation is not only to show and communicate about a mathematical object, but also to work on mathematical objects. Duval (2003) points out that two different types of transformation of semiotic representations may occur in any mathematical activities where one representation may be converted to another (See figure 2.9).

The first one is known as *treatments,* which involves movement of a representation within the same registers where both registers are associated with the exact same mathematical concept. For example, the transformation of Sphere in Cartesian Coordinates $x^2 - y^2 +=$ z as $\rho sin^2 \emptyset (cos^2 \theta - sin^2 \theta)$ in Spherical Coordinates. The second type of treatment, called *conversion*, involves the transformation that is a representation in another register. For example, in the algebraic register, the equation $z = x^2 - y^2$ as $z = r^2 (os^2 \theta - sin^2 \theta)$ in Cylindrical Coordinate and as $\rho sin^2 \emptyset (cos^2 \theta - sin^2 \theta)$ in Spherical Coordinate. Students often struggle to perform these conversions between mathematical registers, more especially, if the conversion does not include algorithms for mapping objects from the source to the target register. Fortunately, technological graphical tools such as GeoGebra, Applets, and MATLAB have offered solutions to this.

To add to this, technological graph tools aid in translation of the 3D solid figures associated in vector integration problems from one coordinate system to another coordinate system and provides students with quick feedback in problem solving in vector calculus**.** This in line with the research questions number 1 and 2; how do rural-based pre-service teachers apply their spatial-visalisation skills in solving vector calculus problems and to what degree do rural-based pre-service teachers' SV skills correlate with their vector calculus achievement. In this regard, MATLAB was employed to serve as a source of spatial reasoning for the rural-based pre-service teachers in other to measure their SV skills against the variables; vector calculus problem-solving skills and vector calculus achievement.

Duval's (2003) theory is established on the basic principle that every mathematical concept is built on representation and this can be in the form of imagery, sketch or diagram. The theory also laid emphasis on the transitions that exist between registers (refer to figure 2.9) in problem-solving situations. Hence, guided by Duval's theory, the solution presented by pre-service teachers was analysed to find the kind of registers which were employed in each problem-solving situation. Furthermore, the theory provides the conceptual framework to analyse what kind of transitions or types of registers rural-based pre-service teachers employed in each problem-solving situation. This guides in analysing rural-based pre-service teachers' conceptual understanding, factual knowledge, and procedure skills which constitute problem-solving skills as well as achievement in vector calculus.

Math Object

(representation)

Treatment A

Treatment B

Cognitive

obstacle

Treatment

Conversion
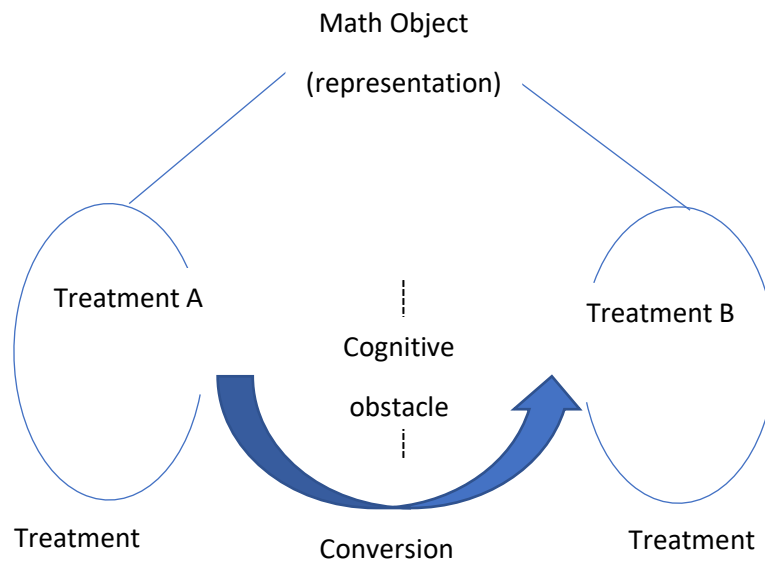
Treatment

Figure 2.9: Transformation processes: treatments are transformation within the same register; conversions occur across registers without changing the mathematical object. Adapted from Duval (2006).

2.7.2 Zazkis et al's. (1996) Visual-Analytical model (VA)

The Visual-Analysis (VA) model developed by Zazkis et al. (1996), highlights the visual and analytical steps used in the problem's solution. The VA model ccounts for the link between the acts of visualisation and analysis or analytical thinking such as problem solving. The model views visual and analytical thinking in complementing each other in vector calculus problem solving. In regards, Zazkis et al's VA model indicates that every mathematics problem solving begins with visualisation follow by analysis. Hence, Zazkis et al's (1996) VA theory equally shed light on Duval's. (2003) theory of semiotic representation. The VA model was employed to scrutinise rural-based pre-service teachers' ability to coordinate visual (e.g., graphs, sketch) with analysis (e.g., algebraic equations) in vector calculus. For, example the activities of employing the MATLAB in performing the translation of 3D shapes associated in vector integration from one coordinate system to another or a sketch, graph or vector diagram to depict a problem have an effect on spatial visualisation skills. In their study, Nilsson and Juter (2011) accounted for processes, movement and links between visualization acts and analytical acts in 3D generalization patterns. It was extended and used by Zazkis (2013) in her doctoral thesis on the V-A-P model, where P refers to the physical situation of the problem.

According to the model illustrated in Figure 2.10, the process would begin with visualization, V_1 which could be any visual representation (drawing, image on the computer screen or mental image). Through the model, the image could be analysed during stage A_1 which consists of some sort of coordination of the objects and processes constructed in step V_1. This analysis may lead to new constructions. In a subsequent act of visualisation, V_2, the student returns to the same ''picture'' object used in V_1, but as a result of the analysis in A_1, the picture may change. As the sequence is repeated, each act of analysis is dependent on the previous act of visualisation. It is also believed that the act is used to produce a new and richer visualisation which is subject to more complex analysis. From a cognitive perspective, it is drawn from the VA model that in any mathematics problem-solving situation there is always a constant coordination between visual and analysis to form a richer visual reasoning. This is an indication that an

interaction in a visual tool environment tends to enhance or enrich spatial-visualisation skills. This aligned with the research question 3; how do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills. The VA theory therefore, provides the conceptual framework to analyse the effect of MATLAB on rural based pre-service teachers' SV skills; also, based on research evidence that ones' spatial reasoning is linked with performance with visual activities such as mental rotation tasks, problem solving, and achievement. Hence, this research undertakes a close scrutiny of the solution presented by rural-based pre-service teachers' ability to coordinate visual representation and analysis in vector calculus problem-solving measures on spatial reasoning skills.
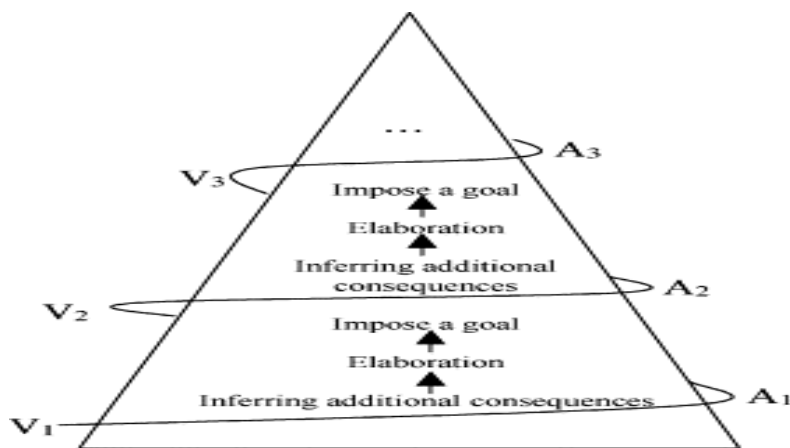
Figure 2.10: The Visualisation-Analysis (VA) model for probing thinking Zazkis et al., (1996: p.447).

Referring to Figure 2.10 and the VA framework, we find performances of visualisation combined with analytical thinking. The difficulty of visualising the projections of 3D objects is one of the reasons that students find it difficult to translate vectors from one coordinate system to another which coordinate problem solving in vector calculus. As suggested by the model, it is likely that students who have well-built schemas for mathematical objects would proceed directly to the visual representations of the functions without going through the process of detailed projections.

Therefore, triangulation of the theories overcomes the weakness of one over the other in addressing the research problems. Hence, the theoretical frameworks: theory of the register of the semiotic representation (TRSR) and Visual-Analysis (VA) model underpin

the role of SV skills in problem solving in vector calculus. Thus, the intersections between students' 3-D modelling, visual abilities, and vector calculus achievement (analytical thinking) can be analysed by using the TRSR and V-A model.

2.8 Chapter summary

In summary, this chapter was a review of relevant literature which interrogated studies of the analysis of MATLAB instruction on rural-based pre-service teachers' spatial-visualization skills and problem-solving in vector calculus. It is argued that spatial-visualization skills have been extensively explored in many disciplines owing to its complexity yet to date; research has not provided us with a common definition as well as its classifications.

It is important to note that the shared link of spatial-visualization skills with mathematical aptitude is well established (Cheng & Mix, 2012; Uttal et al., 2013). For Jansen and colleagues (2013), SV skills are a 'cognitive tool' wielded for learning, whereas Shepard and Metzler (1971) and Wai et al. (2009) hypothesised that, spatial-visualization skills mediate problem-solving and hence, the predictor of achievement in STEM. This has led to the existence of dynamic environments and their integration in mathematics classrooms.

The role of SV skills in cognitive science was discussed and their shared link with mathematics education. In the cognitive science discipline, complex technology tools such as fMRI and EEG are tailored to investigate how the brain handles mathematical tasks (Thomas al et., 2010; Babai, 2010).

While the association of spatial-visualisation skills has been largely applied in some areas in mathematics such as calculus and geometry, we have scanty research which interrogates vector calculus (Gire & Price, 2014; Nguyen & Rebello, 2011).

Chapter three presents the research methodology, research paradigm, validity issues, and instructional design.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Overview

In the previous chapter, previous studies and theoretical models related to the current study were discussed. These among alluded to: spatial skills, spatial-visualization skills, mental rotation skills, computational and mathematical models, technology and mathematics education, the role of spatial-visualisation in problem solving in vector calculus and the conceptual theatrical frameworks underpinning the research which were selected and justified.

The next section therefore provides the highlights of the systematic approach followed in addressing the research problems. These include the choice of research paradigm and research design, sampling method, data collection, data analysis, and ethical considerations. The main objective is the analysis of MATLAB instruction on rural-based pre-service teachers' spatial-visualisation skills and problem solving in vector calculus.

The study is therefore grounded on the works of Duval's (2003) semiotic representation and Zazkis et al's (1996), Visual-Analyser (VA) model and also motivated by the pedagogical method to support the application of dynamic software and computer environments such as MATLAB to support the acquisition of spatial-visualization skills and problem-solving skills.

3.2 Clarification and Justification of the Study

A considerable number of studies have been conducted regarding correlation between students' spatial-visualization skills, problem-solving skills, and achievement employing dynamic environments such as MATLAB, GeoGebra and many more. This justifies the discourse of the analysis of MATLAB instruction on rural-based pre-service teachers spatial-visualization skills and problem solving in vector calculus.

3.3 Research Paradigms

The term 'paradigm' is described as a worldview, belief or an understanding of a phenomenon. Paradigm originated from the Greek word *paradeigma* which means pattern and was first used by Kuhn (1962) to denote a conceptual framework shared by research community which provided them with a convenient model for analysing problems and finding solutions. Kuhn defines a paradigm as: "an integrated cluster of substantive concepts, variables and problems attached with corresponding methodological approaches and tools…" (p. 32). Ibid elaborated further that the term paradigm refers to a culture of research with a set of beliefs, values and assumptions that a research community has in common about the nature and conduct of the research.

According to Briggs and Coleman (2012), a paradigm is network of core ideas that guides thinking and action in research. There are a number of theoretical paradigms such as positivism, constructivism, interpretation, transformation, pragmatism and positivism (Creswell & Creswell, 2017; Mackenzie & Knipe, 2006). Among these theoretical paradigms, positivist and interpretive are mostly employed in research dependent on the nature of the study.

3.3.1 Positivism paradigm

Positivism believes that that only factual knowledge gained through observations (i.e., intelligence), including measurement is truthful. Positivism principles then depend on quantifiable observations leading to statistical analysis. Positivism is in accordance with the empiricist view that knowledge stems from human experience. It has an atomic, ontological worldview that includes discrete, observable elements and events that interact in an observable, determined and regular way (Collins, 2017). The researcher was unbiased throughout the study and no provisions of personal interests which may have an effect on the outcome of the research findings. The researcher however, assumed some elements of the positivist approach to the study in order to ensured pure objectivity throughout. Thus; data collection and interpretation were purely objective. Research findings were observable, quantifiable and were statistically analysed. Independence

implies that the researcher had minimal interaction with research participants during the course of this study, as Wilson (2010) postulated.

3.3.2 Interpretive paradigm

The interpretivism paradigm is based on the thought that there are fundamental differences between the world of natural and society. The main objective of interpretivism is to understand the subjective experiences of those being studied: how they reason and feel, and how they act or react in their usual contexts. Its core assumption is that social actors build meaningful social world structures in which they operate (Crofts, Madden, Franks, & James, 2011). The researcher's position is founded on a theoretical belief that "reality is socially constructed" (Mertens, 2005, p.12) and integrating the idea that "social/settings and relations with participants are important" (Creswell & Creswell, 2017, p.123). In lieu of this perspective, validity or truth cannot be based solely on an objective reality. Interpretivist paradigms describe the qualities or characteristics of a phenomenon. Among includes information on the needs, desires of participants and a variety of other information that is essential for producing what is beneficial for the lives of participants (Madrigal & McClain, 2012).

These further require flexibility, permitting participants to respond to data as it emerges during a session. Identifying patterns and needs when analyzing is important. As MacKenzie and Knipe (2006) stated, both of these paradigms can be extremely effective in identifying factors affecting the areas under investigation in one study and then using this information to quantify how these factors would affect participant preferences.

Pragmatism values both objective and subjective values of knowledge in terms of methodologies focusing on what " works" (Johnson & Onwuegbuzie, 2004; Morgan, 2007) and oriented to the study of real world problems rather than the nature of knowledge (Hall, 2013). A pragmatic approach has been encouraged by several researchers (e.g., Cameron, 2009; Cronholm & Hjalmarsson, 2011; Morgan, 2007). Pragmatism challenges claims by methodological purists that qualitative and quantitative methods represent two different worlds that cannot be integrated. Rather, in view of the different methods' respective strengths and weaknesses, pragmatism views them as complementary (Cronholm & Hjalmarsson, 2011) and interdependent, with the ensuing advantages of

mixing methods outweighing potential disadvantages. The above assumptions and the subsequent paradigms influenced the methodological choices of this research. The researcher found it appropriate to use both qualitative and quantitative (mixed methods research) approaches for this study hence a pragmatism paradigm is the most suited for the study since its ideas embrace positivism and interpretivism paradigms.

3.4 Research Design

Research design refers to the overall plan that the researchers use to integrate the different components of the study in a comprehensible and logical manner, thereby ensuring the study will effectively address the research problems. It constitutes the blue print for the collection, measurement and analysis of data (Labaree, 2013). An experimental design research (refer to figure 3.1) approach allowed for the collection of data using a range of approaches (see section 3.6 for further detail). An experimental design approach was chosen for the study. Experimental research involves the manipulations of one variable to determine a cause-and-effect relationship (Johnson & Christensen, 2008). From this indication, 100 rural-based pre-service teachers who has the same chatracteristics traits in a second year vector calculus class at UNIZULU were randomly assigned to either the control or the experimental groups. This was followed by a pre-test (refer to Appendix 4) to ensure both the experiment groups have prior knowledge on the content area. The control group were taught by the researcher every day for 2 hours for three weeks on vector calculus using a traditional approach (lecturer's notes and chalk). The experimental group was also taught by the same researcher for the same time frame and topics on vector calculus. However, they were given a treatment (i.e., the use of MATLAB in teaching and learning these topics on vector calculus). A post-test (the same as the pre-test) was then conducted from which data were collected in addressing the research questions:

1. How do rural-based pre-service teachers apply their visualisation skills in problem solving in vector calculus?

2. To what degree do rural-based pre-service teachers spatial-visual skills correlate with their vector calculus achievement?

An achievement test served as pre-test and post-test and was administered to both groups as shown in the study design (Figure 3.1).
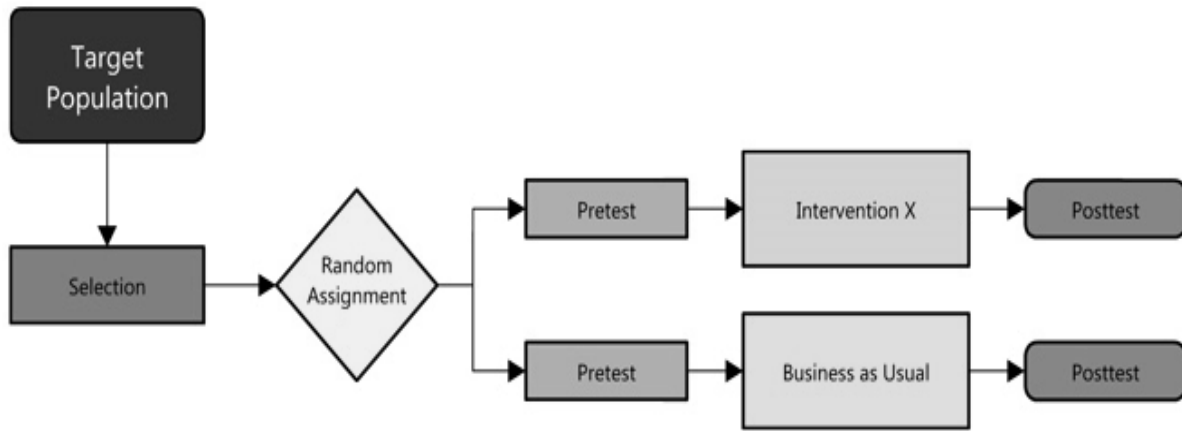
Figure 3.1: A diagram showing experimental research design for the study

3.4.1 Mental Rotation Test and instrumentation

Section A of Purdue spatial-visualisation test/rotations (PSVT/R) comprising of ten items (refer to Appendix 5) was adapted and used in the data collection to analyse the research question 3: "How do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills?" The PSVT/R test was also designed to measure the participants' ability to visualise the rotation of three-dimensional objects. The instrument was chosen because of its higher correlation with similar instruments measuring visualisation such as the Shepard-Metzler tests. According to Bodner and Guay (1997), these tests are "among the spatial test least likely to be confounded by analytic processing strategies" (p.13). The Minnesota Paper Folding Board Test is also designed to measure spatial visualisation but has a weaker correlation with other spatial visualisation instruments and is likely to be confounded by analytic processing (Bodner & Guay, 1997). The format for the PSVT/R is 10 items and for each question an object is pictured in one position then it is shown, in a second image, oriented to a different position. The participants were shown a second object and given five choices, one of which matches the rotation of the original object example.

3.5 Population,

Saunders (2012), defined population as the total number of individuals or objects of similar charateristics from which a sample size is chosen. Furthermore, population refers to the total number of elements (such as; individuals, events, objectives, or items) from which samples are chosen for measurement (cooper & Schindler, 2006). In this study, the population was represented by rural-based pre-service teachers in a second year vector calculus class at UNIZULU. According to the Department of Mathematics, Science, and Technology Education (MSTE) at UNIZULU, 137 students registered for vector calculus for the academic year 2018/2019.

3.5.1 Sampling techniques

A simple random sampling technique was used in the selection of the research participants. In this regard, each rural-based pre-service teacher had equal probability of inclusion in the sample size.

3.5.2 Sampling and Sample size of the study

Saunders, Lewis and Thornhill (2009) described sample size as a group or subset of the population of interest. Hence, sampling is referred to as the selecting of subset of individual participants or objects that represents the total population. This is done to: minimise the cost of the study; to reduce the work load on data collection; and ensure greater accuracy of outcomes. In view of this, Saunders (2010) stated that the procedure provides enough time to collect data, obtain comprehensive data, and enables the researcher to collect data from more demanding situations. Sample size can be calculated using different statistical formula such as the one by Krejcie and Morgan 1970. Glenn (1992), Saunders (2010) and Singh and Masuku (2014), advised that the population's characteristics for calculating a sample size from a given population should be within plus or minus 5% of its true values. Accordingly, for this study, the desired level of precision was +/– 5%. The sample size was calculated based on Krejcie and Morgan (1970) formulas for confidence level of 95%.

$$s = \frac{X^2 NP(1-P)}{d^2(N-1) + X^2 P(1-p)}$$

s = required sample size.

X^2 = the table value of chi-square for 1 degree of freedom at the desired confidence level (3.841).

N = the population size.

P = the population proportion (assumed to be .50 since this would provide the maximum sample size).

d = the degree of accuracy expressed as a proportion (.05)

$$s = \frac{1.96^2 \times 136 \times 0.5(1 - 0.5)}{0.05^2(240 - 1) + 1.96^2 \times 0.5(1 - 0.5)}$$

$$s \approx 100$$

To confirm this result, a Sample Size Calculator was utilised from the following website (http://www.raosoft.com/samplesize.html).

Hence a sample size of 100 pre-service teachers were randomly selected from a total sample space of 137 rural-based pre-service teachers in a second year vector calculus class at UNIZULU.

Table 3.1: Composition of sample

No. of students	Group of students	Breakdown	Percentage
	Experimental Group	50	50%
100	Control Group	50	50%
Total		100	

3.6 Data Collection

Research data was collected in diverse ways (Table 1). There were three tests carried out by all the research participants. A pre-test (see Appendix 4) was first conducted covering vector calculus topics; vector operations, double integral, and triple integral. This was used to collect data from both experimental and control groups. Two-hour lessons each day for three weeks were given by the researcher for both groups using the lecturer's note on vector calculus however, the experimental group was given intervention by using the dynamic software, MATLAB in lesson delivery. This was used in data collection to determine a cause-and-effect relationship between the experimental and the control groups. Finally, a Purdue spatial visualisation test/rotation (refer to Appendix 5) was used in data collection on both groups. The control group served as a baseline in examining the effect of environment dynamic software MATLAB instruction on the experimental group's spatial-visualisation skills.

3.6.1 Data Collection Instruments

Data collection is the process of gathering and measuring information on variables of interest, in an established systematic technique which enables one to effectively address research questions, test hypotheses, and evaluate outcomes. A pre-test, post-test, and Purdue spatial-visualisation test/rotations (PSVT/R) were adopted in the quantitative data collection.

3.6.1.1 Pre-test/posttest

The research participants were randomly assigned to experimental and control groups to ensure all conditions were the same. Baseline assessment test (pre-test) and achievement test (post-test) designs are widely used in experimental research for the purpose of comparing groups and/or measuring change resulting from experimental instruments the dynamic visual tool, MATLAB.

A pre-test (refer to Appendix 4) was administered to both groups to ensure they are of the same baseline (i.e., in terms of prior knowledge). The experimental group was exposed to treatment. They were taught by the researcher for two hours each day for three weeks

using MATLAB as a pedagogical tool for teaching and learning vector calculus. The experimental group were drilled on the use of MATLAB to generate 3D solid shapes such as; cube, cylinder, and cone and translate these 3D shapes from one coordinate system to another (refer to figures 3.3, 3.4 and 3.5). The control group was also taught by the researcher for the same time frame however, lessons on these same sections were delivered solely by using lecturer's notes and a chalkboard. Geometric figures and vector fields were sketched on the board and explained. After, a post-test was conducted (refer to Appendix 4) and data was collected.

3.6.1.2 Content analysis

The qualitative approach: content analysis was employed in analysing the participants conceptual understanding, factual knowledge, and procedural skills required in problem solving in vector calculus before and after treatment. This was achieved through radom selection of three scripts each from both groups and the solutions presented by the participants were weigh against the scheme developed by the researcher for each item through the frameworks.

Thus, guided by the conceptual frameworks of Duval's (2003) theory of semiotic representation and Zazkis et al's. (1996) VA model solution presented rural-based pre-service teachers' problem solving in vector calculus were analysed on the basis of: their ability to switch between registers and how they coordinate visual and analysis in the problem-solving.

3.6.2 MATLAB Intervention for Teaching Vector calculus

The experimental group were given orientation on the operation tools on MATLAB. The aim was to orientate students to MATLAB software: exploring and introducing the different menu options as well as observing tutorials and presentations built into the MATLAB software.

3.6.2.1 MATLAB and Mathematical environment

Students were taught lanching the software double clicking on the MATLAB logo which gets the MATLAB progam started. The symbol ≫ (known as the command prompt) open the mathematical environment for the interactions for performimg basic mathematics operations as shown in figure 3.2.
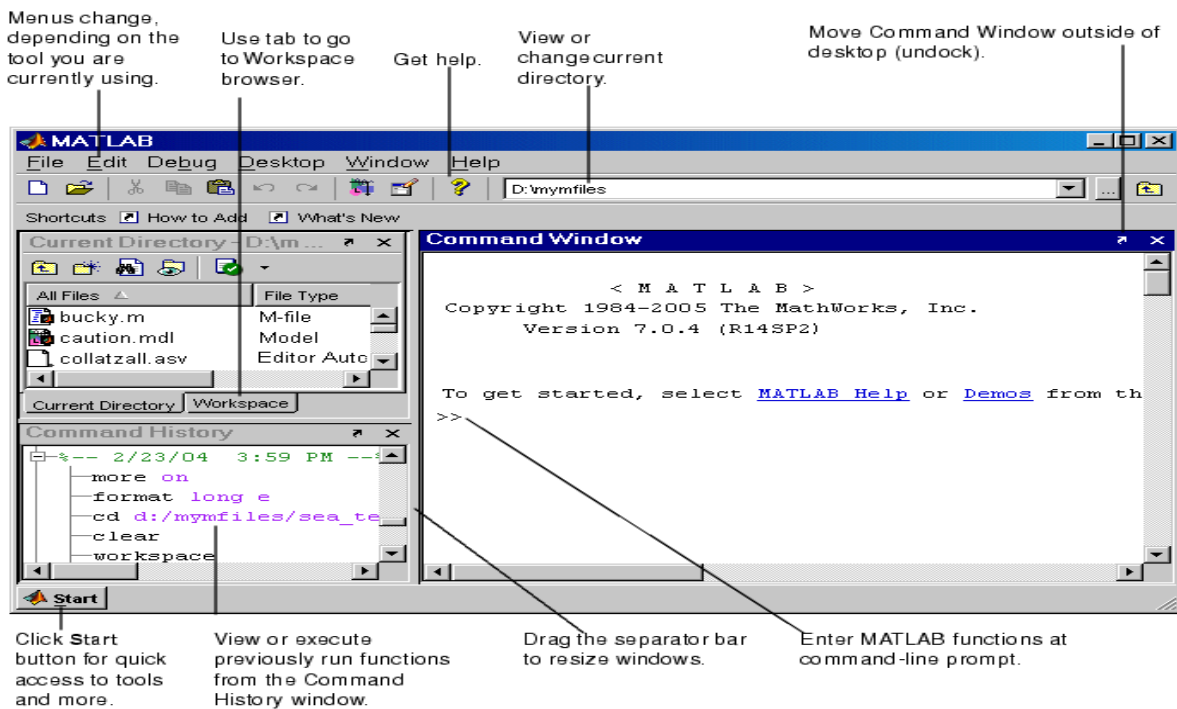
Figure 3.2 MATLAB desktop

They were then taken through the MATLAB systems which consists of five main parts namely: desktop tools and development environment, MATLAB mathematical function library, MATLAB language, graphics and MATLAB external interfaces.

Desktop tools and development environment: this is the set of tools and facilities that help use of the MATLAB functions and files. These include MATLAB desktop and command window, a command history, an editor, and code analyzer and and many more.

The matlab mathematical function library: this consists of vast collections of computational algorithms ranging from elementary functions (e.g., simple arithmetic and trigonometric ratios) to more sophisticated functions (e.g., matrix inverse, matrix eigenvalues, Bessel functions, and fast Fourier transforms).

The matlab language: this are written language by means of composing commands, variables and other objects with Latin characters a, b, …, y, z, numeric numbers 1, …, 9, 0, few additional symbols as _, +, -, *, ^, % to interact with the MATLAB just like in many other programming languages. Besides, the MATLB language or commands can also be written in computer programs such as C and Frotran programs.

Graphics: MATLAB has in-built capabilities that display vectors and matrices as graphs. It contains high-level functions for 2D and 3D data visualization, image processing, animation, and presentation graphics.

The matlab external interfaces: This is a library that allows you to write programs (such as C and Fortran programs) that interact with MATLAB.

Having taken them through above mentioned functions of the MATLAB, the experimental students were able to use the MATLAB to perform functions ranging from; simple arithmetic calculations, integral functions, exponential functions to name but a few. In addition, they were able to display mathematical functions in their graphical representations and data also be to create 2D or 3D dimensional shapes. They were further taken through activities such as: vector operations, plotting and rotating 2D and 3D shapes, and performing integral function which consititute to the vector calculus course.

3.6.2.2 Laboratory Activities

The experimental students were grouped into small groups and demonstrations were given by the researchers on how to lunch the MATLAB by double clicking on the MATLAB logo. They were introduced to the icon and their usage on the MATLAB environment and this was followed by activities relating to vector calculus.

Activity 1
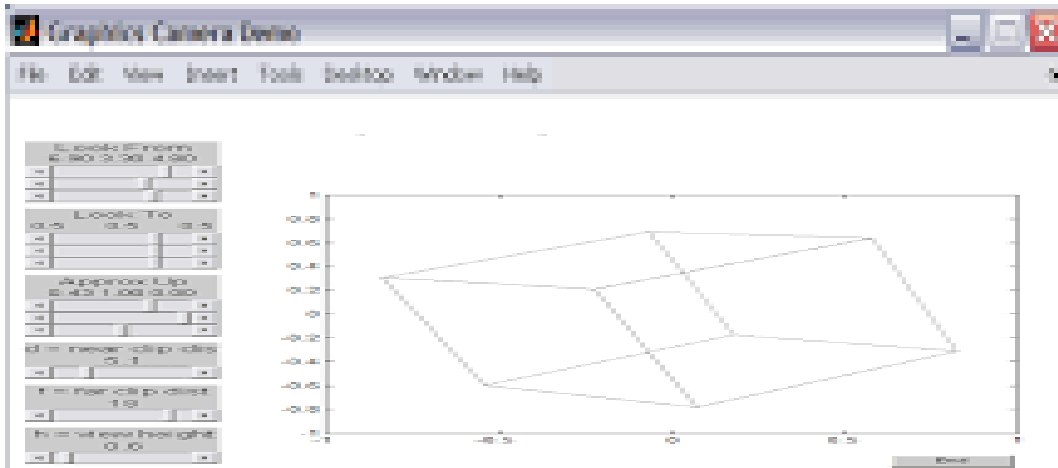
Figure 3.3 MATLAB use to view a cube fom different angles

Activity 2
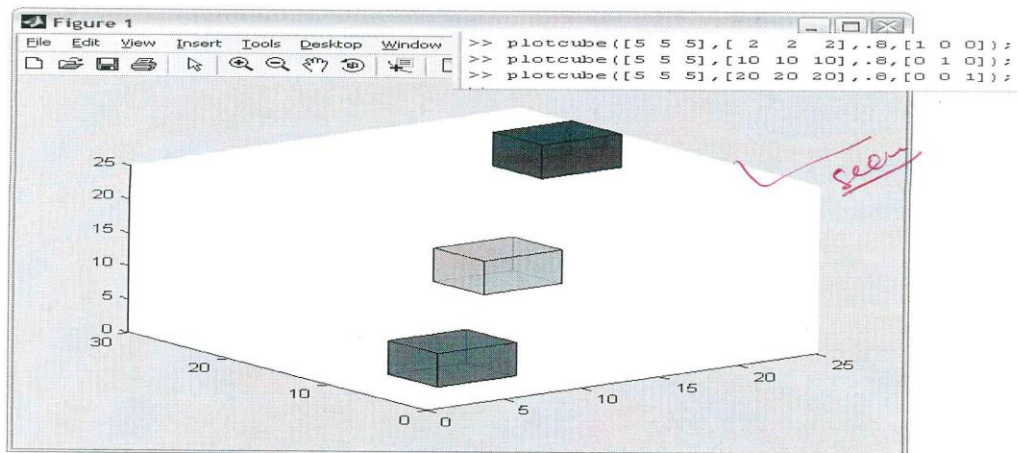
Figure 3.4: A worksheet presented by student (ED14) from the experimental class who used the MATLAB to generate cube boxes.

Acitivities 1 and 2, the experimental students verified, the use of the MATLAB to generate and rotate 3D shapes (e.g., such as a cube). The aim was for the experimental students to visualise, view 3D figures from different angles and make their own conjectures about the properties of 3D shapes. These activities (i.e., refer to figures 3.3 and 3.4) are presumed to have direct link with students' cognitive reasoning such as problem solving.

In the literature review, (see chapter 2 for detail) it was highlighted that every mathematical reasoning is grounded on representation thus; a correct representation of mathematical problem ease in problem solving situation. Furhtermore, it is also theorised that the use of technological visual tools such as the MATLAB as a pedagogical tool in teaching and learning mathematics reduces students' cognitive load. This addresses research question 1, "How do rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus?" and research question 2, "To what degree do rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement?"

Activity 3

```
figure
quiver3(Z,U,V,W)
view(-35,45)
```

[Code adopted from mathworks.com in generating the vector field]
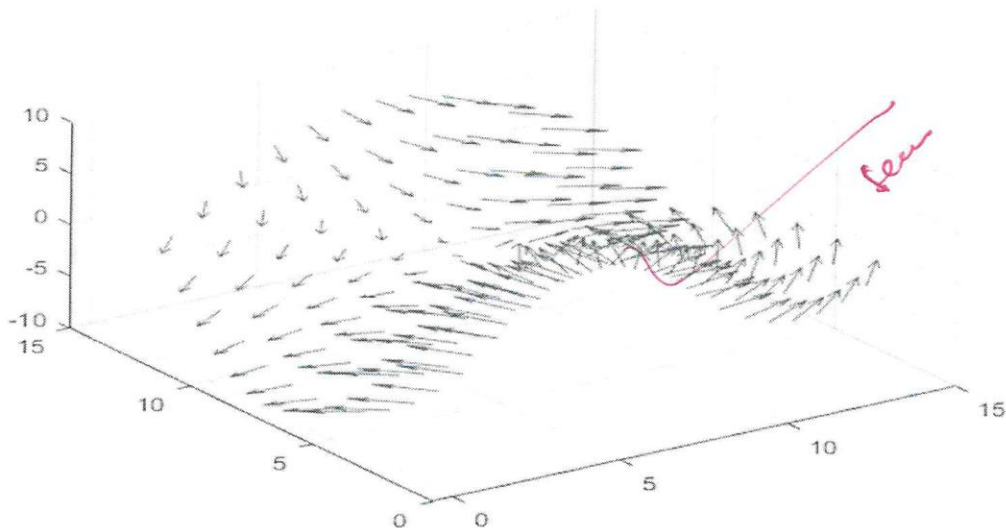
Figure 3.5: A worksheet presented by student (EG015) from the experimental class run the code given to generate vector field.

The aim was to equip the experimental students on how to use the MATLAB to generate vector field from any given vector equations (refer to figure 3.5). This contributes to their ability to visualised vector field and performed vector operations such as additions of

vectors, flux, cross product, and more. The practical assisitance of the MATLAB to generate vector field form vector equation is aimed at improving the experimental students' understanding, confidence and visual reasoning.

Futhermore, the students will be able to visualise or sketch a correct vector field and give correct interpretations of the vector field drawn. This addressed research question 3 "How do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills?"

3.6.3 The Purdue Spatial Visualisation Test: Visualisation of Rotation (PSVT-R)

A standard test of spatial visualisation skills was used as a measure of whether or not there was an effect of the visual tool, MATLAB on the experimental group's spatial-visualisation skills. The PSVT/R is exceedingly used in different fields such as mathematics, physics and engineering in measuring students' visualisation skills. For example, engineering field PSVT/R "is often used as an assessment tool for diagnosing and improving students' spatial visualisation skills in any engineering course that requires basic understandings of visual representation of objects" (Alqahtani, Daghestani, & Ibrahim, 2017, p. 91). There are many measures to test the mental rotation skills. The measures (standard tests) are Mental Rotations Test (MRT), Purdue Spatial Visualisation Test/Rotation (PSVT/R), Online surveys, and Concurrent STEM course grades (Martin-Gutierrez, Trujillo, & Acosta-Gonzalez, 2013). The basic test is the Mental Rotation Test by Shepard and Metzler (1971). A mental rotation test is a measurement tool for mental visualisation and spatial visualisation skills. A mental rotation test required to calculate not only the result of the test, but also how much response time for each question with calculating error rate (Branoff, 2000; Miller & Halpern, 2013). The student decided if two objects are matched or mis-matched or can choose one of three models which one rotated as the original model (Harle & Towns, 2010). However, PSVT/R considers both rotation and spatial skill with visualisation ability. The PSVT/R test consists of three parts that are "development" to see how the pre-service teachers can visualise the folding of 3D objects. The second part is "rotation" to see how the pre-service teachers can mentally rotate the 3D objects. The third part is "visualisation" to see how the student visualises 3D objects from one point of view through a glass cube (Harle & Towns, 2010). However,

the researcher adopted only the section A (i.e., Mental rotation) of PSVT/R since this was most suited to the objectives (refer to appendix 5).

3.7 Triangulation of Method, Data and Theory

This refers to whether the verdicts of a study hold and are "true" on the grounds that study results accurately reflect the situation under discusion, and "certain" on the ground that the results are supported by the evidence. Triangulation is employed in qualitative research to check and establish validity by analysing a research question from multiple perspectives. Guion, Diehl, and McDonald (2011) cited that, it is a common misconception that the objective of triangulation is to achieve consistency between data sources or approaches; in fact, these inconsistencies can probably be given the relative strengths of various approaches. Hence, the triangulation of data collections should be seen as means of strengthting evidence and also an opportunity to uncover richer meaning in the data.

3.7.1 Methodological triangulation

Methodological triangulation involves the use of qualitative and quantitative methods to study the program. Data was collected using different data collection medods namely: quantitative approach (i.e., pre-test, post-test and PSVT/R), and qualitatively, content analysis. Hence, this confirmed the validity of the results.

3.7.2 Data triangulation

Data triangulation involves using more than one method of data collection in order to increase the validity of a study. Data was therefore collected through pretest, posttest, Mental Rotation Test (PSVT-R), and content analysis. During analysis, feedback further showed that there were areas of divergence and areas of agreement.

3.7.3 Theory triangulation

Theory triangulation involves the use of multiple viewpoints to interpret a single set of data. Hence, a mixed method approach was used in data collections. Thus: data was collected using multiple means such as pre-test, post-test, Purdue Spatial-visualisation Test and content analysis. This was done to increase confidence in the research data collection, create innovative ways of understanding a phenomenon, reveal unique findings, challenge or integrate theories and provide a clearer understanding of the problem. Triangulation can however, be used effectively to deepen the researcher's understanding of the underlying issues and maximise their confidence in the findings, even though it is time-consuming, requires planning, and organisation of resources.

3.8 Data Analysis

Analysis of data is a process of inspecting, cleaning, transporting, and modelling data with the goals of discovering useful information, suggesting conclusions and supporting decision-making.

Table 3.2: Mapping research questions to sources of data and data analysis strategy

Research Questions	Data sources	Respondents	Data analysis method
1. How do rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus?	Pre-test/Post-test.	Participants	content analysis/ Independent sample test
2. To what degree do pre-service teachers spatial-visual skills correlate with their vector calculus achievement?	Pre-test/Post-test	Participants	Descriptive statistics /Independent sample test
3. How does Dynamic software environment such as MATLAB influence rural-based pre-service teachers spatial-visual skills?	PVR/T	Participants	Descriptive statistics/Independent sample test.

3.8.1 Independent sample t-test

An independent sample t-test, as the name indicates, allows one to compare two independent groups (i.e., the control and the experimental group) on continuous dependent variables of problem-solving vector calculus, vector calculus achievement, and spatial-visualisation skills. That is, the groups must be independent with different participants in each group and the dependent variable must be continuous (Gravetter & Wallnau, 2012). The independent sample t-test enables the researcher to decide whether there is a significant difference in the dependent variables (i.e., problem-solving vector calculus, vector calculus achievement, and spatial-visualisation skills) between the control group and the experimental group. Table 4 gives the description of research variables used in the research study.

Table 3.3: Description variables used in the study

Independent variables	Descriptions	Dependent variables	Scale of measurement	Descriptions
MATLAB instruction	Using the MATLAB to generate, view and translate 3D objects (e.g. cone, cube, and cylinder) from one coordinate system to another. Plotting vector equation and performing integral functions.	Problem solving in vector calculus	Content analysis/independent sample t-test	Pre-test and post-test content analysis
MATLAB instruction	Using the MATLAB to generate, view and translate 3D objects (e.g. cone, cube, and cylinder) from one coordinate system to another. Plotting vector equation and performing integral functions.	Vector calculus Achievement	Independent sample t-test	Score on pre-test and post-test.
MATLAB instruction	Purdue Spatial-visualisation/rotation test	Spatial-visualisation skills	Independent sample t-test	Purdue Spatial-visualisation/rotation test

3.9 Trustworthiness

The researcher ensured the study's trustworthiness by maintaining validity, transferability, reliability, dependability, and conformability. Trustworthiness is a demonstration that the evidence of the research findings reported is sound and the argument made based on such results is equally strong (La Banca, 2010).

3.9.1 Validity of Survey instruments

Validity refers to the extent to which empirical measure accurately reflects the concept it is intended to measure, producing results that reflect the true variables being measured (Frankfort-Nachmias & Nachmias, 2007). Hence, the validity is raised in the context of the three points: the form of the test, the purpose of test, and the population for whom it is intended. It can be categorised as internal and external validity.

Internal validity: is the flaws within a research such as not controlling some major variables for example the research instruments (e.g. instrument sensitivity, experimental validity, and replicable). The researcher therefore, ensured internal validity is achieved by considering the following validity types:

*Content validity***:** This is the degree to which a study instrument is a representative sample of the content domain measured (Leedy & Ormrod, 2013). The researcher used lectures' notes on the vector calculus topics (i.e., vector operations, double and triple integrations) investigated, ensured the pre-test/post-test were evaluated by lecturers from the Department of Mathematics, Science, and Technology Education (MSTE) at UNIZULU and finally the advice from my supervisor.

Face validity: Is the degree to which a test appears to measure what it purports to measure. The researcher ensured face validity was held low by evaluating the research, the pre-test/post-test and the questionnaire. This was to prevent participants from predicting what the study is to achieve since it may lead to "fake good'' responses.

Concurrent validity: Is the degree to which the score of a test is related to the score of another, already established, test administered at the same time or valid criterion available at the same time. The researcher used similar research conducted using a technological tool, MATLAB and compared the findings with the already established score.

Construct validity: Is the degree to which a measuring instrument provides adequate coverage of the investigative question (Oluwatayo, 2012). The researcher reviewed the literature and the theories to come up with constructs that can be measured.

External validity: is the extent to which the research outcome can be generalised or relate to other situations or people. An important variant of the external validity problem deals with sampling bias. The researcher therefore employed random sampling to avoid bias in the selection of sample size and also to ensure the sample size represents the larger population.

3.9.2 Transferability

Transferability is applying research results to other contexts and settings in order to get at generalised ability. Thus, it is the phenomenon where the findings described in one piece of research are applicable or useful to make generalisations for future research. The researcher provided a detailed description of the study's sites, participants, and methods used to collect data for future studies.

3.9.3 Reliability

Reliability refers to the extent to which test scores are free of measurement error. Salkind (2012) refers to dependable, consistent, stable, trustworthy, predictable and faithful as synonyms for reliability whereas Muijs (2011) advanced the notion that there is always some element of error when researchers measure variables. This understands that no measure or instrument is perfect; each will contain some degree of error. These errors can emanate from individual general skills, attitude, motivation or the way the instrument is designed and administered. Although it is rare to have perfect reliability, the researcher used the following strategies as suggested by Salkind (2012) to ensure that data collection, analysis, and findings were reliable.

Triangulation: A mixed method approach was adopted by the researcher to facilitate validation of data.

Audit: The researcher contacted some experienced lecturers who are lecturing on vector calculus to review the pre-test/post-test. A pilot study was then conducted prior to a larger piece of research to determine whether the methodology, sampling, instruments and analysis are adequate and appropriate.

Researchers' position: The researcher has explained his position and declared his unbiased position involving data collection and analyses.

Code-recode strategy: The researcher coded the data over an extended period of time to ensure consistency.

3.9.4 Dependability

According to Riege (2003) dependability is similar to the notion of reliability in research. The study has dependability by its selection of the sample size, justification, and application of research design, procedures, and methods were clearly explained and its effectiveness evaluated by the researcher and confirmed by my supervisor. Prior to this, the instrument for data collection was piloted to make sure that it pursued relevant information without unintended ambiguity. Care was also taken to ensure that the research process was logical, traceable, and clearly documented in a reflexive manner by giving a detailed account of the research process.

3.9.5 Confirmability

According to Shenton (2004), confirmability is achieved when the researcher provides detailed methodological descriptions, showing how data was collected, constructed, and the theories from it, can be accepted. This was achieved through basing findings on the analysis of the collected data and examined via an auditing process, i.e. the researcher confirmed that the study findings are grounded in the data and inferences based on the data were logical and have clarity, high utility or explanatory power.

3.10 Ethical issues

Ethics is a philosophical term derived from the Greek word ethos, meaning approach or tradition and connotes a social code that conveys moral integrity and consistent values (Partington, 2003). Mouton (2001) believes that science ethics concern what is wrong and what is right in conducting research. For all research studies, irrespective of research designs, sampling, techniques and choice of methods, are subject to ethical considerations (Gratton & Jones, 2010). Henceforth, the following ethical aspects were adhered to in this research study:

- Ethical approval was first obtained following completion of all the required ethical procedures of the University of Zululand Higher Degree Committee and approval was granted (refer to Appendix 3).
- A letter was presented to the participants with the questionnaires to encourage their participation in the research (refer to appendix 1). Information was also provided to the participants concerning the nature of the study, participation

requirements (e.g. activities and duration), confidentiality and contact information of the researcher.

- Providing participants with a participant information sheet. The participants were drilled on the purpose of the research which outlined that they were voluntary participants; they had the right to opt out anytime they wanted without any consequences, and they were made aware they would receive no remuneration for participating in the study. However, they were encouraged to participate by emphasising how the designed instrument is relevant to their syllabus.

- Providing participants with a participant consent form, which confirmed they were willing to participate in the study.

- Confidentiality of the participants was guaranteed by not releasing names or student numbers of the participants and by only allowing the investigator access to the data.

- Participants were assured the outcome of their performances will be made available upon their request.

3.11 Conclusion

The chapter covered the research methodology used in the study. The chapter discussed the research paradigms, research design, target population and sample size, the sample technique used in the study, data collection, data analysis, and measures used to enhance trustworthiness, and ethical issues. The next chapter seeks to present results analysis of the data which shed light to the conclusion results of the research questions through teaching experiment.

CHAPTER FOUR

DATA ANALYSIS

4.1 Overview

This chapter presents a synopsis of the findings of the study based on the analysis of the dataset collected through pre-test, post-test, and Purdue spatial-visualization test (PSVT/R) among rural-based pre-service teachers in a second year vector calculus class at UNIZULU. In the milieu of research evidence available, the dynamic visual environment enhances spatial-visualisation skills and analytical reasoning hence, MATLAB has been employed as a pedagogical teaching tool to analyse its effect on rural-based pre-service teachers' spatial-visualisation skills, problem solving and as well as achievement in vector calculus. From the aforementioned, the chapter is therefore structured according to the order of the research questions under consideration as follows:

1. How do rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus?

2. To what degree do rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement?

3. How do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills?

The description of the analysis of the data collected was interpreted with the adoption of both a quantitative and qualitative approach. A quantitative method was used to analyse the degree to which rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement and the effect of the dynamic software environment, MATLAB, in enhancing rural-based pre-service teachers' spatial-visualisation skills. For the qualitative approach, content analysis was employed to analyse how rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus. In this regard, rural-based pre-service teachers' conceptual, procedural, and factual knowledge required in problem-solving situations was analysed on the solution presented based on the pre/post-tests

administered. The analysis was guided by the conceptual frameworks of Duval (1996), semiotic transformation registers and Zazkis et al's. (2003) visualization-analysis (VA).

4.2 Analysis of rural-based pre-service teachers' prior knowledge on Vector Calculus (Pre-Test).

A sample size of 100 rural-based pre-service teachers in a second year vector calculus class at UNIZULU were randomly selected and grouped into a control group (n=50) and an experimental group (n=50) as detailed in table 1. A pre-test (refer to Appendix 4) was administered to determine whether or not there was significant difference in prior knowledge between the control group and experimental group. This was conducted to determine the knowledge gaps and the misconceptions among the groups. Also, the pre-test was used as a guideline for laboratory instructions. The pre-test items consist of eight items in all.

In question1, rural-based pre-service teachers were to match the equations with their respective graphs (refer to figure 5.1). In all, 72% of the rural-based pre-service teachers were able to match the equations: $y = x^3$ and $xy = 3$ to their respective graphs while nearly 28% of the students were able to match the equations $x^2 + y^2 = 3$; $z = x^2 + y^2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ to their irrespective graphs. Question 2(a) required rural-based pre-service teachers to sketch and describe the vector field of the vector function $\vec{F}(x,y) = x\vec{j}$. Overall, only a few rural-based pre-service teachers (10%) sketched the vector field correctly and the descriptions (refer to figure 4.1). Question 2.1(a) required rural-based pre-service teachers to convert the coordinate $(1, \sqrt{3}, 2)$ in cartesian system to cylindrical coordinates system and question 2.2(b) required rural-based pre-service teachers to sketch the cylindrical coordinates calculated in question 2.1a above (refer to figure 4.3). Interestingly, a large number of the rural-based pre-service teachers (85%) were able to do the conversion by computing the coordinate point $(1, \sqrt{3}, 2)$ in cylindrical coordinate equation however, among these students only 15% were able to sketch the cylindrical coordinates correctly. Question 3.1. 3.2, and 3.3 were based on multivariate calculus. Question 3.1 required them to make a 3D sketch of the algebraic equation $2x + y + z = 8$. In all, nearly 13% of the rural-based pre-service teachers were able to calculate the $x, y,$ and z intercepts and sketched the

3D shape of the algebra correctly. For question 3.2 rural-based pre-service teachers found it difficult to sketch the graph bounded by the region $h(x) = 2x$ and $g(x) = x^2$ (refer to figure 4.5). Finally, the last item of the pre-test question 3.3 equally posed a challenge to rural-based pre-service teachers. They were unable to visualise and give a correct sketch of a cube. However, all those who made a correct sketch to represent the cube, perform the integration of the volume correctly (refer to figure 4.6). The tests (i.e., the pre/post-test and Purdue spatial-visualisation test) were analysed using the statistical method independent sample t-test. This was to analyse whether there was a significant difference between the groups on the MATLAB instruction on rural-based pre-service teachers problem-solving, achievement in vector calculus, and spatial-visualisation skills.

Tables 4.1 and 4.2 give a summary of the group statistics and the independent sample test of the pre-test.

Table 4.1: Group statistics on pre-test

Pre-Test Score	Group	N	Mean	Std. Deviation	Std. Error Mean
	Control Group	50	14.62	3.78	.535
	Experimental Group	50	15.40	3.65	.516

The pre-test was administered to 100 rural-based pre-service teachers constituted of control (N=50) and experimental groups (N=50) as indicated in table 4.1. The mean pre-test score of the control group ($\bar{x} = 14.62, SD = 3.784$) and that of mean pre-test score for the experimental group ($\bar{x} = 15.40, SD = 3.648$). An independent sampled test was further carried out to see whether or not there was significant difference of the pre-test score between the two groups (refer to table 4.2).

Table 4.2: Independent Sample Test for assessing the baseline of the groups

		Levene's Test for Equality of Variances		t-test for Equality of Means					95% confidence interval of the Difference	
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Pre - Test Score	Equal variances assumed	.51	.48	-1.05	98	.29	-.78	.74	-2.26	695
	Equal variances not assumed			-1.05	97.87	.29	-.78	.74	-2.26	695

T-Value significant at $p < 0.05$

From Table 4.2, the mean difference between the groups was 0.78 with a t-value 1.05. However, the p-value (2-tailled) was 0.29 ($p > 0.05$) hence, we have reason to conclude that there was no significant difference between the mean pre-test score of the control group and the experimental group. This meant that control and experimental groups were both on a similar in baseline before the teaching experiment (treatment).

4.3 Evaluation between Computer Pedagogical Teaching Approach and Traditional Approach.

In the review of the literature in chapter two, studies identified a shared link between dynamic environments such as MATIMATICA, CAS, and the MATLAB aid problem-solving skills, achievement, and spatial-visualisation skills. Accordingly, the study employed the dynamic visual tool, MATLAB, as a pedagogical teaching tool in learning vector calculus where data was collected to analyse rural-based pre-service teachers' spatial-visualisation skills, problem-solving skills and achievement in vector calculus. The experimental group had 2-hour lessons on vector calculus with particular reference to vector operation, double and triple integration every day for 3 weeks. Teaching and learning were coupled with laboratory activities such as; the use of MATLAB to generate geometric 3D shapes, viewing 3D shapes in a different view, and many more (refer to figures 3.3, 3.4 and 3.5). For the control group, they were taught for the same durations but employing the traditional approach only (i.e., lectures

were given using lecturer's note and text books only). A post-test (the same as the pre-test) was administered afterward and a sample independent t-test was used to determine whether there was a significant difference in MATLAB instruction on rural-based pre-service teachers' achievement in vector calculus. Content analysis guided by Duval's (2003) semiotic representation theory and Zazkis' et al. (1996) VA was also employed to analyse rural-based pre-service teachers' problem-solving skills. Aside from these assessments, Purdue spatial-visualisation Test/Rotations (PSVT/R) was employed to assess the influence of the dynamic environment, MATLAB, on rural-based pre-service teachers' spatial-visualisation skills.

4.3.1. How do rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus?

This employed a qualitative method thus: content analysis to examine rural-based pre-service teachers' conceptual knowledge, procedural skills, and factual understanding which are prerequisite in problem-solving situations. Three scripts each were randomly selected from the group before and after the teaching experiment. The solutions and explanations provided by rural-based pre-service teachers in arriving at their answers assisted the researcher to examine their conceptual knowledge, procedural skills, and factual knowledge which constitute the problem-solving skills in vector calculus. With the solutions and the explanations and guided by the conceptual frameworks of Duval's semiotic representation and Zazkis et al. (1996) visual-analysis, the researcher was able to closely analyse responses, and this assisted in analysing how they related visual reasoning skills with analytical reasoning. Thus; the researcher provides a solution for each item (pre-test/post-test) and guided by the frameworks (i.e., Duval's and Zazkis et al's ferameworks) a coding structures were developed. Hence, the code structures are described the visual and analysis interactions that take place in each item solutions (i.e., the transfer between registers). The coding structures were then used as schema to analyse the rural-based pre-service teachers conceptual understanding, procedural skills and factural reasoning.

The solutions and responses presented by six rural-based pre-service teachers (i.e., three from each group) were based on the items 2.1a, 2.1b, 2.2(a), 2.2(b), 3.1, 3.2 and 3.3 under examination. The presentation started with the reproduction of the question

followed by solutions given by the student combined with his/her explanations taken to arrive at the solutions. This helped the researcher to give the visual-analysis presented in rural-based pre-service teachers' solutions and also analysed how they relate their spatial-visualisation skills to problem-solving in vector calculus.

The question requires students conceptual understanding, spatial-visualisation skills, and factual knowledge in providing the solution. Firstly, s/he needs to visualise and recall that the vector equation is in the $y-$axis (factual knowledge). Finally, sketching the vector field taking into consideration that the magnitude (length) of the vector increases as it is further away from the origin of the $xy-$plane (thus; conceptual and factual knowledge is required in this situation). This scheme was then compared with the solution presented by rural-based pre-service teacher CG012.

Question 2.1a Sketch and briefly describe the vector fields in 2-space given by $\vec{F}(x, y) = x\vec{j}$.

A solution provided by a rural-based pre-service teacher, CG012
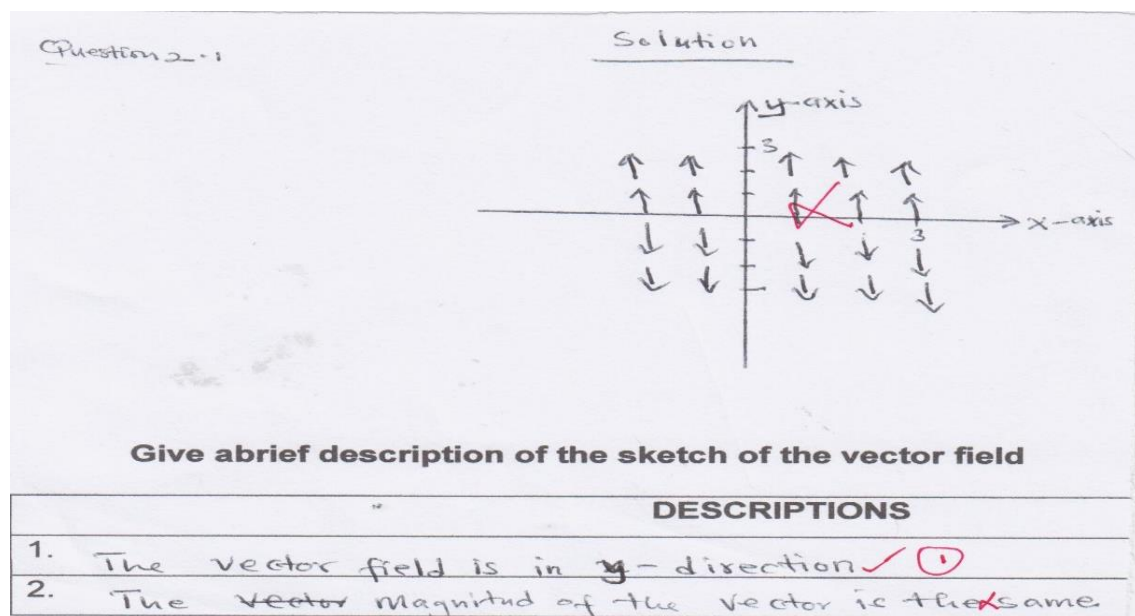
Figure 4.1: CG012's solution for the vector function $\vec{F}(x, y) = x\vec{j}$.

Table 4.3: Analysis of the Visual-Analysis in the solution of the rural-based pre-service teachers, CG012.

Visual	Analysis
V1. Question 2.1a vector field sketch	Analysis
V2. Visual step: setting the Cartesian plane	A1. Algebraic step: reading the vector function
V3. Represent the vector in the y-axis	A2. Read the vector function $\vec{F}(x,y) = x\vec{j}$ on the y-axis.
V3. Sketch for vector field	A3. Error. Wrongful vector field representation for both positive and negative y-axes.
V4. Sketch shows magnitude and direction.	A4. Error. The magnitude of the vector field drawn is the same even as they further away from the origin.

From the analysis on the solution presented by a rural-based pre-service teacher CG012 in table 4.3, s/he was able to adhere to factual knowledge by recalling that the vector value $\vec{F}(x,y) = x\vec{j}$ is in the y-axis. However, s/he was unable to recall that the length of the vector represents the magnitude hence, visual representation vector field must show an increase in length as the vector field is further away from the origin of the xy plane (refer to figure 4.1). This error showed a lack of both conceptual understanding and factual knowledge.

Question 2.1b A girl rides a bicycle from her house in the direction of $80^0 NW$ for $5m$ to the library. After borrowing some few books from the library, she rides back to the school in the direction of $10^0 NE$. The school is $7m$ from the library. Calculate the distance and the direction from the school from her home.

In the problem-solving situation of question 2.1b, it requires rural-based pre-service teachers' spatial reasoning coupled with both conceptual understanding and procedural skills. Firstly, they need a sketch to represent the vector word problem with a sketch and follow by writing the vector equation which requires spatial-visualisation skills and conceptual knowledge. Finally, the substitution of the values of the distance given in the vector equation derived from the word problem requires some procedural

skills (e.g., calculation of the magnitude of the vector). This scheme was then compared with the solution presented by rural-based pre-service teacher, CG022.

A solution provided by a rural-based pre-service teacher, CG022

Give three (3) points brief explaining how you arrived at your answer

	Explanation
1.	I found it difficult to write the mathematical (vector) equation
2.	I normally ignore word problem because it is too callecop...
3.	[5 Marks]

Figure 4.2: CG022's solution for the word problem in vector

Table 4.4: Analysis of the Visual-Analysis in the solution of the rural-based pre-service teachers, CG022.

Visual	Analysis
V1. Question 2.1b Visual step: Wrongful representation of vector diagram to the question	A1. Algebraic step: wrongful writing of equation. A2. Wrongful vector calculation.

From the solution presented and the table of visual-analysis (refer to figure 4.2 and table 4.4), the rural-based pre-service teacher (CG022) failed to represent the word problem hence, that led to his/her inability to write the correct equation. Also, it can be said that s/he lacks the conceptual understanding and spatial-visualisation skills required for the sketching and writing of the vector diagram for the word problem.

Question 2.2a. Convert the Cartesian coordinate $(x, y, z) = (1, \sqrt{3}, 2)$ to cylindrical coordinates.

[Hint: $r^2 = x^2 + y^2$, $tan\theta = \frac{y}{x}$, $z = z$]

In providing the solution for question 2.2a, procedural skills are required. The question is a 'routine type of question' which requires them to write the cylindrical coordinate equations and substitute the Cartesian coordinate points and compute to get the cylindrical coordinates. For question 2.2b, thus sketching the cylindrical coordinate points calculated in question 2.2a above requires students' spatial-visualisation skills. This scheme was then compared with the solution presented by rural-based pre-service teacher EG009.

Solution presented by a rural-based pre-service teacher EG009

Figure 4.3 EG009's solution for the conversion of the Cartesian coordinate $(x, y, z) = (1, \sqrt{3}, 2)$ to cylindrical coordinates and a graph in cylindrical coordinates.

Table 4.5: Analysis of the Visual-Analysis in the solution of a rural-based pre-service teacher, EG009.

Visual	Analysis
2.2a & 2.2b conversion between Cartesian to cylindrical coordinate system. Visual steps	Algebraic steps
2.2a Numerical register: copying the equation and substituting into the cylindrical coordinate equations.	A1. Coordinate points were substituted in the cylindrical coordinate equations $r^2 = x^2 + y^2$, $tan\theta = \frac{y}{x}$, $z = z$. A2. Calculation computed and cylindrical coordinates was found.

2b. V1. Geometric register: Set up the x, y, and y plane.	A3. Algebraic register: Coordinate points plotted on the x, y, and y plane correctly.
V2. Joining the cylindrical coordinate points correctly to form the 3D shape.	

Based on the analysis of the solutions (refer to figure 4.3 and table 4.5) presented by the pre-service teacher (refer to EG009), procedural skills were correctly adhered (e.g., analytical step A1 substituting into the cylindrical equations correctly). Again, conceptual understanding was demonstrated by the student leading to correct conversion between geometric and algebraic registers. Hence, it is justifiable from the analysis of the solutions presented by the student (EG009) that there was coordination between spatial and analytical thinking.

Question 3.1 Sketch the following 3D given by the algebraic equation $2x + y + z = 8$ in the Cartesian coordinate system.

In providing the solution for question 3.1 the student must apply his/her spatial-visualisation skills, conceptual understanding, and procedural skills. They need to find x, y, and z intercepts from which s/he can write the cartesian coordinate points and as such this requires both conceptual understanding and procedural skills. Furthermore, setting up the x, y, z cartesian plane and plotting cartesian coordinate points to form a tetrahedron 3D shape requires spatial-visualisation skills. This scheme was then used to analyse the solution presented by rural-based pre-service teacher, EG043.

A solution and steps provided by a rural-based pre-service teacher, EG043
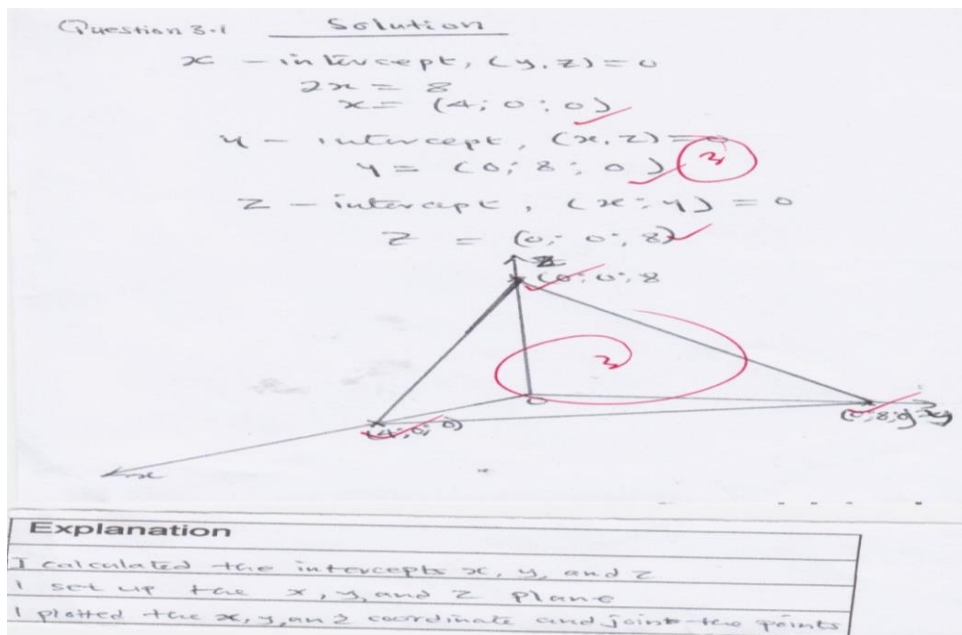
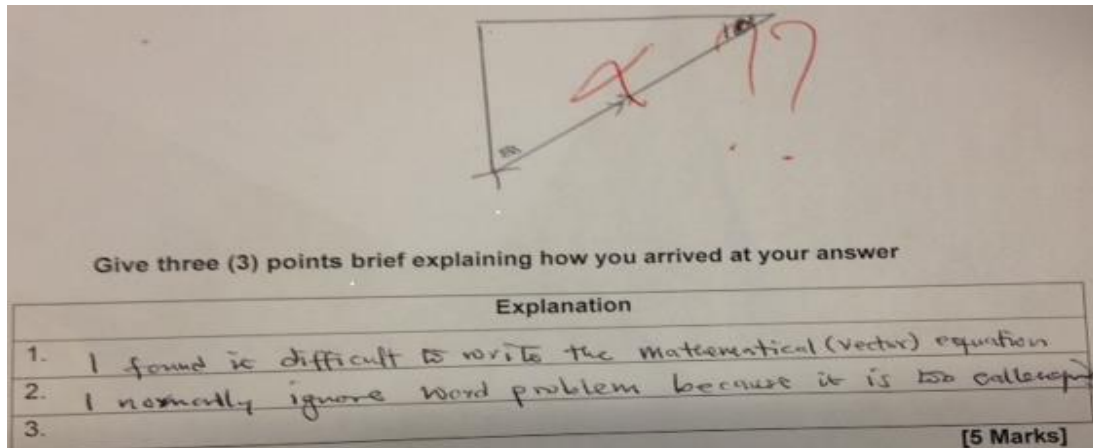Figure 4.4 EG043's solution for sketching the 3D shape of the algebraic equation $2x + y + z = 8$ in the Cartesian coordinate system.

Table 4.6: Analysis of the Visual-Analysis in the solution of a rural-based pre-service teacher, EG043.

Visual (V)	Analysis (A)
3.1 Multivariate calculus	
Visual steps	Analysis
V1. Numerical register: find the coordinate points for plotting the graph of the equation $2x + y + z = 8$.	A1. Algebraic register: Calculation of $x, y,$ and y intercepts.
V2. Geometric register: Correct setup of the $x, y,$ and y cartesian plane.	A2. Calculation of $x, y,$ and y intercepts and writing the coordinate.
V3. Sketch of the coordinates points to form the 3D shape.	A3. Coordinate points indicated on the $x, y,$ and y plane correctly.

Based on the analysis of the solution presented in table 9 above, the rural-based pre-service teacher (EG043), exhibited the connections between numerical, geometric, and algebraic registers. Furthermore, s/he followed the mathematical processes (i.e., calculated the $x, y,$ and y intercepts and wrote the coordinates) once more both conceptual understanding and procedural skills as demonstrated in the calculation and sketching (refer to figure 4.4).

Question 3.2 Sketch the graph of the region R is bounded by $h(x) = 2x$ and $g(x) = x^2$, and evaluate $\iint_R (x + 2y)dydx$ along the R with the interval $0 \leq x \leq 2$.

The solution to question 3.2 requires rural-based pre-service teachers to first recall (factual knowledge) that the equation $h(x) = 2x$ is a straight line graph and $g(x) = x^2$ is a parabola graph and sketch or visualize the region bounded by the two graph functions (spatial-visualisation skills). This should be followed by writing the double integral equation for the region bounded by the graph functions and compute the area which requires both conceptual understanding and procedural skills. This scheme was then used as guide in analysing the solution presented by rural-based pre-service teacher, CG016.

Solution presented by a rural-based pre-service teacher CG016

Give three (4) points brief explaining how you arrived at your answer

Step		Explanation	
1.	I sketch the graph, without the intercept	The graphs is difficult to draw	
2.	The graph $y=x^2$ have no intercepts	I am not I got the intercept correct	
3.			
4.			

Figure 4.5: CG016's solution for sketching the graph of the region R bounded by $h(x) = 2x$ and $g(x) = x^2$, and evaluating $\iint_R (x + 2y)dydx$ along the R in the interval $0 \leq x \leq 2$.

Table 4.7: A close scrutiny of the Visual-Analysis in the solution of the rural-based pre-service teachers, CG016.

Visual (V)	Analysis (A)
3.1 Question on Double integral Visual steps	Analysis
V1. Conversion: Algebraic registrar to geometric registers: $h(x) = 2x$ and $g(x) = x^2$.	A1. Treatment of algebraic functions $h(x) = 2x$ and $g(x) = x^2$ is equated to zero find their intersection.
V2. A correct sketch of $g(x) = x^2$ but incorrect representation of the linear graph $h(x) = 2x$.	A2. Error: Incorrect setup of the double integration. A3. Incorrect calculation and appear not sure. A4. Unable to calculate the intersection of the two functions $h(x) = 2x$ and $g(x) = x^2$ correctly.

A close scrutiny of the solution presented by the rural-based pre-service teacher (CG016) shows that, s/he lacks factual knowledge thus; s/he failed to recall that function $h(x) = 2x$ is a linear equation. Having failed to sketch the functions of $g(x) = x^2$ and $h(x) = 2x$, was unable to set up double integral for the equations $g(x) = x^2$ and $h(x) = 2x$, and finally failing to perform the calculations to get the coordinates for the functions of $(x) = x^2$ and $h(x) = 2x$ was an indication of a lack of conceptual knowledge and procedural skills. Hence, it can be concluded that s/he failed to demonstrate the coordination between spatial and analysis reasoning needed for problem-solving situations (refer to figure 4.5).

Question 3.3 A cube has side of length 4. Let one corner be at the origin and the adjacent corners be on the positive xx, yy and zz axes. If the cube's density is proportional to the distance from the xy-plane, find its mass. [Hint: the density of the cube $f(x, y, z) = tz$ where t is constant.

In providing the solution for question 3.3, it firstly requires a sketch or imagery of the 3D cube shape (spatial-visualisation skills) and one must recall that the dimensions of a cube are equal (i.e., factual knowledge). This is followed by writing the triple integral equation and computing the mass and as such requires students' conceptual understanding and procedural skills. This scheme was to in analysing the solution presented by rural-based pre-service teacher, EG036.

**A solution and explanation provided by a rural-based pre-service teacher, EG036**
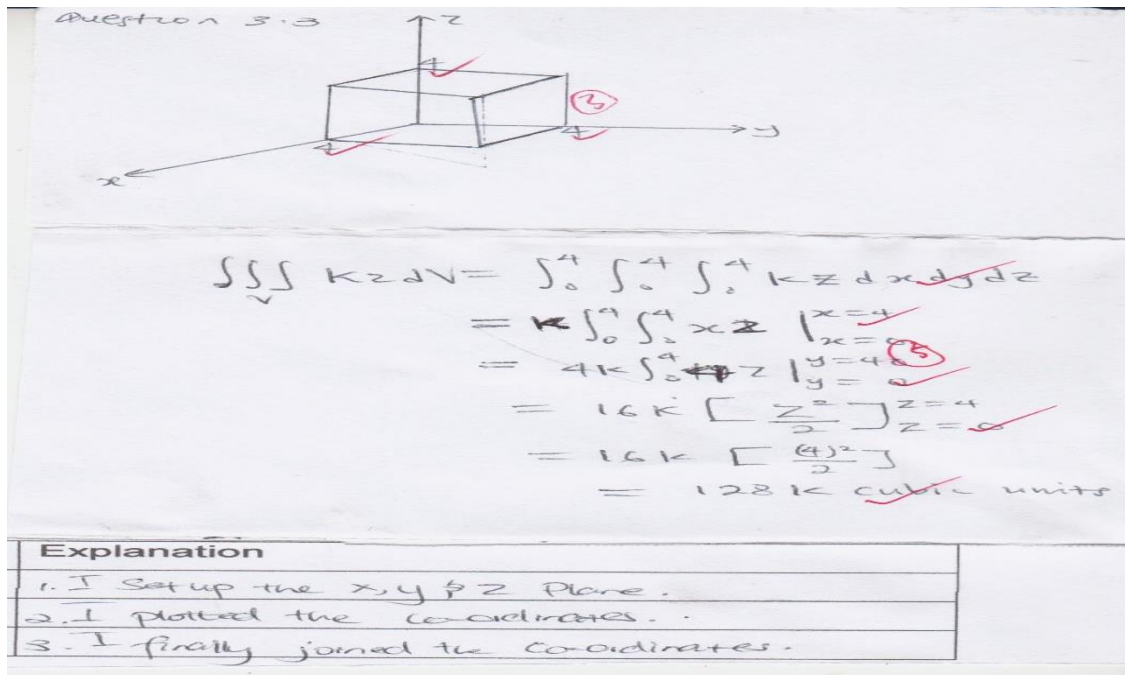
Figure 4.6: student EG036's solution for the calculation the mass of a cube with density of

$$f(x, y, z) = tz$$

Table 4.8: A close scrutiny of the Visual-Analysis in the solution of the rural-based pre-service teachers, EG036.

Visual	Analysis
Q.3.3 Triple integral Visual steps	Analysis steps
V1. Conversion from Algebraic to geometric register.	A1. Treatment in algebraic register.
V2. Correct sketch of cube in the x, y, and z plane	A2. Assigned correct dimension to the cube. A3. Generate triple integral. A4. Correct substitution of lower and upper limits. A5. Correct calculation of the volume of the cube.

Based on the analysis (table 4.8) and the solution presented by the rural-based pre-service teacher EG036 (refer to figure 4.6) s/he demonstrated a high sense of smooth coordination between visual and analytical reasoning. First of all, by sketching correctly a 3D cube figure and indicating the dimensions to represent the problem situations, factual knowledge was demonstrated. Furthermore, s/he has demonstrated both conceptual knowledge and procedural skills having correctly; generated the triple integral equation, indicated lower and upper limits, and performed the calculations of the volume of the cube.

4.3.2 To what degree do rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement?

In the literature review (refer to section 2.2.3), researchers such as: Höffler and Leutner (2011), Smith and strong (2001), Van Garderen and Montague (2003) and Jansen et al. (2013) hypothesised that the ability to visualise or use a correct sketch to represent problem aids in problem-solving situations. Delice and Ergene (2015) support the notion but also further suggested that sketching of graphs/shapes to represent a problem situation is cognitively demanding hence computer software is the solution. The setback from Delice and Ergene (2015) means that computer software (e.g., GeoGebra, Mapel, CAS, MATLAB and many more) is able to perform functions such as; graph sketching, generating geometric figures, vector plots, and many more which promote mathematics problem-solving situations. Hence, MATLAB was employed as a pedagogical tool to assess the degree to which rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement.

The experimental group went through laboratory activities such as; using the MATLAB to generate 3D solid shapes, using plot vector fields, viewing geometric shapes from different views and a region bounded by surface as shown in figures 3.3, 3.4 and 3.5.

Meanwhile, the control group had their lessons during the same time frame however; this was done using the traditional method. The traditional approach seen the researcher sketching vector fields, and drawing 3D figures on the chalkboard and giving explanations about them, and occasionally asking the students questions. Also, the students solved problems on vector calculus individually. Subsequently, a post-test was then administered and comparison between the control and experimental

groups was carried out based on the post-test scores. An independent sampled t-test was carried out to evaluate whether there was significant difference in the post-test score between the two groups (refer to Tables 4.9 and 4.10).

Table 4.9: Summary of group statistics and the Independent Sample Test on the post-test.

Post-Test Score	Group	Number	Mean	Std. Deviation	Std. Error Mean
	Control Group	50	22.20	3.169	.448
	Experimental Group	50	29.12	3.274	.463

From the table 4.9, the mean post-test score of the experimental group was ($\bar{x} = 29.12, SD = 3.27$) and the mean post-test score of the control group ($\bar{x} = 22.20, SD = 3.169$). This showed there was an increase in achievement test after the two teaching experiments (i.e., the use of MATLAB as pedagogical tool in teaching and learning and the traditional approach) compared to the baseline assessment test (pre-test) which was for the control ($\bar{x} = 14.62, SD = 3.78$) and for the experimental group ($\bar{x} = 15.40, SD = 3.65$). An independent sample test was further carried out to find out whether there was significant difference between the groups (see table 4.10).

Table 4.10: Independent sample test for achievement test on vector calculus (Post-test).

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | | | | |
| | | F | Sig. | T | Df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95% confidence interval of the Difference | |
									Lower	Upper
Post - Test Score	Equal variances assumed	.239	.626	-10.739	98	.000	-6.920	.644	-8.199	-5.641
	Equal variances not assumed			-10.739	97.895	.000	-6.920	.644	-8.199	-5.641

T-value significant at $p < 0.05$

Table 4.10 shows a mean difference of 6.920 between the groups and the p-value 0.00. The p-value was lower than the T-value significant ($p < 0.05$) in favour of the experimental group. The inference was that, there were cognitive interactions between the dynamic environment, MATLAB and the students in the course of performing the laboratory activities as such generating geometric figures, rotating and viewing them from different views, generating vector fields, etcetera. Also, it means the laboratory activities mediate coordination between spatial-visualisation and vector calculus achievement. Thus MATLAB promotes the performances of the experimental group in vector calculus achievement test. Hence the use of dynamic environments and computers as pedagogical tools in teaching and learning in traditional classrooms enhances mathematics problem-solving skills and achievement.

4.4 How do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills?

Research evidence from many disciplines supports the claim that dynamic environments such as MATIMATICA, CAS, and MATLAB enhance spatial-visualisation skills. This shared relationship led to the design of some computer games which are tailored in

enhancing spatial reasoning skills. Building of blocks, playing chess, matching of designs, and spatial puzzles are among a few computer programs claimed to have a positive relationship with spatial-visualisation enhancement.

In the field of cognitive science, the ability to accurately predict or imagine orientation of cubes with respect to the time taken is a reflection or measure of one's spatial-visualisation skills. Research has established strong claims on this shared relationship which has led to spatial skills' standard tests such as; Mental rotation task (Shepard & Metzler, 1971); mental paper folding (Shephard & Feng, 1972); and Purdue Visualisation rotation (Guay, 1976). This relationship seems not only to share mental rotation and spatial reasoning skills however, research evidence on dynamic environments such as MATIMATICA, CAS, and MATLAB enhance mental rotation skills ad as well as spatial-visualisation skills.

From the aforesaid, MATLAB was employed as a pedagogical tool in teaching vector calculus for the experimental group. As outlined in chapters 2, 3 and section 4.2.2, experimental groups were taken through computer activities (such as; generating geometric figures, rotating and viewing them from different view, generating vector field, etcetera). Literature evidence available indicates that these activities do not only have an effect on achievement but also, positively affect students' spatial-visualisation skills. Having subjected the experimental group to these activities (refer to figures 3.3; 3.4; and 3.5), the first ten items from the Purdue Visualization Rotation Test was administered to assess the effect of MATLAB on their spatial reasoning (refer to appendix 5).

The control group was used as a baseline since statistical evidence from tables 4.1 and 4.2 indicated that the two groups are of the same prior knowledge. Hence, the standard Purdue Visualisation Rotation Test was then administered for both the experimental and the control groups. The results were compared using the independent sampled t-test to evaluate if there is significant difference in the mean spatial-visualisation skills difference between the two groups (refer to Tables 4.10 and 4.11).

Table 4.11 Group Statistics

Spatial-Visualisation Test Score	Group	N	Mean	Std. Deviation	Std. Error Mean
	Control Group	50	5.86	.86	.12
	Experimental Group	50	6.86	1.18	.17

Table 4.11 shows the control group obtained the following mean score ($\bar{x} = 5.86, SD = .857$) whereas the experimental group obtained the following mean score of ($\bar{x} = 6.86, SD = 1.178$). The independent sample test further revealed that, the mean score difference and the p-values of the control group and the experimental group (refer to table 4.12).

Table 4.12: Independent Sample Test

		Levene's Test for Equality of Variances		t-test for Equality of Means					95% confidence interval of the Difference	
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Spatial-Visualisation Score	Equal variances assumed	13.423	.000	-4.853	98	.000	-1.000	.206	-1.409	-.591
	Equal variances not assumed			-4.853	89.530	.000	-1.000	.206	-1.409	-.591

The findings presented in table 4.11 show that the mean difference between the groups was 1.000 whereas the p-value was 0.00 which is lower($p < 0.05$). An independent sample t-test results from table 4.12 revealed that there was a significant difference between the control group and the experimental group. This tended to see the experimental group performing better in the Purdue Spatial-visualisation skills Test than the control group. This finding relates to earlier studies whose results show that three-dimensional dynamic geometry software can help to develop spatial ability (Oldknow & Tetlow, 2008; Güven & Kösa, 2009; Baki et al., 2011). In this respect, the feature that enables the construction and animation of three-dimensional shapes on a two-dimensional screen could be a key factor in the development of spatial

visualisation skills. This possibility is supported by the frequent use of implementations such as rotating and imagining views from different perspectives in training programs that are directly designed to improve spatial skills. However, the rural-based pre-service teachers in the present study developed spatial visualisation skills in the process of studying vector calculus using MATLAB software. This means that spatial visualisation skills can be developed using dynamic geometry software in the context of a vector calculus course.

4.5. Chapter Summary

This section presents a summary of the analysis of data from the teaching experiment. A descriptive statistics and independent sample t-test were carried out to assess whether or not there was significant difference in the mean pre-test score and the mean post-test score between the control group and the experiment. Again, the independent sample t-test was employed to evaluate the effect of the dynamic environment, MATLAB, on the rural-based pre-service teachers' spatial-visualisation skills. Firstly, there was no significant difference in the mean pre-test score (refer to tables 4.1 and 4.2) between the groups; an indication that both the control group and experimental group were at the same baseline.

The statistical difference between post-test results of the groups again employing the independent sample t-tests indicated there was significant difference between the two groups however, this showed that the experimental group outperformed the control group (refer to table 4.9 and 4.10). Further assessment was carried out to assess the effect of the dynamic environment, MATLAB, on rural-based pre-service teachers' spatial-visualisation skills, employing Purdue spatial-visualisation test/rotation (PSVT/R), showed there was significant difference between the two groups. Again, the independent sample score showed the experimental group performed better than the control group (refer to tables 4.11 and 4.12).

The findings from this study have been found to be consistent with several research evidence of dynamic environments, MATLAB shared a link in enhancing problem-solving skills, achievement in vector calculus, and spatial-visualisation skills. Reflection on the teaching experiment, thus employing MATLAB as a pedagogical tool

in teaching and learning vector calculus, turned out to be more successful than the traditional approach.

Accordingly, to integrate educational technological learning tools in teaching and learning promotes conceptual knowledge, procedural skills, and factual understanding in learning mathematics. This should be reviewed in terms of long practicality engagement with the learning software, visual stimulations, self-instructive, and cognitive development.

CHAPTER FIVE

DISCUSSION OF FINDINGS

5.1 Overview

The chapter presents a discussion of the findings in relation to the research questions, literature review, theoretical frameworks, and their implications in learning vector calculus. The research attempted to analyse MATLAB instruction on problem-solving skills and achievement on vector calculus. Furthermore, the relationship between the dynamic environment, MATLAB, and spatial-visualisation skills was also examined. Guided by Duval's (1996) semiotic representation theory, the level geometric thinking was discussed and Zazkis et al's (1996) Visual-Analysis framework was used to scrutinise visual-analysis of students' solutions presented. The next chapter presents a discussion of the findings based on the research questions;

1. How do rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus?

2. To what degree do rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement?

3. How do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills?

5.2 Discussion

5.2.1 Prior Knowledge Assessment test (Pre-test)

This research investigated the role of spatial-visualisation in problem-solving in vector calculus viz the MATLAB enhancing problem-solving skills, achievement, and spatial-visualisation skills. A descriptive statistics and independent sample t-test were used to determine whether or not there was significant difference between the pre-test. The descriptive statistics (refer to tables 4.1 and 4.2) showed that the mean score of prior knowledge assessment test for the experimental group was (15.40) and the control group was (14.60). Whereas the independent sample t-test (refer table 4.2) showed that, the mean pre-test score difference between the groups was 0.78 and the p-values

(2-tailled) was 0.27 ($p > 0.05$). This means there was no significant difference between the mean pre-test between the experimental group and the control group. Furthermore, this explains that both groups were on an equal baseline in terms of prior knowledge in content area.

5.2.2 Achievement test after the teaching experiment (Post-test)

While learning vector calculus with the MATLAB, the experimental group performed implementations such rotation planes in space, generating 3D solid shapes (e.g., Spheroid, Cube, etcetera), sketching vector fields and viewing geometric shapes from different angles (refer to figures 3.3, 3.4, and 3.5) using the MATLAB; whereas the control group had the same timeframe of teaching and learning the same topics on vector calculus. The researcher drew 3D shapes and vector fields on the chalkboard and gave explanations and exercises. After, an achievement test was administered to the groups to analyse the effects of MATLAB instruction on teaching and learning of vector calculus. The descriptive statistics (refer table 4.9) showed the mean achievement test score of the control group was ($\bar{x} = 22.20, SD = 3.169$) and for the experimental group was ($\bar{x} = 29.12, SD = 3.274$). This showed that there was an increase in the mean score of the achievement test for both groups.

However, the independent sample t-test (refer table 4.10) results further revealed that, mean difference between the groups was 6.920 and the p-value 0.00 which is lower than the ($p < 0.05$) which means that, there was significant difference between the two groups. Even though there was an increase in the mean scores in the achievement test (post-test) for both groups yet, the experimental group tended to outperform the control group in the achievement test as shown in table 4.10. This difference in the groups' mean post-test score may stem from these implementations of the laboratory activities.

A review of relevant literatures in chapter 2 outlined that; manipulations, rotations or imagery of 3D objects, in space and predicting the orientations of cubes are factors that enhance mathematics problem-solving situation and achievement (Van Garderen & Montague, 2003; Smith & Strong, 2001; Ho & Lowrie, 2014; Hoffkamp, 2011; Höffler, 2010; Jansen et al., 2013; Koch, 2006; Mahir, 2009; Wai et al., 2009; Utall et al., 2013). Hence, referring to the mean difference of the achievement test score (refer tables 4.9

and 4.10) and closely scrutinising the solutions and comments presented by both groups, the conclusion drawn was that there was much more improvement in problem solving in the experimental group than in the control group. Furthermore, this also stipulated that in comparison the method of using the dynamic environment, MATLAB, in traditional classrooms in teaching and learning vector calculus was far more effective than the traditional approach.

5.2.2.1 Question 1: **Rural-based pre-service teachers' ability to switch between graph and algebraic registers.**

Question 1 was meant to examine the conversion between two registers thus, graphic and algebraic registers.

The findings revealed that, rural-base pre-service teachers struggled to link algebraic function to graph. For instant, a rural-base pre-service teacher CG 005 (refer figure 5.1) raised the concern that s/he does not constantly work with these algebraic equations: $x^2 + y^2 = 3$; $z = x^2 + y^2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ hence, s/he found it difficult to match them to them with their respective graphs. S/he further claimed that, "it is not easy to memorise all equations with their graph functions."

QUESTION 1

MATCH EACH OF THESE EQUATIONS WITH ITS GRAPHS (A, B, C, D AND E) AND WRITE ONE STATEMENT FOR THE CHOICE OF YOUR ANSWER IN THE BOX PROVIDED BELOW EACH QUESTION.

1.1 $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
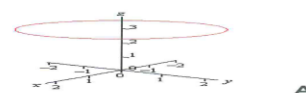
B ~~X~~ I am not sure of the ~~answer~~. It is not easy to memorise all graph functions.

Accordingly, numerous studies have evidenced that students' inability to switch from different coordinate systems is a hindrance to problem-solving in vectors (Mahir, 2009; Kohl & Finkelstein, 2005; Wagner et al., 2012). According to Duval (2006), mathematical thinking requires representation systems (i.e., sketching graphs or geometric figures) and cognitive coordination (i.e., computing of numbers or problem-solving). From this perspective, mental representations are essential for cognitive thinking however, semiotic representations can be much more cognitively demanding of the creative potential in mathematics stems from these transformations. Kösa (2011) asserted that, statistic sketches of 3D objects on 2D paper may be incomplete, thus causing inaccurate interpretations; and even if the drawings are perfectly executed, it is impossible to view or rotate the 3D shapes from different angles, due to the static nature of a drawing. Interestingly, the computer dynamic environment plays a significant role in these conversions. Computer dynamic environments support the

operations of both algebraic, graphic representations of geometric shapes in 2D and 3D interface, allow rotation of 3D shapes in different projections, and conversions between registers.

However, after the laboratory session, the experimental group did not only become more familiar with 3D geometric shapes (such as; planes, 3D shapes such as ellipsoid, cone, cube, hyperboloid, and many more) but they were able to generate and display them on the screen (refer to figure 5.2).
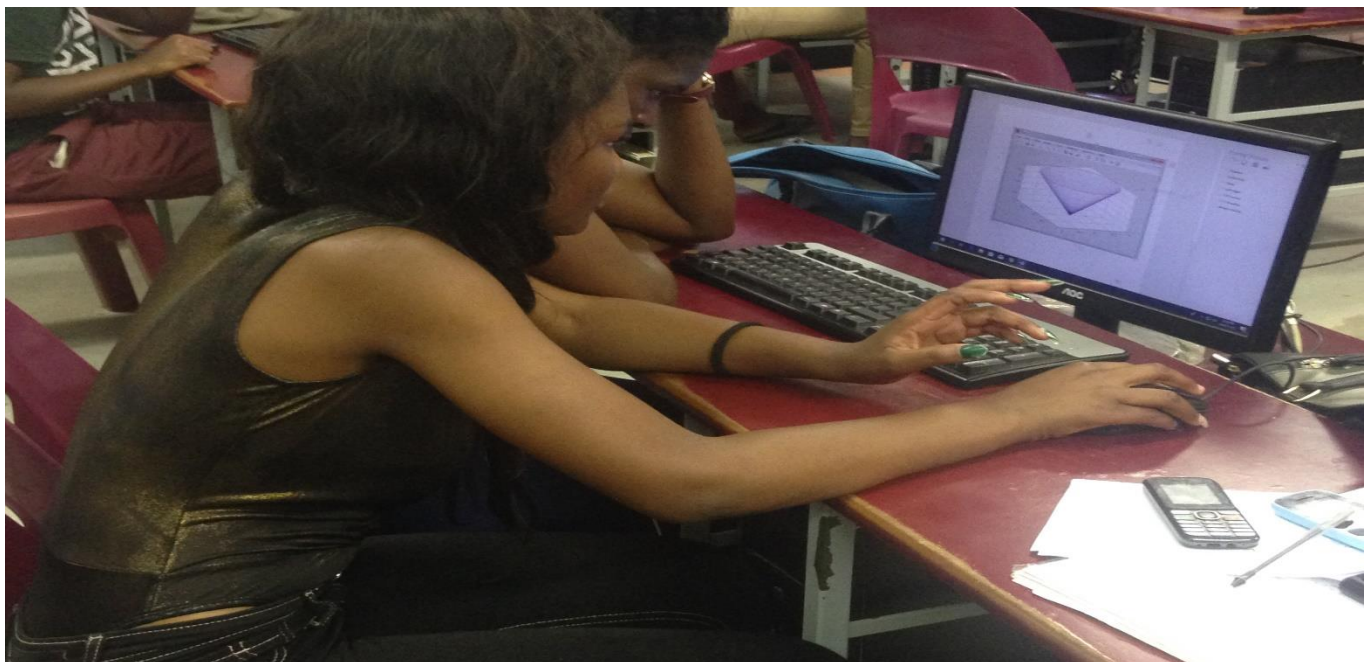
Figure 5.2: Section of students in the UNIZULU HP lab during practical session.

Thus, after the treatment (i.e., laboratory activities) the score btween the two groups on question 1 (thus; in matching the quations to their respective graphs) was analysed and presented in the table 5.1 below.

Table 5.1: The analysis of question 1 in percentage after the teaching experiment.

Question	Percentage score by control group (%)	Percentage score by experimental group (%)

1.1 $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	56	75
1.2 $z=x^2+y^2$	64	87
1.3 $xy=3$	69	89
1.4 $y=x^3$	72	95
1.5 $x^2+y^2=3$	74	94

The results from table 5.1 indicate that, there was significant increase in the scores for the control and experimental groups. This means that students gained from both approaches but however, it appeared the experimental group performed better. The findings aligned with the claim by some researchers such as Arıcı and AslanTutak (2015), Tokpah (2008), Ke (2008), Kebritchi et al. (2010) and Baltacı et al. (2015) that dynamic visual tools (such as: CAS, MATHEMATICA, Mapel, and MATLAB) could enhance mathematics achievement. Hence, based on the statistical results obtained and scrutinising rural-based pre-service teachers' conceptual knowledge through the solutions and comments provided, we have reason to support the claim by Arıcı and AslanTutak (2015), Tokpah, (2008) and Baltacı et al. (2015).

Furthermore, from observations, the interactions between the rural-based pre-service teachers' and the computer graphic interface seems to engage students for longer periods of learning which also confirmed the hypothesis by Kebritchi and colleagues (2010). As theorised by Duval (1996, 2006), every mathematical reasoning is grounded on representations and these representations are transformed within registers. Indeed, the theory explained the transformations that took place thus; a transfer from the mathematical equations (algebraic register) to graphs function (geometric registers). This transformation between registers mediates problem-solving skills and achievement (Mahir, 2009). Accordingly, it owes to the laboratory activities experimental students had gone through enabling them to obtain a higher score compared to the control group in terms of matching mathematical equations to their respective graphs (refer to table 5.1). In summary, the findings give a convincing report to support the research claim that dynamic visual tools are technology cognitive tools and as such their interactive environment enhances cognitive reasoning as theorised by Ke (2008). Hence, the use of a dynamic visual tool as a pedagogical tool

to enhance problem-solving skills, achievement, and spatial-visualisation skills can work equally well with rural-based pre-service teachers.

5.2.2.2 Question 2a: How do rural-based pre-service teachers apply their spatial-visualisation skills in solving vectors problems?

Question 2a was meant to evaluate how rural-based pre-serviced teachers integrate spatial-visualisation skills and analytical reasoning in sketching vector diagrams. It was found that even though rural-based pre-service teachers had gone through some fundamentals of vectors in high school yet, they had difficulty in handling vector operations (such as; sketching and describing vector diagrams, addition and subtraction of vectors. A close scrutiny of the solution presented by the rural-based pre-service teacher EG012 on sketching vector field for the vector equation $\vec{F}(x,y) = x\vec{j}$ and the descriptions given thus: "vector field lines are not to get closer to each other", "the same distance is maintained between vector field lines", and "vector field lines move in upward directions and downwards in the Cartesian plane." From these descriptions, the rural-based pre-service teachers failed to understand that the closer the vector field lines indicate the strength of the vector field and failed to represent the length of the vector according to vector magnitude (refers to figure 4.1).

Question 2b was meant to examine rural-based pre-service teachers' ability to convert word problems to algebraic equation. Surprisingly, most students did not attend to question 2b and among were some of the explanations picked up: for instance, the rural-based pre-service teacher (CG005) indicated that, "I always find it challenging to extract mathematics equation correctly from word problems". In addition, s/he made a conclusive comment which drew more attention claiming that "I don't waste my time on word problems" (refer to figure 4.2).

As outlined in sections 2.5 and 2.5.1, relevant studies have evidenced against students' ability to draw correct sketches to represent vectors and also to interpret vector diagrams correctly (Nguyen & Meltzer, 2003; Gire & Price, 2014; Dray & Manogue, 2003). Also, other researchers such as: Mahir (2009); Kohl and Finkelstein (2005) and Wagner et al, (2012) further the argument claiming that, students struggled to convert vector word problems into vector equations. Accordingly, the claims by the students 'CG005' (refer to figure 4.2) give a convincing reason to support the research claim that students struggle in switching from word problems to algebraic equations.

This was also affirmed by Gire and Price (2014) who claimed that students' inability to sketch vector diagrams and interpret them correctly may be as a result of instructors relying mainly on traditional approaches of instruction. The idea that can be drawn from Gire and Price's (2014) claim is that even though vector tasks require high spatial-visualisation skills yet, fewer tasks are designed in this effect. This further explains that less attention is paid to graphical functions.

Besides, this also affirms the relationship between spatial-visualisation skills and mathematics aptitude. Piaget and Inhelder (1966) hypothesized that spatial-visualisation skills is a cognitive tool which enhances problem-solving situations. In physics education, researchers Nguyen, Gire and Rebello (2010) suggested that students' problem-solving skills can be enhanced across multiple representations. Accordingly, the experimental group having gone through the laboratory activities using MATLAB to: generate vector plot; to displace some word problems into mathematical equations; and to perform vector operations (such as; addition and subtraction of vectors, dot product, and cross product of vectors) they were able to make their own conjectures. Among these were: vector fields do not cross each other; the strength of vector field is represented by many close loops; and the magnitude of a vector is displaced by the length.

A reflection on Zazkis et al's (1996) theory of visual-analysis highlights the coordination that exists between spatial-visualisation skills and analytical reasoning. The theory further elaborated that, analysis starts with coordinates of an object (visualization) and as it continues richer visuals are formed which help in problem-solving situations. Hence, the experimental group's ability to make their own conjectures on vector fields may stem from the cognitive interactions between them and the dynamic software environment.

5.2.2.3 Question 3: How do rural-based pre-service teachers apply the spatial-visualisation skills in solving integral calculus?

Questions 3.2 and 3.3 were meant to examine rural-based pre-service teachers' coordination between registers in problem-solving situations. The analysis of question 3.2 revealed that most of them, nearly 89%, performed the calculation correctly but somehow they struggled with the sketch of the 3D shape. From this indication, it can be said that they are conversant with routine tasks which demand procedural skills. However, it was not the case for question 3.3 thus; nearly 62% of the students failed to recollect that a cube has all sides equal. It was also observed that few of them who drew a correct diagram to represent the cube were able to write the integral equation (refer to figure 4.6). Here were a few excerpts of explanations given by the students EG036:

Steps explained by the rural-based pre-service teacher (EG036) to question 3.3.

1. I found it challenging to draw the 3D shape of the cube.

2. Also, only the length of the cube was given in the problem

3. Hence, it was difficult to generate the triple integral.

However, a follow up on the same student's scripts after s/he having gone through the laboratory activities showed that a different explanation was given thus;

1. I firstly sketched a cube in the x, y and z plane.

2. This guided me to set up the integral function.

3. I then performed the integration to get the volume of the cube (refer to figure 4.6).

From the rural-based pre-service teacher's (EG036) excerpts it can be deduced that, students are more comfortable performing routine task such as working with mathematics equations but feel reluctant or unable to work in graph functions. Henceforth, they struggle in switching between registers (e.g., geometric register to algebraic register) perhaps this may be too cognitive demanding as hypothesised by Sweller (1999). Accordingly, this affirms Habre's (2002) and Trigueros' (2004) assertion that students are reluctant to work in geometric registers.

Furthermore, it can be concluded that rural-based pre-service teachers who were able to draw a correct sketch (3D shape of cube) to represent the problem accurately wrote the integral equation and performed the integration procedure (refer to figure 4.6). However, the reverse is the case giving reason to support Van Garderen and

Montague (2003) claim that students' difficulties in problem-solving owes to their inability to visualise mathematical concepts, manipulate and interpret geometrical shape meaningfully. To add, from the analysis of the solution and the excerpts, it can be said that an accurate sketch of the 3D cube shape to represent the problem guided s/he in the problem-solving situations (refer to figure 4.6).

Reflecting on Duval's (1996) theoretical framework of semiotic representation, the theory outlined that a mathematics concept is based on representations and this consists of coordination between registers. From this perspective, it can be said that there was coordination between the 3D cube sketched by the student (geometric register) and the computation of the integral equation (algebraic register). Accordingly, an accurate representation coupled with factual knowledge of a cube (i.e., a cube has all sides equal) demonstrated in the sketched graph (refer to figure 4.6) may have aided in writing the correct computation of the mass of the cube. Zazkis's et al. (1996) visual-analysis model explains that in a problem-solving situation the step begins with visualisation and coordinates with analysis. The two variables (i.e., visual and analysis) complement each other in every stage resulting in producing richer visualisation skills which facilitate a better problem-solving situation. Hence, the reason behind all rural-based pre-service teachers (e.g., as we have seen in the solution presented by the student EG036 after treatment) who accurately used a sketch to represent the question tended to present the correct solution. Surprisingly, from the statistical evidence (refer to tables 4.9 and 4.10) and the content analysis of the solutions presented, the experimental group performed much better than the control group in switching between registers. Hence, the conclusion can be drawn is that the dynamic visual environment, MATLAB, mediates in the conceptual understanding, procedural skills, and factual reasoning. However, the exact mechanism explaining this association between visual and analysis was not accounted for in the scope of this research study.

5.2.2.4 Students' performance against PSV/RT

The main aim of this assessment was to assess the influence of dynamic software, MATLAB instruction, on rural-based pre-service teachers' spatial-visualisation skills. The experimental group went through laboratory activities such; as generating vector

fields, generating and viewing geometric figures different viewpoints, etcetera with the MATLAB in teaching and learning of vector calculus (refer to the figures 3.3, 3.4, and 3.5). For the control group, geometric shapes were sketched on the chalkboard and explained along with teaching and learning of vector calculus.

This was followed by an assessment test to assess the effect of the MATLAB on the pre-service teachers' spatial-visualisation skills. The section A of the PSVT/R (i.e., visualization of rotations) comprising of 10 multiple questions was adapted and used in this effect. A descriptive statistics and independent sample t-test were used to examine whether or not there was significant difference between the control and experimental groups. The statistical scores revealed that there was significant difference between the groups and the experimental group performed better in the PSVT/R than the control group (refer tables 4.11 and 4.12).

In review of relevant literature, several research findings have evidenced that computer-based 3D visualisations support spatial-visualisation skills (Chirstou et al., 2007; Oldknow & Tetlow, 2008). Generating and viewing geometric figures in space, cubes arrangement, and predicting the orientations of cubes are factors identified to have a link with spatial-visualisation skills (Cheng & Mix, 2014; Uttal et al., 2013; Arıcı & AslanTutak, 2015). This may stem from the features that enable the creation and animation of 3D display on the screen of dynamic visual tools that could play a major role in the development of spatial-visualisation skills. Apparently, the results obtained from Purdue spatial-visualisation/rotation test revealed that the experimental group developed spatial-visualisation skills while learning vector calculus with the dynamic visual tool MATLAB (refer to table 4.9 and 4.10). This finding seems to support the theory by Cheng and Mix (2014), Uttal et al. (2013) and Arıcı and AslanTutak (2015) that spatial-visualisation skills are trainable through dynamic visual tools in the context of manipulation and orientation of geometric shapes.

In addition, Shepard and Metzler (1971) made strong arguments about an association between physical manipulation or rotations of the cubes (i.e., mental rotation task) and spatial-visualisation skills. They further claimed that these two variables, mental rotation tasks and spatial-visualisation skills both share a link with mathematics problem-solving skills. Researchers Hubbard, Piazza, Pinel and Dehaene (2005), Umilta, Priftis and Zorzi (2009) and Park and Brannon (2013) have deepened the

argument claiming that there are interconnections among mental rotation, spatial-visualisation, and mathematics achievement. Accordingly, the research findings based on the content analysis, achievement test (refer to table 4.9 and 4,10), and PSVT/R (refer to 4.11 and 4.12) showed that MATLAB, as a pedagogical tool, has a positive effect on pre-service teachers' spatial-visualisation skills, problem-solving skills and achievement.

In Duval's (1996) conceptual framework, every analytical reasoning starts with representation and this exists between at least two registers. In principle of the framework, every mathematics problem-solving starts with geometric register to algebraic register (e.g., graphic register to analytical register). Zazkis and colleagues (1996) also argued along the same line of thought thus, according to their V-A model mathematics problem-solving begins with a visual step (refer to figure 2.11). Furthermore, Zazkis and the colleagues postulated that as the interactions between visual and analysis continue to get richer, visuals are generated and this mediates accurate problem solving situation.

Drawing from both theories, it can be deduced that individual's spatial-visualisation skills measure mathematics aptitude. Comparatively, both statistical mean post-test score (refer tables 4.9 and 4.10) and the Purdue spatial-visualisation test score (refer table 4.11 and 4.12) have seen the experimental group perform much better than their control group. Hence, from the statistical and content analyses (refer to EG043 and EG009), the study supports the claim by: Hubbard, Piazza, Pinel and Dehaene (2005), Umilta`, Priftis and Zorzi (2009) and Park and Brannon (2013) that there is a shared link between the dynamic visual tool, MATLAB, spatial-visualisation, problem-solving, and achievement.

Hence, from the results obtained from the baseline assessment through statistical results and content analysis, rural-based pre-service teachers poorly apply their spatial-visualisation skills in problem-solving situations. However, the analysis on using the MATLAB instruction in teaching and learning vector calculus revealed that;

- The degree rural-based pre-service teachers apply their spatial-visualisation skills correlate with their vector calculus achievement and
- The dynamic software environment such as MATLAB influences rural-based pre-service teachers' spatial-visual skills.

5.3 Summary

This section gives a recap on the discussion of findings. Duval's (1996) semiotic representation theory and Zazkis et al's (1996) VA frameworks were applied to analyse the solutions presented by the students' solutions. Overall, there was no achievement difference (i.e., no significant difference) on the pre-test administered to the groups ($0.27 > 0.05$). After the teaching experiment (i.e., MATLAB as pedagogical teaching tool against the traditional method of teaching), two assessments were administered the post-test and Purdue spatial-visualisation test. The experimental and control groups observed an increase in achievement however, the independent sample test showed there was significant difference ($0.00 < 0.05$) which favours the experimental group (refer to table 4.11 and 4.12). Also, employing Zazkis et al's VA in analysing student's solutions it has been found that majority of the experimental students (84%) were able to draw a sketch to depict the problem compared to the control group (26%).

The effect of the MATLAB instruction on rural-based pre-service teachers' spatial-visualisation was examined using the Purdue spatial visualisation test. Again, an independent sample test indicated that there was significant difference ($0.00 < 0.05$) between the groups and this also tended to favour the experimental group. Hence, the findings reviewed a relationship between the dynamic environment, MATLAB, problem-solving and achievement in vector calculus, and spatial-visualisation skills. Furthermore, from these findings the researcher made a summary, conclusion, limitations and recommendations which are further discussed in the next chapter.

CHAPTER SIX

SUMMARY, LIMITATIONS, RECOMMENDATIONS, AND CONCLUSIONS

6.1 Research Summary

The research employed a mixed method coupled with 100 second year vector calculus students at UNIZULU to investigate the "analysis of MATLAB instruction on rural-based pre-service teachers' spatial-visualisation skills and problem solving in vector calculus". The participants were randomly put into an experimental group (n=50) and control group (n=50).

Relevant literature on the role of spatial-visualisation skills and mathematics aptitude were revisited and among them were, but not limited to, the works of Cheng (2016), Stieff and Uttal (2015), Miller and Halpern (2014), Jansen et al. (2013), Wai et al. (2009), Ke (2008), Wade et al. (2017), Price et al. (2013) and Nguyen and Rebello (2011). Furthermore, the views of researchers from different disciplines such as mathematics education, physics education, and cognitive scientists on the role of spatial-visualisation skills as a basis of learning were revisited (sections 2.2.2, 2.2.3 and 2.2.4). According to Ke (2008), the association between spatial-visualisation skills and mathematics aptitude has led to the development of computer dynamic software (e.g., GeoGebra, MATLAB, CAS and many more) and they serve as a technology 'cognitive tool'.

The notion of spatial-visualisation skills as a measure or reflection of ability in many learning disciplines has been accepted by researchers from different disciplines, which was discussed in detail in section 2.1. For mathematics psychologists Shepard and Metzler (1971) and Jansen et al. (2013), the ability to correctly predict the orientations or assemble cube boxes has direct connections with mathematics problem-solving. Other researchers such as Wai et al. (2009), Uttal et al. (2013), and Stieff and Uttal (2015) insisted that the students' achievement in STEM is dependent on their spatial-visualisation skills. This is based on the notion that the ability to visualise or correctly represent a problem with a sketch/diagram aids in problem-solving situations.

The views of cognitive scientists on mental rotation and mathematics aptitude were also revisited to deepen the understanding of the association between spatial-visualisation skills as a basis of learning. They, Ho and Lowrie (2014), Hoffkamp (2011) and Wai et al. (2009) hypothesised that one's ability to correctly predict the orientations of rotated cubes (mental rotation skills) measures his/her spatial-visualisation skills and this predicts his/her mathematics achievement (section 2.2.2, figure 2.2). Farmer and colleagues (2013) extended the notion that the association of spatial-visualisation/mental rotation skills and mathematics aptitude shared links with family social economic status.

In the review of literature in chapter 2, it was also noted that other studies matching the association of mental rotation skills with achievement found some inconsistencies in gender disparities (Sipus & Cizmesija, 2012). This inconsistency was attributed to factors such as hormonal difference (Pintzka et al., 2015) and different cognitive strategy selections Kozhevinkov (2007). However, one of the progressive achievements of research was the discovery that spatial-visualisation skills are trainable. Thus, research studies have provided enough evidence that spatial-visualisation skills can be enhanced through selected computer games such as blocks arrangements, chess, and many more giving hopes of bridging the disparities in ability between genders (Mix & Cheng, 2012; 2014; Kebritchi et al., 2010).

Neuroimaging research was revisited to understand the cognitive processes that take place during learning. fMRI technology revealed that similar areas are activated when the human brain processes both spatial tasks (e.g., mental rotation tasks) and number tasks (section 2.2.4) giving much evidence to the association between spatial-visualisation skills and mathematics aptitude. The use of complex technological tools such as eye-tracking and EEG help to uncover cognitive processes that are taken when learning. Lindström et al. (2015) mentioned that eye-tracking techniques are employed in enhancing problem-solving skills.

Having mentioned earlier the essence of dynamic visual tools in learning in section 1.1, the integration of dynamic visual environments in mathematics education was further elaborated in section 2.4. Researchers such as Wai et al. (2009) and Mix and Cheng (2012) evaluated the key pedagogical uses of digital technologies in relation to

effective mathematics learning and practical ideas for teaching and learning mathematics and their findings revealed that technology dynamic visual tools;

- stimulate students' interest in mathematics,
- promote student and teacher interactions,
- help students to build their own prior knowledge and reasoning abilities,
- introduce mathematics instructional activities and occupy students in activities for long periods of time,
- enhance students' spatial-visualisation skills and conceptual knowledge,
- promote problem-solving skills,
- promote student-centred learning (i.e. students work on their own and are able to develop their own understanding),
- students visualize mathematical concepts and explore mathematics in multimedia environments which can foster their conceptual understanding.

In considering the aforementioned role of computer dynamic environments in learning and teaching, MATLAB has been employed as a pedagogical teaching tool in experimental teaching. MATLAB contains learning features which have been in the domain of discussion by some researchers. For instance, Andreatos and Zagorianos (2009) asserted that the MATLAB environment contains applications that foster teaching and learning. Andreatos and Zagorianos (2009) further claimed that the use of MATLAB as an instructional tool in traditional classrooms enhances students' problem-solving skills, achievement, and spatial-visualisation skills.

However, it seems this shared link between spatial-visualisation and mathematics aptitude was not effectively explored in vector calculus as lamented by some researchers such as Rebello, Engelhardt and Singh (2012), Dray and Manogue (2003) and Hinrichs, Singh, Sabella and Rebello (2010). The issues raised range from students' inability to sketch vector diagrams correctly and the addition of vector components, an inability to interpret vector diagrams correctly, difficulty in interpretation of vector operators use in vector calculus, students might be comfortable with performing algebraic operations but fail to convert one register to another (section 2.5.1-2.5.4). Gire and Price (2014) attributed these to instructors paying less attention to graphical problems.

The findings arising from the literature review provided a conceptual and theoretical context and direction for the investigation of the present study. For the purposes of this study, a qualitative content analysis approach was used to examine the effect of the use of MATLAB as a pedagogical tool on students' conceptual knowledge and procedural skills in problem-solving vector calculus. The theoretical frameworks: Duval's (1996) theory of register of semiotic representation and Zazkis et al's (1996) visual-analysis model were chosen as a tool to clearly and precisely specify both the independent and the dependent variables under investigation, guide in unpacking the research questions, arrive at more objective conclusions, and achieve high levels of reliability in the gathered data (section 2.5.1 and 2.5.2).

The research focus was to employ MATLAB as a pedagogical tool in teaching and learning vector calculus and weighing its effects on rural-based pre-service teachers' problem-solving, achievement, and spatial-visualisation skills as presented in chapters 2 and 3. Henceforth, the experimental group went through laboratory activities such as generating geometric shapes, rotating and viewing objects at different angles, generating vector plots, generating graphs, and performing vector operations. The control group had the same timeframe for learning and teaching on vector calculus however, they were taught using the traditional approach (i.e., lecturer notes, textbooks, and chalk).

The data set collected was analysed and further discussed in chapters 4 and 5 respectively. The results tend to agree with the association of MATLAB enhancing conceptual knowledge and procedural skills required in problem solving in vector calculus, achievement in vector calculus, and spatial-visualisation skills. Rural-based pre-service teachers in the experimental group, after having gone through the laboratory activities, were able to; generate geometric 3D shapes, plot graphs, plot vector fields, write integral equations from algebraic words, and systematically perform analytical steps.

The main objectives of the study were to analyse whether or not MATLAB instruction has an effect on students' problem-solving skills, achievement in vector calculus, and if it enhanced their spatial-visualisation skills. Thus, to address the research questions; how do rural-based pre-service teachers apply their spatial-visualisation skills in problem solving in vector calculus? and to what degree do rural-based pre-service

teachers spatial-visualisation skills correlate with their vector calculus achievement? An independent sample test was adopted to check whether or not there was significant difference between the experimental and control groups. The p-value was 0.00 which is lower than $p < 0.05$ revealing there was significant difference among the groups. This favoured the experimental group and indicated that after the treatment, MATLAB enhanced their performance (refer to tables 4.9 and 4.10). A separate assessment was conducted employing 10 items (i.e., rotation) from Purdue spatial-visualisation/Rotation test (PSVT/R) to assess the dynamic visual tool MATLAB's influence on rural-based pre-service teachers' spatial-visualisation skills. Again, the independent sample test showed the $p-$value was 0.00 which is lower than $p < 0.05$, thus there was significant difference between the groups. This also tended to favour the experimental group confirming that MATLAB enhances rural-based pre-service teachers' spatial-visualisation skills (see table 4.11 and 4.12).

Reflecting on Mahir's (2009) claim that students found difficulty in visualising and sketching space figures, and had difficulties in translating between graphical and algebraic representation in 3D when solving vector problems, the outcome of the study revelead that employing MATLAB in traditional classrooms in teaching and learning vector calculus enabled rural-based pre-service teachers to translate from one coordinate system to another without difficulty. It was also observed that they were able to make their conjectures, visualise mathematical concepts, and explore mathematics in multimedia environments which fosters their conceptual understanding. In my own observations, the MATLAB stimulated rural-based pre-service teachers' interest and promoted student and teacher interactions, leading to performance in problem-solving skills, achievement in vector calculus and enhanced students' spatial-visualisation skills.

6.2 Conclusion

The conclusion was sourced from the examination of the literature, theoretical framework and from empirical conclusions obtained from the research findings.

6.2.1 Conclusion based on a review of literature and theoretical frameworks

The examination of literature guided by the research questions: how do rural-based pre-service teachers' apply their spatial-visualisation skills in problem solving in vector calculus, to what degree do rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement, and how do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visualisation skills?

First of all, analysis of rural-based pre-service teachers' solutions presented in the pre-test revealed that they have several difficulties when learning and teaching vector calculus. Among them were: they struggle in visualising and sketching space figures; translating between graphical and algebraic representation in 3D; they were unable to describe vector plots; they had a challenge with solving vector word problems; performing vector operations; and were unable to set up double and triple integral equations. A reflection on the reseach questions:

1. How do rural-based pre-service teachers apply their visualisation skills in problem solving in vector calculus?

2. To what degree do rural-based pre-service teachers' spatial-visual skills correlate with their vector calculus achievement?

3. How do dynamic software environments such as MATLAB influence rural-based pre-service teachers' spatial-visual skills?

For research question 1, the analysis of the solution and statistical results showed that rural-based pre-service teachers have poor spatial reasoning skills and hence struggle with problem-solving in vector calculus. A lack of spatial activities (e.g., chess, computer games, toys games) in the early age could be the reason for poor spatial-visualisation skills.

In response to research question 2, from the analysis of the solution and the statistical results it was found that rural-based pre-service teachers' spatial-visualisation skills correlate with their vector calculus achievement. Thus; the employment of the dynamic visual tool as a pedagogical tool in teaching and learning of vector calculus enhanced achievement (refer to tables 4.9 and 4.10). This is an indication that the immediate solution to improving rural-based pre-service teachers' problem-solving and achievement in vector calculus should be to extend the use of technological teaching tools in rural-based universities.

Finally, the statistical results for research question number 3 showed that the instruction of MATLAB significantly influenced the rural-based pre-service teachers' spatial-visualisation skills. From these indications, it is suggested that the use of technological teaching tools should be an introduction course for every learning area especially for rural-based pre-service teachers.

From these findings based on the three research questions under consideration, the present study reached the following conclusions;

- From a pedagogical point of view, the dynamic software environment, MATLAB promotes the conceptual understanding, procedural skills, and factual reasoning needed in problem-solving situations.
- The use of MATLAB to generate figures (i.e., cube, cylinder, cone, spheres), and rotate and view in different angles enhance problem-solving situations as theorised by Van Garderen and Montague (2003).
- The dynamic software mediates the coordination between spatial and analytical reasoning which are essential for mathematics problem-solving.
- The dynamic software eases in the conversion between geometric register to algebraic registers. Hence, the evidence supports the claim that dynamic visual tools are seen as a technological 'cognitive tool' as hypothesised by Shepard and Metzler (1971).
- The dynamic visual tool aroused students' curiosity leading to engaging rural-based pre-service teachers' for longer periods of time. The long interaction periods with the dynamic visual tool helped them to create their own mathematical understanding.

- Rural-based pre-service teachers' spatial-visualisation skills were enhanced through the dynamic visual tool and as such this stimulated their interest and they were able to work with less monitoring.

- The dynamic visual tool, MATLAB, created a conducive learning environment which fostered high student and teacher interactions.

- Rural-based pre-service teachers were able to make their own conjectures and accurate descriptions of vector plots.

- The dynamic software, MATLAB, enabled rural-based pre-service teachers to make connections between graphical, geometric, algebraic, and numerical representations.

The dynamic visual tool has potential for enormous impact in teaching and learning vector calculus however it is recommended that;

- Mathematics instructors design activities that foster spatial-visualisation skills.

- Duval's semiotic representation framework and the VA-framework provided insight into students' cognitive difficulties as students hence, the use of dynamic visual tools instruction in mathematics classroom should be extended to rural-based universities.

- The use of computers and technology needs to inculcate and should form an integral part of the mathematics courses.

6.4 Recommendations

The study showed that rural-based pre-service teachers struggle in visualising and sketching space figures, translating between geometric and algebraic registers, were unable to describe vector plots, perform vector operations, and unable to set up double and triple integral equations hence, the recommendation for areas of further study are;

1. Why do rural-based pre-service teachers struggle with vector operations such as vector arithmetic, dot and cross products, gradient, divergence, and curl, which are the fundamentals when learning vector calculus?
2. Why do rural-based pre-service teachers experience difficulties in switching between field vectors and field lines?
3. Why do rural-based pre-service teachers find it difficult to sketch or visualise a 3D mathematical object in space for a given algebraic expression, interpret it with understanding, and use the 3D object as an aid to write out the integral function?
4. Why do rural-based pre-service teachers find it easy to evaluate a given vector integral using analytical techniques for integrations but struggle to visualise and transform it from one coordinate system to another?

6.5 Limitations

There were three limitations for the current study. Frist, the University of Zululand is situated in a rural part of Kwazulu-Natal, South Africa and as such, most of the students have poor computer skills. Research has evidenced graphic interface interaction and computer activities (such as: arranging blocks, toys games, chess, and many more) enhance spatial reasoning. Most rural-based pre-service teachers were unfortunate in that they were not exposed to these computer games and spatial activities. To add, technology integration in the mathematics classroom is mostly focused on urban-based universities while neglecting rural-based universities.

Finally, the current study focused only on rural-based pre-service teachers in a second year vector caculus class at the University of Zululand, Kwazulu-Natal in the republic of South Africa. For a larger view, other rural universities situated in other provinces could be considered for future research in this area.

6.6 Conclusion

From the emperical evidence gathered from the research results, we found that the use of MATLAB as a pedagogical tool in traditional classrooms mediates ;

- Problem-solving skills in vector calculus,
- Achievement in vector calculus, and
- Spatial-visualisation skills.

Hence, we can conclude that the use of computer and technology dynamic software can be equally employed in rural-based universities in enhancing mathematics problem-solving skills, achievement, and spatial-visualisation skills.

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Appendices

Appendix 1: Participant Information Sheet and Consent Form

PARTICIPANT INFORMATION SHEET AND CONSENT FORM

TITLE OF THE RESEARCH STUDY

Analysis of MATLAB Instruction on Rural-based Pre-Service Teachers' Spatial-visualisation Skills and Problem-Solving in Vector Calculus.

NAME OF THE RESEARCHER: G Amevor

ADDRESS: Department of Mathematics, Science, and Technology Education, University of Zululand, Private Bag X1001, KwaDlangweza, 3886

CONTACT NUMBER AND EMAIL: 0765005820, radixma@yahoo.com

RESEARCH SUPERVISOR: PROF A BAYAGA & CO-SUPERVISOR: PROF M. BOSSE

INVITATION

You are invited to take part in a research study. Please take time to read the following information. Your participation is entirely voluntary. You are free to decide whether or not to participate. Questions are welcome on anything you are not clear about or if you would like more information. Before you decide it is important that you understand the objectives of the study and what it involves.

1. **What is the purpose of the study?**

The purpose of this study is to investigate the role spatial-visualization skills in problem solving in vector calculus. The MATLAB is employed as pedagogical teaching tool to support the claim that mathematical software enhances spatial reasoning. This is to ascertain the fact that the ability to imagine or draw a sketch/diagram that depicts a problem aids in problem solving situations. Finally, the researcher will be able to establish whether or not if there exist association between the variables spatial-visualization, problem-solving, and achievement in vector calculus.

2. **What will happen to me if I take part?**

Since the research is aligned with the module of the department of mathematical sciences UNIZULU, participating in the research will serve as benefit of gaining the in-depth of the content of vector calculus. This will also enable you to decide which teaching methodologies best suit your study.

3. **Will this not waste my time for my own study?**

No. Since the research is aligned with their module outline it will rather add up to your content knowledge on vector calculus and you been of the advantage of choosing between the teaching methodologies that best suit you study.

4. **Are there any risks / benefits involved?**

There are no risks involved. In terms of the mathematical software, it is installed on the computers in the various labs of UNIZULU for students to use. The use of the program should help you solve problems in 3D vector calculus, matrices and systems of differential equations. The benefits are benefits students can gain when s/he decide to take part in this study.

5. **Do I have to take part?**

It is up to you to decide whether or not to take part. You are free to withdraw at any time and without giving a reason. A decision to withdraw will not have any consequences on your main study.

6. Will I be paid for taking part?

There is no payment or tips or whatever in taking part in this study and as well as there are no costs involved in the participation.

7. Will my information be kept confidential?

All the information obtained will be kept confidential. You will receive a participant number and remain anonymous. The information obtained will only be used for the dissertation and for publishing a research article.

8. What will your responsibilities be?

You should be aware once you accepted to part-take in the research you will be randomly selected to a control group or experimental group. If it happens you belong to the experimental group, you have to attend a lab session for explanation on how to use the mathematical software, the MATLAB to plot 3D shapes and translation of these shapes from one coordinate system to another coordinate system. Main while, the researcher will make time available after the research to take the control group through the use of the MATLAB since the research program have not make provision for them on lab session.

9. When do I start?

You indicate your willingness to take part by signing the declaration to participate. Ethical clearance is already obtained from the university, UNIZULU. Upon waiting from the response from the dean of faculty of science and agriculture, a permission will be obtained to get asses to use one of the university labs for the lab sessions. After these, a time will be arranged that will be convenient to everyone participating.

Appendix 2: Declaration by the participant/Researcher

UNIVERSITY OF ZULULAND
RESTRUCTURED FOR RELEVANCE

Declaration by the Participant

I have read the participant information consent form and I have agreed to take part in a research title "Analysis of MATLAB Instruction on Rural-based Pre-Service Teachers' Spatial-visualisation Skills and Problem-Solving in Vector Calculus".

 Signed at (place) ……………………………………………………………………..on

(date)…………………………………..2018

……………………………………………………………

Signature of participant

Declaration by the researcher

I declare that I followed duly the ethical procedures of the university, UNIZULU in choosing my sample target, explaining the objectives of the research, and encouraging them to participate after creating the awareness and benefits of the study.

Signed at (place)……………………………………on(date)………………….2018

…………………………………………………………

Signature of researcher

Appendix 3: Ethical Clearance Certificate

UNIVERSITY OF ZULULAND
RESEARCH ETHICS COMMITTEE
(Reg No: UZREC 171110-030)

RESEARCH & INNOVATION
Website: http://www.unizulu.ac.za
Private Bag X1001
KwaDlangezwa 3886
Tel: 035 902 6731
Fax: 035 902 6222
Email: DlaminiA@unizulu.ac.za

ETHICAL CLEARANCE CERTIFICATE

Certificate Number	UZREC 171110-030 PGM 2018/494						
Project Title	THE ROLE OF SPATIAL-VISUALIZATION IN PROBLEM SOLVING: APPLICATION IN SECOND YEAR VECTOR CALCULUS IN UNIZULU						
Principal Researcher/ Investigator	A Godfred						
Supervisor and Co-supervisor	Prof A Bayaga			Prof M Bosse			
Department	Mathematics, Science and Technology						
Faculty	Education						
Type of Risk	Med risk: Data collection from people						
Nature of Project	Honours/4th Year		Master's	x	Doctoral		Departmental	

The University of Zululand's Research Ethics Committee (UZREC) hereby gives ethical approval in respect of the undertakings contained in the above-mentioned project. The Researcher may therefore commence with data collection as from the date of this Certificate, using the certificate number indicated above.

Special conditions:
(1) This certificate is valid for 2 years from the date of issue.
(2) Principal researcher must provide an annual report to the UZREC in the prescribed format [due date- 28May 2019]
(3) Principal researcher must submit a report at the end of project in respect of ethical compliance.
(4) The UZREC must be informed immediately of any material change in the conditions or undertakings mentioned in the documents that were presented to the meeting.

The UZREC wishes the researcher well in conducting research.

Professor Gideon De Wet
Chairperson: University Research Ethics Committee
Deputy Vice-Chancellor: Research & Innovation
28 May 2018

CHAIRPERSON
UNIVERSITY OF ZULULAND RESEARCH ETHICS COMMITTEE (UZREC)
REG NO: UZREC 171110-30

2 9 -05- 2018

RESEARCH & INNOVATION OFFICE

UNIVERSITY OF ZULULAND
RESTRUCTURED FOR RELEVANCE

INSTRUCTIONS: DO NOT WRITE YOUR NAME OR YOUR STUDENT NUMBER.

INFORMATION GATHER WILL BE USED FOR ONLY BE USED FOR RESEARCH PURPOSE.

RESEARCHER: AMEVOR GODFRED

PRE-TEST/Post-test

Title: Analysis of MATLAB Instruction on Rural-based Pre-Service Teachers' Spatial-visualisation Skills and Problem-Solving in Vector Calculus.

.

Participant Number: …………………………………

Time Allowed: 1 Hour   **Score: 50 MARKS**

ATTEMPT ALL QUESTION

QUESTION 1

MATCH EACH OF THESE EQUATIONS WITH ITS GRAPHS (A, B, C, D AND E) AND WRITE ONE STATEMENT FOR THE CHOICE OF YOUR ANSWER IN THE BOX PROVIDED BELOW EACH QUESTION.

1.1 $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

A

1.2 $z = x^2 + y^2$

B

1.3 $xy = 3$

C

1.4 $y = x^3$
D

1.5 $x^2 + y^2 = 3$

E
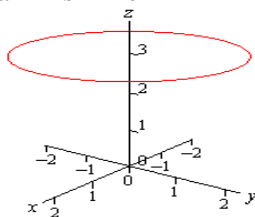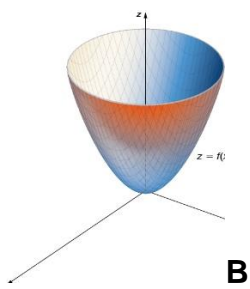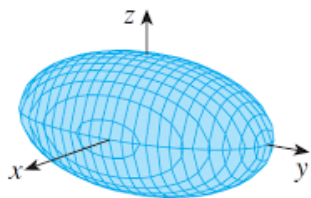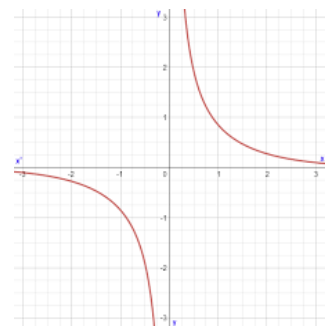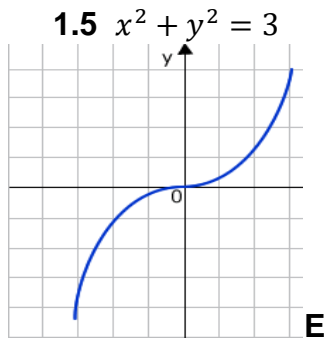
[2x5= 10 Marks]

QUESTION 2 (vector fields)

2 a Sketch and give a brief describe the vector fields in 2-space given by $\vec{F}(x, y) = x\vec{j}$

Give a brief description of the sketch of the vector field

DESCRIPTIONS
1.
2.

[4 Marks]

2.1b A girl ride a bicycle from her house in the direction of $80^0 NW$ for $5m$ to the library. After borrowing some few books from the library, she rides back to the school in the direction of $10^0 NE$. The school is $7m$ from the library. Calculate the distance and the direction from the school from her home.

Solution

Give three (3) points brief explaining how you arrived at your answer

Explanation
1.
2.
3.

[5 Marks]

2.2a. Convert the Cartesian coordinate $(x, y, z) = (1, \sqrt{3}, 2)$ to cylindrical coordinates.

[Hint: $r^2 = x^2 + y^2$, $tan\theta = \frac{y}{x}$, $z = z$]

2.2b Give of the sketch of the graph of the cylindrical coordinates calculated in QUESTION 2.2a above

----

Give three (3) points brief explaining how you arrived at your answer

Explanation
1.
2.
3.

[7 Marks]

QUESTION 3 (Multivariate Calculus)

3.1 Sketch the following 3D given by the algebraic equation $2x + y + z = 8$ in the Cartesian coordinate system.

Give three (3) points briefly explaining how you arrived at your answer

Explanation
1.
2.
3.

[7 Marks]

3.2 Sketch the graph of the region R is bounded by $h(x) = 2x$ and $g(x) = x^2$, and evaluate $\iint_R (x + 2y)dydx$ along the R with the interval $0 \le x \le 2$.

Solution

Give three (4) points brief explaining how you arrived at your answer

Explanation
1.
2.
3.
4.

[8 Marks]

3.3 A cube has sides of length 4. Let one corner be at the origin and the adjacent corners be on the positive xx, yy and zz axes. If the cube's density is proportional to the distance from the xy-plane, find its mass. [Hint: the density of the cube $f(x, y, z) = tz$ where t is constant.

Solution

Give three (4) points brief explaining how you arrived at your answer

Explanation
1.
2.
3.
4.

[8 Marks]

THANK YOU FOR THE PARTICIPATION!!!!!!!!!!

Researcher: G. Amevor   **Supervisor: Prof. A. Bayaga**

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Appendix 5: Purdue Spatial Visualisation Test

SECTION A: VISUALISATION OF ROTATIONS

Directions: This instrument is designed to measure how well you can visualize the rotation of three-dimensional objects. There are 20 questions in this section. Shown below is an example of the type of question. You are to:

1. Study how the object in the top line of the question is rotated;
2. Picture in your mind what the object shown in the middle line of the question looks like when rotated in exactly the same manner;
3. Select from among the five drawings (A,B,C,D or E) given in the bottom line of the question the one that looks like the object rotated in the correct position.
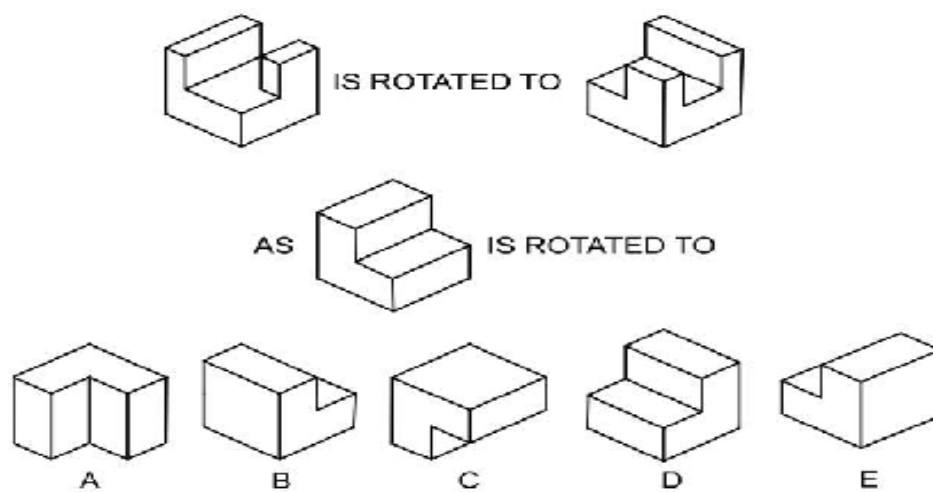
What is the correct answer to the example shown above?

Answers A, B, C, and E are wrong. Only drawing D looks like the object rotated according to the given rotation.

Remember that each question has only one correct answer.

Bel Lettres
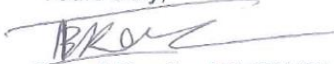## Editing Services

6 December 2018

To whom it may concern

Re: Language editing of Master's dissertation in the Department of MSTE, Faculty of Education, University of Zululand for Mr A. Godfred.

Title of Dissertation: 'Analysis of MATLAB instruction on rural-based pre-service teachers' spatial-visualisation and problem-solving skills in vector calculus' by Mr A Godfred

This letter serves to confirm that I, Isabel Rawlins did language editing in the dissertation named above. I therefore grant permission for the document to be sent for external examination.

Yours truly,

Isabel Rawlins MACW (Rhodes)