Errors and misconceptions related to learning algebra in the senior phase – grade 9

by

PHILILE NOBUHLE MATHABA

20053405

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Supervisor: Prof. A. Bayaga

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ABSTRACT

Algebra is a mathematical concept that explains the rules of symbol operations, equations, and inequality. Algebra is a combination of logic and language; hence common mistakes and conceptions are either attributed to logic or language problems, or both. There is also ongoing debate about the fact that learners come to class with different ideas that result in errors and misconceptions when they solve algebraic equations and expressions. Based on this debate concerning both errors and misconceptions in solving algebraic equations and expressions, the purpose of this study was to investigate the errors and misconceptions committed by learners when learning Algebra. The study answered the following research questions: What are the types and the sources of errors and misconceptions committed by Grade 9 learners in Algebra learning? How do the types and the sources of errors and misconceptions influence errors in Grade 9 learners' cognition when learning Algebra? Which strategies work to avoid errors? What are the sources of the errors and misconceptions in Algebra? Unlike the predominant existing studies, which are urban-based, this study was based in rural schools in the King Cetshwayo District of UMlalazi and Mtunzini Municipality. The structure of the observed learning outcome (SOLO) theory was adopted to observe, examine and analyse learners' misconceptions in rural-based secondary schools.

Methods

In line with the research objectives, both interpretivist and positivist paradigms were used for this study. The participants in the study comprised 100 Grade 9 learners from a rural school in KwaZulu-Natal. Focus group interview questions, as well as questions appropriate to the Curriculum Assessment Policy Statement (CAPS) for the General education and Training (GET) band or Senior Phase, (counting 50 marks in total) were used to collect data. These instruments were aimed at addressing the main objectives of the study. The criteria used for this study included Koch's error method of analysis, as well as descriptive statistics and thematic content analysis. Distribution and interpretation analyses were used in evaluating the types of error and sources of errors using the two research instruments (the test and the focus group interview).

The sources of data were triangulated and interpreted to answer the research questions.

The findings revealed that Grade 9 learners committed various types of error and displayed a number of misconceptions when solving Algebra problems. The types of error and misconceptions included cancellation errors, computational errors, problem-solving errors and careless errors. The findings of the test revealed that most of the learners were operating on level one of the SOLO model theory. As the SOLO model explains, level one is when the learner is operating in terms of one idea which is related to the solution. The strategies highlighted by the SOLO model include one-structure, many-structure and relational strategies. The results revealed that learners had a lack of conceptual understanding of algebraic rules, they were unable to recollect and apply algebraic rules properly and route means of performing algebraic functions, lack of adequate knowledge algebraic rules; lack the conceptual understanding of algebraic rules; unable to apply algebraic rules; and route learning in performing algebraic tasks. The study recommended re-teaching and re-learning of the concepts of the word-problem, equations with fractions as well as the concept of ratio.

Keywords: errors, misconceptions, Algebra, and learning, cognition and Algebra, sources of errors, strategies for avoiding errors

DEDICATION

I dedicate this research to my husband Nkanyiso Excellent Mathaba, my parents Thiyekile Biyela (uMamkhize) and Gcinangaye Joseph Biyela, my in-laws Fikile Mathaba (uMangema) and Phenious Neh Mathaba, my sons Yakhani, Aqotho and Siyenzelwa and my daughters Sisanda and Amahle.

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DECLARATION

I, PHILILE NOBUHLE MATHABA, hereby declare that this dissertation, entitled **'ERRORS AND MISCONCEPTIONS RELATED TO LEARNING ALGEBRA IN THE SENIOR PHASE –GRADE 9'** is my original work and that all resources that I have used or quoted have been indicated and acknowledged using complete references. It has been submitted for the degree of Master of Education at the University of Zululand. It has not been submitted for any degree or examination in any other university.

__________________________ 16 Jan 2019

Candidate's signature Date

________________________ ________________________

Supervisor's signature Date

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LIST OF ABBREVIATIONS

ANA	Annual National Assessments
APFE	Action Plan for Education
BODMAS	Bracket, Of, Division, Multiplication, Addition, Subtraction
CAPS	Curriculum Assessment Policy Statements
CES	Counter-examples strategy
CL	Cooperative learning
CN	Cognitive neuroscience
DBE	Department of Basic Education
DBEAPR	Department of Basic Education Annual Plan Report
DBEDRANA	Department of Education Diagnostic report on Annual National Assessment
DD	Developmental dyscalculia
DOE	Department of Education
ECD	Early child development
FET	Further Education and Training
FGI	Focus group interview
FGIA	Focus guided instructional approach
FP	Foundation Phase
GFET	General Further Education and Training
GIA	Guided instructional approach
ITC	Technology – integrated
KNEC	Kenya National Examinations Council

KZN	KwaZulu-Natal
LCD	Lowest common denominator
MAN	Mathematical Association of Nigeria
MST	Mathematics, Science and Technology
NCSER	National Council for School Evaluation Report
NCTM	National Council of Teachers of Mathematics
NCS	National Curriculum Statement
NCF	National Curriculum Framework
NPFA	National Protocol for Assessment
NRC	National Research Council
PBL	Problem-based learning
RSA	Republic of South Africa
SACMEQ	Southern and Eastern African Consortium for Monitoring Education Quality
SASE	South African School Evaluation
SBA	School-based assessment
SMT	School management team
SOLO	Structured observed learning outcome
TIMSS	Trends in International for Mathematics and Science Study

CHAPTER ONE: AN OVERVIEW OF THE STUDY

1.1. Background of the study

In mathematics, a mistake or a blunder a learner makes when solving a mathematical problem, either by computing wrongly or mishandling variables, is referred to as an error. Matuku (2017) claims that mathematical errors result when a learner fails to recall the correct procedure from his/her long-term memory and consequently applies an incorrect schema in problem-solving situations. Matuku (2017) further states that to avoid errors and misconceptions, learners' existing schemas should be ready to accept and store new knowledge by accommodating it and relating it to existing knowledge. Researchers (Makonye 2012, Makonye & Fakude, 2016) believe that misconceptions are based on learners' flawed conceptual understanding and erroneous principles, which led to various types of error. Naturally, if the learner's conceptual knowledge is built on false ideas, then the new knowledge contains misconceptions. However, it is not easy for the learner to give up their wrong beliefs and principles, which may be deeply embedded in their schema, leading to learners making errors consistently. In 2017, the National Senior Certificate Examination Diagnostic Report (NSCEDR, 2017) reported that Grade 12 learners had in recent years performed poorly in Mathematics. The figures below indicate the percentages of learners who achieved 40% and above in the final National Senior Certificate examination:

- 35,1% in 2014
- 31,9% in 2015
- 33,5% in 2016, and
- 35,1% in 2017

These results indicate that learners of Mathematics, of which Algebra is part, have performed poorly in South African schools in recent years, particularly at the end of Grade 12. Of the errors identified in particular in these years, it would seem that learners struggle with substitution and binomial expansion (NSCEDR, 2015). Ncube (2016, p. 1) believes that "learners experience difficulties in understanding algebraic concepts and they fail to manipulate algebraic concepts according to accepted rules, procedures or algorithms". The Department of Basic Education (DBE) (NSCEDR, 2017) reported that just over 51, 9% of learners achieved 30%

and above in 2017; 51, 1% in 2016, 49, 1% in 2015 and 53,5% in 2014. DBEANAR, (2014 p.9) reported that the average of Grade 9 performance in three years which were evidence on learner difficulties in Algebra (see page 23 and page 24). Learners achieved 13% in 2012, 14% in 2013 and 11% in 2014.

There are various reasons attributed to these pass rates. One that has been suggested is that learners approach Algebra with prior theories that have been constructed from their everyday experiences, which in essence, necessarily a bad idea. However, the challenge is that sometimes these theories are based on misconceptions (Egodawatte, 2011). Additionally, if learners are cognitively attached to these misconceptions, it is often difficult to change their minds.

However, this may not be the only cause of learner difficulties in Algebra. Other suggested causes include a lack of adequate support for effective teaching and learning in the subject, and careless mistakes on the part of the learners.

Thus, it is suggested that these misconceptions may be the result of learners' memorisation of mathematical rules or the use of procedures without connection and understanding (Makonye, 2012). These gaps serve as a motivation to investigate misconceptions and errors related to learning Algebra in the Senior Phase. Hence, this study investigated errors and misconceptions related to learning Algebra in the Senior Phase. The study further focused on the possible causes of learner errors and misconceptions when learning Algebra, as well as a discussion on identifying possible strategies for avoiding errors and misconceptions when learning Algebra. Vague

1.2. Introduction

Algebra is a branch of Mathematics that employs symbols to represent numbers, and the manipulation of these symbols is based on a set of rules and theories (Egodawatte, 2011; Mangorsi, 2013; Matuku, 2017; Moodley, 2014; Ncube, 2016). Algebra is composed of polynomial equations, algebraic expressions and algebraic properties (Mangorsi, 2013). This implies that learners need to have the conceptual, procedural and factual understanding of these rules and theories in order to perform algebraic operations. As asserted by Owusu (2015, p. 11), learners are not "blank slates"; this implies that learners come to the classroom with the knowledge they acquire from everyday experiences. Others such as

Amirali and Halai (2010) and Ali (2011) partly support this idea, arguing that these everyday experiences may contribute to errors and misconceptions when learning Algebra. Thus, while errors and misconceptions may be related in some way, there is a distinction between them (Egodawatte, 2011; Luneta, 2015; Matuku, 2017). In the current study, an error is understood as a blunder or deviation from the correct answer that is a non-recurring event, whereas a misconception is an error made as a result of ideas that are based on incorrect facts and thus it is likely to recur (Egodawette, 2011; Makonye, 2012).

Because a distinction can be made between an error and a misconception, researchers have found reason to suggest that learners' mistakes, blunders, deviations and ideas built on incorrect facts are related to their learning of Algebra (Aygor & Ozdag, 2012; Bohlmann, Prince, & Deacon, 2017; Egodawatte, 2011; Iddrisu, Abukari, & Boakye, 2017; Mdaka, 2011). It is thus suggested that errors and misconceptions committed by learners when learning Algebra may be as a result of the following, among others: lack of conceptual understanding of algebraic rules; inability to recollect and apply algebraic rules properly; lack or inadequate knowledge of algebraic rules; lack of conceptual understanding of algebraic rules; inability to apply algebraic rules; and rote learning when performing algebraic tasks (Aygor & Ozdag. 2012; Bohlmann et al., 2017; Iddrisu et al., 2017).

Research (Gumpo, 2015; Mashazi, 2014 Moodley, 2014; Ncube, 2016) that interrogates learners' errors and misconceptions when learning Algebra is generally limited to the Senior Phase grades (i.e., Grade 9), even though Algebra forms part of the basic rules of Mathematics, the learning of which starts at the elementary level. Algebraic errors and misconceptions may be exacerbated by socioeconomic issues such as the difference between urban and rural schools. Urban school classrooms are equipped with technological tools which aid the teaching and learning of Algebra in contrast to rural schools which do not have much in the way of technological resources (Cravens, 2011; Lucas, 2012; Ntsohi, 2013). Research has evidenced that computer-aided programs can enhance learners' conceptual understanding of Algebra (Ntsohi, 2013). In rural settings, the teaching of Algebra is based solely on chalkboard and textbooks, making it difficult for learners to construct their own inferences. Hence, rural learners' inability to make conjectures may result in errors and misconceptions when learning Algebra.

Some researchers (Egodawette, 2011; Gumpo, 2015; Luneta, 2015; Pournara et al, 2016) have argued that if learners have a conceptual understanding of algebraic rules and theories from the lower grades, such as Grades 7, 8 and 9, errors and misconceptions may be dealt with before they reach the higher grades of 10, 11 and 12. Drawing from the Chief Examiner's report of the NSC (2016), it would seem that many of the errors learners commit are a result of misconceptions when performing simple algebraic functions; this leads to them losing marks. This study therefore investigated the errors and misconceptions that occur when learning Algebra, focusing particularly on Grade 9 learners. As a result of the problems that the researcher experiences daily, she interrogated learners' errors and misconceptions related to learning Algebra. This study was carried out in quantile 2, rural-based schools (poor schools), which experience a lack of resources and/or facilities. In order to effectively investigate learners' errors and misconceptions related to learning Algebra, attention was also given to the causes of these errors and misconceptions and how they may be avoided.

1.2.1. State of errors and misconceptions in South Africa – National report

The National Senior Certificate Examination Diagnostic Report (NSCEDR, 2015) revealed that, among other things, learners were unable to find a general term in sequences and made errors in the simplification of exponents. The NSCEDR (2017, p. 153) reported that algebraic skills were very poor and most of the learners lacked fundamental and basic Mathematics competence, which should have been acquired in the lower grades. The errors found in these years (2014 to 2017) were mostly related to learners' struggle with Algebra. Learners are reported to have difficulties in simplifying expressions with negative numbers, for instance $-(-2y-5)^2$; huge issues were reported with word problems, learners were reported to struggle with working with inequality signs such as <, >, ≤ and ≥; learners experienced difficulties in rounding off numbers; problems with understanding the meaning of a 'square root' which includes working with square numbers; and difficulties in working with negative numbers.

Other challenges included that learners lacked knowledge on how to use formulae, with some writing formulae incorrectly. In addition, there would seem to be a lack of conceptual understanding in the drawing of graphs because candidates drew exponential graphs instead of straight graphs. Because the report indicated an increase in learner errors over the four years (2014, 2015, 2016, and 2017), there are still gaps in knowledge that teachers need to fill when teaching Algebra.

Additionally, it would seem that these errors and misconceptions originate in the lower grades (8 and 9). If these errors were subsequently corrected in the Senior Phase, instances of errors made by learners in the NSC examinations could probably be reduced. The report emphasised that the errors originated from a lack of basic skills and Mathematics competence inculcated in the lower grades. The NSCEDR (2015) reported a similar notion that “the algebraic skills of the candidates were poor. They struggled with Mathematics in Grades 11 and 12 because learners could not do basic Mathematic skills for grades 8, 9 and10. If the problem could be fixed, learners will perform much better in the grade 12 examination” (p. 151). The report further mentions that “problem-solving” and “non-routine” are important issues that form a basic part of the teaching and learning process in the classroom. The report emphasised that the best way to avoid errors and misconceptions is for the teacher to have adequate content knowledge. Thus, a good teacher should be aware of learner errors and determine the sources, instead of coming up with various strategies to avoid any type of error.

Gabriel et al. (2013), Umalusi (2015) and Matuku (2017) agree with the notion that errors and misconceptions in the Grade 12 NSC are caused by a lack of fundamental skills. Umalusi has since been working hand in hand with the DBE to ensure quality in all types of assessment, including the implementation of national and school-based assessment to avoid irregularities. The learners’ errors are analysed after the assessment is done. Teachers then meet for moderation every term where the subject advisors check learners’ scripts, the marking thereof and the marks, and then sign to confirm quality. Umalusi has come up with these strategies to support the DBE in learning about the errors that are committed by Grade 9 learners when solving Algebra problems. However, there are still gaps because the types of error made by learners persist.

1.3. Statement of the problem

The study of Algebra is abstract in nature and is guided by a set of rules and theories which need to be adhered to when performing algebraic operations.

However, errors and misconceptions surface when these rules and theories are overlooked or even misapplied. The chief examiner's report, which has drawn considerable attention from researchers such as Egodawatte (2011), Sasman (2011), Mamba (2012), Makonye and Luneta (2014), and Pournara, Hodgen, Sanders, and Adler (2016), indicates that Grade 12 learners are unable to recollect algebraic rules and apply them correctly. However, there is also demonstrable evidence that suggests that a number of these studies interrogating learners' errors and misconceptions are directed mainly at the higher grades (10, 11, and 12), with few addressing errors and misconceptions in the Senior Phase (i.e., Grades 8 and 9). The poor and poorest schools in rural settings have recorded the reasons why learners struggle to learn Algebra as poverty and a shortage of the materials and technological tools required to support teaching and learning, for example calculators and computers. There is a difference between learning Algebra in rural and urban schools, as in the urban areas, parents are working, and are thus able to buy school materials for their children. Urban school children generally come to school with full stomachs, whereas in the rural schools they very often come to school hungry. Teaching in rural schools is chalkboard based, whilst in urban schools computers, laptops and even cell phones (tablets) are used to access mathematical programs such as GeoGebra.

The aforementioned factors, therefore, prompted the researcher to fill the literature gap on what is known about learner errors and misconceptions in the learning of Algebra in a rural context, particularly in the Senior Phase. It is believed that giving attention to learners' errors and misconceptions in the Senior Phase will reduce the strain put on the FET phase educators who sometimes have to correct misconceptions in Algebra that have been acquired in the lower phases, by re-teaching the previous grade work. It has been suggested that if learners' errors and misconceptions when learning Algebra are given attention, such as that required in the Senior Phase, that is Grade 9, these could be minimised in the higher grades (i.e., Grades 10–12). Thus, the current study was built on the aforementioned problems and hence the need for the current research.

1.4. The South African position on misconception and errors – Annual National Assessment (ANA) as a strategy for avoiding errors

The Annual National Assessments (ANA) is an approach designed to observe the "level and quality of basic education" (Kanjee & Molo, 2014, p. 93). The ANA was initiated in 2010 by the DBE (Kanjee & Molo, 2014) and was introduced nationally in 2011 (Van de Berg, et al 2015). The reason for the introduction of the ANA was to fill gaps and ensure the quality of education in all schools (Kanjee & Molo, 2014; Van de Berg, 2015 et al). The ANA process is based on tests that learners write, the results of which are evidence of learner performance. The scripts are marked and moderated formally by qualified teachers and are monitored by the DBE (DBEANAR, 2014).

An analysis of the ANA in 2014 was done and the results revealed poor performance among Grade 9 learners generally (DBEANAR, 2014 p.9). Average Grade 9 results were 13% in 2012, 14% in 2013 and 11% in 2014. The document reveals that errors and misconceptions displayed by learners show that they were "unfamiliar with mathematical terminology [and] … properties, and often use them incorrectly", and that "basic algebraic skills have not been mastered" (p. 11). Questions on Algebra were included in the ANA paper from 2012, 2013 and 2014. Table 1.1 below displays the questions based on algebraic equations and expressions in Grade 9 tested in different years to assess learners and analyse the types of error they commit. These questions were adopted in this study because as they address the research questions**.**

Table 1.1: ANA questions and learners' responses in three years. Learner errors in 2012, 2013 and 2014

2012	2013	2014
Learner activity: ***Factorise fully:*** $\boldsymbol{8p^2 + 4p^2}$ ***errors: 1*** $\boldsymbol{12p^5}$ ***error: 2*** $\boldsymbol{12p^3 - 5p^2}$ ***activity: solve for*** $\boldsymbol{x}$ $\boldsymbol{x^2 - 2x = 0}$ ***errors: 3*** $\boldsymbol{2x = 0 + x^2}$ ***error 4:*** $\boldsymbol{x^2 = 0 + 2x}$ **Activity:** ***error 5:*** $\boldsymbol{4ab(5a^2b^2 - 2ab - 3)}$ $\boldsymbol{20ab + 8a - b}$ $\boldsymbol{28 - 1}$	$6a^2 + 12a^2 + 18a^{12}$ ***error: 1*** $= 0$ ***error: 2*** $\frac{-1}{x} = \frac{8}{x}$ $x = 8$	The diagnostic report analysis 2014 revealed Foundation Phase errors

As reflected in Table 1.1, the report revealed errors and misconceptions in questions involving Algebra. Thus, poor performance is linked to learner difficulties experienced with algebraic expressions and equations. For instance, the Department of Basic Education Diagnostic Report on ANA Report (DBEDRANA, 2012) revealed the following errors done by learners:

DBEDRANA (2012, p. 34) reported that the "specific standard procedures to manipulate quadratic expressions and equations required learners to practice regularly as a strategy or tool to solve complex problems". Errors arising in this regard were said to be due to the lack of procedure that originated from poor conceptual knowledge. The problem-solving process involves working with conceptual knowledge while adequately following a procedure or performing steps to reach a solution.

Building on DBEANAR (2014, p. 11) and the results of the "Trends in International Mathematics and Science Study (TIMSS) and The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), fourth (IV) plans were made to support the developments in improving the numeracy and literacy abilities in all learners". The errors that the DBE revealed were supposed to work as a tool for teachers to raise awareness of the types of error committed in Grade 9. A good teacher learns through learners' errors and devises strategies for avoiding them. Thus, the DBEDRANA (2012) suggested that intervention involves the ability to apply conceptual knowledge, ability to work with concepts such as laws of exponents, multiplication, addition, subtraction, and distributive property, as well as the importance of training teachers so that they in turn can train learners in those skills, procedures and concepts that are linked and are not be regarded as separate. Hence, the need for the current research in the Senior Phase and in Grade 9 in particular.

1.5. The purpose of the study

The purpose of this study was to investigate the errors and misconceptions related to learning Algebra in the Senior Phase by

- exploring the possible causes of learners' errors and misconceptions when learning Algebra
- Identifying possible strategies for avoiding errors and misconceptions when learning Algebra in the Senior Phase.

1.6. Objectives

The objectives of the study were to

- explore the possible causes of learners' errors and misconceptions when learning Algebra
- Identify possible strategies for avoiding errors and misconceptions when learning Algebra in the Senior Phase.

1.7. Research questions

The current study investigated the errors and misconceptions related to learning Algebra in the Senior Phase by considering the following research questions:

- What are the types and the sources of errors and misconceptions committed by Grade 9 learners in Algebra learning?
- How do the types and the sources of errors and misconceptions influence errors in Grade 9 learners' cognition when learning Algebra?
- Through which strategies and sources could the errors and misconceptions relating to Algebra be avoided?

1.8. Definitions of key terms

Mamba (2012) defines Algebra as a field comprising several aspects, including abstract arithmetic, language, and the tools for the study of functions and modelling aspects. Mamba (2012) further explains that learning to understand, appreciate and use mathematical language is a crucial part of Algebra learning**.** This suggest that the correct use of mathematical symbols, rules, and mathematical language forms the key component of Algebra.

Misconception

Misconception is explained as a combination of a lack logic and the learner relating his/her ideas to a false understanding already established, thus leading to misconceptions when learning Algebra.

Errors

As highlighted by Khalo and Bayaga (2014), errors surface when learners carelessly fail to relate mathematical concepts and display a lack of awareness or an inability to check answers given. Thus, errors are caused by a learner's failure to apply algebraic rules or inability to connect new topics successfully to his/her prior knowledge.

Learning

Sarwadi and Shahrill (2014) argue that learning takes place when learners acquire and apply knowledge in a problem-solving situation. Generally, learning takes place when the mind discovers new knowledge based on prior knowledge.

1.9. Dissertation outline

Chapter one

Chapter one covers the introduction, background, problem statement, aims of the study, research questions, and definitions of terms, as well as outlining the thesis.

Chapter two

Chapter two covers the theoretical framework including theories of the study, the types of error when learning Algebra, the sources of errors, and teaching and learning strategies for avoiding errors and misconceptions as indicated in the literature. This chapter also discussed the application of cognitive neuroscience when learning Algebra.

Chapter three

Chapter three covers the research design, research methods used to conduct the study, sampling, data collection methods, methods of data analysis, validity, reliability and research ethics.

Chapter four

Chapter four covers the data analysis.

Chapter five

This chapter discusses the results of the study.

Chapter six

This chapter covers the conclusions, recommendations and limitations of the study.

CHAPTER TWO: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1. Introduction

As mentioned in Khanyile (2016), a literature review is a guide for the researcher about what has been studied about the topic. In addition, the literature review aids in informing the researcher about the thinking, ideas, research questions, methodologies, analysis and findings of similar past research. Thus, reviewing literature helps in locating the gap(s) in the field of interest that new research may fill (Creswell, 2012). For this reason, this chapter reviews literature based on the research questions and the topic under study. The intention of the chapter is also to discuss the theoretical framework in relation to the research questions and how the theory was used in understanding the study.

2.2. Review of the literature

The review of the literature was largely based on the ongoing debate regarding the types of error and misconception when learning Algebra, the sources of these, as well as strategies for avoiding errors and misconceptions when learning Algebra. This, for the most part, was addressed by taking cognisance of both the phase and the setting in terms of either being a rural or an urban setting.

2.2.1. Error types and misconceptions learners display when learning Algebra

As theorised by Booth and Koedinger (2008), learners' misconceptions are related to the types of error they commit when learning Algebra. In the support of, Durkin and Rittle-Johnson (2015) have advanced the notion that errors made by learners when performing algebraic operations are the manifestations of their understandings built on false ideas. What is drawn from Durkin and Rittle-Johnson (2015) is that errors and misconceptions generally surface when learners integrate their prior knowledge with a new topic in Algebra. The observation made by Durkin and Rittle-Johnson (2015) is important to investigate for various reasons. For instance, in their study, Cangelosi, Madrid, Cooper, Olson, and Hartter (2013) pointed out that underdeveloped knowledge or understanding built on false experience results in persistent errors in performing algebra tasks. Another reason to investigate this is that prior studies probing learners' errors and misconceptions when learning Algebra identify a number of misconceptions learners display when performing algebraic operations. For instance, in the past four years, Makonye and Hantibi (2014) have attempted to classify errors into categories, namely, systematic errors, careless (random) errors, and transformation errors.

As explained by Makonye and Hantibi (2014) and in support of Luneta and Makonye (2010), systematic errors occur when, for example, the learner is given a positive smaller number to be subtracted from a negative bigger number or vice versa. S/he consequently ignores or fails to apply the rule that states that the answer takes the sign of the bigger number. These errors are based on a faulty line of thinking and are likely to recur since they were built on false ideas.

Case in point 1: $12x – 15x = 3x$

Careless/random errors**:** In this case the learner carelessly gives an answer to a mathematical task without applying his/her knowledge, or better still, the learner provides an answer which has no analytical reasoning (Makonye & Khanyile, 2015; Dlamini, 2017).

Case in point 2: $15x – 13x = 3x$

In most cases, after cross-checking, the learner is able to correct or rectify this mistake on his/her own. A transformation error occurs where a learner mishandles arithmetic signs, for instance a multiplication sign is taken as an addition sign and vice versa, or multiplication is confused with a subtraction action sign. Also, when these arithmetic signs are combined in a function, learners fail to observe the rules for their ordering, which normally goes by the acronym BODMAS, meaning "bracket of, Division, Multiplication, Addition, and Subtraction".

Makonye and Hantibi (2014) observed transformation errors in their study where learners used addition instead of multiplication; for instance, in the case of $-3 \times -5 = -8$. It is important to investigate these types of problem because it is argued that learners might well understand there is a problem, but they lack the procedure for solving it, resulting in them going for a wrong solution. As supported by Abdullah, Abidin, and Ali (2015), transformation errors often occur when the learner already has an idea of what is expected in the question but fails to recognise operations that involve mathematical Algebra. Sanders (2017) suggests that "semiotic" intervention can be used as an aid or reminder (BODMAS rule) in the use of signs to mediate learning (p. 12). Thus, as argued by Muschla, Muschla, and Muschla (2011), when these arithmetic signs (×, ÷, – and +) are combined in a function, learners fail to observe the rules for their ordering which are directed by the BODMAS. As Muschla et al. (2011) explain, the BODMAS rule helps in simplifying by grouping symbols and exponents. Accordingly, this rule assists both learners and teachers to avoid mishandling signs when solving equations.

Case in point 3: $3 \times 4 - 3 = 3$

In this case, the learner failed to apply the rules for ordering arithmetical operations (i.e., BODMAS), thereby considering the subtraction sign first before the multiplication sign.

Case in point 4: $15t \times 20t = 35t$

Here, the learner carelessly treated the multiplication sign as an addition sign. An error like this is often corrected by the learner by means of cross checking. Makonye (2011) reviewed the type of error, however in this claim 5 + 2 = 10, the learner treated the addition sign as a multiplication sign. For example: 5 + 2 = 10. Makonye (2011) opines that these types of error are the result of distraction or forgetfulness and are not meant to arrive at an unsystematic answer but are due to carelessness, sloppiness or oversight.

As outlined in the blueprint of the South Africa's Department of Basic Education (DBE 2012), learners' errors and misconceptions are a result of a poor foundation in the earlier grades. The errors committed are consequently perpetuated by learners as a result of them building undeveloped ideas or being unable to link these ideas to the learning of new algebra topics. Mahlabela (2012) maintains that in order to understand the types of error learners commit, one has to address the methods or strategies that the learners use to arrive at the incorrect solutions. What can be drawn from Mahlabela's (2012) claim is that the solution provided by learners gives a clue to the basis on which they build their ideas, and also displays the types of error and misconception in this regard.

It may also be concluded that the types of error and misconception learners display when learning Algebra differ in classification. For instance, the researchers

(Gumpo , 2015; Dhlamini & Kibirige ,2014) various studies, the use of the equal sign, conjoining, operation signs and errors associated with commutative and distributive properties are ignored or not taken into consideration.

Case in point 5: Use of the equal sign

The use the equal sign (=) has significance in the equations and also gives meaning. Thus, there is a difference between the values of the equal signs when we sum two numbers, for instance, 4 + 5 = 9. This means that the addition of these two numbers is equal to 9; however, when one is given the task $4x + 5 = 9$, it necessitates solving for x. Researchers such as Stephens et al. (2013) and Sanders (2017) explain the three uses of the equal sign as operational, relational-computational and relational-structural. In respect to this concept of considerate equal sign or the meaning of an equal sign, it has an important role to play in the solution of equations (Gumpo, 2015). This means that the equal sign is an action sign; however, whether or not the right procedure will be chosen depends on how the learners chooses to use it.

When explaining the study of Gumpo (2015), Sanders (2017) suggests that the equal sign creates the "intended reaction" in solving the equation but does not hinder the procedure that a learner performs (p. 12). Gumpo (2015) suggests that in solving equations, learners must understand the meaning of the equal sign in order to use it correctly. Thus, in operations, this author suggests three different strategies for using the operation sign for a conceptual understanding of algebraic operations. These include *relational-computational,* where the left-hand side must be equal to the right-hand side; the *relational structure* where the learner needs to think and provide a solution regarding an empty box or an empty space or any variable representing an unknown number. Lastly, the operational sign is seen as a signal for operating numbers. These strategies will help learners to understand the meaning of the equal sign in order to avoid errors and misconceptions that may result in learners making mistakes when solving equations.

Case in point 6*:* Conjoining of terms as an error

Researchers have found conjoining of terms in their studies (Alshwaikh & Adler, 2017; Gumpo, 2015; Mashazi, 2014; Ncube, 2016; Pournara et al., 2016). An example of conjoining terms, as suggested by Mashazi (2014), Gumpo (2015) and

Pournara et al. (2016), reveals that learners simplified $3x + 5$ as $8x$ and $x^2 + 4 + 2$ as $6x^2$. Similarly, the review of Ncube (2016) revealed where a learner conjoins $2x + 3y = 5xy$ and $2x + 4 = 6x$. This could happen in an addition calculation when a learner can incorrectly simplify binomial with binomial. Alshwaikh and Adler (2017) found a type of conjoining in their study when the learner simplified $(x + 2)(x + 4)$ as $2x + 6$ or as $2x + 4x = 6x$. This error reveals that a learner added variables and also added numbers to solve the problem which led to an error. Contributing to this, the conjoining of terms is a common error in learners, particularly in the secondary schools. In fact, demonstrable evidence prior to the work of Alshwaikh and Adler (2017), as suggested by Ncube (2016), indicates that conjoin errors are a result of a lack of understanding of the algebraic expression.

In **case in point 7**- Arithmetic operation errors are associated with commutative properties

In a multiplication of numbers, other learners also apply this case in the division, which is not applied: e.g.

$2 \times 6 = 6 \times 2 = 12,$

$\frac{2}{6} = \frac{6}{2}$ an error $\frac{2}{6} \neq \frac{6}{2}$

This rule applies when adding and multiplying, but not when subtracting and dividing. This type of error occurs when learners incorrectly generalise an idea, ignore accurate information, or consider false information as correct.

Operational errors associated with distributive property

$5(2x + 2) = 20$ $5(4x) = 20$, $5 \times 4x = 20$, $20x = 20$. The learner then divided by 20 on both sides and obtained the wrong answer $x = 1$. The answer was wrong because the learner did not follow the procedure to remove brackets. If learners do not remove the brackets in this type of operation, distributive property has an effect on the solution to an equation.

For example, here a learner worked this out as follows: $2(x - 4) = 2$, $2x + 8 = 2$, $2x = 10$, $x = 5$

In fact, the learner made a mistake in the second stage, $2x + 8 = 2$. This should have been $2x - 8 = 2$. The learner failed to transpose correctly by changing the sign of the term when transposing it from one side to the other, or misapplied the mathematical rule. Ncube (2016) states that the error caused by distribution law is overgeneralisation, for example the learner calculated as follows: $4(2x + 4) + 5(2x + 4)$ as $4 \times 2x + 4 \times 4 + 5 \times 2x + 5 \times 4$. This error is due to a lack of procedural knowledge.

Case in point 8: Operation signs ignored or ignoring a letter

In the equation $2x + 2x - 8 = 12$, the learner seemingly just added the coefficients of x, and the constant 8 and carelessly ignored the minus ('-') sign (Gumpo, 2015; Mashazi, 2014). This situation also pertains when learners fail to link new knowledge to existing knowledge (interference). Malahlela (2017) claims that the delivery of new knowledge to learners depends on what knowledge exists cognitively. Thus, if the new and the old knowledge do not connect, it is probable that there will be no learning in which interference occurs.

Case in point 9: Interference of new knowledge

Arrange like terms: $4y + 5 + 2y$

Learner error: $4y - 2y + 5 = 0$

Correct solution: $4y + 2y + 5 = 6y + 5$

In this example of the **interference of new knowledge,** the learner treated the algebraic expression as an equation because it has a variable. In the simplification of, $x + x$ as $2x$ and $x \times x$, as $x^{2,}$ it is then possible that when expressions like $x + x$ are revisited, some learners will think of the newly learnt concept of exponents, hence the new knowledge causes them to commit errors in their operations (Mashazi, 2014; Gumpo, 2015). In their research, Dhlamini and Kibirige (2014) hypothesised that errors and misconceptions very often surface when learners handle fractions. Dhlamini and Kibirige (2014) further argue that one of the major challenges learners encounter is when they are asked to determine the lowest common denominator of a fraction of which the denominators are variables.

Case in point 9: Fraction errors

Simplify: $\frac{5}{r} + \frac{4}{2pr}$

Learner error: $\frac{5}{r} + \frac{4}{2pq} = \frac{9}{2pqr}$

In the above case of fraction error, the learner applies the rule by multiplying the denominators to get the common factor; however, s/he carelessly adds the denominators. This error may also be as a result of the learner not being able to work with variables. In their studies, Mhakure, Jacobs, and Julie (2014) and Khanyile (2016) established that one of learners' biggest struggles was that involved in solving fractions; this also applies to Grade 9 learners. This evidence reveals that learners do not give up on committing errors, instead it is a continuous process. Khanyile (2016) suggests that fractions should be taught in the lower grades so that learners are taught the basics earlier. In support of Khanyile (2016), Makonye and Khanyile (2015) suggest that learners' inability to handle variables could lead to error types such as cancellation errors, incorrect use of the mathematical rules, and errors in factorisation.

Case in point 10: Cancellation errors

In this case, the learner cancelled at random, especially when the letters were the same, without following any mathematical procedure or cancellation rule.

For example:

$$\frac{3b^2 + a^2}{3b^2 - 2a^2} \times \frac{3a^2 + 4b}{3a^2 + 4b} = -2$$

Cancellation errors occur when the learner incorrectly cancels because the variables look alike. No knowledge relating to solving the algebraic expression is displayed, instead the learner looks for similar variables, and then merely cancels them without following the rule or procedure.

A study on understanding the errors and misconceptions in elemental Algebra, conducted by Makonye (2016), with a focus on Grade 10 learners, suggests pedagogical interventions for decreasing errors. However, despite such interventions, the study found that the cancellation error persisted. In his review, Makonye (2016) suggests that the cancellation errors were due to "senseless cancelling" where learners cancelled similar variables without following mathematical rules (p. 296). This also emphasises the fact that learners have difficulties in solving fractions. These types of senseless cancellation error, as Makonye (2016, p. 296) states, result from a lack of procedural knowledge. Mulungye, O'Connor, and Ndethiu (2016) claim that learners fail to follow the correct mathematical steps and apply their conceptual knowledge. In support of Mulungye et al. (2016), Fisher and Frey (2012) and Riccomini (2014) also note that these errors originate when learners do not understand the concept itself. Thus, algebraic cognition is largely dependent on procedural skills.

Case in point 11*:* Misunderstanding of factors

Factorise: $x^2 + 2x + 1$

$(x + 2)(x + 1)$

$x = -2$ or $x = -1$

The learner mixed up the factors, writing down erroneous factors that do not connect to the equation.

Case in point 12: Inability to find a common factor

Factorise: $2x + 4$

$x(2 + 4)$

Correct solution: $2(x + 4)$

The learner did not know the common factor and instead wrote an incorrect common factor. Learners misunderstand the factors,

Case in point 13: Use of incorrect mathematical rule

$\frac{2x + 1}{2x + 1}$

$$= \frac{3x}{3x}$$

= 1

Learners used the wrong procedure but nevertheless got the right answer. In this regard, Bush (2011) postulates that errors are a regular facet of the learning process. What Bush (2011) means is that errors committed by learners serve as a guide for educators and instructors to identify the sources of the errors and misconceptions when learning Algebra. Other researchers such as Adu-Gyamfi, Bossé, and Chandler (2015), Adu-Gyamfi, Stiff, and Bossé (2012) and Bossé, Adu-Gyamfi, and Cheetham (2011) also found the following types of error: interpretation errors, preservation errors and implementation errors. Additionally, some errors, explained as errors which happen in translation, are as follows**:**

Implementation error: Adu-Gyamfi et al. (2012) argue that learners write coordinates or points wrongly and use operational signs such as addition, multiplication, division and subtraction incorrectly, or incorrectly add a negative sign to a number. These errors happen when the learner is translating from a table to an equation, an equation to a table and the like, for instance:

Case in point 14: Writing a given equation as a table: $x = 2y + 1$

Table 2.1: Example of learner activity (implementation error*)*

x	-5	-6	-7	**0**	**1**	**2**	**3**
y	-6	-7	-8	0	0	0,5	1

The learner got the wrong answer here, instead, they left it as it is without dividing both sides by two in order to remove the two from the y. When working with positive numbers the other coordinates are correct but when working with negative numbers, incorrect coordinates or points are obtained. Regarding interpretation errors, the word ‘interpretation’ refers to ‘explanation’. In this example, the learner is unable to interpret or identify what happened or what was the cause of the situation, and when learners do the translation, they show their misunderstanding by the use of incorrect points, that is, $x\ nd\ y\ values$ (Adu-Gyamfi et al., 2012).

Case in point 15: Writing a given table as an equation.

For example:

Table 2.2: Example of learner activity (interpretation error)

x	-3	-1	0	1	2	3
y	-1	0	1	2	3	4

Suppose a learner's answer to the activity in Table 3 is the equation $y = -x + 1$. This does not correspond with the table. For example, the learner has used x values instead of y values which is the exact opposite of what was needed for the above tables (1; 2) and (2; 3) (see Table 2, 1, point 4 and 5) for the equation of $y = x + 1$. This is correct for the table (2.2) which was given above; however, the learner also wrote the wrong coordinates (2, 1) and (3, 2) for the equation $y = -x + 1$.

Preservation error: This type of error is shown by learners when not confirming all attributes of the source (Adu-Gyamfi et al., 2012). Calculations are done correctly, but not all of them. The points on the graph are drawn correctly, but there is no link between the graph and the points

Case in point 16: Suppose the learner was given $3y + 4x = 4$

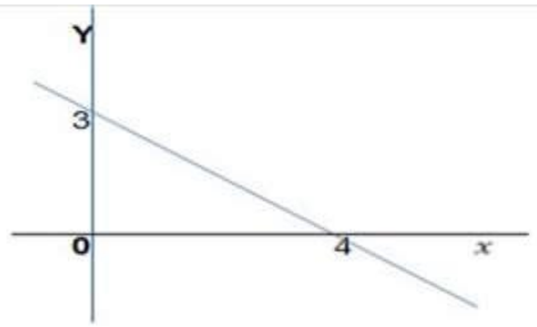


Figure: 1 an example of learner activity to display Preservation Error.

Figure 2.1: An example of learners' activity to display preservation error

In case in point 16, a learner puts a 3 as the y-intercept and a 4 as the x intercept without showing any calculations. S/he interpreted the line of x (horizontal values) and y values (vertically) correctly. Accordingly, s/he looked at the numbers in the equation and inserted them as the x intercept without making any calculations. So, the line in the graph shows the correct interpretation but the wrong x and y intercepts. Additionally, the learner drew a graph and put the x and y intercepts in right place but used the wrong points. Consequently, there is no connection between the intercepts and the graph. Dlamini (2017) found that other learners solved the problem correctly but lost marks or made errors because they failed to use a calculator when calculating the answer.

2.2.2. Sources of errors and misconceptions in Algebra

This section discusses the possible sources of errors and misconceptions when learning Algebra in the Senior Phase. It should be noted that this study focused on Grade 9 learners. The possible errors and misconceptions when learning Algebra include lack of procedural and conceptual knowledge, lack of connecting new knowledge with old knowledge, lack of interpretation, lack of emphasis by the teacher, overgeneralisation, oversimplification, overspecialisation, as well as error caused by translation.

2.2.2.1. Lack of procedural and conceptual knowledge

There is no single accepted definition for errors or misconceptions. The standard ones as adopted below come with some variations, as noted by Mulungye et al. (2016, p. 31), Egodawatte (2011) and Tweed (2014). Mulungye et al. (2016, p. 31) are of the opinion that errors "are mistakes in the process of solving a mathematical problem algorithmically, procedurally or by any other method". Egodawatte (2011), on the other hand, claims that algebraic learning built purely on procedural skills without considering conceptual understanding, could lead to errors and misconceptions. What may be drawn from Mulungye et al. (2016) and Egodawatte (2011) is that both procedural knowledge and conceptual knowledge can lead to errors and misconceptions when performing algebraic operations. Hence, it may be proposed that educators as a source of knowledge must have a way of changing learners' conceptual knowledge and design strategies when dealing with errors and misconceptions among learners. Tweed (2014) thus postulates that altering the learner's conceptual framework is the most important solution for overcoming and addressing errors and misconceptions when learning Algebra. In support of Tweed (2014), Schwartz (2010) believes that when a learner's conceptual knowledge has changed or developed in dealing with equations or expression involving division, errors may result when solving algebra problems using different procedures such as division, repeated subtraction, repeated addition or number lines, or using objects and modelling the action of division.

Tweed (2014), Schwartz (2010) and Hodgen, Foster, Marks, and Brown (2018) recommend that learners should organise their knowledge, procedures and concepts so that they are able to retrieve the knowledge and apply it. Thus, the learner must learn the concepts first and then the procedure follows. Drawing from the emphasis of the National Council of Teachers of Mathematics (2014), it is argued that after the learner has learnt the concept, the concept supports the procedure. Thus, in support, Schwartz (2010) emphasises that the learner is less likely to forget concepts than procedures – this holds for both human behaviour generally and when the learner is learning algebra.

If learners fail to write down their calculations when doing algebra tasks and constructing theories, it is difficult for teachers to identify the sources of errors and misconceptions, as they are unable to see what is going on in the learners' minds. As a source of knowledge, the teacher finds ways to change learners' conceptual knowledge and design strategies in order to deal with their errors and misconceptions. Fisher and Frey (2012) and Riccomini (2014) mention that procedural and conceptual errors are caused by a learner's lack of knowledge or a misunderstanding of the concept itself. Egodawatte and Stoilescu (2015) emphasise that errors which originate from a lack of meaning are procedural and structural errors. In the study by Zakaria (2010, p. 107), it is argued that when learners do not understand the meaning of certain word roots or origins, they do not understand the terms used and misinterpret what the question requires. Zakaria (2010, p. 107) argues further that this is caused by educators' lack of emphasis when teaching factorisation and other concepts.

2.2.2.2. Lack of factual knowledge as a source of error in Algebra

There are several debates regarding the causes of error and types of factual knowledge in Algebra (Adu-Gyamfi et al., 2015; Brown, Skow, & the Iris Centre, 2016; Cooper, 2015; Fisher & Frey, 2012; Madzorera, 2015; Riccomini, 2014). In Brown et al.'s (2016) study it is suggested that factual errors occur when a learner lacks factual information, for example vocabulary, digit identification and place value identification. For instance, learners do not have an understanding of the meaning of terms like numerator, denominator, most significant common factor, least common multiple or circumference, and also do not know mathematical formulae, for example the area of a square or a perimeter. In addition, Brown et al. (2016), Fisher and Frey (2012) and Riccomini (2014) mention that factual errors are caused by a learner's lack of knowledge or a misunderstanding.

In addition, when learners lack information on or understanding of the concept itself in algebra, this will result in errors, such as a failure to recognised formulas, thus leading to the wrong solution or mishandling operation signs that constitute the basics of mathematical algebra. In the study by Madzorera (2015), the sources of errors and misconceptions in word problems were shown to be a lack of vocabulary, and symbolic and metacognitive skills. Adu-Gyamfi et al. (2015) argue that the primary cause of many learners committing implementation errors is a lack of mathematical knowledge. Adu-Gyamfi et al. (2015) opine that preservation errors occur when the source and the target representations do not correspond semantically because the primary attribute or property of the target representation is not adequately accounted for. Cooper (2015) identifies a number of factors that cause errors in Algebra, such as not seeking help, lack of practice, insufficient prior knowledge, not asking questions and difficulty paying attention.

2.2.2.3. Failure to connect new knowledge with old knowledge

Egodawette (2011, p. 8) explains schema as the form which "allows an individual to organise similar experiences in such a way that the individual can easily recognise additional similar experiences". What this means is that schemas help learners to connect their new knowledge to old knowledge. Mashazi's (2014) examination of learners' errors, using Grade 9 as respondents, found that learners who could not interpret letters are unlikely to solve algebraic expressions. Problems include lack of interpretation, inability to articulate new knowledge with existing knowledge, ignoring the letters, and replacing letters with numeric values which indicate new knowledge that is not linked to the existing schema. Moodley (2014) supports the notion of Mashazi (2014) and Egodawette (2011, p. 8) by revealing that learners interpret letters differently, which leads to conjoin errors and ignoring letters. On the other hand, Gore's (2016) work on errors made by learners in solving simultaneous linear equations and causes of errors revealed that learners struggle to solve linear equations because of errors committed by substitution, elimination, transposing, removal of brackets, omissions of brackets and incomplete multiplication. Gore (2016) concludes that is due to a lack of connection between old knowledge and new knowledge.

It is also argued that this is often caused by the various theories learners embrace in their mind which led to them failing to connect new ideas with old ones, as well as their attitudes towards the subject of Mathematics. In a study by Brijlall and Ndlovu (2013), which examined learner difficulties in learning calculus (Algebra), it is emphasised that learners fail to link new Algebra topics with the ones they have learnt in the previous grade. Linking old and new knowledge issues were particularly true with Grade 11 work in which the concepts of minima and maxima were some of the sources of error, in addition to failure to link the old topic with a new topic. Brijlall and Ndlovu (2013) thus point out that:

> The minima/maxima schema was partially assimilated into their cognitive structures, but at times they failed to coordinate it with other existing schemas, such as function and gradient, which were vital in solving optimization problems (p. 16).

It is also necessary to note that there are topics that learners learn in Grades 10 and 11 that are not incorporated in the Grade 12 syllabus, but do form part of the Grade 12 examinations. Thus, as learners of Algebra learn, they fail to link the new Algebra topic with the old ones, thus leading to errors. For example, when learners learn the difference of two squares, they sometimes confuse this with factorisation.

In response to such confusion associated with say, factorisation, and the common core state standards for Mathematics (2010) highlight standard algorithms and suggest that there are more chances for learners to improve their thinking. This is initially in the earlier grades to help Algebra learners. Gardee and Brodie (2015) argue that in the process of conflicts between new knowledge and existing knowledge the logic is to wait for restructured schema to accept or connect in order to work and produce correct solutions. Thus, the errors and misconceptions will only be corrected if learners are able to revise old concepts before learning a new concept.

2.2.2.4. Lack of interpretation as a source of error in Algebra

It is also argued that when solving Algebra problems learners are unaware that they are making errors. Many researchers have found conjoining of terms to be an error. For instance, when examining conjoining terms, i.e. $3x + 5 = 8x$, it is found that the origin or the cause of this error is the interpretation of the expression, which is "add three times x and five" or as an object (Egodawatte 2011; Gumpo, 2011; Mashazi, 2014; Mdaka, 2011; Mulungye et al. 2016; Pournara et al., 2016). Thus, the interpretation is very important for the learner when engaged in algebraic equations or expressions that require a reasonable level of interpretation in order to acquire the standard meaning of the expression. Gumpo (2011) suggests that confusion between an equation and an expression may reflect lack of knowledge of the two concepts. Thus, a learner would not be able to do Algebra without knowing the terms or concepts. Hence, "errors displayed due to misconceptions [that] learners have about [a] topic indicate the incorrect interpretation of a mathematical idea as a result of a learner's personal experience or incomplete observation" (Mdaka, 2011, p. 3).

2.2.2.5. Lack of emphasis or knowledge by the teacher

Zakaria (2010) identifies the types of error that occur when learners complete the square and argue that the cause of an error is a lack of emphasis by teachers on understanding the language of mathematical Algebra. Poor emphasis by the teacher leads to failure to learn new topics. In addition, teachers may fail to ensure that every learner grasps the necessary skills before continuing with the new topics. As a result of Zakaria's (2010) assertion, it is argued that this cause of errors when learning Algebra originates from a lack of prior information. Even though there is knowledge in the learners' minds, as the learners do not come to class as empty vessels, this information may be erroneous to use to acquire new knowledge. Zakaria (2010) suggests that one cause of errors is a lack of emphasis on the part of educators when teaching factorisation.

In addition, a teacher needs to reorganise previous work in learners' minds and come up with strategies to teach learners to understand new work. For instance, Kenya National Examinations Council (n.d.) emphasises that causes of learners' errors when solving word problems include inadequate coverage of the syllabus content. Also, some of the teachers do not finish the syllabus, some of them struggle to teach certain other concepts, and ignore or skip others. When concepts are overlooked, it results in errors during examinations. It is also argued that learners' knowledge generally depends on the teachers' knowledge. Thus, learners' knowledge depends on the strategy used by the teacher in delivering a lesson and the conceptual knowledge of the teacher. When the teacher does not have conceptual knowledge, learner errors may not be corrected. Therefore, teachers need to develop and improve their conceptual knowledge in order to correct and avoid learner errors.

2.2.2.6. Overgeneralisation as a source of error when learning Algebra

Typically, other causes of errors and misconceptions are committed by overgeneralisation, oversimplifying and overspecialisation of Algebra equations or expressions. This is emphasised by Mulungye et al (2016) who argues that "overgeneralization of number and number properties might be the single most important underlying cause of learners' misconceptions".

Case in point 19

Overgeneralisation: Solve for: $(x+8)(x+2)=10$ *The error :*

x + 8 = 10 or x + 2 =10, which leads to an error x = 2 or x = 8. This rule should not be applied in this case; it should only be applied if the equation is equal to 0, for example in this equation (x + 8)(x + 2) = 0.

In these examples continue as x = -8 or x = -2.

The most common example is $(t + s)^2$, error is $t^2 + s^2$

The error is as a result of overgeneralisation of the distributive property. In this case, the learner misinterpreted the bracket; thus, the error is $t^2 + s^2$. The learner thus misapplied the rules and lacked conceptual understanding. In the study done by Makonye (2011), it was revealed that learners confuse the commutative law of addition and subtraction. Makonye (2011) argues that multiplication and division are also an example of overgeneralisation.

For instance, $2 \times 3 = 3 \times 2 = 6$ but

$$\frac{2}{3} \neq \frac{3}{2}$$

This normally happens in multiplication but not in division. As Makonye (2011) states, the use of operational signs the author mentioned that there is a different when operating using numbers, as the multiplication sign is not operated as a division sign. It is important for the learner to understand that a positive sign does not work as a negative sign and multiplication does not work as division. It was also found that some learners think that $\frac{0}{0} = 1$ because they know that dividing same numbers or same variables gives 1 but do not know that division by zero is undefined. In addition, other learners generalise equations because they lack procedural knowledge for solving the equation. Khanyile (2016) argues that overgeneralisation when simplifying expressions results in a cancellation error. As discussed (refer to case in point 10), cancelling similar variables instead of following mathematical rules is a common problem in the Senior Phase (i.e. Grades 7 to 9).

2.2.2.7. Oversimplification as a source of error when learning Algebra

Misconceptions tend to hinder learning when learners must interpret new experiences. Mdaka (2011) believes that learners become “emotionally and intellectually attached to their misconceptions because they have actively constructed them. Hence, they find it difficult to accept new concepts which are unfamiliar and dissimilar to their misconceptions” (Mohyuddin & Khalil, 2016). Tweed (2014) argues that many misconceptions apparent in Algebra are rooted in misconceptions about arithmetic, for example common fractions and decimal fractions, and the magnitude of negative numbers. Sanders (2017) reveals learner error when solving equations wrongly, for example:

4x - 8 = 5.

4x = 8 - 5

4x = 3

x = 4 - 3

x = 1

This learner has wrongly generalised the concept. The error is caused by an erroneous understanding of transposing. It appears that learners lack the skill of procedure or the steps required when dividing both sides. This failure or confusion is the origin of the mistake in this problem. Having algorithmic skills without being clear on the concept is one of the reasons for hitches in mathematical Algebra (Zakaria, 2010). Oversimplification refers to over calculating the equation or expression, such as fractions, when a learner gets an answer of $\frac{10}{4}$, but continues solving as $\frac{5}{2} = 2\frac{1}{2}$. This is particularly true when writing solutions as a mixed fraction.

2.2.2.8. Overspecialisation as a source of error when learning Algebra

Suppose that a learner is given different triangles to name. In triangle 1, no angles are given/allocated but an indication of the equality of sides is given, which shows equal angles, also as the equilateral in both angles and sides as in triangle 2. *1.*

2

Figure 2.2: Example displaying overgeneralisation

A learner often fails to see that all angles are equal if all sides are equal (triangle 1). In triangle two, they can see that $60^\circ + 60^\circ + 60^\circ = 180^\circ$, therefore it is an equilateral triangle. Usually, in these cases, the questions are related to algebraic equations because in other questions learners are asked to find an unknown angle if they are given other angles. In responding to such questions, learners need to identify the type of figure first before performing the equation, which is done by adding all given angles and equalising with the sum of all angles in the triangle (180

°). Usually, unknown angles are named by any variable. For instance, in triangle 1, let us assume that angle 1 is x; this means that all angles are equal because the figure is an equilateral, then this could be $x + x + x = 180°$, hence, $x = 60°$

2.2.2.9. Inattentiveness, failure to read and understand

Chege (2015) identified factors that prevent learners from solving word problems correctly. These include learners' inattentiveness and failure to read and understand proper algebraic mathematical operations. Additionally, it is argued that failure to understand a problem and weak semantic skills involving symbols and meanings of terms, as well as vocabulary, are the main factors that cause errors when solving word problems. Others causes include not concentrating while the teacher is busy teaching, ignoring lessons claiming that they already know the work, the holding of theories that block their mind from grasping or mastering new concepts, and quickly forgetting what the teacher teaches them, which is sometimes caused by overconfidence.

According to Gore (2016), errors and misconceptions should not be understood as a problem but observed as an opportunity to reflect and learn. Supporting this, Mathematics teachers should recognise an error and take it as a driver for finding solutions. This is helpful for directing learners' minds and what to teach, and what strategies to use for solving learner errors .Many researchers (Dlamini, 2017; Dhlamini & Kiribige, 2014; Gumpo, 2015; Khanyile, 2016; Mahlabela, 2012; Makonye & Hantibi, 2014; Makonye & Khanyile, 2015; Mashazi, 2014; Ncube, 2016) have identified different kinds of errors and causes of errors in their studies. Some of these include that the learner has to understand the language of algebraic Mathematics, such as simplify, factorise, expand, and solve, in order to understand the concepts, the similarities and the differences between those concepts before solving an algebraic task. Furthermore, researchers such as Mdaka (2011), Hansen (2011) and Khalo and Bayaga (2014) emphasise that diverse actions may cause errors in Algebra, including inattentiveness, misinterpretation of symbols or text and not checking the answers carefully.

Case in point 20

In factorisation, the learner must understand that brackets have to be introduced. For example, factorise $20x^2 - 2x + 10$

Expected answer: $2(10x^2 - x + 5)$

Error 1: $18x^2 + 10$

Error 2: $18x^3 + 10$

Error 3: $2x(10x - x + 5)$

The error here is caused by the exponents. The learner does not understand the concept of exponents. Subsequently, the learner has to revise relevant previous knowledge which, in this case, is the laws that govern exponents. Luneta and Makonye (2010) state that learning difficulties exist whenever a learner fails to grasp a concept or an idea owing to a lack of prior knowledge, as well as inadequate knowledge about the concept to be acquired. The various sources of mathematical Algebra errors can also be identified by researching tools to use for reproducing, recollecting, finding, treating, conducting, and sorting the information in Algebra tasks. Mamba (2012) states that in Algebra, errors and misconceptions originate during the introduction of new concepts. Additionally, this happens when a learner fails to link a new concept with an existing concept, that is, the prior knowledge. This also happens when the old information is erroneous, hence the new knowledge does not work effectively.

Case in point 21

Simplify: $(x + 3)(x – 3)$

Expected answer: $x^2 - 3x + 3x - 9$

Solution: $x^2 - 9$

Error: $x^2 - 3x + 3x + 9$

Solution of error: $x^2 - 6x + 9$

In the above example, the learner thinks that $(x – 3)^2$ is the same as $(x + 3)(x – 3)$. In this case, the learner is confused by the new concept, the difference of two squares, which is introduced together with the familiar concept of binomial expansion. This error may be caused by the method of teaching. It is very important for the teacher to revise previous work so that will be easier for the learners to master the new topic.

2.2.2.10. Errors caused by translation

Various studies have focused attention on errors in translation (Adu-Gyamfi et al., 2012; Adu-Gyamfi et al., 2015; Bossé et al., 2011; González-Calero, Arnau, Puig, & Arevalillo-Herraez, 2013; Molina, Rodríguez-Domingo, Canadas, & Castro, 2017. What is found in these studies is that in secondary schools, learners have difficulties or make errors when translating between the algebraic symbolism and the verbal representation.

As a part of the school curriculum, learners are expected to express mathematical Algebra ideas accurately, communicate their algebraic thinking, solve problems in Algebra using models and also interpret them as well (Molina et al., 2016). When learners fail to do that as a part of the curriculum, errors may occur and there are various sources of such errors in Algebra. Bossé et al. (2011) identified factors in translation and opined that when a learner translates from the verbal to the symbol, it often results in the manifestation of implicit and unrelated or confusing data. Molina et al. (2017) thus argue that the struggle in translation may be influenced by the existence of a certain kind of context implicit in the verbal representation given. On the other hand, Adu-Gyamfi et al. (2012) claim that attribute-situated errors are caused by the misrepresentation of the algebraic mathematical structure of the problem situation. There are also encoded errors in the implementation construct, which are caused by an algorithmic misstep or incorrect treatments in the algebraic register (Adu-Gyamfi et al., 2012).

2.2.2.11. Errors caused by a lack of basic skills in Algebra

Maharaj, Brijlall, and Narain (2015) argue that in Algebra, learner's lack of basic skills and knowledge in Algebra functions and reasoning, and the use of symbols, and connectives are causes of error. Pournara et al. (2016) claim that when learners of Algebra make errors and harbour misconceptions, those new errors are related to the new procedure. Moreover, they depend on how the new procedure has been taught by the teacher. They are also caused by poor understanding of algebraic mathematical facts and lack of basic skills in a lower grade. The Department of Basic Education (DBE, 2012, p. 12) states that most of errors done by learners in solving Algebra were based on poor understanding of basics and foundation competencies taught in early grades.

This emphasises that learners often do not learn the basic skills in Algebra in the lower grades, which leads them to make many errors, for instance in relation to operational signs such as (×), (-), (+), or (÷), which are taught in the lower grades and the meaning of the equal sign.

2.2.3. Strategies for avoiding errors and misconceptions when learning Algebra

This section addresses learning and teaching strategies, approaches and methods for avoiding errors and misconceptions in the learning of Algebra. This includes workshops that are organised by the DBE to support teaching in providing learners with a quality education.

2.2.3.1. A Mathematics strategy for building or filling gaps so as to avoid errors in Algebra

Mdaka (2011) emphasises that there are conceptions and preconceptions that learners of different ages and backgrounds bring with them to the Mathematics classroom. However, if preconceptions are misconceptions, teachers need knowledge of strategies that are likely to be fruitful in reorganising the learners' understanding. This will help teachers to avoid the errors that Grade 9 learners commit and the misconceptions they have when they solve Algebra. Mamba (2012) points out that errors and misconceptions originate when learners are introduced to new concepts in mathematical Algebra. In the above example, the learner thinks the $(x - 3)^2$ is the same as $(x + 3)(x - 3)$.

The teacher's knowledge is a learner's source of information. Errors may be caused by a lack of information on the teacher's part or an incorrect teaching strategy that was used to teach. The teacher, therefore, needs to revise the previous work and come up with strategies to reorganise previous work in the learners' minds before starting new work, as well as to show the differences and similarities between the two concepts. To dispel misconceptions and show learners the differences and similarities between the two concepts, teachers may have to ask questions before commencing with new work. This is not about learning how to get the answer but is a matter of knowing the logic of procedures when learning Algebra. Algebra is abstract and when a learner lacks understanding in the use of terminology or fails to concentrate when the teacher is teaching, errors and misconceptions may arise.

Researchers such as Hughes (2011), Cease-Cook (2013), Mudaly and Naidoo (2015) and Washing (2018) have reviewed studies on the effect of concrete representational abstract (CRA). The CRA is the teaching strategy that help teachers in order to teach learners fraction and easily understand.

Mudaly and Naidoo (2015) indicate that the used of CRA is best for the effective teaching of Mathematics. Hughes (2011) supports the notion that in teaching learners fractions, CRA encourages or motivates learners who have difficulties. In a review by Cease-Cook (2013), it is also argued that the use of CRA in solving equations using the inverse operation is best for instructional learning of mathematics. Washing's (2018) review reveals that the CRA may be effective for learners with disabilities. This indicates that the use of CRA is best for the teaching and learning of Algebra learners with difficulties in secondary schools. The errors and misconceptions committed by Grade 9 learners in this study also took account of CRA usage.

2.2.3.2 Schematic approach

Scheme learning is a teaching approach that helps learners develop skills in solving difficult concepts such as word problems. Powell (2011, p. 1) argues that "[i]n Mathematics, students can use schemas to organise information from a word-problem in ways that represent the underlying structure of a problem type. Pictures or diagrams, as well as number sentences or equations, can be used to represent schemas".

Many researchers (Ahmad, Tarmizi, & Nawawi 2010, Fagnant & Vlassis, 2013; Murtini, 2013; Powell, 2011; Raoano, 2016; Sepeng & Webb 2012; Van Klinken, 2012) suggest a schematic approach as a strategy for learning Algebra. For instance, Van Klinken (2012) argues that a schematic approach is the right approach for teaching word problems. Furthermore, this approach helps both teachers in giving them the direction on teaching difficult concepts and learners (conceptualised semantically). In support of Van Klinken (2012), Murtini (2013) discovered that a scheme learning approach improved skills when learners solve Algebra word problems. In the study by Sepeng and Webb (2012), it was discovered that the use of a schema-based strategy has a good effect on developing learners when solving problems such as word problems. Fagnant and Vlassis (2013) a year later emphasised that this approach has a good effect on learners' development.

2.2.3.6. Pedagogical strategies and tactics

Researchers such as Ramlia, Shafieb, and Tarmizi (2013) and Dlamini (2017) recommend pedagogical strategies and tactics for avoiding errors. Such pedagogical strategies and tactics on the part of the teacher can help learners to avoid errors and misconceptions when learning Algebra. Such tactics may include fun learning, effective communication, and problem-based instruction, a constructivist approach, real-life applications, technology-integrated learning and learner-centred learning. The errors and misconceptions that Grade 9 commit when solving Algebra problems may be highlighted in a study by evidence from learners' scripts and from focus group interviews. Most errors are due to a lack of procedural and conceptual knowledge. The strategies discussed in this section will help both teachers and learners to overcome errors and misconceptions.

2.2.3.8. Counter-example strategy (CES)

Klymchuk (2012) mentions that CES is straightforward, quick and efficient to use, making it easy to see when a given statement is incorrect. The use of CES can indicate whether a hypothesis is wrong before proving it using another method (Klymchuk, 2012). Researchers such as Klymchuk (2012) and Dlamini (2017) explain that when using CES, every learner should pay attention to every detail of the statement and should observe the order of words and symbols that were used in the statement. Errors and misconceptions should be corrected, and learners should acquire a solid conceptual understanding and procedural knowledge in order to able to solve algebra equations and expressions. CES is a working strategy use with learners that have difficulties in counting or calculating Algebra. This strategy was revealed to overcome computational errors.

2.2.4. Algebra content in South Africa: What does the government say about the Algebra curriculum in South Africa?

This section addresses the curriculum and the learning of Algebra in the South African context, particularly with regard to the teaching and learning of algebra in the Senior Phase. This includes learner and teacher support, the school and the learning environment such as the classroom, early child development and educational policies. The topic will also address educational policies such as, the CAPS document for teacher guidance in the teaching and learning process, as well as national policy on assessment protocol. This section discusses the teaching and learning of Algebra, as the teacher cannot teach without the policies and documents that serve as a guide in the process of teaching and learning. If the teacher follows the CAPS document and applies the departmental rules correctly, the teaching process will proceed effectively. When the teacher teaches well the learner benefits and, in the teaching of Algebra, errors and misconceptions on the part of Grade 9 learners may be avoided.

2.2.5. Learners' performance in Algebra

Diagnostic Reports (2015 to 2017) show gaps in learners' performance in terms of the learning and teaching of Mathematics. Learners and teachers need more development in order to improve education in the Republic of South Africa (RSA). There is a need to rebuild plans for supporting learners and teachers in teaching and learning. In this report (NDRP, 2015), it would appear that all teachers need teacher professional development and a plan for improving results. One plan that was introduced in 2015 was the National Diagnostic Report for Learner Performance (NDRP). The NDRP (2015) was used in 2016 to fill gaps in schools, support teachers in developing their activities and lastly building up school-based assessment (SBA). The report (NDRP, 2015) states that in 2015, 49,1% of learners achieved 30% and above, while 31,9% achieved 40% and above. This indicates that learners made errors when doing Algebra; such errors involve algebraic concepts such as factorisation, solving involved unknown values etc. Diagnostic Reports (2016) indicates that the subject improvement plan was developed on the basis of diagnostic analysis of learner's responses for common errors and misconceptions. The NDRP (2017) was used to fill gaps in schools 2018, supporting teachers in developing their activities and building up SBA. Tables 5 and 6 show learners' Mathematics performance in the NCS examinations from 2012 to 2017. The types and sources of errors found in learners' scripts reveal that learners are struggling in Algebra. The National Senior Certificate Examination Diagnostic Report (NSCEDR, 2015, 2017) indicates the types of error found in those years, as discussed in the table below.

Table 2.3: Adapted from National Senior Certificate Examination Diagnostic Report, 2015 (p. 150)

Year no	Wrote no	Achieved at 30% and above	% achieved at 30% above	No achieved at 40% and above	% achieved at 40% and above
2012	225874	121970	54.0	80716	35.7
2013	241509	142666	59.1	97790	40.5
2014	225458	120523	53.5	79050	35.1
2015	263903	129481	49.1	84297	31.9

Table 2.4*:* Diagnostic report 2017 (Adapted from NSCEDR, 2017, p.151)

year	No wrote	% achieved	%	% achieved	%

		at 30 above		at 40 above	
2014	225458	120523	53,5	79050	35,1
2015	263903	129481	49,1	84297	31,9
2016	265912	136011	51,1	89119	33,5
2017	245103	127197	51,9	86096	35,1

The National Senior Certificate Examination Diagnostic Report (NSCEDR, 2015) reported the following errors and misconceptions in Algebra: errors in writing quadratic formulae and also substitution, as well as in binomial expansion, inability to find the middle term, errors in finding the square root and in squaring, treating inequality as an equation, misunderstandings regarding the inequality sign, interval notation errors, inability to identify the domain and the range, misunderstandings regarding the nature of roots of quadratic equations and patterns (NSCEDR, 2015).

2.3. Theoretical framework (SOLO model)

A theoretical framework specifies the researcher’s assumptions and beliefs about how the many ideas of research may be viewed. This section discusses the theory used in analysing errors and misconceptions regarding algebra in Grade 9 learners in secondary schools.

There are many theories, such as behaviourism, cognitivism, and constructivism, and the spiral theory of learning that are relevant for the teaching and learning of Algebra. Among them is the SOLO model. The SOLO model was developed by Biggs and Collis in 1982 and has been revised by many researchers (Biggs & Collis, 2014; Lian & Yew, 2012; Lucander, Bondemark, Brown, & Knutsson, 2010; Makonye, 2011; Na’imah, Sulandra, & Rahardi, 2018; Frame, 2017). Frame (2017) claims that the model helps to observe the building of new knowledge in learners.

The work of Biggs and Collis (2014), namely, the SOLO model was used in the current study as an approach, methodology, and technique to engage in assessing and observing the quality of learning. For the current study, the SOLO model was used based on the information in Table 2.2. Thus, Table 2.2 serves as a guide in analysing the way learners learn Algebra.

Table 2.5: SOLO model structures and explanation on levels by (Lian & Yew, 2012)

Levels of thinking	Explanations on the level of thinking.
One structure	There is one piece of information that is related to the question.
Many structures	Most correct information but not all of them and not entirely interrelated with one another and no clear explanation that is related to the question.
Relational	All information is correct about a question.

The SOLO theory was used in the current study to classify errors and contribute to our understanding of Algebra learning, using questions structured on three levels, as indicated in Table 2.5. According to the SOLO model, a learner's response depends on two structures which, in the current case, were cognitive development and levels of response. In this study, the SOLO model was also used to examine levels of thinking in the causes of errors and misconceptions among Grade 9 learners in Algebra, to explore the possible sources of errors and misconceptions in Algebra and to identify strategies for avoiding error and misconceptions when learning Algebra. The three levels of thinking as applied in the current study are shown from the lower to the upper levels as follows:

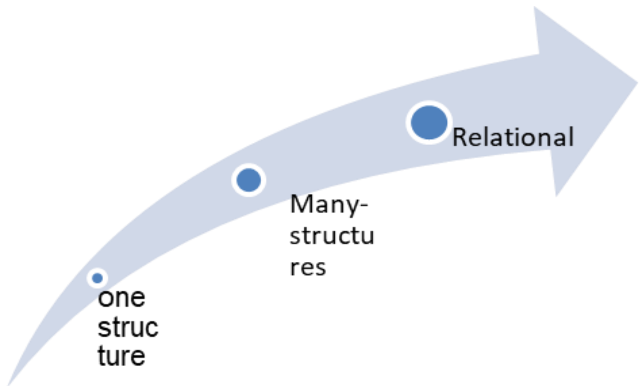


Figure 2.3: The stages in SOLO model theory

These stages indicate the levels of thinking and levels of response to the questions that were used in this study to examine Grade 9 learners' level of thinking when solving and learning mathematical Algebra problems. The current study examined learners' responses using the SOLO model, as have other researchers (Biggs & Collis, 2014; Frame, 2017; Lucander et al., 2010; Lian & Yew, 2012; Makonye, 2011). In 2010, in their study, Lucander et al. (2010) asked 32 students to write a test with 35 students in the control group, to examine the way the SOLO model assisted dental students to develop a broad approach to learning in dentistry. Students were assessed on learning outcomes in a summative assessment. Lucander et al. (2010) concluded that SOLO was able to promote and develop a more profound methodology for learning in dentistry.

2.3.1 Application of the SOLO model in the current research

In support of the work of Lucander et al. (2010), a separate research study was conducted by Makonye (2011), who applied two types of structure, namely unistructure and multistructure. The study concluded that the SOLO model is advantageous when analysing the levels of learner thinking and observing how they work when solving a mathematical problem. Makonye (2011) claims that levels of thinking depend on the need of the particular subject, which in this case is in mathematical algebra According to Lian and Yew (2012), the SOLO model depends on two features, that is, cognitive development and level of responses by the learner. Thus, in the current research, the model was applied as a means to assess learners' learning outcomes and to classify the value of the response, which could be concluded from the structure of the answer as reflected in the mathematical task assigned and the research question. Na'imah et al. (2018) recently used the SOLO model in their study to examine learners' skills of multilateral FD students to solve Pythagoras problem and its structure. The finding reveals the ability of multi-structural level FD students could use some of the information provided for problem solving. The authors revealed that in the process the answers that were given were not accurate and, lastly, the scaffolding efforts to increase the level of FD students' ability from multi-structural level to relational level consist of reviewing, restructuring, and developing contextual thinking.

Building on the works of Lian and Yew (2012), Na’imah et al. (2018) and Lucander et al. (2010), the current study adopted three structures of learner understanding based on the SOLO model. These three structures were adopted for the current study to respond to the types of error and misconception that Grade 9 learners commit in responding to Algebra problems. The choice of the model was also intended to contribute to our understanding of the sources of errors and misconceptions, well as the strategies for avoiding errors and misconceptions in the Senior Phase. In Lian and Yew’s (2012) work, it is argued that the level of structures should be reduced to three, namely, uni-structural, multi-structural, and relational. The structures adopted for the current study took the form of one structure, many structures and relational. In the study by Lian and Yew (2012), the SOLO model was used to reverse and combine superitems which were designed in such way that any super item that presents a correct response to an answer indicates thinking ability. For the current study too, the SOLO model focused on the thinking levels of Grade 9 learners when they respond to the questions relating to Algebra, in which case the analysis depended mostly on the learners’ responses. To unpack the three research questions, the SOLO model was used to examine, observe and analyse Grade 9 learners’ responses to questions in Algebra. In the study, the model was adopted in order to assist in conducting the research in secondary schools in King Cetshwayo District of uMlalazi and Mtunzini Municipality.

It is important to recognise, however, that when Biggs and Collis (1982) developed SOLO theory, there were five structural levels: 1) pre-structure level, meaning no response (no thought); 2) one-structure level meaning one response (little thinking); 3) many-structure level (more thought) meaning that most responses are related to the answer; 4) relational level (much thought) meaning all correct answers; and 5) extra-extended level, meaning exceptional response. For the current study, the researcher excluded two levels, namely, pre-structure (no idea) and extra extended (adequate idea) because the examination was based on learners’ types of error committed and the misconceptions learners displayed when they solving Algebra problems, sources of those errors and misconceptions, as well as strategies for avoiding errors and misconceptions in the Senior Phase. For the current study, the levels of thinking were examined when learners responded to

Algebra questions such as equations with fractions, word problems, and translation from an equation to a table or graph.

Many studies on error analysis in Algebra have included equations and expressions, see for instance the works of Mashazi (2014), Herholdt and Sapire (2014), Gumpo (2015), Ncube (2016), Khanyile (2016) and Mashazi (2014) who explored the thinking underlying Grade 9 learner errors in introductory Algebra. In essence, what may be drawn from the studies of Mashazi (2014), Herholdt and Sapire (2014), Gumpo (2015), Ncube (2016), Khanyile (2016) and Mashazi (2014) is that various aspects contribute to learners' errors, including task instructions, new knowledge, ignoring the letters, and replacing letters with numeric values. However, Mashazi (2014) further recommends that by pointing out to teachers the types of error that committed by learners in our classes, teachers may be assisted to observe those errors and address the sources of errors and misconceptions, as well as design new strategies for overcoming those errors. A study by Lumbala (2015) focused on algebraic graphs and consequently concluded that learners have difficulties working with algebraic graphs. The type of errors found were related to coordinates, intercepts, domain and range, asymptotes, and identification, drawing and function errors.

2.4. Koch error analysis

To complement the SOLO model, Koch error analysis was used as the second theory for the current study. Koch (2015) classifies errors into five types, namely, careless, computation, precision, problem-solving and unpreparedness. The current study used this classification to describe the types of error found in Grade 9 algebra.

2.4.1. Careless error

Koch (2015) explains that careless errors occur when learners write down the wrong numbers and do not follow mathematical procedures or directions for what is expected in the answer. It is also argued that the learner fails to pay attention during the lesson perhaps as a result of tiredness. Researchers such as Matuku (2017), Salihu (2017), and Agustyaningrum, Abadi, Sari, and Mahmudi (2018) have explored careless errors in their studies. Matuku (2017), for example, explains that careless errors are the mistakes that learners commit when solving mathematical problems carelessly even though they know the correct solution. This is sometimes the result of failing to concentrate during the lesson or when solving the mathematical problem. The current study is thus guided by a suggestion by Agustyaningrum et al. (2018), who claim that learners may be unable to solve mathematical problems regardless of having the conceptual knowledge for the given concept. Agustyaningrum et al. (2018) list the following as the causes of learners' careless errors:

- copying a wrong number
- reading algebraic problems inappropriately
- dropping the sign somehow somewhere, either positive or negative,
- writing untidily or messily,
- inability to follow the procedure
- labelling wrongly
- Changing a given number by writing the wrong one.

2.4.2. Computation error

Koch (2015) explains computational errors as calculations that may be done wrong in a mathematical problem when the learner works with operational signs. Agustyaningrum et al. (2018) and Koch (2015) claim that learners commit errors when working with operational signs, as they misuse addition sign, subtraction, multiplication, and division sign. The current study is also guided by the fact that some errors in computation may be caused by a lack of adequate ability in both English and algebraic terminology, as well as the gap between arithmetic and Algebra (Salihu, 2017). The current study was also guided by the view of Makonye and Fakude (2016), who claim that the causes of errors in subtraction operations that relate to negative integers are due to poor proficiency in English. Thus, the current study takes cognisance of the language of teaching and learning in Mathematics as part of learning Algebra.

2.4.3. Precision error

The current study was guided by Koch's (2015) explanation of what precision entails. Koch (2015) explains that in precision, learners reveal confusion when solving concepts, untidiness, dropping signs and forgetting signs when calculating, either addition or subtraction. In addition, other units may disappear – this could be either a variable or a number – and a lack of labelling and notation may also be a problem.

2.4.4. Problem-solving error

As a guide to the current study, Koch (2015) explains that learners fail to follow proper mathematical rules. Moreover, some do not complete the required steps. Thus, in this study, the researcher was cognisant of what Arum, Kusmayadi, and Pramudya (2018) suggest, that is, that learners do not evaluate or double-check their problem-solving technique when they are solving mathematical problems.

2.4.5. Unpreparedness

In regard to unpreparedness, the current study was guided by Koch's (2015) claim that this involves the learner not finishing mathematical problems for some reason, for instance failure to seek help; lack of formative assessment such as free quizzes or quick checks; and lack of corrections on the work. This view also guided the current study where it was observed that learners fail to complete mathematical problems, which leads to errors because the work is generally given to the learner to provide them with practice. If the learner fails to complete the mathematical problem, they fail to practise as a part of learning Mathematics.

2.5. Cognition in learning Algebra

A study of development psychology was conducted by Fuchs et al. (2016) to assess children's levels of development in algebraic knowledge versus solving word problems. In part, the intent was to assess computational accuracy and fluency as necessary skills. The children were evaluated in terms of their ability to calculate in early stages (early grade e.g. grade 2) word problems and their number knowledge at the start of Grade 2; calculation accuracy and calculation fluency at the end of Grade 2; and pre-algebraic knowledge and word problem solving at the end of Grade 4. The research revealed that understanding of language was critical for the word problems at the pre-algebraic knowledge stage. Additionally, the findings revealed that the pathways in the development of these forms of fourth-grade Mathematics performance are more alike than different but demonstrate the need to fine-tune instructions for strands of the Mathematics curriculum in ways that address individual students' foundational Mathematics skills or cognitive processes.

Another study in psychology was conducted by Chimoni and Pitta-Pantanzi (2017). The objective was to assess the relationship between algebraic thinking and abilities in the central reasoning processes. In this study (Chimoni & Pitta-Pantanzi, 2017), 190 learners aged 13 to 17 were assessed in both algebraic thinking and central reasoning processes. The first part of the study evaluated the cognitive systems such as spatial-imaginal, causal-experimental, and qualitative analytic and verbal-propositional while the second part evaluated algebraic thinking. The findings revealed that all the tested cognitive processes support learners' algebraic thinking. Another study by Zhang (2018) looked at the development of reasoning in Algebra learning, by taking into account content knowledge and the cognitive skills of Algebra learning. The findings revealed that the diagnostic assessment mode could affect learners' progress in Algebra learning in terms of both content knowledge and thinking skills.

2.5.1. Developmental dyscalculia (DDs)

Developmental dyscalculia (DD) is related to learners who suffer from computational problems when solving Mathematics (Laurillard, 2016). In support of Laurillard (2016), Skagerlund and Traff (2016) and Filippo and Zoccolotti (2018) suggest that DD leads to difficulties in solving problems and poor performance in Mathematics. Recent studies by Skagerlund and Traff (2016) also examined DD in children with different profiles of mathematical deficits and different aspects of processes.

2.6. Chapter summary

The literature review in this research displayed the types of error as well as misconceptions in Algebra learning in Senior Phase learners. The types of error and misconception committed by learners include careless errors (use addition as a multiplication), computation errors (multiplying wrongly), i.e. $2(x - 2) = 2x - 2$, problems in solving problems including conjoining errors, cancellation errors and the like. The literature revealed the possible sources for those errors which included lack of procedural and conceptual knowledge, lack of factual knowledge or vocabulary, lack basics skills, lack of emphasis by the teacher, errors caused by exponents, as well as oversimplification, overspecialisation and overgeneralisation. The review of the literature also revealed strategies for avoiding these types of error when learning Algebra such as CES, CRA, departmental workshops for helping teachers, scheme learning and the like. Koch error analysis theory was discussed in terms of careless errors, computation errors, problem-solving errors, and errors involving precision and unpreparedness. The SOLO model was discussed as one structure – referring to the idea of thinking displayed by the learner, many structured, that is, many ideas revealed by the learner and relational, meaning that learners revealed all ideas when thinking. Theories relating to the content of Algebra in South Africa and the Department of Education (i.e. DBE-ANA) were also discussed. The following chapter outlines the research methodology and the research design for the study.

CHAPTER THREE: RESEARCH METHODOLOGY

3.1. Introduction

Research methodology involves a set of specific techniques for selecting cases, measuring and observing aspects of social life, gathering and refining data, analysing data, and reporting the results (Neuman, 2011). This also includes the population and sample data collection to address the research objectives of the study, which were used to examine the causes and sources of errors and misconceptions, and how misconceptions can be avoided when learning Algebra. Thus, the current chapter focuses on paradigms, the research design, the method used to collect data and the analysis of the study findings in response to the research questions, as reflected in chapter one and addressed in chapter two.

3.2. Philosophical paradigm

"The concept of paradigm originated from the Greek word *paradeigma* which denote pattern" (Owolabi, 2017, p. 104). Mathematically, a philosophical paradigm refers to a pattern + thinking = number of patterns that aids certain actions or movements. Drawing from this scientific assertion, the researcher is guided by a series of ideas about the world. On the other hand, Neuman (2014) asserts that a research paradigm is the total thinking and expectations in a worldview. Kivunja and Kuyini (2017) in support of Neuman (2014) suggest that a research paradigm creates the thinking involving the beliefs and values in relation to the way the researcher views the world. Drawing from Kivunja and Kuyini (2017) in support of Neuman (2014), the current research was guided by the views of Grix (2010), who mentions that a research paradigm in academic research directs the view on the field of study.

Based on different worldviews, Maree (2011) identified three different types of research paradigm – positivism, critical theory and the interpretive paradigm. In the current study, both positivism and interpretivism were adopted due to the nature of the research questions, as reflected in chapter one and interrogated in chapter two.

3.2.1. Positivism

Positivist procedures relate to predictions of the perceptible, as well as the explanation of truths and their relationships (Neuman, 2011). Thus, positivists believe that the truth is factually given and is measurable using effects which are autonomous of the researcher – thus knowledge is impartial and measurable. As guided by Neuman (2014), the current study combines deductive logic with empirical observations of individuals for identifying and verifying patterns in error and misconception analysis.

3.2.2. Interpretivism

Neuman (2011) opines that an interpretive paradigm refers to someone who obtains the understanding of research from experiences and skills gained over time. This could be due to the different research approaches used in the investigation of human behaviour, environments and situations. Essentially, the researcher is very much involved in seeking to find out opinions on how and why learners commit errors and form misconceptions (Neuman, 2011). This was guided by the researcher who collected the data by means of a focus group interview during which algebraic questions were asked. In line with the nature of the interpretivist paradigm, the results were video recorded and field notes were taken. Learners consequently revealed various opinions, attitudes, experiences and characteristics that helped address the problem statement. Thus, in line with Creswell (2012), the current study was guided by the characteristics of interpretivism; as a thorough understanding of a central phenomenon of errors and misconceptions in algebra, which were based on learners' experience, analysis of data for description and themes using text analysis and interpreting the larger meaning of the findings.

3.2.3. Mixed method research (MMR)

In general, MMR is defined as gathering, analysing and interpreting data using both quantitative and qualitative approaches. Creswell (2014) and Creswell and Clark (2011) state that MMR combines two approaches, namely, quantitative and qualitative. These approaches when intertwined yield a collective form of data. This assists the researcher to compare the data to confirm the results or findings of the study. In mixing approaches when conducting the study, the researcher decided to administer each form of data individually, decide the sequence in which data would be collected, decide on how to combine the data, and whether to use theory to guide the study (Creswell & Clark, 2011). The way in which data are combined depends on the nature of the inquiry and the philosophical outlook of the person conducting the research.

Cameron (2011) identifies different ways of using MMR to include social sciences, education and political science. When the two approaches are used to gather data, differences can be noticed regarding data gathering methods, employment of logic, different research paradigms, methods of analysing data and ways of presenting the research findings (Neuman, 2014). Creswell and Plano Clark (2011) claim that an MMR approach enables a higher degree of understanding to be formulated than if a single approach were adopted for specific studies. Accordingly, the current research employed an MMR approach to investigate learners' errors and misconceptions when learning Algebra. The findings were interpreted (qualitative + quantitative =mixed method) while quantitative results were analysed using the SPSS software and qualitatively interpreted. Both qualitative and quantitative results were triangulated.

Table 3.1: Mix-methods research (Cameron, 2015, p. 4)

Quantitative (Positivist)	Qualitative (Interpretivist)
Objective reality	Subjective reality
Causal	Meanings
Detached	Human intentions
Samples/populations	Personally involved
Contrived	Study cases
Variables	Actors in natural settings
Numerical	Verbal & pictorial data
Statistical	Generalise case findings
Impersonal	Group

The focus group interview was conducted in a group of six learners per school, the questions were asked orally and in the form of group, but the test assessed individuals. The results obtained using both approaches were comparable. Accordingly, in the data Grade 9 learners revealed similar types of error and misconception as well as the sources of such errors.

3.3. Research design

The research design is the framework and planning procedure which demonstrating the order of how research will take place. Both research methods (qualitative and quantitative) have different types of designs that are applicable, and those designs are also divided into categories, for example experimental and non-experimental (Owolabi, 2017; Brink, Van der Walt, & Van Rensburg, 2014). In the current study a non-experimental design was adopted. There are many types of non-experimental design, but this study employed survey design for gathering descriptive information. The nature of the population and the required information was also considered before the researcher chose the design of this research.

3.3.1. Survey of the research

Neuman (2011) emphasises that survey research methods involve the process of acquiring information which records answers from different groups of respondents. Survey research is well defined as the gathering of data from a sample of individuals through their answers to problems (Check & Schutt, 2012). Data gathering using this method traditionally involves dealing a large population. In the current study, survey research was aimed at obtaining data from a large sample of individuals. Based on Fowler (2014), who lists why a survey design is required, the current study considered the use of a survey because it was easy to direct even remotely via the internet, cell phones, by post and email, as well as the fact that it is relatively low cost. The data were gathered from 100 Grade 9 learners at schools in the King Cetshwayo District by directing an Algebra test counting 50 marks. The test was used to identify the types and sources of errors and misconceptions that Grade 9 learners commit when doing Algebra and the strategies used to avoid these errors and misconceptions.

3.3.2 Target population and sample size

Brink, Van der Walt and Van Rensburg (2012) explain that a target population is the total population to which the researcher intends to generalise his/her findings. The target population for this study was 100 Grade 9 learners ranging in age from 13 to 16 years from five secondary schools in the King Cetshwayo District of the

KwaZulu-Natal province. Each class comprised 20 learners. For the calculation of the sample size of 100 (see Instrumentation and setting for data collection in Part one).

Instrumentation and setting for data collection

The test comprised four questions which were based on Algebra. Question one covered algebraic fractions, question two graphs, tables, expressions and equations, question three words problems, and question four on equations and expressions related to the concept of ratio, height, length and area (see Appendix A). The test counted out of 50.

Part One

The Algebra test was written by the classes in the five schools. To obtain the sample for the study, the researcher used a sample size calculator with a confidence level of 95% and a confidence interval of 9, giving a sample size of 91. However, for ease of reporting and better reliability, the researcher sampled 100 learners. In addition, purposive sampling and convenience sampling were used to select the five schools. Thus, non-probability sampling was used to identify the five schools selected in the King Cetshwayo District. Accordingly, two schools were located in uMlalazi, and three schools in Mtunzini. The researcher requested the sampled learners to write the test (Appendix A) in order to identify the types of error committed and misconceptions displayed by Grade 9 learners and the sources of these when solving Algebra problems.

Part two

Part two of the research was carried out in the same five schools and the same learners were used for interview purposes. In order to compare the findings, six learners per school (giving a total of 30) were selected for the focus group interview. Focus group interviews were employed to complement the quantitative data obtained through the use of question for the test. The focus group interview schedule contained questions concerning Grade 9 Algebra. Learners answered the questions orally, and the information was recorded using pen and paper (participant observation). The information was also recorded using videotape. All research questions were addressed both in part one and part two. The sampled learners were interviewed using the focus group interview schedule (see Appendix C) to obtain the types of error and misconceptions displayed by Grade 9 learners and the sources of such errors and misconceptions when solving Algebra problems.

Focus group interviews- From five schools, six learners were chosen per school

1. School one: Six learners purposely/conveniently selected to participate in focus group interviews.
2. School two: Six learners purposely/conveniently selected to participate in focus group interviews.
3. School three: Six learners purposely/conveniently selected to participate in focus group interviews.
4. School four: Six learners purposely/conveniently selected to participate in focus group interviews.
5. School five: Six learners purposely/conveniently selected to participate in participate in focus group interviews.

3.3.3. Sampling procedures and methods

Sampling refers to a way of choosing sample of objects in the group of events, such as people or any other elements used in the study to fulfil the needs of a researcher. The group of the object in this study that was used in order to fulfil the need of the researcher was the learners in King Cetshwayo District. Neuman (2011) emphasises that a population is all respondents who meet the criteria for investigation about what the researcher seeks to establish. The population for this study was all secondary schools in King Cetshwayo District. Hence, the researcher selected a sample of 100 learners in this District using a sample size calculator (as indicated in Instrumentation and setting for data collection in part one). There were 22 circuits in King Cetshwayo District, but in this study only two circuits were selected as a sample, namely, uMlalazi and Mtunzini. In total, in the Mtunzini and uMlalazi circuits there are 140 schools; thus, five schools were selected as a sample of all schools in the King Cetshwayo District. Subsequently, the three schools were selected from the Mtunzini circuit, and two schools from the uMlalazi circuit. The researcher's reason for selecting King Cetshwayo District was that the District had had underperforming schools in Mathematics for several years preceding this research. The importance of selecting these learners related to the research questions: the researcher sought to classify the types of error and misconception displayed by Grade 9 learners when solving Algebra problems and observing the sources of those errors. Lastly, the study sought to classify strategies for avoiding those errors.

3.4. School settings

School one

The school is located in uMlalazi circuit in the King Cetshwayo District. The school has one principal and two deputy principals, one for administration and the other for academic matters. The school is divided into four departments – commerce, communication, science and technical – which are headed by four heads of department. At the time of the study, the school had an enrolment of 855 learners, with a mixture of 34 temporary and permanent educators. There were four Mathematics educators teaching Grades 8 to 12 and there were 141 Grade 9 Mathematics learners. In addition, the school had two other staff members, an administration clerk and a learner support agent (LSA). Only 20 learners were selected to write the 50-mark test (which formed the quantitative part of the study). In addition, six learners were selected for focus group interviews (which formed the qualitative part of the study). The school used a timetable based on a five-day cycle with one break. Classes started at 8:00 and ended at 14:30. The school is a quintile 2 school, which means the school is situated in a rural area, is poorly served regarding resources and the parent body is largely unemployed; however, this school does not fall into the poorest quintile – quintile 1. The school is a public, no-fee school and is geographically located in the Mvuntshini area.

School two

As indicated, the second school also falls under the uMlalazi circuit of the King Cetshwayo District. At the time of the study, the school had one principal and no deputy principal. The school had two departments, namely, commerce and science, which were headed by two heads of department. The school had an enrolment of 255 learners, with a mixture of 11 temporary and permanent educators. There were two Mathematics educators teaching Grades 8 to 12, with 41 Grade 9 Mathematics learners. The school used a timetable based on a five-day cycle with one break. Classes started at 8:00 and ended at 14:30. Of the 255 learners, 20 learners only were selected to write the test and six of these were chosen for a focus group interview. The school falls into quintile 1, the poorest quintile (see school 5). The school is geographically located in the Enyezane area.

School three

This school is situated in the Mtunzini circuit in King Cetshwayo District. At the time of the study, the school had one principal and one deputy principal. There were four departments – commerce, communication, science and general – but three HODs, with no HOD in the commerce department. The school had an enrolment of 597 learners, with a mixture of 19 temporary and permanent educators. There were four Mathematics educators teaching Grades 8 to 12 and 114 Grade 9 Mathematics learners in the school. Two other staff members and admin clerk and an LSA were also present. Only 20 learners were selected to write the test (quantitative), while six learners were selected for focus group interviews (qualitative). The school uses a timetable based on a five-day cycle with one break. Classes start at 8:00 and end at 14:30. The school falls inti quintile 1 (see school 5) and is geographically located in the Makhilimba area.

School four

This school is situated in the Mtunzini circuit in King Cetshwayo District. At the time of the study, the school had one principal and one deputy principal. The school included three phases – Intermediate, Senior and Foundation – each with an HOD. The school has an enrolment of 561 learners, with a mixture of 19 temporary and permanent educators. Two Mathematics educators were teaching Grades 7 to 9 and there were 29 Grade 9 Mathematics learners. In addition, there was one nonteaching teaching staff member, an administration clerk. Out of the 521 learners, 20 were selected to participate in the achievement test (quantitative). In addition, six learners were selected for the focus group interviews (qualitative). The school uses a timetable based on a five-day cycle with one break. Classes start at 8:00 and end at 14:30. The school falls into quintile one (see school 5). The school is geographically located is in the Hemfane area.

School five

This school is situated in the Mtunzini circuit in King Cetshwayo District. At the time of the study, the school had one principal and no deputy principal. There were three departments, namely, commerce, science and general, but the general department had no HOD although the other two did. The school had an enrolment of 302 learners, with a mixture of 15 temporary and permanent educators. Two Mathematics educators taught Grades 8 to 12 and there were 68 Grade 9 Mathematics learners. There was one additional staff member, an admin clerk. Only 20 learners were selected to write the 50-mark test (quantitative) and six learners were selected to participate in the focus group interviews (qualitative). The school uses a timetable based on a five-day cycle with one break. Classes start at 8:00 and end at 14:30. The school falls into quintile 1, meaning that it an impoverished school. It is situated in a rural area and the parent body is unemployed; in addition, there is a lack of teaching and learning resources. The school supported by the Department of Basic Education with funds (no-fee school meaning learners do not pay school fees) and resources, that is, textbooks. The school is geographically located in the Ensingweni area

3.5 Sampling techniques

A sample is a small number of individuals selected from the population participating in a study. De Vos, Strydom, Fouche and Delport (2014) claim that sampling refers to choosing a small number of units from a population in order to obtain information. There are two different kinds of sampling, namely, non-probability and probability sampling. In this study, non-probability sampling was used to select a sample from the population. In the sample of 100, twenty learners were selected per school for the test and in a sample of 30, six learners were selected per school for the purpose of focus group interviews (see Instrumentation and setting of data collection in part one).

There are different kinds of non-probability sampling. In this study purposive and convenience sampling were used to select the sample from the population. Purposive and convenience sampling are discussed below.

3.5.1. Purposive sampling

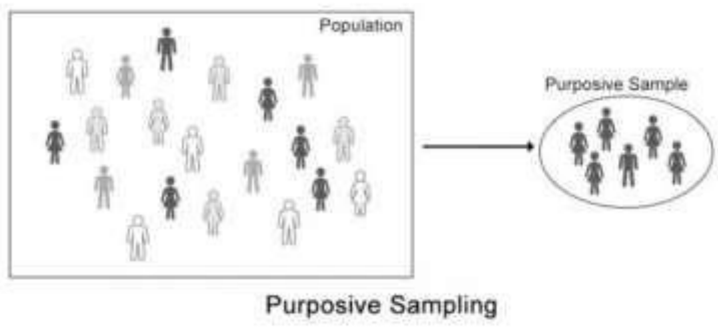


Figure 3.1: Purposive sampling (research-methods.net)

Researchers (Etikan, Musa, & Akassin, 2016; Saunders, Lewis, & Thornhill, 2012) believe that purposive sampling is a non-probability form of sampling in which the sample population is chosen for the specified purpose of the researcher. The authors (Etikan, Musa, & Akassin, 2016; Saunders, Lewis, & Thornhill, 2012) use other terminologies to explain purposive sampling such as judgement, selectively and subjective. The researcher is working in one of the schools as a teacher of Mathematics in Grades 8 to 12, hence this school was selected purposely to classify Grade 9 errors when learning Algebra. The purpose of the study is to overcome errors and misconceptions done by Grade 9 in solving Algebra. Tanujaya, Mumu, and Margono (2017) emphasise that purposive sampling is a non-random procedure that needs no theories or participants instead, the researcher chooses what known requirements, and lastly sets out to invention people who are able and willing to deliver the information with knowledge or experience.

3.5.2. Convenience sampling

Convenience sampling, also known as accidental sampling, is a type of non-random sampling where participants in the target population reveal specific practical characteristics, for example easy approachability, physical propinquity, convenience at a given time, or willingness to participate, that encompass the aim of the study (Etikan et al., 2016). Etikan et al. (2016) mention: “Convenience samples are sometimes regarded as ‘accidental samples’ because elements may be selected in the sample merely as they happen to be situated, spatially or administratively, near to where the researcher is conducting the data collection” (Etikan et al., 2016, p. 2). “Convenience sampling is affordable, easy and the subjects are readily available” (Etikan et al., 2016, p. 2).

The researcher targeted rural learners in high schools in King Cetshwayo District. The five selected secondary schools were not far from each other, and were situated at Umlalazi and Mtunzini Circuits. This saved time and costs regarding the research. As the researcher was a teacher at one of the selected schools, she had easy access to the selected learners. It was thus convenient for the researcher to approach and communicate with the parents, as well as the teachers of the selected learners as they were colleagues of the researcher.

3.5.3. Sampling frame

A sample frame is a set of data used to classify a sample population for statistical purposes (Mthethwa, 2015). Neuman (2011) opines that a sampling frame presents the characteristics of the population from which the researcher collects data for the research investigation.

Table 3.2*:* The sample frame for all sampled schools Part one (quantitative): Test

School 1	School 2	School 3	School 4	School 5	Total number of all sampled

20 learners purposely/conveniently selected	20 learners purposely /conveniently selected	20 learners purposely/conveniently selected	20 learners purposely/conveniently selected	20 learners purposely/conveniently selected	100

As highlighted in the Instrumentation and setting of data collection in part one, in obtaining the sample for this study, the researcher used a sample size calculator with a confidence level of 95% and a confidence interval of 9, giving a sample size of 91. However, for ease reporting and better reliability, the researcher sampled 100 learners.

3.6 Data collection instruments

Choice of data collection method depends on the research method adopted for the study. Questionnaires are suitable for a large population if time and funds are limited (Maree, 2011). In the current study, a self-designed test, based on the research questions, was used to collect data from learners (see details on research questions in chapter one). The first part of the research comprised a test counting 50 marks which was subsequently used to analyse errors and misconceptions displayed by Grade 9 learners when solving Algebra problems, as well as the sources of errors and misconceptions and strategies for avoiding those errors. The second part consisted of a focus group interview, which focused on examining the types of error and misconception displayed by Grade 9 learners and, lastly, how those errors and misconceptions can be avoided.

3.6.1. Test

In general, a test is a formal assessment given to learners to assess them. A test is usually an individual task that is invigilated by the teacher until learners' finish writing. The researcher developed a test counting 50 marks which was written by the Grade 9 learners to principally determine and evaluate learners' errors and misconceptions in two conceptual areas of Algebra, namely, expressions and equations learnt in Algebra in the Senior Phase. The test consisted of questions that tested their knowledge of the aforementioned concepts in Algebra.

Validity

In order to ensure validity, the test was moderated by three teachers (moderated by colleagues) with more than five years' working experience in Mathematics in Grades 8 to 12. It was also checked in the University of Zululand in the Department of Education (Maths, Science and Technology) by the research assistant and my supervisor.

3.6.2. Focus group interview (FGI)

Nieuwenhuis (2014) opines that an interview is a discussion between two people, a researcher (interviewer) and a participant, in which the researcher asked the participant questions in order to collect data and to learn about philosophies, principles, theories, views, opinions and behaviours of the participant. The type of interview employed in this study was a focus group interview. Abawi (2013) identifies the characteristics of a focus group interview as follows:

- It is a structured discussion with the purpose of stimulating conversation around a specific topic.
- The discussion is led by a facilitator (researcher) who poses questions in response to which the participants give their thoughts and opinions.
- The discussion gives the possibility of cross-checking one individual's opinion against the other opinions gathered.

A focus group interview is more than a question and answer session. In a group situation, members tend to be more open, and the dynamics within the group and the interaction can enrich the quality and quantity of information needed. Six learners were selected from each of the five schools chosen as a sample. The learners were tested on Algebra and questions were posed in both IsiZulu and English to observe whether they understood well and responded correctly or incorrectly or gave no response, in other words, whether they understood the language used to ask the question. The questions were based on equations with fractions, expressions in the concepts of ratio, height, length and area, graphs and tables, as well as word problem. This instrument was meant to response on errors and misconception committed by grade 9 learners in solving Algebra, sources of errors and then strategies to minimuse those errors and misconceptions.

3.6.3. Participant observation

Participant observation provides a description, framework, causation and validation, which means that it is often advantageous to include it in a mixed method study (Guest, Namey, & Mitchell, 2013). In this study, the researcher observed the participants (the learners) during the study in a focus group interview. All of these observations were videotaped. The observations were done to obtain data on types of error and misconception that Grade 9 learners display when solving Algebra problems and their sources as well as the strategies for avoiding those errors. It was observed that Grade 9 learners commit errors and display misconceptions when solving Algebra problems, even when they clearly understand the questions as explained in both English and isiZulu. Some of the errors found in the focus group interview were problem-solving errors, indicating a lack of conceptual knowledge, and procedural errors and computation errors.

3.6.4. Code-switching during focus group interviews

Code-switching (refer to Appendix C) refers to when a presenter uses two or more languages in presenting particular content or a topic. The DBE (2010) emphasises that code-switching is a switch from one language of instruction to another language of instruction during teaching and learning. Chikiwa and Schafer (2014) claim that learners tune out when teachers use code-switching. Chikiwa and Schafer (2017) experience the use of codeswitching strategy in teaching mathematics and found that teachers lack planning skills and the materials to teach, using both borrowed code-switching (BCS) and transparent code-switching. There are different kinds of code-switching, for instance intra-sentential and inter-sentential switching. The use of code-switching in this study was meant to respond to the types of error Grade 9 learners make when they respond to Algebra questions. A focus group interview with six learners per school was meant to respond to the types of error and misconception that are prevalent in Grade 9, the sources of and the strategies for avoiding them. The questions (refer to Appendix C) was done using isiZulu and English in order to compare the results of a test and focus group interview (both IsiZulu and English). In this study, the researcher was also testing code switching see whether it works for teaching learners to learn for understanding and to avoid errors and misconceptions. The code-switching in this study was done to address the research question, as highlighted in chapter one, which sought to classify the types of error and misconception committed by Grade 9 learners in solving Algebra problems, the sources of these and strategies for avoiding them.

3.6.4.1. Intra-sentential switching (Table 4.4 and Appendix C)

It has been argued that speakers change from one language to another within the same sentence (Kebeya, 2013). Consequently, a sentence will be made up of two or more languages. In intra-sentential switching, the researcher enters the environment with entrenched languages in the code-switched material. The matrix language is the critical language of code, interchanging different sounds in an embedded language or languages which is the central language that plays a lesser role (Kebeya, 2013).

Thus, contributing to this, the intra-sentential switching researcher was done to change the English language to the isiZulu language in one sentence during questioning in the focus group interviews. This was done to respond to the three research questions, namely, types of error committed by Grade 9 learners, sources of errors and misconceptions in the Senior Phase and strategies for avoiding errors and misconceptions in the Senior Phase.

3.6.4.2. Inter-sentential switching (Table 4.4 and Appendix C)

Kebeya (2013) mentions that a speaker changes from one language to another in two sentences; additionally, an individual's speech is divided into sentences where one sentence is in one language while the other sentence is in an entirely different language.

In this study, inter-sentential switching occurred in the form of switching from English to IsiZulu in two sentences. In this language change, the sentence was built in English and repeated in IsiZulu so that the respondents understood the requirements of the questions in both the language of learning (English) and the mother tongue (isiZulu). Switching was done in the focus group interviews to examine types of error and misconception committed by Grade 9 learners in responding to Algebra questions, the sources of errors and misconceptions in the Senior Phase and strategies for avoiding errors and misconceptions in the Senior Phase.

3.6.4.3. Code-switching in the study

Focus group interviews were adopted in this study as per the explanation (see Appendix C) and both intra-sentential and inter-sentential switching were adopted in this study, whereby the presenter (researcher) used both English and isiZulu. IsiZulu is the mother tongue of most of the learners, and English is the medium of instruction. Code-switching was used by the researcher in determining the participants' ability to understanding the algebraic expressions and equations on a Senior Phase level (Grade 9) when their mother tongue language was used. IsiZulu (the home language) is not generally used as an instructional language since English is the medium of instruction. Home language refers to the language that is spoken most frequently at home by a learner (DBE, 2010).

Furthermore, the language of learning and teaching (LOLT) refers to the language medium in which learning and teaching, including assessment, takes place. The underlying principle of the Language in Education Policy (LiEP) is to maintain the use of the home language as the LOLT (especially in the early years of learning) while providing access to an additional language(s). The LiEP makes the following stipulations:

> All learners shall be offered at least one approved language as a subject in grades 1 and 2. From grade 3 onwards, all learners shall be offered their LOLT and at least one additional approved language as a subject. All language subjects shall receive equal time and resource allocation.

Learners must choose their LOLT upon application for admission (DBE, 2010).

The code-switching in this study examines the types of error committed by Grade 9 learners when they respond in Algebra, the sources of errors and misconceptions in the Senior Phase and strategies for avoiding errors and misconceptions in the Senior Phase. The importance of code-switching in this review was to measure and examine its use as an approach in learner's response to Algebra problems and comparing learners' responses in both focus group interview (English and Zulu languages) and a test. The important of code-switching in this study was to

- observe whether learners understood concepts better in their mother tongue
- understand the effect on the performance when they talk in their own language
- the effect of the medium of instruction on learner errors and misconceptions (see Table 4.4 on the data analysis – chapter 4)

3.6.5. Triangulation

Yeasmin and Rahman (2012) explain that in the social sciences, triangulation is the permutation of two or more theories, sources of data, techniques or investigations in one study of a single phenomenon or a particular concept, and it may be employed in both quantitative (validation) and qualitative (inquiry) studies. Yeasmin and Rahman (2012) further this notion by saying that "triangulation is a process of confirmation that intensifies validity by incorporation several viewpoints and methods that offer a chance for researchers to validate their investigation results, lastly it permits the investigators to be more self-possessed about their research results". Mertens and Hesse-Biber (2012) emphasise that triangulation is a measurement method that is used by surveyors to trace an item in space depending on two well-known ideas in order to triangulate on an unknown fixed point in that same space. In line with Yeasmin and Rahman (2012) and Mertens and Hesse-Biber (2012), the researcher, therefore, collected data from various sources using different data collection methods. Thus, data were collected from different school settings (quintiles). A quantitative approach in the form of an achievement test and a qualitative approach in the form of an interview were employed in data collection. Also, the researcher employed two different theoretical frameworks – Koch's theory of error analysis and the SOLO model to address the three research questions.

3.7. Reliability of survey instruments

According to Creswell (2014), "reliability refers to whether scores to items on an instrument are internally consistent (i.e. are the item responses consistent across construct), stable over time (test-retest correlations), and whether there was consistency in test administration and scoring" (p. 247). The researcher collected the data twice using different instruments such as focus group interviews and a test. Similar results were obtained in both the test and the interviews under similar conditions (in a school environment). Both the results of the test and the focus group interviews were considered to be reliable because learners were found to have committed errors and harbour misconceptions when solving algebra problems in the senior phase – hence, the findings are linked to the research questions. The findings revealed that Grade 9 learners committed errors and misconceptions when solving Algebra problems, that there were different sources of those errors (one of the causes was a lack of conceptual knowledge in solving equations with fractions). For instance, one piece of written work shows the type of error that was committed by a Grade 9 learner in solving an Algebra problem (refer to Figure 4.7 learners 1). The source of errors and misconceptions with the strategy for avoiding the error type (refer to Case in point 1) on error analysis. This indicates that the test was reliable because the results were found to be linked and to respond to the research questions. The results of the FGI (refer to learner 4) revealed that grade 9 learners commit errors and misconceptions to learning Algebra.

3.8. The validity of the survey instrument

Neuman (2011) identifies four types of validity: criterion, construct, content and face validity. Validity is the ability of a research instrument or instrument for data collection to measure a research variable effectively or the degree to which a variable is measured well by a research instrument (6 &Bellamy, 2012). Experts' view on a research instrument is a standard measure of its validity in Algebra research. The test was validated by Algebra experts from the University of Zululand in Department of Education in Mathematics, we well as the heads of department and the five teachers who have more than ten years' experience in teaching mathematics in the Senior Phase. (Refer to the analysis on pages 163 and 164.) This was done to respond to the types and sources of errors and misconceptions committed by Grade 9 learners in response to Algebra problems, as well as the strategies applied to avoid those errors.

In ensuring face validity, the researcher contacted experienced educators who had taught mathematics in the Senior Phase for many years. In terms of content validity, this type of validity considers whether the data instruments cover all the content concerning the variables they are intended to measure. Content validity was achieved by adapting past questions from previous Grade 9 Algebra papers obtained from the South African Department of Basic Education (DBE). Furthermore, the DBE approved textbooks for teaching mathematics were used as guidelines when setting the achievement test, and this test was then moderated by experienced teachers teaching mathematics in the same phase.

3.9. Ethical issues

The University of Zululand ethical guidelines and policies regarding plagiarism, participants and non-participants' indicator content was observed in this research. In this study, the researcher considered confidentiality of information, and respect for intellectual property and copyright. The researcher followed the ethical considerations provided by the University of Zululand (UNIZULU) research office. Data from respondents was used only for the study, and no names were mentioned in the research report. Approval was obtained from UNIZULU (Appendix 0) ethics review office.

Approval from the five schools was also obtained to conduct research in the schools. Informed consent of principals (Appendices I) and parents/guardians was obtained using relevant documentation (Appendices J and K). These documents include invitation letters to the principals to conduct the research in their schools, invitation letters to students for their participation, and informed consent forms to parents/guardians for their children's participation in the study. Only learners whose parents/guardians had granted permission were tested and interviewed. The approval from Department of Education (see appendix H) was obtained to conduct research in the schools included letter from parents and the principals of the schools. The response letters from principal and parent did not attached in the appendix of this document for the case of anonymity and privacy because their names appear on the letters.

Participation was voluntary, and participants had the right to withdraw from the study at any time. During the reporting and discussion of data, none of the participants, schools or communities was identified (pseudonyms were used), and participants were not judged or evaluated on their participation or nonparticipation. All the data that were collected had the names removed before analysis and reporting. The researcher introduced herself to the learners before the test and the focus group interviews to make them feel more comfortable and to communicate freely.

3.10. Data presentation and analysis

Data analysis generally involves capturing data in an electronic file by using a statistical software package and performing appropriate statistical analysis. Statistical analysis methods (frequency tables, bar graphs and percentages) are procedures for manipulating data so that the research questions can be answered, usually by identifying important patterns in quantitative research (6 &Bellamy, 2012).

Table 3.3: Data Presentation and Analysis

Research questions	Approach	Sources of data	Methods of data analysis
What are the types and the sources of errors and misconceptions committed by Grade 9 learners in Algebra learning?	Both qualitative and quantitative (see chapter 3)	Test, interview, literature review (appendices A and C)	Descriptive statistics, thematic content analysis (chapter 4 in 4.6.1.1) (see Figure 4.16)

How do the types and the sources of errors and misconceptions influence errors in Grade 9 learners' cognition when learning Algebra?	Both qualitative and quantitative (see chapter 3)	Test, interview, literature review (appendices A and C)	Descriptive statistics, thematic content analysis (chapter 4 in 4.6.2.1)
Through which strategies and sources could the errors and misconceptions relating to Algebra be avoided?	Both qualitative and quantitative (see chapter 3)	Test, interview, literature review Which appendices (appendices A and C)	Descriptive statistics, Triangulation. Quant +qual =mixed method (and chapter 4)

The research objectives were analysed based on descriptive statistics, thematic content, and multiple regression, as explained in Table 3.3 above. Data analysis generally involves capturing data in an electronic file by using a statistical software package and performing appropriate statistical analysis. Analysis methods are procedures for manipulating data so that the research questions may be answered, usually by identifying important patterns in quantitative research, as are used in this study (6 & Bellamy, 2012).

Table 3.4: Descriptive Statistics

Research objectives of Algebra	Descriptive
Examine the causes of misconceptions among Grade 9 learners in Algebra	Frequency distribution Measures of central tendency
Explore the possible sources of misconceptions	Frequency distribution Measures of central tendency
Identify strategies to avoid misconceptions when learning Algebra	Frequency distribution Measures of central tendency

After the data were collected using both the test and the focus group interviews on Algebra expressions, equations, graphs and tables, descriptive statistics and thematic content analysis were used as methods of data analysis. Frequency distribution and the bar graph were used in an analysis to evaluate the types of error and misconception that Grade 9 learners commit when they are solving Algebra, as well as possible sources of those errors as declared in the literature.

The data were interpreted (focus group interviews) for triangulation (test results and focus group interview results). The purpose was to respond to errors and misconceptions that Grade 9 learners commit when solving Algebra problems by identifying the sources of those errors and the strategies for avoiding those errors. The results of the quantitative data analysis will be presented using frequency counts and percentages, tables and charts (see Appendix E – focus group interview).

3.10.1. Descriptive statistics

Descriptive statistics is a way of summarising and describing data. Statistics is divided into two types – measures of central tendency (giving some sense of the central value of a data set) and measures of dispersion (giving a measure of how spread out that data set is). Measures of central tendency include the mean, mode, median and range of the data. Measures of dispersion include the range, variance and standard deviation of the data. In this study, the descriptive statistics included frequency distributions with minimum and maximum value, mean percentages and standard deviation. The results of the quantitative data analysis will be presented using frequency counts and percentages, tables and charts.

Bertram and Christiansen, 2015 affirms that descriptive statistics transform a set of data into either a visual overview such as a table or graph, or into a few numbers that summarise the data. Accordingly, in this study tables, bar graphs (see data analysis) and pie charts (Appendix E) were used to illustrate the distribution of errors and sources of errors, while bar graph were also used for quantitative data. The descriptive analysis was meant to respond to the types of error and misconception Grade 9 learners commit in solving Algebra problems, as well as the sources of those errors and the strategies for avoiding them.

3.10.2. Thematic content analysis

Neuman (2011) argues that content analysis is measured as an addition and not as a substitute for the personal examination of documents. Alhojailan (2012) opines that thematic analysis delivers the chance to code and classify data into themes; it is considered the most suitable for any study that seeks to discover using clarification. Furthermore, it adds a systematic element to data analysis and allows the researcher to associate analysis of the frequency of a theme with one of the whole content. Such a thematic analysis provides an opportunity to code and categorise data into themes. The thematic content analysis that was used in the study focused on the sources of data, namely, test questions, focus group interviews and the literature review to answer the research questions: What type of errors and misconceptions do Grade 9 learners display in simplifying Algebra? Moreover, what are the sources of errors and misconceptions of Algebra expression? (See Table 4.4).

3.10.2.1. Code and coding

Speech coding is a critical technology for "digital cellular communications, voice over Internet protocol, voice response applications, and video conferencing systems" (Gibson, 2016, p. 1). There are different types of coding:

- "simultaneous coding, which applies two or more codes within a single datum
- vivo code keeps the data rooted in the participant's language/taken directly from the participant and is indicated by quotes
- descriptive coding (which summarises the primary topic of the excerpt)
- initial code (phrases derived from an open-ended coding where first impressions are recorded), as well as
- Process code, which is a word or phrase that captures action" (Saldana, 2015, page 3-5*).*

In this study, the researcher used focus group interviews (qualitative data) in part two of the study. However, the study used both a qualitative and a quantitative approach to compare and confirm the findings of the study. Coding was done in vivo coding, where the data were taken directly from the participant and were indicated by quotes. In vivo coding, data are rooted in the participant's language as the learners speak isiZulu and English (see code-switching) (Appendix C). This was meant to respond to the learner errors and misconceptions when solving Algebra problems in the Senior Phase, the sources of these, as well as strategies for avoiding them.

3.10.3. Interpretation and distribution analysis

The errors and sources of errors and misconceptions were presented using tables, frequencies and percentages. The bar graph represents the distribution of errors in line with the theoretical framework in the literature and also the distribution of errors as discussed in the literature by different researchers.

3.11. Summary

To conclude, this chapter revealed the methodology and research design for the study. The sample and sampling procedures were discussed. The instruments that were used to collect data, both qualitative in the form of an achievement test and qualitative in the form of focus group interviews, were discussed in the methodology section. The validity and reliability of the instrument and various ethical issues were also highlighted and discussed. In the next chapter, the researcher discusses the analysis of the data and the findings of the study.

CHAPTER FOUR: DATA PRESENTATION AND ANALYSIS

4.1. Introduction

This chapter focuses on the analysis, presentation and interpretation of data from five schools using both qualitative and quantitative methods. In the current research, data analysis involved the process of making sense out of the data, and included consolidating, reducing and interpreting what learners had said and what the researcher had seen. Furthermore, it was the process of making meaning (Kubheka, 2013). The purpose of using both qualitative and quantitative data in this study was to investigate the errors and misconceptions learners display in Algebra, identify mistakes and misconceptions learners have in response to algebraic expressions, as well to explain how learners' Algebra errors link with their misconceptions. The information obtained from the respondents was collated, coded and analysed using descriptive statistics (frequency counts, percentages and tabulation) and thematic content analysis. The presentation, interpretation and analysis of data follows the form of the research questions:

- What are the types and the sources of errors and misconceptions committed by Grade 9 learners in Algebra learning?
- How do the types and the sources of errors and misconceptions influence errors in Grade 9 learners' cognition when learning Algebra?
- Through which strategies and sources could the errors and misconceptions relating to Algebra be avoided?

4.2. Findings regarding the quantitative (test) and the qualitative (focus group interviews) investigations

Both Tables (4.1 and 4.2) indicate the number of learners who responded and the nature of their responses, including those who responded correctly. Some incorrect responses are also indicated as the performance in the task that was given. The tasks were based on Grade 9 Algebra and included graphs, tables, equations, expressions, rectangular prism (surface area) and the right-angled triangular prism (perimeter, area etc.).

Table 4.1: Data on Learner Performance in the Quantitative Investigation

Question	Number: wrote	correct response	incorrect response	no response	Explanation based on the number of learners and the number of responses
1.1 $\frac{5x}{7x-7}=\frac{5}{3x-3}$	100	0	82	18	In this question, there was no correct response. Most respondents gave an incorrect answer. A few did not respond.
1.2 $\frac{1}{x}-\frac{1}{x+4}=\frac{-4}{x^2-16}$	100	0	76	24	No correct response. Most respondents gave an incorrect answer. A few did not respond.
1.3 $\frac{x}{x-1}+\frac{x}{2x+1}=\frac{3}{(2x+1)(x-1)}$	100	0	80	20	No correct response. Most respondents got an incorrect answer. A few did not respond.
1.4 $\frac{3}{x^2-1}+\frac{2}{x+1}=\frac{1}{x-1}$	100	0	86	14	No correct response. Most respondents gave an incorrect answer. A few did not respond.
Question 2					
2.1 2.1.1 Write a set of ordered pairs in the form of a values in the table values relationship depicted by graph. (graph see Appendix A)	100	61	39	0	Many learners got this question right. They simply drew a table and set up both values correctly and drew a graph. A few learners drew the graph incorrectly from the correct table.
2.1.2 Determine equation for the above graph	100	2	89	9	Many learners responded incorrectly. Many errors and misconceptions were found in this question. Some learners did not respond at all.
2.1.3 If x = 6, determine the output	100	2	83	15	Many learners responded incorrectly. Many errors and misconceptions were found in this question.
2.2 Given equation $y=2^{x+1}$ 2.2.1 Draw a table of values for the values of $x=$ 0, 1, 2, 3, 4	100	20	25	55	Many learners did not respond at all to this question. Some gave the correct answer and others gave wrong answers. Errors and misconceptions were found.
2.2.2 Draw an accurate graph for the relationship generated by an equation.	100	51	45	04	Most learners drew the graph correctly, but others got it wrong. A few did not respond. Some errors were found.

Table 4.1: Data on learner-performance in quantitative (cont'd)

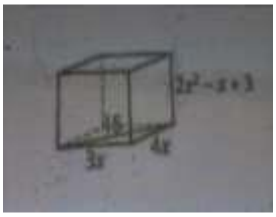
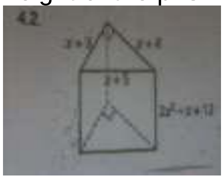
Question	Number: wrote	correct response	incorrect response	no response	An explanation based on the number of learners and the number of responses
Question 3					
3. Mary's mother is three times as old as she is. Five years ago, her mother was four times as old as her. How old as Mary? 3.1 indicate the situation with a table and write the equation then solve for x	100	12	83	05	Most learners gave an incorrect answer, while a few gave the correct answer. Only five did not respond at all. Many errors and misconceptions were found in word problems.
3.2 A rectangle has a length $3x \quad (x-2)$ and breadth 3.2.1 Write down an expression for its perimeter.	100	11	82	07	Most learners responded incorrectly but a few got the answer right. Just seven did not respond. Many errors and misconceptions were found in writing of expressions.
3.2.2 What is the length if the perimeter is 68 m?	100	00	92	08	None of the learners got the correct answer and a few learners did not respond.
3.2.3 What is its area?	100	01	85	14	Most learners got incorrect answers. A few did not respond.
3.2.4 Write the ratio of the length to breadth if the area is 72 square cm.	100	00	85	15	None of the learners got the correct answer and a few learners did not respond. Errors were found.
Question 4					
4.1 4.1.1  The measurements of rectangular prism are shown in the sketch. The length of the diagonal of its base is equal to 15 cm Write an equation and solve for x	100	00	98	02	No correct response; many errors and misconceptions found in equations.

Table 4.1: Data on learner-performance in quantitative (cont'd)

Question	Number

	wrote	correct response	incorrect response	no response	An explanation based on the number of learners and the number of responses
4.1.2 Determine the numerical value of the height of the prism 	100	00	96	04	No correct response. Four learners did not respond.
The measurements, in mm, of a right-angled triangular prism, are shown in the sketch. 4.2.1 Write an equation in terms of x and then solve the equation.	100	00	96	04	No correct response. Four learners did not respond at all.
4.2.2 Calculate the total surface area of the prism.	100	00	83	17	No correct response. 17 did not respond at all.

4.2.1. The data presented from quantitative (test) analysis (see Table 4.1 Appendix: A)

Table 4.1 represents learners' responses to the test. The responses were classified as 'no response', 'responded correctly' and 'responded incorrectly'.

4.2.1.1. Introduction

Table 4.2 shows the learners' performance in all questions. It was confirmed that learners have difficulties in solving Algebra problems. This is indicated by their performance as presented in terms of percentages in learners' responses (see Table 4.1). The low performance, which is indicated by the learner's response (Table 4.1), may have led to errors and misconceptions in solving a mathematical algebraic problem. The low percentages (see Table 4.1) involve algebraic fractions, tables, graphs, word problems, and equations and expressions that involved the concept of ratio, height, area and length.

4.2.1.2. Interpretation of data pertaining to question 1 (test)

Question 1 was divided into four questions as 1.1, 1.2, 1.3 and 1.4 and was based on equations with fractions (common fractions). This question subsequently revealed that learners experience problems in solving algebraic fractions. The learners' responses were classified as either correct, incorrect or no response. In the test, there were no correct (0%) responses to 1.1. Most (82%) respondents gave an incorrect answer and a few (18%) did not respond at all. In question 1.2, there were no correct (0%), responses, 76% of responses were incorrect 24% gave no response.

In question 1.3, there were no correct responses (0%), 80% gave an incorrect response and 20% gave no response. In the last question in question 1, that is, 1.4, no correct (0%) responses were given, 86% were incorrect and 14% gave no response. The learners’ performance (indicated by percentages) in questions 1.1 to 1.4 indicate that learners struggle to solve algebraic fractions. The low performance (see Table 4.2) indicates that Grade 9 learners commit errors and hold misconceptions when solving fractions. The types of error and misconception in algebraic fractions were analysed according to Koch error analysis, with the errors being categorised as either careless, precision, problem-solving, computation or preparedness, as discussed in the theoretical framework. The cognitive levels were analysed according to the SOLO model where the levels of thinking are classified based on a structure where one idea corresponds to the response (refer to learner 1 figure 6), many-structures indicates many ideas respond to the question (learner 2 Figure 4.104.10) and lastly relational, which indicates that all ideas respond to the question (none).

4.2.1.3. Interpretation of data pertaining to question 2 (test)

Question two was divided into five sub-questions that were based on graphs, equations, expressions and tables. In question 2.1.1, 61% of learners gave the correct answer. In this question they were merely required to draw a table and set up both values correctly, and then draw the graph. Other learners (39%) drew the graph incorrectly even though they had the right table and thus responded incorrectly. In 2.1.2, many (89%) learners responded incorrectly, with just a few (2%) responding correctly, while 9% did not respond at all. In 2.1.3, many 83% learners responded incorrectly. In 2.2.1, many (55%) learners did not respond at all in this question, while 20% of them responded correct, and other responded incorrectly (25%). In 2.2.2, most learners (51%) drew the graph correctly, but others (45%) got it wrong. A few (4%) learners did not respond at all. In both 2.1.1 and 2.2.2, the performance was better compared with other responses in question two (2.1.2 and 2.1.3). This indicates that these learners (51%) had attained the skill to draw graphs and to translate from the graph to the table (61%), however, in 2.1.1

39% did not respond and 2.2.2, 48%, thus indicating that they lack skills in translating from equations to graphs. These skills involve those relating to computation and substitution from equations. Like the errors in question 1, question 2 was also analysed according to Koch error analysis with errors being classified as careless, precision, computation, problem-solving or preparedness, as discussed in the distribution of errors and sources in SPSS results. The cognitive levels were analysed according to the SOLO model. The SOLO structures adopted were one-structure, meaning that one idea revealed in the response; many-structures, meaning many ideas but not all of them correspond with the question as highlighted in the analysis, and lastly relational, which indicates all ideas are responding to the question.

4.2.1.4. Interpretation of data pertaining to question 3 (test)

Question 3 was divided into four questions that were based on the concept of word problems. The learner was required to solve and interpret the word problems applying the skills they had learnt in class. As mentioned in the performance explanation of question 1, the responses were classified into three categories, namely, correct, incorrect or no response. In 3.1 most learners (83%) arrived at incorrect answers; a few (12%) got the correct answer, and 5% did not respond at all. In 3.2.1, most (82%) of the learners responded incorrectly but a few (11%) responded correctly, while 7% did not respond at all. In 3.2.2, no learner (0%) responded correctly, with most (92%) responding with a wrong answer, while a few (8%) o did not respond. In 3.2.3, most learners (85%) responded incorrectly, a few (14%) did not respond and (1%) gave the correct response. In 3.2.4, no learner got the correct answer, and a few learners did not respond at all. This performance indicates that Grade 9 learners commit errors and have misconceptions when attempting to answer word problems. The performance was abysmal and learners require special attention in order to correct those errors.

4.2.1.5. Interpretation of data pertaining to question 4 (test)

Question 4 was divided into four questions which addressed algebraic equations and expressions, including the concepts of length, ratio, height, and the area of a rectangular prism and a right-angled triangular prism. Performance was inferior in all questions in question 4, indicating that Grade 9 learners commit errors and have misconceptions when they perform algebraic equations and expressions pertaining to the concept of length, height, area and ratio. No correct responses (0%) appeared in learners' scripts in all four questions that involved the concept of ratio, area, height and length. Moreover, many learners did not respond at all. In the first sub-question of question 4, 98% gave an incorrect response and 2% gave no response. There were no right answers. In the second sub-question, 96% gave an incorrect response and 4% no response (no correct answers); in the third sub-question, 96% gave incorrect responses and 4% no response (no correct answers) and in the last question in question 4, 83% gave incorrect responses and 17% gave no response (no right answers). This implies that learners have difficulties in working with those concepts. Accordingly, attention would be paid to correcting the errors and misconceptions of Grade 9 learners in Algebra, as highlighted in the analysis of the distribution of errors and sources in the SPSS results. According to the Koch error analysis, error types such as careless errors, computation errors, problem-solving errors, precision errors and errors caused by unpreparedness were found in learners' scripts. The literature revealed that such errors are due to a lack of conceptual knowledge and procedural knowledge, a lack of factual information, a lack of ability to connect new knowledge with old knowledge, inability to translate from graphs, table or equations, lack of interpretation and lack of basics skills that should have been learnt in the lower grades.

Table 4.2: Data on Learner Performance obtained from the Focus Group Interview Appendix B

Questions	No: responders	Responded correctly	Responded incorrectly	No response at all	An explanation based on the number of responses
1.1 If you look at the question How you can solve for x ?	30	03	20	07	Most of the learners responded incorrectly. Few didn't answer.

1.3 How can you solve for in x algebraic equation with fractions? *Sizoyithola kanjani I value ka x (yilenamba esingayazi?)*	30	00	18	12	Most learners responded incorrectly, and others did not answer the question at all.
2.1.1 How can you find set of ordered pairs? *Sizowathola kanjani ama points***?**	30	23	00	07	Most of the learners answered correctly. Some of them did not answer the question.
2.1.2 How can you determine the equation in the graph that you are given. *Sizoyithola kanjani equation sisebenzisa graph?*	30	03	19	08	Most of the learners did not get the equation. Few did not respond.
2.2 If you are given the equation how you can draw the table using given $x\,values$? *Uma unikiwe equation sodweba kanjani graph?*	30	17	03	10	Most of the learners did respond on the draw of the table and explained $values\ of\ y$.
3.1 How can you solve or simplify word-problem? *Sizolidweba kanjani tabular nalenamba emele u x ku word-problem?*	30	00	16	14	None of the respondents answers correct. Others did not respond.
3.2.1 How can you write an expression for it perimeter? *Sizoyithola kanjani pherimitha?*	30	03	24	03	Most learners responded incorrectly. Very few did not respond at all.
3.2.2 How can you find length if a perimeter given is 68? *Sizobuthola kanjani bude bendawo?*	30	00	14	16	None of the learners got answers correct. They responded incorrectly, and others were silent.
3.2.3 How can you find the area? *Indawo sizoyitola kanjani?*	30	00	15	15	None of the learners got answers correct. Half of the responded incorrectly, and half did not answer.
3.2.4 How can you find the ratio of the length to breadth if the area is given as 72cm^2 *Sizoyithola kanjani ratio?*	30	00	08	22	None of the learners answered correctly. Most learners were silent, and few were incorrect.
4.1.1 How can you find an equation? *Sizoyithola kanjani equation?*	30	00	15	15	None of the learners got answers correct. Half of the responded Incorrect and half did not answer.
4.1.2 How can you find height? *Sizoyithola kanjani height?*	30	00	14	16	None of the learners got answers correct. They responded incorrectly Moreover, others were silent.

4.2.1.6. The data presented in focus group interviews (Table 4.3 Appendix: B)

4.2.1.6.1 Introduction

The data was presented and classified according to three categories as a response, correct response and incorrect response. The Table 4.2 showed learners low performance in percentages. These learners' response from the focus group interview revealed the similar results as the test learner-performance in percentages in all questions (1, 2, 3 and 4). In Table 4.1 and 4, 2 it was confirmed that learners struggled to solve Algebra problems. The responses revealed inadequate performance which indicates that Grade 9 learners commit errors and misconceptions when solving algebraic problems. The thematic analysis revealed learner errors and misconceptions in solving Algebra, the sources of those errors and strategies. These were analysed according to Koch error analysis and SOLO theory.

4.2.1.6.2 Data presentation in question 1 (focus group interview)

Question 1 addressed algebraic fractions. The learner was expected to find an unknown value in an equation containing fractions. It was confirmed that learners had difficulties in solving Algebra. The poor performance was indicated by percentages (refer to Figures 4.2, 4.3, 4.4 and 4.5). The percentages show the performance of learners in responding to Algebra questions in equations with fractions. Accordingly, 32% of learners did not respond at all, while 6% gave correct responses and 62% incorrect responses. This performance indicates that Grade 9 learners made errors and had misconceptions when solving fractions. The types and sources of errors and misconceptions in algebraic fractions were analysed according to Koch's error analysis. Accordingly, the errors were placed into categories such as careless, precision, problem-solving, computation and preparedness. The thinking levels were analysed according to the SOLO model in which the levels of thinking are classified as structures, i.e. (one idea is corresponding to the response), secondly many-structures indicates the many ideas respond to the question, and lastly relational, which indicates all ideas correspond to the question.

4.2.1.6.3 Data presentation in question 2 (focus group interviews)

In question 2, translation between graphs, equations and tables was addressed. The low percentages of performance (Table 4.3) revealed that Grade 9 learners commit errors in solving Algebra problems; however, other learners (42%) responded correctly although the percentage of correct responses to question 2 was a bit higher than other questions such as question 1, where low performance was indicated as only 6% responded correctly. In question 3 3% responded correctly and 0% in question 4. Moreover, 32% of learners did not respond at all to question 1, and 27% gave wrong answers to question 2. As highlighted in the data presentation and the analysis results given by SPSS software in question 1, the types of error that Grade 9 commit in solving and learning Algebra were classified into categories using Koch's theory and SOLO theory (as discussed above).

4.2.1.6.4 Data presentation in question 3 (focus group interviews)

This question addressed word problems. The learners were expected to interpret and solve unknown values and simplify equations. The data on question 3 show that 47% of learners responded incorrectly, 51% gave no response and 2% only gave the correct response. The data indicate that learners have difficulties in solving Algebra problem. As indicated by the application of the SOLO model, one, many and relational was applied to learners' responses. Errors were classified into groups using Koch's theory and the SOLO model, as discussed in the data presentation for questions 1 and 2. The strategies for avoiding errors are classified in the next section.

4.2.1.6.5 Data presentation in question 4 (focus group interviews)

Question 4 was based on algebraic equations and expressions involving ratio, length, height and area in relation to a right-angled rectangular prism and a rectangular prism. The data indicate learner difficulties in solving Algebra involving these concepts. As indicated in the data presentation, the data were presented in terms of three categories – correct, incorrect and no response. Accordingly, 52% of learners gave no response, 48% gave an incorrect response and none of the learners gave the right answer to this question. These low percentages pertaining to learner performance imply that Grade 9 learners committed errors in solving Algebra which need special attention. The classification on the types of error were analysed using both Koch's theory and the SOLO model as discussed in chapter two.

4.3. Comparison of on the test (quantitative) responses and focus group interview (qualitative) responses

In equations with fractions (question 1), there was no difference in the responses. No correct (0%) answers were obtained in the test; however, in one FGI 6% gave the correct response. The low percentage revealed similar results, 0% (test) and 6% (focus group interview), indicating similar learner difficulties in solving algebraic fractions which need special attention. In graphs and tables, learners showed correct responses in both the qualitative (42%) and the quantitative (61%)

approach. Learners were able to display and draw the graph correctly; however, 27% gave incorrect responses in the focus group and 39% in the test. Learners did not respond at all; this is indicated by the 0% obtained in the test. Further, during a focus group interview 31% did not respond, which revealed similar performance in both test and focus group interview. The learners were able to find ordered pairs. However, other learners (45%) in the test struggled to draw graphs.

A high percentage (89%) of incorrect responses was indicated in both the test (89%) and a focus group interview. The results of both focus group interview and test indicate a similar problem in terms of percentages. The percentages in the performance of learners found in word problems shown (2%) in a focus group interview and 2% in a test. Difficulties were experienced in finding the perimeter, area, ratio and length in both approaches. Both test and oral indicate the same problems. For instance, learner 2 in Figure 4.15 struggled to find the length, used the correct method but failed to substitute and continue with other steps.

4.4. Error analysis in a Qualitative data

4.4.1. Error analysis using Koch's procedure and the SOLO model

In this section, the purpose is to analyse errors and misconceptions using learners' response. The SOLO model was used to review each piece of information in the learner's response. The model was applied according to Lian and Yew's (2012) stages that were review by in this study. The errors and misconceptions committed by Grade 9 learners when they are solving Algebra problems were analysed using Koch error analysis in terms of careless errors, computation errors, problem-solving errors, precision errors and error caused by unpreparedness. As highlighted by researchers (Lian & Yew, 2012; Na'imah, Sulandra & Rahardi, 2018; Putri, Mardiyana, & Saputro, 2017) in the literature, solo structure is useful for analysing learner cognition. This study did not consider either pre-structure or extra-extended structures because of the nature of the research questions. The Koch's procedure for error analysis was used to analyse errors in all questions (1, 2, 3 and 4 – comprising Algebra equations with fractions) according to the three research questions in this study. Learner 2 (Fig. 4.2) – one-structure, learner 2 (Fig. 4.6) – many-structures and learner 2 (Fig. 4.10) were selected to answer the research questions so this was an applicable SOLO model theory, as highlighted in the application of the theory.

Learner 1

(Question 1): Learner Activity no. 1, solve for x $\frac{5x}{7x-7} = \frac{5}{3x-3}$

Expected answer: $15x^2 - 15 = 35x$

$15x^2 - 50x + 35 = 0$

$3x^2 - 10x + 7 = 0$

$(3x - 7)(x - 1) = 0$

$x = \frac{7}{3}$ or $x = 1$

Learner's response no. 1 (question 1)

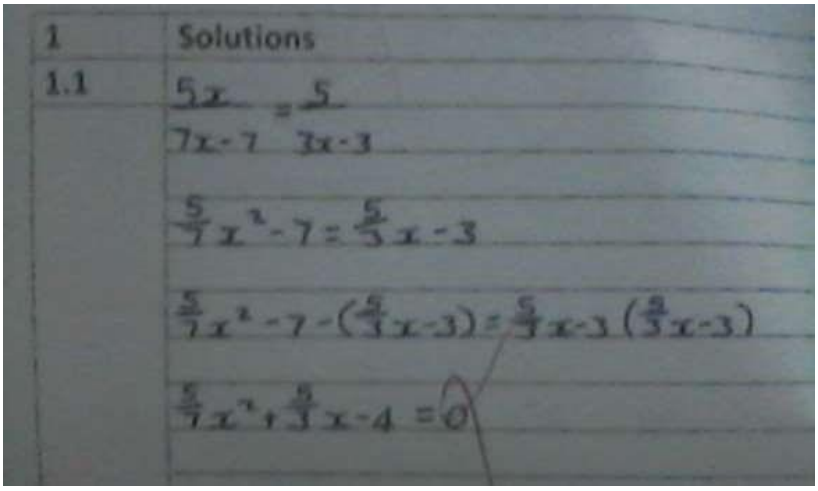


Figure 4.1: Learner 1's written work showing problem-solving (Koch's theory)

Case in point 1: Error analysis

In Figure 4.1, the learner failed to complete all the steps. Another error was shown when the learner mishandled the signs, as revealed by the use of multiplication instead of division, for instance when the learner computed ($x \div x = x^2$). Koch's theory classifies this type of error as a careless error. On the other hand, Makonye (2011) believes that this type of error is usually caused by distraction and is thus a careless error (Koch's theory). Makonye and Hantibi (2014) maintain that the learner knows what the problem requires, however they failed to apply the procedural knowledge, which meant the learner ended up with the wrong solution. However, in Makonye's (2011) review, it was found that learners used addition instead of multiplication. In this study, the learner used division instead of multiplication. Cooper (2015) maintains such errors are committed because the learner has failed to pay attention during teaching and learning. What can be drawn from this (Cooper, 2015; Makonye & Hantibi, 2014) is that although the learner would seem to have an idea about how to approach the problem when able to handle variables, however failed to handle the sign as a basic of mathematics ($x \div x = x^2$) in Figure 4.1 learner 1. Researchers suggest that teachers and learners should use the BODMAS rule to solve such problems (Muschla et al., 2011; Sanders et al., 2017). On the other hand, Sanders et al. (2017) maintain that the BODMAS rule is a semiotic intervention which serves as recap support.

Learner 2: In the same activity:

Learner response no. 2 (question 1)

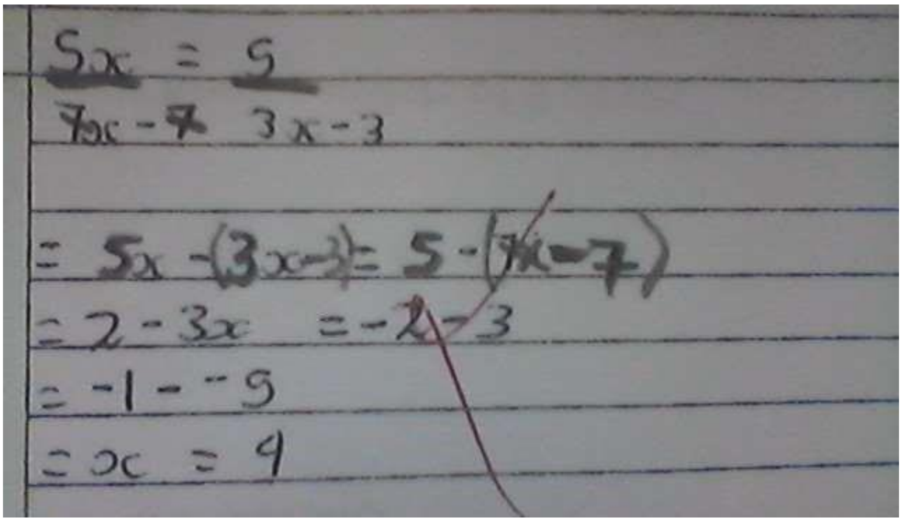


Figure 4.2: Learner’s written work showing computation error (Koch’s theory)

Case in point 2 – error analysis

In line with Koch’s theory this error was analysed as a computation error because the learner failed to operate the sign in the problem. This learner failed to understand the meaning of brackets, which in fact require the same operation as a multiplication sign. It is suggested that teachers should emphasise the meaning of brackets in an equation or expression.

Learner activity: 2 $\frac{1}{x} - \frac{1}{x+4} = \frac{-4}{x^2-16}$

Expected answer: $\frac{1(x+4)-1(x)}{x(x+4)} = \frac{-4}{x^2-16}$

$$\frac{x + 4 - x}{x(x + 4)} = \frac{-4}{x^2 - 16}$$

$4(x^2 - 16) = -4(x^2 + 4x)$

$4x^2 - 64 = -4x^2 - 16$

$8x^2 + 16x - 64 = 0$

$x^2 + 2x - 8 = 0$

Learner's response no. 3 (question 1)

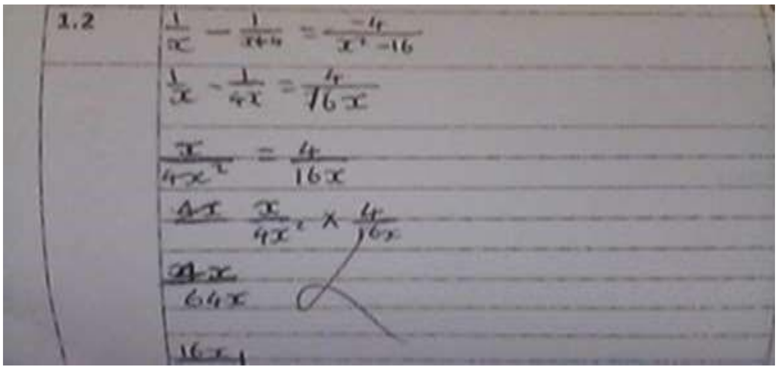


Figure 4.3: Learner's written work showing problem-solving (Koch's theory)

Case in point 3 – error analysis

The learner (learner 3 Fig. 4.3) did not follow mathematical rules and procedure due to lack in completing all the steps. As highlighted in the analysis in case 1 (learner 1 Fig. 4.1) the learner does not show an understanding of the concept but instead wrote all the mathematical steps wrong when solving the problem. This learner failed to conceptualise. The conjoining errors, as suggested by researchers (Gumpo, 2015; Mashazi, 2014; Pournara et al., 2016; Ncube, 2016) are shown in the solution as $x + 4 = 4x$ and $x^2 – 16 = 16x$. Ncube maintains that these types of error are caused by a lack of knowledge regarding the concept of algebraic expressions. Researchers believe that these errors are caused by interpretation of expressions, for instance $9x + 4$ as add nine times and 4 or as an object (Egodawette, 2011; Gumpo, 2011; Makonye, 2016; Mashazi, 2014; Mdaka 2011; Moodley, 2014; Mulungye et al., 2016; Pournara et al., 2016). In this study, it is suggested that teachers need to emphasise the meaning of the sign in the expression and the meaning of the number that is bonded with the variable, such that 9 should be multiplied by x. This means that there is no relationship between 9 and 4 to operate the sign in between. Schlemann (2013) suggests that Grade 9 learners should stop overlooking operational signs in the expression as a symbol of joining terms. Cancelling errors as suggested by Mhakure et al. (2014) and Makonye (2016), appeared in the solution when the learner revealed the line on top, which indicated that $4x$ cancelled with another $4x$. This was due to cancelling without meaning "senseless cancelling" (Makonye, 2016, p. 296). As suggested, this involves cancelling similar variables without following procedure. Khanyile (2016) suggests generalisation in order for the learner to understand and recognise the characteristics of the expressions and equations. Mahakure et al. (2014) suggest fusion in the teaching of fractions so that the learners become familiar with the different structures of equations and fractions.

Learner no. 4: on the same Activity: Learner's response: no. 4 (question)

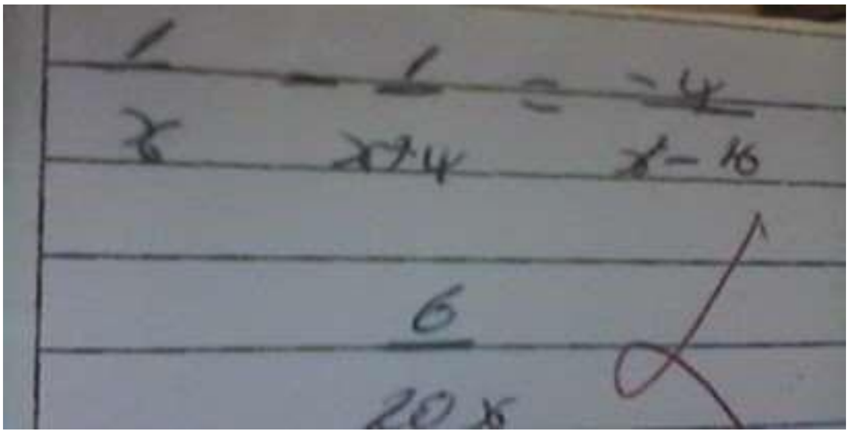


Figure 4.4: Learner's written work showing problem-solving (Koch's theory)

Case in point 4 – error analysis

Learner 4, as reflected in Figure 4.4, added both denominators and numerators. This is a common fractional error committed by learners when they are unable to handle variables (Khanyile, 2016; Mahakure et al., 2014; Makonye & Khanyile, 2015). It may be deduced from the Figure 4.4 that the error was caused by a lack of knowledge of the concept itself. As a result, there was poor conceptual and poor procedural knowledge as discussed in the literature by researchers (Egodawatte & Stoilescu, 2015; Fisher & Frey; 2013 Riccomini, 2014). The lack of knowledge leads to poor algebraic problem-solving. As a result, Fisher and Frey (2012) and Riccomini (2014) claim that procedural and conceptual errors are due to a learner's failure to understand the concept. Egodawatte and Stoilescu (2015) mention that this type of error is usually caused by the concept having little meaning for the learner, which leads to procedural errors. Hodgen et al. (2017) suggest that learners need to develop or restructure their previous knowledge so that they are able to apply and retrieve knowledge.

4.3.2 Error analysis of question 2 (both Koch's theory)

Learner no. 1 (question 2): Activity 1 (question 2)

Consider this graph

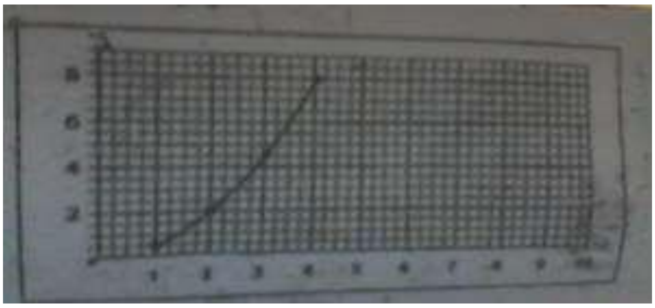


Figure 4.5*:* Learner's response in the drawing of a graph

Write a set of ordered pairs in the form of a table of values for the relationship depicted by the graph.

Learner no. 2 (question 2)

Expected answer: (1, 0.5) (2, 2) (3, 4.5) (4, 8)

Learner's response:

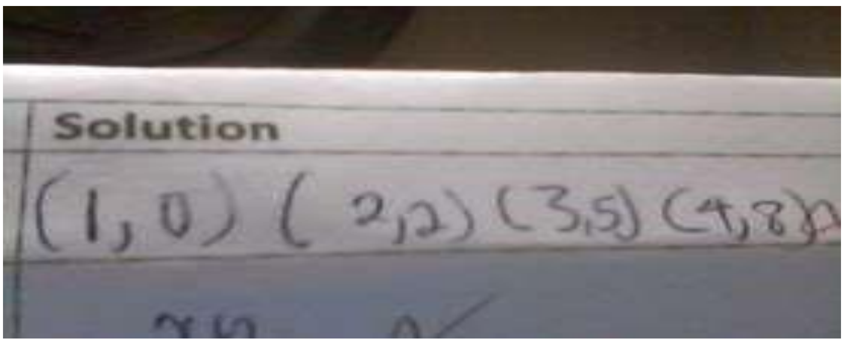


Figure 4.6: Learner's written work showing careless errors (Koch's theory) and many structures (SOLO theory)

Case in point 5 – error analysis

According to Koch's theory and as reflected by the work of learner 1 in Figure 4.1, some errors are classified as a careless error as discussed in detail in learner 1 (Fig. 4.1) in question 1. The analysis pertaining to learner 1's answer in Figure 4.1, reflects a case where a learner used a division sign as a multiplication sign. However, this mishandling of operational signs was due to the learner's careless when reading the graph and observing decimal fractions as single numbers. The commas were ignored and taken as a single number. In addition, the learner assumed 0, 5 to be 0 and 4, 5 as 5 (learner 2, Fig. 4.6). As evidenced by the figure the learner mishandles the decimal fractions in the graph and considers them a single number. These types of problems point to the need for learners to pay attention when observing measurements, points and graphs. The graph paper allows learners to clearly observe numbers which are clearly displayed.

Another case in the same activity – Learner no. 3 (question 2)

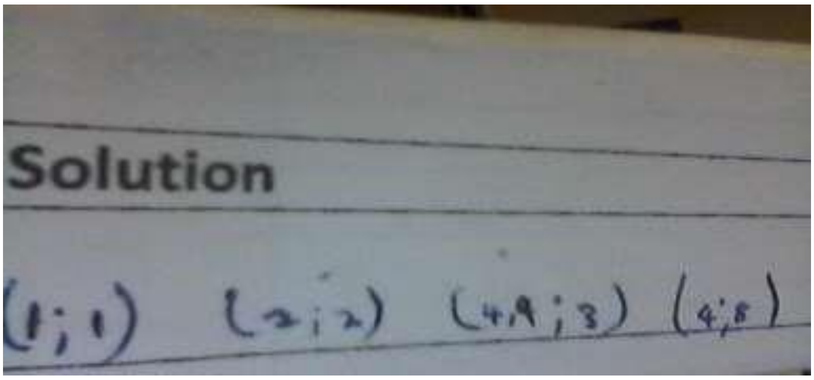


Figure 4.7: Learner's written work showing careless errors (Koch's theory) and many structures (SOLO theory)

Case in point 6 – error analysis

According to the SOLO model and as reflected by learner 2 in Figure 4.6, a many-structures thinking level is evident because the learner reveals several correct answers but not all of them and these are not entirely interrelated. Thus, the points (2; 2 and 4; 8) were considered to be correct, but the points (1; 1) and (4, 9; 3) were wrong (learner 3 – Fig. 4.7). This reveals a misconception when reading the graph in relation to translating from table to equation, the equation to the table. Adu-Gyamfi et al. (2012) claim that this type of error is an implementation error where learners display disordered coordinates. In the current situation, it was a case of misusing operational signs such as addition, multiplication, division and subtraction or wrongly adding a negative sign to a number. This thus led to a misrepresentation of the algebraic structure of the problem. Encoding errors in the implementation are caused by an algorithmic misstep or as a result of incorrect treatments in the algebraic register. However, the learner did not show any relevant calculation as evidence, s/he wrote only the points. The researcher assumed that the error was caused by the algorithmic missteps as revealed by 1, 1 and 4 .9, 3.

Koch's theory states that when learners fail to operate signs in calculating coordinates this error is a computation error, which occurs when a learner used operational signs such as adding, subtracting, multiplying or dividing incorrectly. It is thus suggested that the learner lacks interpretation (Maharaj et al., 2015; Pournara et al., 2016). Mashazi (2014) and Gumpo (2015) thus argue that such computation errors are sometimes caused by a failure to link new knowledge with existing knowledge. Cooper (2015), in support of this notion, claims that this is caused by the short supply of previous knowledge. Re-teaching and re-learning are suggested for avoiding this type of error.

Activity no 2 (question 2) Learner no 4 (question 2): Determine the equation for the graph (see Appendix A)

Expected answer: $y = \frac{x^2}{2}$

Learner's response

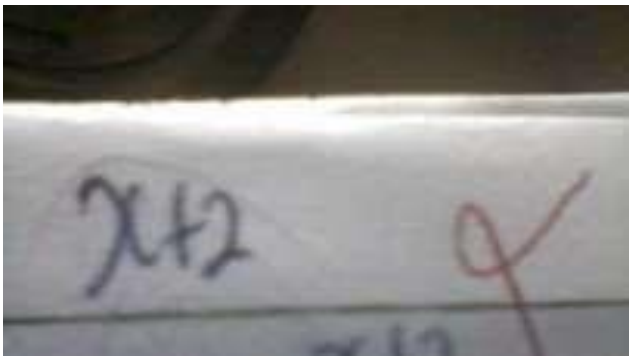


Figure 4.8: Learner's written work showing precision errors (Koch's theory) and one-structure thinking (SOLO model)

Case in point 7 – error analysis

It would seem that learner 4 in Figure 4.8 has committed a precision error, as classified by Koch's theory. The learner wrote an expression instead of an equation, and the plus sign was supposed to be a division sign. The variable was expected to be a square. According to Koch's theory, precision errors happen when the learner forgets or drops parenthesis, which is a division sign; there is a missing unit of the variable, so an expression becomes an equation. Note that a graph has both x and y values but the learner shows only the x in the equation, thus ignoring the y. The errors caused demonstrate a lack of ability to distinguish between equation and expression, as well as incorrect notation. This has been emphasised by Adu-Gyamfi et al. (2012) in the literature when they argued that three types of error are found: Firstly, interpretation errors which happen when learners are interpreting the questions and show misunderstanding by using incorrect symbols. Secondly, implementation errors which occur when learners disorder coordinates. This occurred in the current situation where operational signs such as addition, multiplication, division and subtraction or wrongly adding a negative sign to a number were observed. The third type of error was a result of unpreparedness on the part of the learner, which was indicated by the fact that the learner solution was incomplete (Koch's theory).

Learner no 5 (question 2): Activity 3 Expected answer

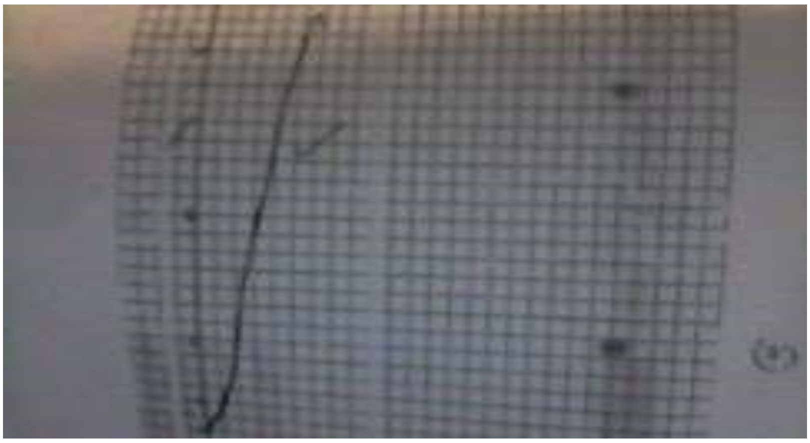


Figure 4.9a: Learner's response

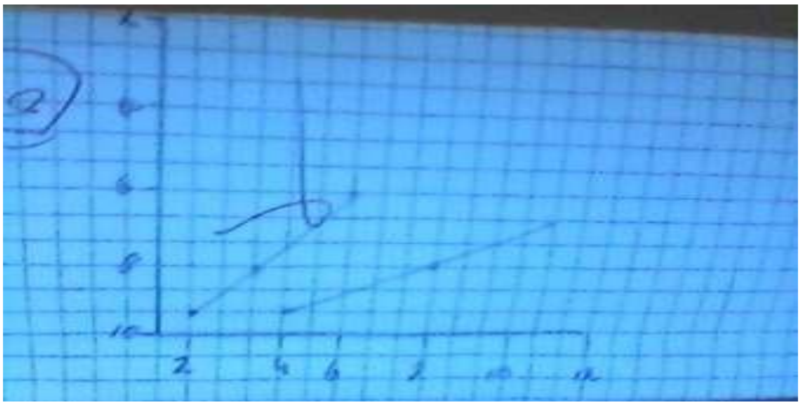


Figure 4.9b: Learner's written work showing Problem-solving error (Koch's theory)

Case in point 8 – error analysis

Case in point 8 involves an error analysis of learner 5 in Figure 4.9, which revealed a problem-solving error as suggested by Koch's theory. The theory indicates that the learner was unable to substitute values from a given equation. The learner shows a lack of procedural and conceptual knowledge in the drawing of graphs. As discussed in the literature (Adu-Gyamfi et al., 2012), learners fail to translate from equation to graph. As indicated in Figure 4.5, learner 1 revealed an error in translating from an equation to a graph. The graph was not connected with the equation. An implementation error can also occur because learners write down the wrong coordinates in this situation and use operational signs wrongly, such as addition, multiplication, division and subtraction or wrongly add a negative sign to a number (Adu-Gyamfi et al., 2012). On the other hand, Koch's theory specifies that learners do not follow proper mathematical rules. It is suggested that to address this error, learners could practice, revise properly using graph paper for the drawing of graphs and ruler could be used. however, this could happen if learner able to handle operational signs, substitute from the equation and computation (Koch's theory).Because if the learner given the equation to do translation from equation to graph, s/he have to apply the skill of calculation and substitution before draw the graph accurate using graph paper.

4.4.2. Error analysis of question 3 (both Koch's theory and the SOLO model)

Learner 1 (question 3): Activity 1 (question 3)

Activity 1 (question) required the learner to indicate a situation with a table and write the equation and then solve it: Mary's mother is three times as old as she is.

Five years ago, her mother was four times as old as her. How old as Mary? (See Appendix A)

Table 4.3: Expected answer for a given activity in question 3 (word problem)

	Age	
	Now	**5 years ago**
Mary	x	x - 5
Mary's mother	3x	3x - 5

Expected solution:

Set up an equation for five years ago. 4 Mary's age multiplied by 5 years ago. Equal to 3x (Mary's mother) multiplied by 5 years ago (3x – 5)

$4 \times (x - 5) = 3x - 5$

Solving the equation to find the value of x

$4x - 20 = 3x - 5$

$4x - 3x = -5 + 20$

$x = 15$

Mary is currently 15 years old.

Learner's response:

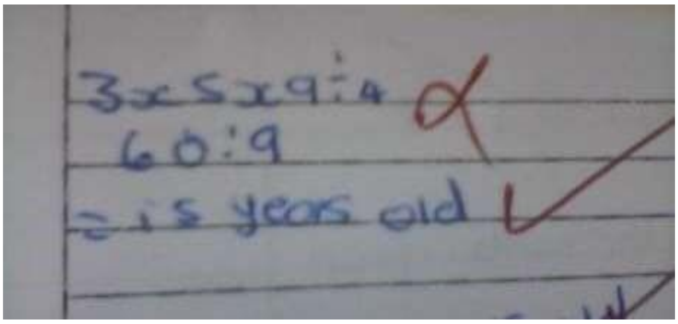


Figure 13: Learner's written work showing one-structure (SOLO model)

Case in point 9 – error analysis

Error analysis of Case in point 9 as reflected by learner 1 shows no evidence as to how the learner reached the correct solution; however, there is evidence to show that the learner used the wrong method but reached the correct answer. This error (wrong method but correct answer) is discussed in the literature. Makonye and Khanyile (2015) for example state that learners are sometimes unable to handle variables and this could lead to errors such as incorrect use of mathematical rules. This happens in particular when a learner arrives at the correct answer by using the wrong procedure. Many researchers suggest reading strategy for teaching word problems (Limond, 2012; Murtini, 2013; Sepeng & Webb 2012; Van Klinken, 2012). As the literature review revealed, a schematic approach may be used as a strategy in learning words problems in Algebra. This could be discussed to help both teacher and learner (Ahmad, Tarmizi, & Nawawi, 2010; Fagnant & Vlassis, 2013; Murtini, 2013; Powell, 2011; Raoano, 2016; Sepeng & Webb, 2012; Van Klinken, 2012).

Learner no 2 (question 3) Learner activity no 2 (question 3)

What is the area? (See Appendix A)

Expected answer:

$length \times breadth$

$A = 3x \times (x - 2)$

$A = 3x^2 - 6x$

Learner's response:

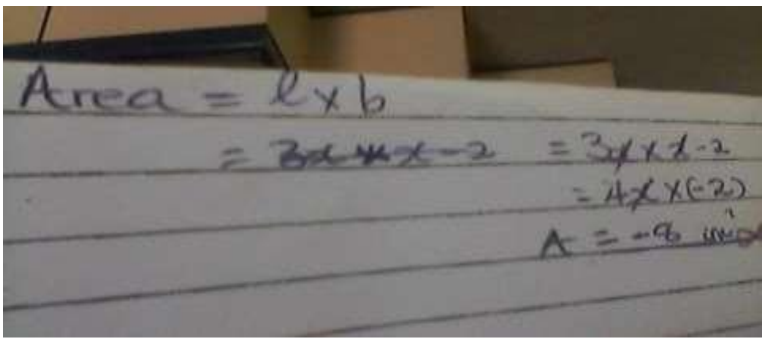


Figure 4.10: Problem-solving error – cancellation error (Koch error analysis)

Case in point 10 – error analysis

An error was shown by learner 2 in Figure 4.10 when they multiplied $3x \times x\ as\ \ 4x$ incorrectly, It was deduced that the learner only multiplied the first term because the bracket was missing. However, other correct ideas were indicated in the learner's work. Unfortunately, a cancellation error resulted in the learner continuing to multiply wrongly thereafter. As guided by Koch's model, it is noted that this type of error is a precision error. This occurred because the learner wrote the work too messily and dropped the parenthesis (bracket), which indicated multiplication.

4.4.3. Error analysis of question 4 (both Koch's theory and the SOLO model)

Learner no 1(question 4)

Activity 1: *A rectangle has length and breadth* $3x(x - 2)$*. Write down an expression* for its perimeter (see Appendix A)

Expected answer: Perimeter = 2(length) + 2(breath)

$p = 2(3x) + 2(x\text{-}3)$

$p = 6x + 2x - 4$

$p = 8x - 4$

Learner's response

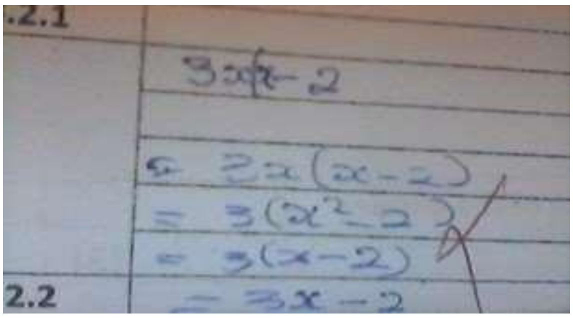


Figure 4.11: Learner's written work showing a problem-solving error and a computation error (Koch's theory)

Case in point 11 – error analysis

Learner 1's work (Fig. 4.11) reveals a problem-solving error (Koch's theory). The learner failed to follow the correct algebraic mathematical rule as discussed regarding learner 1 in Figure 4.1. Brown et al. (2016) suggest that the error type discovered for learner 1 in Figure 4.11 suggests a factual error. There is also a computation error (Koch's theory) where the learner multiplied incorrectly, for instance 3 × -2 = 2, where the learner multiplied only the first term. Brown et al. (2016) believe that this error is caused by failure on the part of the learner to attribute meaning and a lack of knowledge (failure to recognise formula) regarding vocabulary, value and digit identification. Revision is required and the learner needs to practise formulae to avoid factual errors.

Learner no. 2 (question 4) Activity no. 2 (question 4)

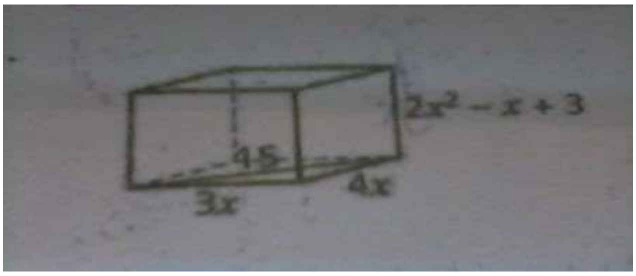


Figure 4.12: Learner's activity rectangular prism

The measurements of rectangular prisms are shown in the sketch. The length of the diagonal of its base is equal to 15 cm, as shown in the figure.

Write an equation and solve for x.

Expected answer: $3x^2 + 4x^2 = 15^2$

$9x^2 + 16x^2 = 225$

$25x^2 = 225$

$x^2 = 9$

$x = 3$ or $x = -3$

Learner's response:

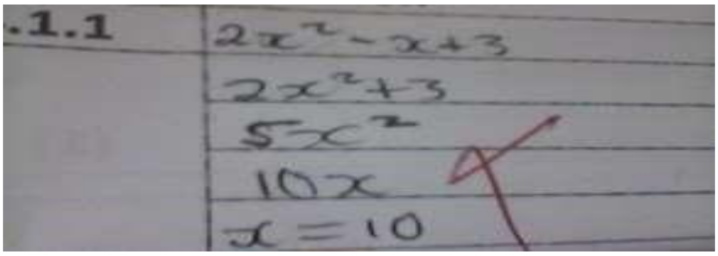


Figure 4.13*:* Learner’s written work showing problem-solving error (Koch’s theory)

Case in point 12 – error analysis

Learner 2 (Fig. 4.13) revealed a problem-solving error (Koch’s theory). The learner was supposed to use Pythagoras’ theorem but failed to follow the mathematical rule and chose to conjoin terms, as discussed (learner 3 Fig. 4.3 and learner 2 Fig. 4.13).

Learner no. 3 (question 4) Learner’s response to the same activity

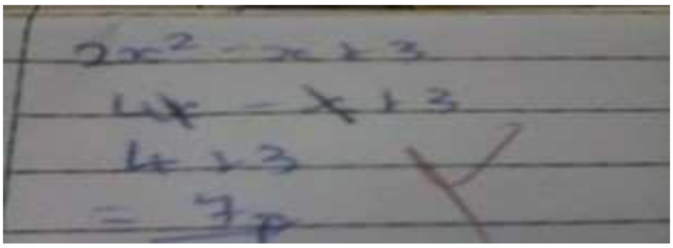


Figure 4.14: Learner’s written work showing problem-solving error (Koch’s theory)

Case in point 13 – error analysis

Learner 3 (Fig. 4.14) revealed a problem-solving error (Koch’s theory) as the learner did not follow the proper mathematical rule; instead the learner used the wrong method. Pythagoras’ theorem should have been applied in the solution. The learner simply cancelled similar variables without following the rule. Accordingly, a cancellation error is highlighted (learner 3 – Fig. 4.3 and learner 2 – Fig. 4.10).

Learner no. 4 (question 4)

Activity: Determine the numerical value of the height of the prism (see Appendix A).

Expected answer: height = $2x^2 - x - 3$

$h = 2(x^2) - 3 + 3$

$h = 18 - 3 + 3$

$height = 18$ cm

Learner's response:

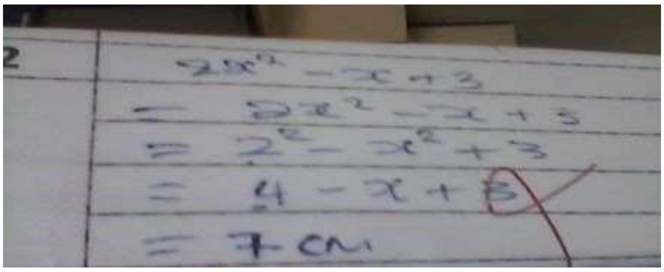


Figure 4.15*:* Learner's written work showing a careless error and a precision error (Koch's theory)

Case in point 14 – error analysis

Learner 4 (Fig. 4.15), according to Koch's theory, made a careless error (see also learner 1 – Fig. 4.1) as they did not follow instructions. In this case, the learner used the correct method but the wrong substitution. According to Koch's theory, this was a precision error, where the learner dropped the variable x. When the learner forgets this variable or drops it, either because they are tired or forgetful or are in a hurry, this might also constitute a careless error.

4.5. SOLO model and Koch's error analysis in a focus group interview

Table 4.3 portrays the data obtained from a focus group interview. The analysis of that data was based on the five steps of Koch's error analysis and the level of thinking based on the SOLO model.

4.5.1. Explanation based on Table 4.3 (oral responses in focus group interview)

The focus group interview was done according to the three levels of thinking suggested by Lian and Yew (2012). These included one structure, many structures and relational. The focus group interview was conducted in both English and isiZulu, as discussed in the methodology section. Koch's theory was used to classify errors and misconceptions into five groups, including problem-solving, precision, computation, careless and unpreparedness errors as reflected in Table 4.4.

Table 4.4: SOLO model and Koch's theory in a focus group interview (qualitative)

Questions	Expected answer	Learner response in an FGI	Explanation based on Koch theory and SOLO model theory
1.1 $\frac{5x}{7x-7}=\frac{5}{3x-3}$ **If you look at the question How can you find value of** x**?** **(***Sizomthola kanjani* $\boldsymbol{x}$ ($\boldsymbol{uyilenamba\ esingayazi}$)?	There is no like expression. Learners need to multiply each term by those denominators, then multiply by removing brackets then simplify (Making equation to be simple).	**Learner 1** I will multiply $5(7x-7)=5x(3x-3)$ Then left like that. **Lerner 2** *We are going to start by adding*$5+5$.	**Learner 1:** **One-structure -** One piece of information was correct. However, the learner failed to continue. This is a lack of a procedural error. **Learner 2** According to Koch theory, this error was classified as problem-solving the error.
2.1.1 How can you find set of ordered pairs? 2.1.1 How can you find set of ordered pairs? ($\boldsymbol{sizozithola}$) $\boldsymbol{kanjani\ izinamba\ ezinobudlelwano\ phakathi\ ka\ x\ no\ y}$) ?	Write each $x\ values$ with the corresponding $y\ values$.	**Learner 3** By looking at the points then draw a table with two columns and two rows put inputs upwards and put output downwards. My points are (1;0,5) ,(2;2), (3;4,5) and (4;8)	**Learner 3:** **Relational** — All answers are correct. The table need not be drawn. Only ordered pairs were requested in a question. This is not related to Koch theory.
2.2 If you are given the equation how can you draw the table using given $\boldsymbol{x\ values}$ **? 2.2.1 If you are given the equation how**	Substitute with x values in an equation then draw and complete table.	**Learner 4** I will draw the table and put x values on top and use equation by putting $x\ values\ to\ get\ yvalues$	**Learner 4** — Many ideas show an understanding of the concept.

can you draw the table using given$x\ values$? *(Sizolidweba kanjani tebula sisebenzisa izinamba esizinikiwe zika $\boldsymbol{x}$ siligcwalise futhi?)*			
3.1 How can you solve or simplify word-problem?	The figure is a rectangular prism so the learner must find the value of x by pythagorus theorem where $r = 5, x = 3x\ , y = 4x$ x^2	***Learner 5*** *I will add all the sides* $2x^2 + 3x + 4x - x = 15 + 3$ $2x^2 + 7x^2 - x = 18$ $7x = 18$ $x = 9$	**Learner 5** Problem-solving (Koch error analysis) This is not related to SOLO model theory because according to this study learner response assessed thinking levels in which this case shows no evidence of thinking.
3.2.1 How can you write an expression for it perimeter? *(Sizoyithola kanjani pherimitha?)*	Learners must look at what kind of figure was given, which is a rectangle. Learners must know the properties of rectangles. $p = 2(l \times b)$.	**Learner 6** **Ngiqale ngabheka rule ye perimeter ngase ngisebenzisa formula** ethi $p = 2n + 2b$ *then I substitute* $p = 2(3x) + 2(x - 2)$ $p = 6x + 2x - 2$ $p = 8x - 2$ **Learner 7** Learner response: I'll use formula furthermore $A = \frac{1}{2} \times l \times b$ then stopped there.	**Learner 6** **Many-structure** — Several correct answers but not all of them and not entirely interrelated with one another. Correct formula but the wrong correct answer is $p = 8x - 4$ **Learner 7** The wrong formula was used by the learner which lead to computational error (Koch error analysis)
3.2.3 How can you find the area?	: Learners must look at what kind of figure was given. Here is a rectangle; learners must know the properties of a rectangle. The formula to be used: $\boldsymbol{A} = \boldsymbol{l} \times \boldsymbol{b}$.	Learner 8 *I used the formula* $\boldsymbol{Area} = \boldsymbol{l} \times \boldsymbol{b}$ $\boldsymbol{A} = \boldsymbol{3x} \times (\boldsymbol{x} - \boldsymbol{2})$ $\boldsymbol{A} = \boldsymbol{3x^2} - \boldsymbol{2}$	Learner 8 Computation error (Koch error analysis).And show one idea in the use of formulae.

Learners failed to solve the problem (Koch's theory), there was no evidence of correct mathematical rules in learners' response (see L1, L2, L5 and L7 in Table 4.4).

There is no evidence of thinking in learners' responses, meaning that learners lack conceptual knowledge and procedural knowledge. As highlighted in the data presentation and the analysis, most of the errors found included learners neglecting to complete certain mathematical steps, leaving blank spaces and solving problems using completely the wrong steps. In regard to L7's problem-solving error (Koch's theory), there is no relevant information which shows understanding;

instead the learner wrote down the wrong formulae. L8 was able to recognise the formulae but failed to compute.

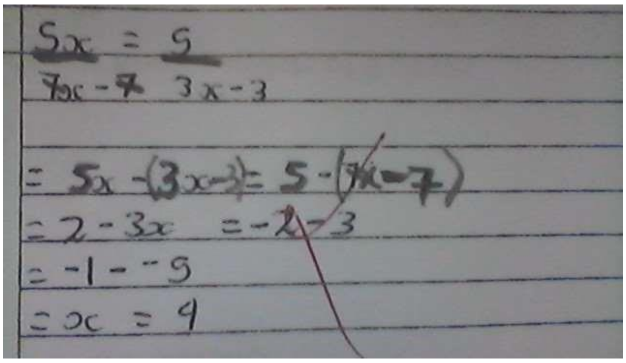
Computation error (Koch's theory). L6 revealed the correct formula and correct substitution but the wrong answer $p = 8x + 2$. This indicates computation errors (Koch's theory) where the learner failed to multiply.

4.5.2. Application of SOLO model in a focus group interview (see Table 4.4)

L1 demonstrated a one-structure response, which means one piece of information was correct. However, the learner failed to continue with the other steps. L3 and L4 revealed the relational structure of the SOLO taxonomy, meaning all answers are correct. The table need not be drawn, as only ordered pairs were requested in the question. There is evidence of learner understanding as indicated by the knowledge of the concept, however, there was some missing information. L6 indicated many-structures of the SOLO taxonomy, meaning s/he provided several correct answers but not all questions were answered correctly, and they were not entirely interrelated with one another. Correct formula and correct substitution were present but the answer wrong answer was given $p = 8x + 2$. This learner shows more than one idea, meaning s/he displays many structures according to SOLO taxonomy. This learner (L6) only lacks multiplication.

4.5.3. Application of SOLO model to a test

There was one correct idea in learner's response that was related to the expected answer of learner 2 (Fig. 4.2). The learner showed little information that indicated knowledge of the concept. One-structure appeared in the idea of cross multiplication which was included by the learner in the solution when inserting brackets. However, the learner inserted a negative sign that was not necessary and this caused an error in the calculation.



The error was caused by inserting a negative sign instead of removing the brackets by multiplying. According to SOLO model, this is applied in terms of one-structure which means one idea shows understanding (refer to Fig. 4.2 learner 2 – learner's written work shows one-structure in terms of the SOLO model).

The measurements of rectangular prisms are shown in the sketch. The length of the diagonal of its base is equal to 15 cm, as shown in the figure. Write an equation and solve it for x.

Expected answer: $3x^2 + 4x^2 = 15^2$

$9x^2 + 16x^2 = 225$

$25x^2 = 225$

$x^2 = 9$

$x = 3$ or $x = -3$

Referring to learner 2 Figure 4.6, the learner's thinking levels indicate many structures in terms of the SOLO model, as the learner was able to find certain points, of which points 2, 4, and 8 are correct. However, there was an inability to read decimal numbers from the graph. It is recommended that learners apply the mathematical rule and use graph paper on which to draw graphs.

Building on the SOLO taxonomy, it is believed that this learner has an idea. In which case as one-structure (SOLO theory) (learner 4 – Fig. 4.8), the learner indicated $x + 2$ as an expression when displayed a variable of x but failed to square it and put a wrong sign "+" instead of "÷" sign.

The number (2) and the variable of x showed that thinking had been applied. However, this was expected to be an equation. The correct expected equation was supposed to be:

$$y = \frac{x^2}{2}$$

In this expression, the sign should be changed to a division sign, the variable squared and then the expression equalised with y, which will produce the correct solution. This error could be dealt with by means of practice and revision because the learner has an idea of the concept but failed to follow the correct steps. It is thus believed that this learner has an idea, in which case a one-structure (SOLO theory) which was indicated in this which $x + 2$ is the expression when a learner displayed a variable of x but failed to square it and put a wrong "+" instead of "÷" sign.

Another learner wrote the correct formula with the correct substitution, but the answer was wrong, for example learner 2 (see Fig. 4.10). Many structures which several correct answers appear but not all of them and they are not entirely interrelated with one another. Accordingly, these findings support some of the claims in the literature reviewed in chapter three. Brown et al. (2016) maintain that procedural errors occur when mathematical steps are inappropriately enacted. According to researchers (Brown et al., 2016; Egodawatte, 2011; Fisher & Frey, 2012; Hodgen et al., 2017; Mulungye et al., 2016; NTCM, 2014; Riccomini, 2014; Egodawatte and Stoilescu, 2015), procedural errors occur when the learner fails to correct all mathematical steps.

4.6. Interpretation and distribution of errors and their sources

This section consists of tables which explain the types of error and misconception that Grade 9 learners commit when they solve Algebra, the sources of such errors and misconceptions, and explanations based on strategies that will help in avoiding the errors found in the study. This section applies descriptive statistics and measures of dispersion in the form of the score, the mean and the standard deviation, which was highlighted in chapter three on data collection.

4.6.1 Statistics and frequencies

The table represents the scores in the distribution of errors which revealed the mean and the standard

Table: 4.5 Score of learners' errors

Statistics		
Score		
N	Valid	100
	Missing	0
Mean		2.72
Std. Deviation		3.346

The total number of learners in the study amounted to 100, as highlighted in the methodology section. The standard deviation was 3,346 and the mean score of was 2.72 for the data on l errors and misconceptions that Grade 9 learners commit when they solve Algebra problems. This means that the standard deviation is close to the mean. The following table represents the frequency including percentages, valid percentages and the cumulative frequencies.

Table 4.6: Scores: frequencies, cumulative frequency, and percentage

	Frequency	Percent	Valid Percent	Cumulative Percent
0	50	50,0	50,0	50,0
1	1	1,0	1,0	51,0
2	4	4,0	4,0	55,0
3	16	16,0	16,0	71,0
4	1	1,0	1,0	72,0
5	5	5,0	5,0	77,0
6	4	4,0	4,0	81,0
7	3	3,0	3,0	84,0
8	7	7,0	7,0	91,0
9	6	6,0	6,0	97,0
10	2	2,0	2,0	99,0
11	1	1,0	1,0	100,0
Total	100	100,0	100,0	

The frequency variables are as follows; Careless Error, CompError, Problem solving, Error Precision Unpreparedness /ORDER=ANALYSIS.

4.6.1.1. Interpretation of the types of error committed by learners

The following tables indicate the types of error found in the study when learners responded to Algebra questions. The errors were classified according to the Koch error analysis as careless errors, computation errors, problem-solving error, precision errors and errors resulting from lack of preparedness. Most of the errors in learners' scripts were problem-solving errors, in terms of which the learner lacked procedural and conceptual knowledge. Ncube (2016 p. 10) supports this notion by saying that "Errors are caused by misconceptions and the latter is attributed to lack of conceptualization and understanding".

The following table presents the number of learners that were tested in order to answer research questions on the type of errors (e.g. careless errors, commutation errors, and problem-solving errors, precision errors and unpreparedness). The total number of learners was 100, accordingly there were 100 scripts, as indicated in the methodology in chapter three.

Table 4.7: The statistics and identification of variables

		Careless Error	Computational Error	Problem-solving Error	Precision Error	Unpreparedness
N	Valid	100	100	100	100	100
	Missing	0	0	0	0	0

The following table presents the careless errors. The frequency of such errors was found to be 13%. This indicates that a few learners (13%) made careless errors.

Table 4.8: The frequency and percentage of Careless errors

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	87	87.0	87.0	87.0
	Yes	13	13.0	13.0	100.0
	Total	100	100.0	100.0	

Of the 100 learners, 49% committed computation errors. These learners revealed problems when doing calculations using operational signs.

Table 4.9: The frequency and percentage of computational errors

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	51	51.0	51.0	51.0
	Yes	49	49.0	49.0	100.0
	Total	100	100.0	100.0	

The following table represents a percentages of the learners committed problem solving errors. Out of the 100 learners, many (86%) committed problem-solving errors, which were revealed by learners not following the mathematical rule and failing to undertake the mathematical rules and procedures in solving Algebra.

Table 4.10: The frequency and percentage of Problem-solving Error

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	14	14.0	14.0	14.0
	Yes	86	86.0	86.0	100.0
	Total	100	100.0	100.0	

Precision errors are revealed by learners when they make errors in solving mathematical steps by writing too untidily or dropping the parenthesis or other signs. Of the 100 learners, 37% committed precision errors when solving the problems in the test.

Table 4.11*:* The frequency and percentage of precision errors

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	63	63.0	63.0	63.0
	Yes	37	37.0	37.0	100.0
	Total	100	100.0	100.0	

Errors linked to unpreparedness were committed by learners when they failed to prepare themselves for the mathematical task. This was indicated by not completing problem, leaving out or skipping certain mathematical steps in the problem. Out of 100 learners, 73% showed unpreparedness in the test.

Table 4.12: The frequency and percentage of Unpreparedness

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	27	27.0	27.0	27.0
	Yes	73	73.0	73.0	100.0
	Total	100	100.0	100.0	

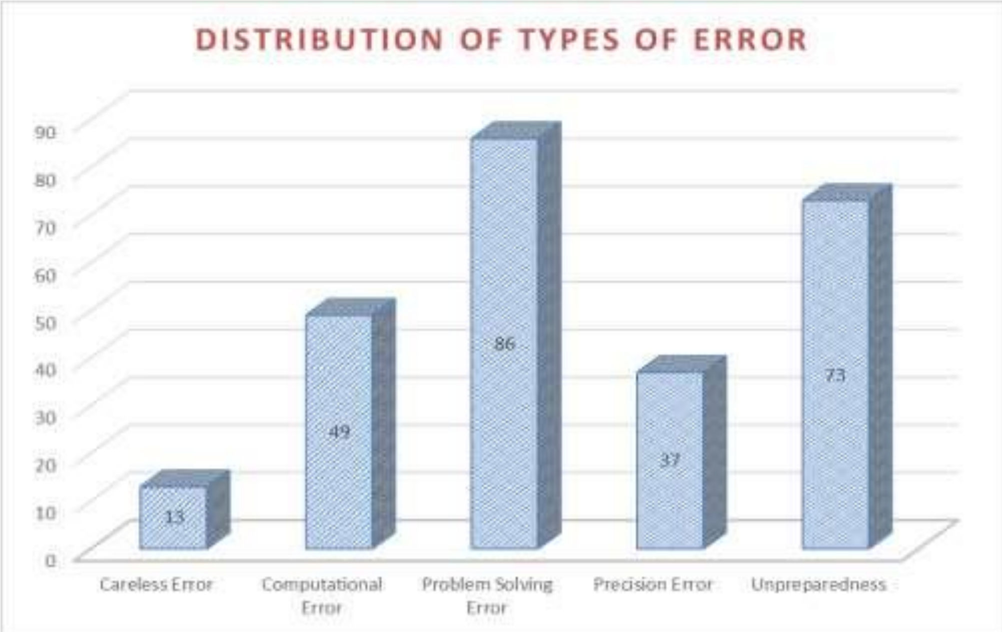


Figure 4.16*:* Distribution on types of error

Figure 4.16 represents the outcomes of learner errors and misconceptions as highlighted in the tables above. Accordingly, 13% of learners committed careless errors, 49% committed computational errors, 86% failed to solve mathematical problems including not the following directions, 37% of learners committed precision errors and 73% committed errors caused by unpreparedness, which was revealed by learners' inability to complete a mathematical problem.

4.6.2. Frequencies

Table 4.13 presents the sources of errors in Grade 9 learners when solving algebra problems. The frequency of variables is as follows: procedural and conceptual; factual knowledge; failure in connection; lack of interpretation; lack of emphasis; overgeneralisation; oversimplification; overspecialisation; inattentiveness; and error translation. These are possible sources of errors in the types of errors found when grade 9 learners solving algebra.

Table 4.13: Statistics in learner sources of errors

Statistics

		Lack of Procedural and Conceptual Knowledge	Lack of factual Knowledge	Failure in connecting new knowledge with old knowledge	Lack of interpretation	Lack of emphasis by the teacher	Overgeneralisation	Oversimplification	Overspecialisation	Inattentiveness, failure to read and understand	The error caused by translation	Lack of basic skills
N	Valid	100	100	100	100	100	100	100	100	100	100	100
	Missing	0	0	0	0	0	0	0	0	0	0	0

The table shows the possible sources of errors in this study, including the number of learners who were assessed. These sources of errors are accompanied by various strategies aimed at avoiding the types of errors and misconceptions committed by Grade 9 learners when solving Algebra problems.

4.6.2.1. Interpretation of the sources of errors when learning Algebra

The causes of errors above displays indicated 99% of learners lack procedural and conceptual errors.

Table 4.14: Frequency and percentage on lack of procedural and conceptual knowledge

	Frequency	Percent	Valid Percent	umulative Percent
	1	1.0	1.0	1.0
	99	99.0	99.0	100.0
Total	100	100.0	100.0	

Eighty-three per cent of learners displayed a lack of factual knowledge or vocabulary. This is indicated by learners' failure to use the correct formula for calculating the area, the perimeter and the like.

Table 4.15: Frequency and percentage on lack of factual information

	Frequency	Percent	Valid Percent	Cumulative Percent
Valid No	17	17.0	17.0	17.0
Yes	83	83.0	83.0	100.0
Total	100	100.0	100.0	

Ninety-nine per cent of Learners displayed an inability to connect new knowledge with old knowledge

Table 4.16: Frequency and percentage show inability to connect new knowledge with old knowledge

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	99	99.0	99.0	99.0
	Yes	1	1.0	1.0	100.0
	Total	100	100.0	100.0	

Fifty-four per cent of learners revealed a lack of interpretation

Table 4.17: Lack of interpretation

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	46	46.0	46.0	46.0
	Yes	54	54.0	54.0	100.0
	Total	100	100.0	100.0	

The following table indicating the result of lack of emphasise by the teacher and the results revealed no evidence of learners show lack of emphasis by the teacher.

Table 4.18: Lack of emphasis by the teacher

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	100	100.0	100.0	100.0

The following table indicating the result Oversimplification and the results revealed no evidence of learners show Oversimplification.

Table 4.19: Oversimplification

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	100	100.0	100.0	100.0

There is no learner shown Oversimplification. The following table indicating the result on Overgeneralisation and the results revealed no evidence of learners show Overgeneralisation.

Table 4.20: Overgeneralisation

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	100	100.0	100.0	100.0

There is no learner’s revealed overgeneralisation. The following table represent Overspecialisation.

Table 4.21: Overspecialisation

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	100	100.0	100.0	100.0

The following table represent Inattentiveness, failure to read and understand. Of the learners interviewed, 32% revealed inattentiveness, failure to read and understand the question, while 68% were not committed.

Table 4.22: Inattentiveness, failure to read and understand

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	68	68.0	68.0	68.0
	Yes	32	32.0	32.0	100.0
	Total	100	100.0	100.0	

Thirty-one per cent of learners made mistakes caused by errors in translation

Table 4.23: Errors in translation

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	69	69.0	69.0	69.0
	Yes	31	31.0	31.0	100.0
	Total	100	100.0	100.0	

Some errors were caused by a lack of basic skills; these are referred to as computation errors. Sixty-one per cent of learners lacked basic skills

Table 4.24: Lack of basics skills

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No	39	39.0	39.0	39.0
	Yes	61	61.0	61.0	100.0

Total	100	100.0	100.0

Figure 4.17 presents the distribution of sources of error. This includes 99% lack of procedural and conceptual knowledge, 83% lack of factual knowledge, 1% connection of new knowledge with old knowledge, 54% lack of interpretation, 0% overgeneralisation, 0% oversimplification, 0% overspecialisation, 32% inattentive, failure to read and understand, 31% errors in translation and 61% lack of basic skills.

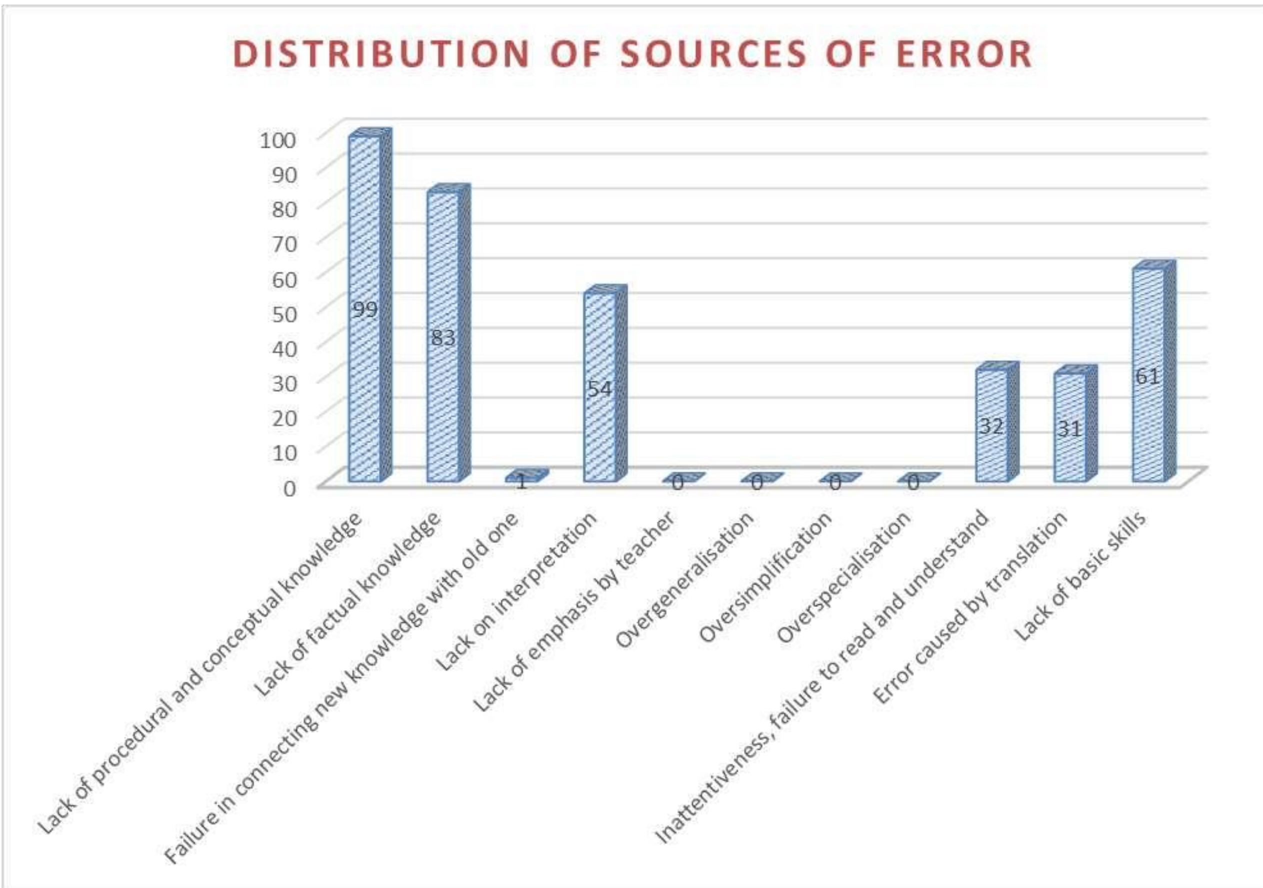


Figure 4.17: Distribution of errors sources

4.7. Summary

Chapter 4 revealed the errors and misconceptions committed by Grade 9 learners in solving Algebra problems. The errors and the misconceptions found in this study include careless errors (13%), for example learners treat a division sign as a multiplication sign; problem-solving errors (86%) where learners revealed a lack of conceptual and procedural knowledge, for example cancelling without following the correct procedure and conjoining terms. Other frequent errors when learning Algebra include computation errors (49%), where learners fail to multiply and get the wrong answers due to a lack of basic skills, errors caused by a lack of precision (37%), where learners illegibly and thus overlook things like parentheses, as well as unpreparedness (73%) on the part of learners when they leave solutions incomplete or leave blank spaces in answers. There is a need for these learners to be given some type of intervention. They need to revisit and revise the concepts and formulae to avoid errors. Other strategies (CRA method of teaching, counter-examples in computation errors – see Klymchuk (2012)) are recommended. In the analysis, it appears as though some learners did not learn certain important concepts, such as word problems and algebraic fractions. If the errors and misconceptions discussed here are corrected in Grade 9, those learners stand a good chance of passing Mathematics when they write the final examination at the end of Grade 12. The next chapter discusses the findings of the research.

CHAPTER FIVE: DISCUSSION OF THE FINDINGS

5.1 Introduction

The current chapter summarises the results and discusses the findings of the study. It focuses on learners' scripts when answering questions on Algebra tasks. These tasks included solving equations and expressions and translating equations, graphs and tables. This chapter is guided by the research objectives, the research questions and the results of the study using the analysis of the learners' scripts. It is also guided by the findings, the literature and the theoretical framework.

5.2 Discussions of results

5.2.1 Error types and misconceptions, sources and strategies in avoiding errors and misconceptions in learning Algebra

Question 1 was based on equations with algebraic fractions, where learners were assessed on their ability to solve the value of an unknown variable (x). The equation with fractions was to respond on learner errors and misconceptions, sources of errors based on equations with fractions, as well as strategies for avoiding those errors in some equations with fractions. Reflection on Koch theory of error analysis suggested that most of the learners did not follow algebraic rules, problem-solving error (Koch error analysis), careless errors, computation errors, precision and unpreparedness directions which resulted in them writing wrong numbers. Learners struggled in algebraic fractions. For instance, learners used operational signs wrongly and they was no evidence of being able to solve the given tasks in each step (e.g. learner 1 in Figure 5) In writing wrong numbers, and use operational signs wrong e.g. a learner compute: $x \div x = x^2$. As discussed by Makonye and Hantibi (2014), ibid suggest, such errors and misconceptions arise as a result of mishandling operational signs.

Errors may also be as a result of carelessness, in that learners tend to use division and multiplication interchangeably. Also, errors may be the result of a lack of algebraic basics skills (see DBE, 2012, p. 12; Maharaj et al., 2015; Pournara et al.

2016) as displayed in the work of learner 4 in Figure 4.4. This examples shows that the addition of numerators and denominator is a significant challenge. A discussion on conceptual errors, as reported by Dhlamini and Kibirige (2014), suggests that misconceptions are commonly related to such errors made when learners mismanage fractions. At other times too, learners cancelled out similar variables or numbers without following the applicable rule (learner 3 – Fig. 4.3; learner 2 – Fig. 4.10; learner 3 – Fig. 4.14) and the addition of numerators and denominators.

Similarly, conceptual errors are discussed by Makonye and Hantibi (2014) and Dhlamini and Kibirige (2014), supported by others such as Fisher and Frey (2012), Tweed (2014), Schwartz (2010) and Brown et al. (2016). It is argued that conceptual errors are a result of misconceptions or a faulty understanding of the underlying principles and ideas connected to the mathematical problem, for example the relationship among numbers, characteristics and properties of shapes (Brown et al., 2016). In addition, errors of overspecialisation occur, some of which may be the result of a lack of conceptual understanding where learners develop an overly narrow definition of a given concept or when applying rules such as cancelling similar variables without following the mathematical rule. Most of the conceptual errors found in the current study were caused by not following proper algebraic rules, for instance, in a co-joining error another learner: ($2x^2 + 3 = 5x^2 = 10x$) and cancelling similar variables ignoring the mathematical rule.

Some other errors identified include the co-joining of terms which appears in the learner's response as follows: $x + 4 = 4x$ and $x^2 – 16 = 16x$. Conjoining errors were identified by Gumpo (2015) in the literature. In this study, learners found conjoining errors where learners ignored the sign and joining the variables with numbers e.g. $4x$ .is differ with $4 + x$. It is recommended that teachers and learners revisit the basic skills or foundation skills so that they have a solid foundation (operational signs ×, ÷, + and -) in Mathematics. Mastering basic skills will help learners to overcome errors and misconceptions and this will give them a good chance of passing Mathematics in Grade 12. In order for learners to progress, they could revisit the concepts of fraction and the application of the rule for fractions to overcome errors and misconceptions in regard to this concept.

Learners also need to relearn algebraic rules and concepts such as terms because they are unable to follow mathematical rules as displayed that when they conjoin the terms (learner 3 – Fig. 4.3; learner 2 – Fig. 4.13). In some cases, terms are separated by a plus or a minus sign in expressions and the value of the term is always the same (constant number). Thus far, the evidence is that learners lack monomial, polynomial, binomial and trinomial concepts. Moreover, learners failed to separate terms. A monomial has one term, bi means two so binomial means two terms, tri means three so trinomial means three terms, and poly means many, so polynomials mean many terms. As highlighted in the literature, other researchers (Gumpo 2015; Mashazi, 2014) suggest that the reason for conjoining terms in Algebra is a lack of ability to distinguish between numbers and variables.

Reflection on the SOLO model theory suggests that most of the learners only achieved one-structure thought processes, where one piece of information indicates a single thought, leading to the learner failing to complete the other mathematical steps. The one-structure of SOLO taxonomy indicates that learners have few ideas in conceptualising algebraic concepts. The types of errors found in learners' scripts was related to the literature and is said to be those errors which are termed as procedural errors by researchers such as (Egodawatte, 2011; Hodgen et al., 2017; Riccomini, 2014, Stoilescu, 2015). Procedural errors are caused by a lack of knowledge when performing certain steps in a mathematical process, for example regrouping and decimal placement (Brown et al., 2016; Egodawatte, 2011; Egodawatte & Stoilescu, 2015; Fisher & Gray, 2012; Hodgen et al., 2017; Mulungye et al., 2016; Riccomini, 2014).

Despite using the correct algebraic rule (learner 2 – Fig. 4.10, L6 and L8 – Table 4.3), those procedural errors were caused by failure to continue following the algebraic steps, for example in the case of learner 2 in Figure 4.2, where the learner followed the rule of multiplying algebraic fractions by cross multiplying but failed to proceed correctly. The error here was caused by negative signs $5x – (3x – 3) = 5 – (7x -7)$, in which case there was no need for negative signs. The learner was supposed to have removed the negative signs $5x (3x – 3) = 5 (7x -7)$ because s/he had already put a bracket there which accounted for multiplication. This was the result of a computation error (Koch's theory of error analysis)

Many errors associated with a lack of procedural knowledge were found in the study and could have been avoided, as suggested by authors such as Egodawatte (2011), Fisher and Frey (2012), Riccomini (2014) and Egodawatte and Stoilescu

(2015) For example, learner 2 (Fig. 4.15) used the correct formula but failed to remove brackets with multiplication, thus revealing a lack of factual information (Brown et al., 2016). Learners showed that they had learnt the concept of equations and expressions that involve the use of formula for calculating the area. However, they failed to apply the correct mathematical rules. Learner 2 (Fig. 4.10) evidenced an error in computation (Koch error analysis) when computing $3x \times x$ as $4x$. The error originated from the bracket $3x(x - 2)$; only the first term was multiplied and the multiplication of the second term was ignored.

To avoid the sources of errors and errors resulting from a lack of procedural knowledge, learners need to develop and organise their procedures (Mulungye et al., 2016; Egodawatte, 2011), however the teacher as the facilitator should plan this development and attempt to change their beliefs and perceptions to avoid rooting these errors in their minds. On the other hand, Tweed (2014) postulates that altering the learner's conceptual framework is the best solution for overcoming and correcting errors and misconceptions when learning Algebra. Hughes (2011) recommends the CRA method of teaching when teaching fractions and claims that this method is useful when teaching learners with difficulties, as when using these methods learners are encouraged and motivated. Cease-Cook (2013) supports this idea, saying that the use of CRA in solving equations using inverse operations is the best means of instruction. Also, teachers are advised to use CRA when teaching fractions so that Grade 9 learners avoid errors and misconceptions when solving Algebra problems.

5.2.2 Findings for question 2

Question 2 was intended to examine learners' ability to translate between graphs, tables and equations. Learners were unable to differentiate between algebraic expressions such as $x + 2$ and an equation such as $= \frac{x^2}{2}$ (see learner 3 – Fig. 4.8). Rather, the learner was supposed to write the equation $= \frac{x^2}{2}$.

Instead, the, learner wrote $x + 2$, which is the expression, as a solution.

Brown et al. (2016) indicate one of the factual errors that arise when learners are unable to recognise formulae. Such an error, guided by Koch's theory, is that shown when the learner (Figure 4.11) was unable to recognise the formula for finding the perimeter; instead they used the formula for finding area. In responding to the question, the learner was supposed to use the formula for finding perimeter. Brown et al. (2016) mention that factual errors are usually the result of that lack of vocabulary, and value and digit identification.

Other learner (Fig. 4.11) revealed computation error when the learner failed to remove brackets by multiplication.

The learners' responses to the graphs reflect that they have difficulty in drawing graphs. Learner 4 (Fig. 4.9 b) evidenced error when merely drawing two straight lines instead of computing the points first and drawing an exponential graph. Reflection on the SOLO model, other learners got many-structures where they indicate several ideas in their thinking. For example (learner 2 Figure 4.6), they got some of the ordered pairs but not all of them.

Reflection on SOLO model the learner revealed one-structure, one piece of information with a written of expression instead of an equation, the learner wrote variable x and wrote 2 but failed to write a division sign in between x and 2 and also failed to square the variable. In addition, there was a missing y variable on the left hand side of the equal sign in order to show that this is an equation as highlighted (learner 4 – Fig. 4.8). As Brown et al. (2016) state in the literature, this is the result of factual errors which result from a lack of vocabulary. Furthermore, this learner (learner 4 – Fig. 4.8) does not understand the difference between the terms *expression* and *equation* as discussed in the literature. For example, a plus sign was used instead of a division sign and a variable need to be a square. Learner displays no knowledge of mathematical terms, such as *numerator* and *denominator*, nor of algebraic equations and algebraic expressions.

Teachers are advised to revisit and retrain learners in basic mathematical skills as well as to drill them in the use of mathematical algebraic language in order to master mathematical terms. This could help them to understand the algebraic language, which could lead to them obtaining a better understanding of Mathematics. When solving equations with fractions, a denominator must not be equal to zero, as dividing by zero is undefined. Moreover, learners must understand mathematical concepts such as *like terms* and *unlike terms*. Only like terms have identical or similar variables that can be added or subtracted. Terms that do not have similar variables are unlike terms of which cannot be added or subtracted. As Klymchuk (2012) states, CES is recommended in computation so that learners may avoid Grade 9 errors and misconceptions when responding to Algebra questions or problems. This could help learners to avoid computation errors.

5.2.3 Findings for question 3

Question 3 consisted of word problems aimed at testing the learner's ability to solve the value of x. The word problems were chosen to highlight the types of error committed by Grade 9 learners when solving Algebra problems. As guided by the theory, problem-solving errors (Koch error analysis) were found which revealed learners' failure to follow proper mathematical rules and failure to complete all steps. As discussed in the literature, conceptual errors are those that are caused by faulty ideas. Learner 2 (see Table 4.3) responded in the focus group interview as follows: *"we are going to start by adding 5 + 5*". In addition, in the test learner 4 (see Fig. 4.4) used the wrong method to solve the Mathematics problem, adding numerators instead of following the rule for solving the fraction. This misconception needs special attention in order to be resolved. This type of error should be addressed by re-learning and re-teaching the concept. The learners in question reveal no evidence of knowledge.

Researchers (Gumpo, 2011; Egodawatte, 2011; Mashazi, 2014; Mdaka, 2011; Mulungye et al., 2016; Pournara et al., 2016) have emphasised that learners' failure to transform words into equations or expressions could cause errors in Algebra. As was seen in this research, some of the learners failed to interpret word problems and translate them to equations or expressions because they misunderstand the concept of word problems (see Appendix a, 3.1). In this regard, one of the learners responded 3 x 5 x 9 ÷ 4 = 60:9; however, they did end up getting the correct answer, that is, 15 years old (refer learner 1) but failed to interpret the word problem. As highlighted in the literature, researchers such as Egodawatte and Stoilescu (2015) believe that this is caused by the lack of meaning attributed to the concept itself. Most of the errors found in question 3 were caused by procedural and conceptual errors, as highlighted by learner 1. This learner answered 3 x 5 x 9 ÷ 4 = 60:9, thus indicating a lack of conceptual and procedural knowledge.

As discussed in the theoretical framework, Koch maintains that this is a problem-solving error as the learner failed to solve the mathematical problem correctly in all steps, which indicates a failure to conceptualise. Other researchers discussed in the literature (Fisher & Frey, 2012; Riccomini, 2014) state that learners' lack of procedural and conceptual knowledge is caused mainly by learners' lack of knowledge or misunderstanding the concept itself. Teachers must prioritise time to teach these concepts and provide extra work and extra classes for practice and revision. These types of error are avoided when the teacher understands learner errors and misconceptions, and then designs an excellent strategy for avoiding such errors. A learner's conceptual knowledge depends on the teacher's knowledge.

Good teachers have a way of correcting learners' errors and overcoming them. This notion is in line with the literature review where Gore, (2016) claims that teachers have to look at the types and causes of errors and then develop strategies for use in teaching for understanding. Many researchers (Limond, 2012; Murtini, 2013; Sepeng & Webb, 2012; Van Klinken, 2012) have suggested strategies for teaching and learning word problems. In a review, Limond (2012) suggests reading strategies in the Algebra class and found that a four-cornered and a diamond-shaped organiser are suitable for teaching learners to give details in their responses to word problems and improving learner reading skills.

A similar notion is expressed by Ahmad et al (2010), Powell (2011), Van Klinken (2012), Sepeng and Webb (2012), Murtini (2013), Fagnant and Vlassis (2013) and Raoano (2016). They suggest a schematic approach as a learning strategy for avoiding errors and misconceptions in learning Algebra. Van Klinken (2012) also recommends a schematic approach to avoiding errors and misconceptions when teaching word problems. With regard to 3 x 5 x 9 ÷ 4 = 60:9, learner 1 failed to conceptualise and indicated lack of conceptual and procedural knowledge. Van Klinken (2012) claims that the schematic approach will give teachers a way to teach word problem, as learners struggle with these problems and this will helps learners to conceptualise semantically. Murtini (2013) suggests scheme learning in avoiding errors and misconceptions and mentions further that the strategy improved skills when learners are required to solve Algebra word problems.

In the study by Sepeng and Webb (2012), a schema-based strategy was used to improve learners' problem-solving performance in Algebra word problems. Fagnant and Vlassis (2013) highlighted that the approach influences learner's development. Reflection on SOLO model theory, one of the learners (learner 1) 3 x 5 x 9 ÷ 4 = 60:9 = 15 years old got one-structure .Correct answer but wrong procedure; it looks like the learner guessed the correct answer. Learner reveals one root of information showing thinking. Accordingly, these findings support some of the claims in the literature reviewed in chapter two, however, believe that was a problem-solving error (Koch theory) where this learner lack both conceptual and procedural knowledge which include failure to think or lack in conceptualisation.

As discussed in the literature (Brown et al., 2016), learner 7 made factual errors (see Table 4.3) by failing to recognise the correct formula; instead they used the formula for finding perimeter. Learners wrote wrong formulae showing that they do not know the required mathematical formulae. In this study, many formulae were required to be used for example, an error here was caused by the use of the wrong mathematical formula. Learners thus need to revise formulae and re-learn basic mathematical skills such as multiplication in the removal of brackets to avoid computational errors.

Another learner revealed lack of procedural knowledge when recognising the correct formula and wrote it correctly in his/her script, but failed to write correct answer, for example (learner 2 – Fig. 4.10): A = $\ell \times b = 3x \times x - 2 = 4x \times - 2 = -8x^2$ Accordingly, these findings support some of the claims in the literature reviewed in chapter three. Lack of procedural knowledge was displayed in the inappropriate enactment of mathematical steps (Brown et al., 2016). According to researchers (Brown et al., 2016; Egodawatte, 2011; Fisher & Frey ,2013 ; Gray, 2012; Hodgen et al., 2017; Mulungye et al., 2016; NTCM, 2014; Riccomini, 2014; Stoilescu, 2015), lack of procedural knowledge occurs when the learner fails to action all mathematical steps correctly. However, SOLO taxonomy classifies this level of thinking A = $l \times b$ = $3x \times x - 2$ = $4x \times -2$ = $-8x^2$ as many-structures, which is explained in theory as several correct answers but not all of them and not entirely interrelated with one another. In this case, the learner wrote the correct formula and substituted correctly, but failed to give the correct solution (see learner 6 in Table 4.3). Learner mixed two languages in the sentence (see code-switching); in IsiZulu learner 6 responded that "*Ngiqale ngabheka rule ye perimeter ngase ngisebenzisa formula ethi* $p = 2n + 2b$ "*then I substitute"*.

$$p = 2(3x) + 2(x - 2)$$

$$p = 6x + 2x - 2$$

$$p = 8x - 2$$

However, in theory, Koch error analysis classifies this error as a computational error where learner fail to calculate using operational signs; this points to a lack of basic Mathematics skills, as discussed in the literature. Here the learner failed to remove brackets by multiplication. Learner indicated that he/she struggled to solve words problems. This kind of error needs a teacher. Learners cannot avoid those types of error alone as they would seem to have missed the concept. This becomes a problem when those learners progress to higher grades and continue to make these errors, as these have not been addressed. Eventually, these learners will not pass Mathematics in Grade 12. It is therefore important that they relearn and revise word problems.

5.2.4 Findings for question 4

Question 4 covered equations and expressions, with the learner being tested on their ability to solve unknown values in diagrams that included a square, a cube, and a rectangular prism. Learners were also tested on their skills pertaining to the use of formulae for length, perimeter, height and the like. One of the learners (learner 2 – Fig. 4.13) indicated:

$2x^2 + x + 3 = 2x^2 + 3 = 5x^2 = 10x$ and the solution was $x = 10$. They were supposed to use Pythagoras' theorem, as indicated in the analysis:

Expected answer: $3x^2 + 4x^2 = 15^2$

$9x^2 + 16x^2 = 225$

$25x^2 = 225$

$x^2 = 9$

$x = 3$ or $x =-3$;

Instead, the learner used the wrong expression, $2x^2 + x + 3$ instead of $3x^2 + 4x^2 = 15^2$ and continued by multiplying the exponent with base $4x = x + 3$. S/he then committed a precision error by dropping the x variables to operate signs 4 + 3 = 7 correctly. In this question, four other learners also revealed co-joining terms (Gumpo, 2015; Mashazi, 2014). As highlighted in the literature those types of errors, shown as $2x^2 +3 = 5x^2$, appear in learners' scripts where an addition sign is used as a symbol for joining terms. However, $2x^2$ and 3 are two different terms, and thus there is no solution for it. It is an expression, and there is no mathematical rule for summing a number and a variable. The addition only works with numbers and not with variables.

$5x^2 = 10x$ – this solution involves another concept which is not applied in a lower grade. In Grade 12, the concept of the derivative appears in the topic of Calculus. $10x \rightarrow x = 10$. This shows that the learner lacks conceptual and procedural knowledge which includes failure to conceptualise, which is revealed the equal sign having no meaning. When do we use the equal sign?

Similarly, Researchers (Gumpo, 2015; Dhlamini and Kiribige 2014) claim that there is a difference between the principles for using equal signs. The researchers further mention that when we sum up two numbers, for instance, 2 + 4 = 6, the addition between of two numbers (2 and 4) means are equal. However, this does not exist or apply in variables, e.g. $x + y \neq xy$, as the learner in this study found $2x^2 +3 = 5x^2$, which is against the mathematical rule. Other researchers (Gumpo, 2015; Mashazi, 2014; Pournara et al., 2016) make a similar claim when explaining co-joining of terms. The authors further say that the addition of a number with a variable is the result of a co-joining error. These errors are caused by a lack of conceptual and procedural knowledge, as discussed in the literature by many researchers (Brown et al., 2016; Hodgen et al., 2017; Mulungye et al., 2016).

Reflection on the SOLO model (show many structures in learner 2 Fig.4.10) revealed that learners have difficulty with these types of question. Most of the learners (refer to Tables 4.1 and 4.2) performed poorly in this question. One of the learners (learner 2 – Fig. 4.10) got many structures, meaning many ideas were available in learner's response and in their thinking. The learner recognised the formula for finding the area and substituted correctly but failed to do the calculation correctly. Another learner in a similar case revealed computation error as follows (Table 4.3 learner 7):

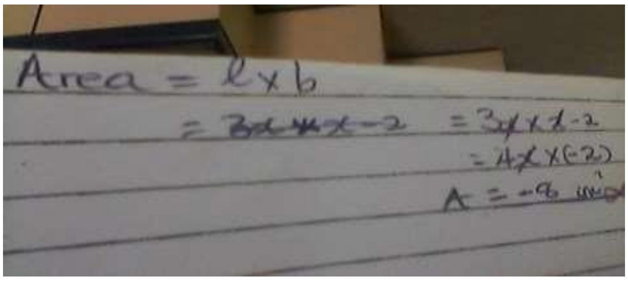
Expected answer: Perimeter = 2(length) + 2(breath)

$p = 2(3x) + 2(x\text{-}3)$

$p = 6x + 2x - 4$

$p = 8x - 4$

Wrote correct formula with correct substitution by failed to compute as indicated by the computation error (Koch's theory) and ended up with the wrong solution.



With reference to learner 2 (Fig. 4.10), s/he also revealed correct formula for finding area and substituted correctly but failed to insert brackets in order to do multiplication. Instead s/he merely cancelled a similar variable without following the rule. Cancelling variables without following procedure is unacceptable according to mathematical rules. This is the wrong method for solving this type of problem. In line with Koch's theory, learners experience problems if they do not follow of the mathematical rule. Similarly, Makonye and Khanyile (2015) identify a type of error called a cancellation error. Cancellation errors occur when cancellation is done without following the mathematical cancellation rule. In the test, it would seem that learners at random and do not follow the mathematical rule when they cancel. Such errors are caused by a lack of conceptual and procedural knowledge, as discussed in the findings pertaining to question 1.

In this regard, Tweed (2014) claims that *cancel* or *cancelling* is a term that causes confusion in both addition and division in operations of mathematical Algebra. Often this results in learners finding their solution to be zero or one. Contributing to this, when learners see similar variables or numbers they simply cancel them out without following the procedure or mathematical rule. To avoid cancellation errors, learners need to relearn the concepts, with teachers clarifying when and where cancel should be applied. Learners must understand that they must not cancel without following the rule, as $3x$ and x are different terms, which have to be multiplied to get a solution. Teachers must support learners by giving them extra work to practise this process and thus avoid cancellation errors. In algebraic equations learner must learn that when adding a number and its additive inverse the answer is zero; when multiplying a number and its multiplicative inverse, the answer is one. In ordered pairs, pairs of values are connected by a certain rule, and substitution requires replacing a variable with a number value. One of the learners (learner 4 – Fig. 4.15) calculated the answer using the correct method but substituted incorrect, thus this was a procedural error.

5.3 Findings

The findings were considered in the light of Koch's error analysis and the SOLO model. The findings revealed that 13% of errors were careless errors that include the use of a division sign as a multiplication sign, in other words carrying out the wrong operation; 49% were computation errors, which include failure to calculate accurately, subsequently getting the wrong answer or arriving at the wrong solution; 86% were problem-solving errors that were due to a lack of conceptual and procedural knowledge, for example conjoin error, cancellation errors, and computation errors. However other steps were correct. Other types of error that are found in learners' response include the use of correct formulae and correct substitution but incorrect computation, thus resulting in the wrong solution. 37% of precision errors were committed where learners dropped signs, dropped variables and wrote too untidily, ending up with the wrong answer. 73% of errors were caused by unpreparedness, which was due to the failure to complete algebra problems and thus leaving blank spaces in scripts. The SOLO theory revealed the level of thinking, indicating one-structure (equations with fractions, word problems and equations and expressions that include the concept of ratio, height, length, area and perimeter). Many-structures only in equations and expressions involving translation through an equation to table and table to the graph.

5.4 Summary

Chapter 5 contained a discussion of the findings of the study. Errors and misconceptions committed by Grade 9 learners when solving Algebra problems were discussed in line with Koch's error analysis and the SOLO model. In this chapter, careless errors, computation errors, and problem-solving errors, precision errors and errors resulting from unpreparedness were discussed. The levels of thinking (one-structure and many-structures as revealed by the learners in their responses to Algebra solutions) were also discussed in connection to Koch's theory and the literature itself. The next chapter concludes they study by providing a summary, discusses the limitations of the study and makes a number of recommendations.

CHAPTER SIX: SUMMARY, LIMITATIONS, RECOMMENDATIONS, AND CONCLUSIONS

6.1 Introduction

This chapter presents a summary, the conclusions and the limitations of the study and makes a number of recommendations.

6.2. Research summary

As highlighted in chapter one, the objectives of this study were to investigate the types errors and misconceptions related to learning Algebra in the Senior Phase, explore the possible causes of learners' errors and misconceptions when learning Algebra, and identify possible strategies for avoiding errors and misconceptions when learning Algebra in the Senior Phase.

The literature review was directed by two theoretical frameworks, namely, Koch's theory which classifies the types of error and their sources, and the SOLO model, which was used to analyse the cognitive levels of thinking. The study adopted a descriptive survey design. The sample size numbered 100 and the unit of analysis was Grade 9 learners from 13 to 16 years of age. To acquire the sample size required for the study, the researcher used a sample size calculator with a confidence level of 95% and a confidence interval of 9, giving a sample size of 91. However, for easy reporting and better reliability, 100 learners were sampled. The test was based on Algebra questions (see Appendix A).

Data were collected in five schools (20 learners per school) in King Cetshwayo District. The analysis of the quantitative data (test) was done using descriptive analysis (tables representing the frequency and percentages of types of error and sources of errors in solving Algebra problems). A bar graph was used to represent the distribution on types of error and sources of errors when learning Algebra. The qualitative data (obtained from focus group interviews) were analysed using content analysis and were collected in the same five schools. The sample size for the interviews was six learners per school. This chapter aims to conclude and summarise the study and make recommendations based on the study findings.

The types of error were classified according to Koch's theory of error analysis as careless, computation, problem-solving, precision and unpreparedness errors, as discussed in the literature. In this study, learners committed 13% of careless errors, 49% of computation errors, 86% of problem-solving errors, 37% of the learners revealed precision, and 73% revealed unpreparedness. Some of learners did make two or more of those errors at the same problem or at the same time, meaning that percentages do not add up to 100%.

The sources of errors, which was the second main aim of the study, were classified using literature sources, as a lack of conceptual and procedural knowledge; lack of factual knowledge; lack of connection between new knowledge and old knowledge; lack of interpretation; lack of emphasis by the teacher; overgeneralisation; oversimplification; overspecialisation; inattentiveness and failure to read and understand; errors caused by translation (graph to equation, graph to table, table to graph, equation to table, and table to equation) and, lastly, a lack of basic skills.

The possible sources of these errors mentioned above in the first research aim included a lack conceptual and procedural knowledge (revealed by 99% of learners); a lack of factual knowledge (83% of learners); lack connection between new knowledge and old knowledge (1% of learners); lack of interpretation (54% of learners); lack of emphasis by the teacher (0% of learners); over-generalisation (0%); over-simplification (0%); overspecialisation (0%); inattentiveness, failure to read and understand (32%); errors caused by translation (graph to equation ,graph to table , table to graph equation to table and table to equation) (31%) and lastly, learners' lack basic skills (61%).

As discussed in chapter four, question 1 of the test was based on equations with fractions. In a focus group interview, 32% did not respond at all, 6% responded correctly, and 62% responded incorrectly. In question 2, 31% did not respond, 42% responded correctly, and 27% responded incorrectly. In question 3, 47% not responded, 2% correctly and 51% were incorrect. In question 4 no learners responded correctly, 48% responded incorrectly, and 52% of the learners did not respond at all (Appendix E). The results of the focus group interview indicate poor performance by learners. Data from the focus group interviews was applied to answer the research questions on the different types of error made by learners in Algebra in Grade 9, as well as the possible sources of those errors possible strategies for avoiding them. This indicates that Grade 9 learners commit errors in solving Algebra. The results indicate poor performance in the test.

The third main aim of the study was to determine possible ***strategies for avoiding*** careless, computation, problem-solving and precision errors and errors caused by unpreparedness. Results were interpreted according to various researchers reviewed in the literature.

6.2.1. The types of error and misconception experienced by Grade 9 learner and the sources of these, as well as strategies for avoiding them

The first objective of the study was to examine the types of error made by Grade 9 learners in responding to Algebra problems. This section presents a summary of the types of error learners made as per the study results.

$x \div x = x^2$

6.2.1.1. Careless errors

About 13% of the learners were found to have committed careless errors in this study. In solving algebraic fractions, learners' test results also revealed careless errors. The learners' responses indicated that they misused operational signs, and there was no evidence to show that learners were able to solve the given tasks in each step, as they wrote done numbers incorrectly, and misused operational signs, e.g. on learner computed $x \div x = x^2$.

As stated by Makonye and Hantibi (2014), careless errors arise due to the mishandling of operational signs. They may also occur as a result of careless use of the division sign as a multiplication sign, using the addition sign as a multiplication sign and the like. The DBE (2012, p.12), as well as Maharaj et al. (2015) and Pournara et al. (2016) believe that these careless errors may be due to lack of basic algebraic knowledge. Learner 4, for example, failed to divide, as directed to do so by the division sign, instead carrying out multiplication.

6.2.1.2. Problem-solving errors

Problem-solving errors were the most common errors found in the learners' scripts. Problem-solving errors occur when learners fail to follow the directions for solving mathematical problems. In this study, 86% of learners committed such errors. The source of this type of error is a lack of conceptual and procedural knowledge, which results in learners failing to solve the problem. All mathematical steps in the problem were wrong (see figure 4.3) because the learners lack the conceptual knowledge and procedural knowledge (Egodawatte & Stoilescu, 2015; Fisher & Frey, 2013; Riccomini, 2014).

In question 1, in which learners were required to solve equations containing fractions, one of the learners was found to be adding numerators and adding denominators (see Figure 4.4, learner 4). Other researchers such as Mahakure et al. (2014), Makonye and Khanyile (2015) and Khanyile (2016) observed a similar type of error and believed that these errors are caused by mishandling variables, where a learner fails to perform or solve a mathematical problem. The possible sources of the errors are identified in the literature by various researchers (Egodawatte & Stoilescu, 2015; Fisher & Frey, 2013; Riccomini, 2014) as a lack of conceptual and procedural knowledge. Learners' test results revealed evidence of poor reasoning or failure to conceptualise .Koch classified a problem-solving error as one where the learner ignores or fails to follow analytical procedures and rules.

In addition, Egodawatte and Stoilescu (2015) believe that errors are caused by a lack of conceptual meaning. To resolve this type of error amongst learners, Hodgen et al. (2017) recommend that learners need to restructure their existing knowledge to prepare themselves for new knowledge, analytical procedures and concepts so that they can retrieve and apply knowledge. Thus, there is a need for teachers to emphasise that when dealing with fractions, numerators and denominators cannot be added.

Other problem-solving errors in mathematics include cancelling without following the procedure which is caused by a lack of knowledge of both concepts and procedures. Makonye and Khanyile (2015) identify a cancellation error as a different type of error and mention that learners cancel willy-nilly without following mathematical rules. When solving Algebra, there are rules for cancellation. Therefore, cancellation errors are caused by a lack of conceptual and procedural knowledge, as discussed in the results for question 1.

Another error type is one that is the result of a failure to follow proper algebraic rules, for instance a conjoining error (Alshwaikh & Adler, 2017; Gumpo, 2015; Mashazi, 2014; Ncube, 2016; Pournara et al., 2016). Learners revealed conjoining errors in this study by displaying the joining of terms using the plus sign as a joiner

($2x^2$ +3 = $5x^2$ = $10x$). See Figure 4.13, where learner 2 co-joined terms as x + 4 = $4x$ and x^2 – 16 = $16x$. As stated (Gumpo, 2015; Mashazi, 2014), the source of this type of error is an inability to translate between the numbers and variables. Learners' failure to interpret word problems and translate them to equations indicates a lack of conceptual and procedural knowledge; seen in particular in one of the learner's response (3 x 5 x 9 ÷ 4 = 60:9).

6.2.1.3. Precision

Koch (2015) describes lack of precision as being caused by learners' disarray when solving problems caused by untidiness, dropping signs when calculating and forgetting signs, either addition or subtraction. In this study, errors resulting from lack of precision were found in 37% of learners' scripts. Koch (2015) indicates that lack of precision causes units, either variables or numbers, to disappear as a result of lack of labelling and notation. Consequently, precision errors were prevalent in learners' scripts in this study.

6.2.1.4. Unpreparedness

Unpreparedness on the part of learners was to be seen in the empty spaces left by learners instead of responding to the questions. Others failed to complete mathematical steps because of lack of knowledge, thus leaving solutions incomplete. About 73% of the learners had empty spaces and incomplete solutions in their scripts, thus indicating unpreparedness (Koch error analysis).

6. 2.1.5. Computation errors

About 49% of the learners committed computation errors. For example, in the following equation an error was caused by the negative sign. Learner 2 (Fig. 4.2) computed $5x$ - ($3x$ - 3) = 5 – ($7x$ – 7). In this case, there was no need for negative signs, as the learner was supposed to have removed them: $5x$ - ($3x$ - 3) = 5 – ($7x$

– 7). This occurred because the learner had already inserted a bracket, which accounted for multiplication. Hence, a computation error occurred (Koch's theory of error analysis). Other learners failed to compute multiplication (learner 7 – Table 4.3) as follows:

Perimeter = 2(length) + 2(breath)

$p = 2(3x) + 2(x-3)$

$p = 6x + 2x - 4$

$p = 8x - 4$

The learner failed to remove bracket by multiplication; $2(x – 3) = 2x - 4$, not sure where 4 comes from, which was incorrect (learner 7 – Table 4.3). In which the learner multiplied errors and misconceptions when responding to Algebra questions or problems.

2(x-3) as 2x - 4 .The expected answer was $2x - 6$. There is consequently a need for teachers and learners to revisit Mathematics foundation skills (operational signs ×, ÷, + and -). Klymchuk (2012) suggests CES in computation so that learners can avoid such errors and misconceptions when responding to Algebra questions or problems.

6.3. Recommendations

6.3.1. Teacher development and support

The errors and misconceptions committed by Grade 9 learners when solving Algebra problems can be avoided with guidance from effective educators, who can also assist in addressing the sources of such errors. Educators could develop or improve the learners' algebraic reasoning with regard to the mathematical algebraic content featured in the South African curriculum. There is a need to develop and support teachers' conceptual knowledge so they may in turn support learners. If teachers' conceptual knowledge is sufficient and they are well-prepared to teach, learners' conceptual understanding and knowledge could be improved. Learners need to improve their conceptual understanding and competency when learning Algebra so as to avoid misconceptions and errors.

The researcher believe in presenting and discussing the findings with other teachers to each of the five schools that were used as a sample of all schools in

King Cetshwayo district so as to make the principals of the schools and mathematics teachers teaching (grade 8-12) to be aware on types of errors committed by learners in grade 9. The sources of those errors and misconceptions will be presented and discussed as revealed on the thesis. The strategies will be also mentioned and presented as displayed on the document. Hopefully those teachers including the researcher will also contribute the information on types of errors, sources and strategies during departmental workshops to help other teachers to come up with new strategies on dealing with careless errors, computation errors, problem solving errors, precision errors and unpreparedness errors.

It is recommended that teachers, schools, circuits and districts, and all other stakeholders, should be made aware of the types of error committed by learners, and the misconceptions they hold that led to these errors when solving Algebraic problems. Awareness in these errors and misconceptions will strengthen efforts to find solutions for minimising these misconceptions in the early grades. The results of this study may be of benefit to Grade 9 teachers in that they may assist them to find ways to overcome learner errors even beyond Grade 9.

6.3.2. Future research

The results indicate that errors and misconceptions displayed by the Grade 9 learners who participated in this study were similar to those displayed in other studies in the literature However, the current study did not determine whether there is a relationship between the misconceptions and errors and specific contextual issues. Future research could explore the relationship between misconceptions and/or errors, and language and culture (how learners experience mathematics in their everyday life).

6.6. Limitations

The study focused only on the errors and misconceptions that Grade 9 learners display when solving algebraic equations and expressions in the District of King Cetshwayo, KwaZulu-Natal province in the Republic of South Africa. A more complete analysis of learner misconceptions and errors could have been obtained had other districts and provinces been included.

6.7. Summary

This chapter summarised the study, outlined the conclusions of the research, made a number of recommendations for all stakeholders, as well as suggestions for future research. The limitations of the study were also outlined in this chapter.

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APPENDIX A: TEST

Mathematics Test

Grade 9

Marks: 50

Date: 22/09/2017

Solve for: x

1.1 $\frac{5x}{7x-7}=\frac{5}{3x-3}$ (4)

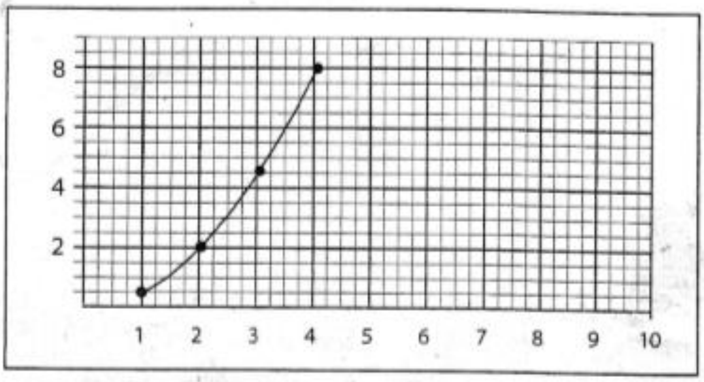
1.2 $\frac{1}{x}-\frac{1}{x+4}=\frac{-4}{x^2-16}$ (5)

1.3 $\frac{x}{x-1}+\frac{x}{2x+1}=\frac{3}{(2x+1)(x-1)}$ (5)

1.4 $\frac{3}{x^2-1}+\frac{2}{x+1}=\frac{1}{x-1}$ (5)

Question 2

2.1 Consider this graph:



2.1.1 Write a set of ordered pairs in the form of a table of values for the relationship

Depicted by the graph. (2)

2.1.2 Determine the equation for the above graph. (2)

2.1.3 If $x = 6$, determine the output value. (1)

2.2 Given the equation $y = 2^{x+1}$

2.2.1 Draw a table of values for the following values of $x = 0, 1, 2, 3, 4$ (3)

2.2.2 Draw an accurate graph for the relationship generated by the equation. (3)

Question 3

3.1. Mary's mother is three times as old as she is .Five years ago her mother was four times as old as her. How old is Mary? Indicate the situation with a table and write the equations then solve for an unknown. (4)

3.2. A rectangle has length $3x$ and breadth $(x - 2)$.

(a) Write down an expression for its perimeter (1)

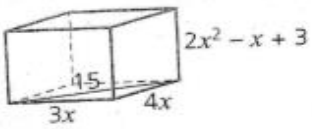
(b) What is the length if the perimeter is 68m? (1)

(c) What is its area (1)

(d) Write the ratio of the length to breath if the area is 72 square centimetres. (1)

Question 4

4.1.

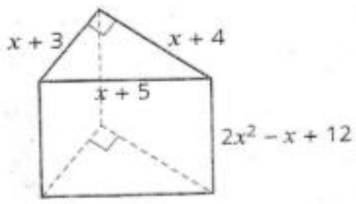


The measurements of rectangular prisms are shown in the sketch. The length of the diagonal of its base is equal to 15 cm (as shown).

4.1.1 Write an equation and solve for x. (3)

4.1.2 Now determine the numerical value of the height of the prism. (3)

4.2.



The measurements, in mm, of a right-angled triangular prism are shown in the sketch.

4.2.1 Write an equation in terms of x and then solve the equation. (3)

4.2.2 Calculate the total surface area of the prism (3)

APPENDIX B: Test memorandum

Mathematics Memorandum grade 9

Memorandum

Question 1

1.1 $\frac{5x}{7x-7} = \frac{5}{3x-3}$

$(5x)(3x-3) = 5(7x-7)$

$15x^2 - 15x = 35x - 35$

$+ 15x^2 - 50x + 35 = 0$

$3x^2 - 10x + 7$

$(3x-7)(x-1) = 0$

$x = \frac{7}{3}$ or $x = 1$

(4)

1.2 $\frac{1}{x} - \frac{1}{x+4} = \frac{-4}{x^2-1}$

$\frac{1(x+4)-1(x)}{x(x+4)} = \frac{-4}{x^2-16}$

$\frac{x+4-x}{x(x+4)} = \frac{-4}{x^2-16}$

$4(x^2-16) = -4(x^2+4x)$

$4x^2 - 64 = -4x^2 - 16$

$8x^2 + 16x - 64 = 0$

$+ x^2 + 2x - 8 = 0$

$(x-2)(x+4) = 0$

$x = 2$ or $x = -4$

(5)

1.3 $\frac{x}{x-1} + \frac{x}{2x+1} = \frac{3}{(2x+1)(x-1)}$

$$\frac{x(2x+1)+x(x-1)}{(x-1)(2x+1)} = \frac{3}{2x^2-2x+x-1}$$

$$\frac{2x^2+x+x^2-x}{2x^2+x-2x-1} = \frac{3}{2x^2-x-1}$$

$$\frac{3x^2}{2x^2-x-1} = \frac{3}{2x^2-x-1}$$

$$(3x^2)(2x^2-x-1) = 3(2x^2-x-1)$$

$+2x^2 - x - 1$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x^2 - 2 = 0$$

$$(x-1)(x+1) = 0$$

$$x = 1 \; or \; x = -1$$

(5)

1.4 $\frac{3}{x^2-1} + \frac{2}{x+1} = \frac{1}{x-1}$

$$\frac{3(x+1)+2(x^2-1)}{(x^2-1)(x+1)} = \frac{1}{x-1}$$

$$\frac{3x+3+2x^2-2}{x^3+x^2-x-1} = \frac{1}{x-1}$$

$$\frac{2x^2+3x+1}{x^3+x^2-x-1} = \frac{1}{x-1}$$

$$1(x^3+x^2-x-1) = (x-1)(2x^2+3x+1)$$

$$x^3+x^2-x-1 = 2x^3+3x^2+x-2x^2-3x-1$$

$$2x^3+x^2-2x-1 = x^3+x^2-x-1$$

$$x^3 - x = 0$$

$$x(x^2-1) = 0$$

$$x(x+1)(x-1) = 0$$

$$x = 0 \; or \; x = -1 \; or \; x = 1$$

(5)

Question 2

2.1.1 (1,0.5) (2,2) (3,4.5) (4,8) (2)

2.1.2 $y = \frac{x^2}{2}$ (2)

2.2.2 If $x = 6$ $y = \frac{6^2}{2}$

$y = \frac{36}{2}$ (1)

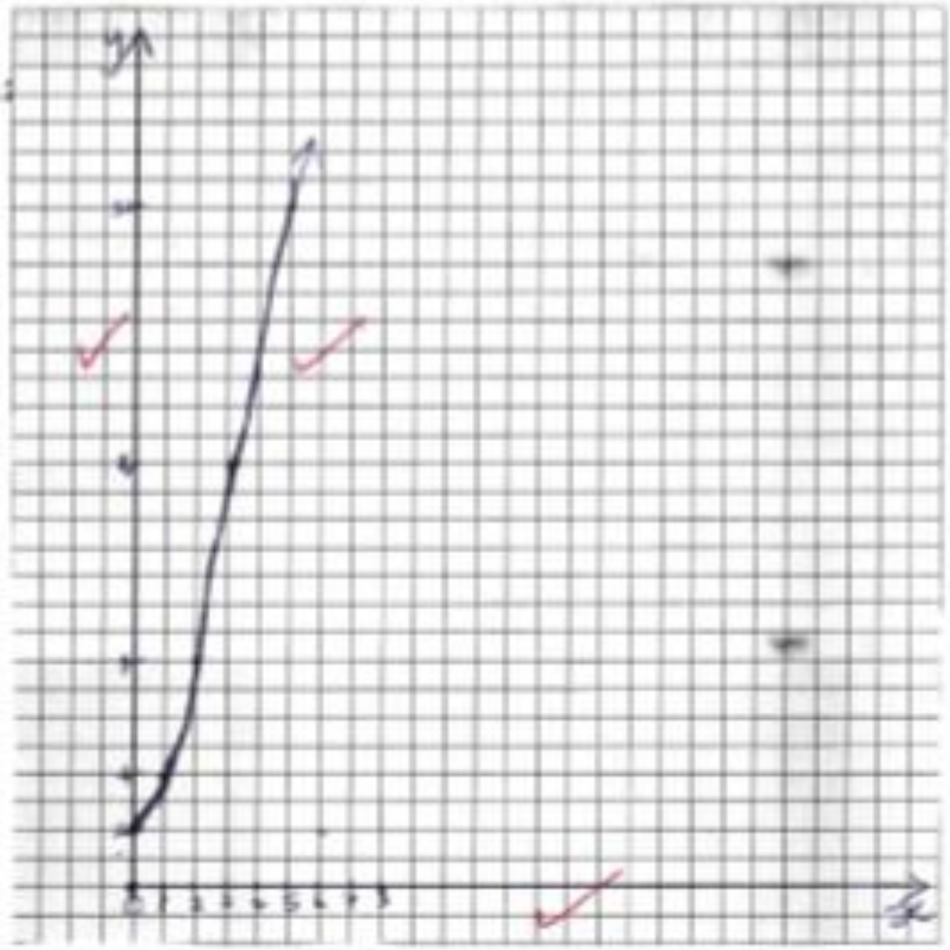
$y = 18$

2.2.1 $y = 2^{x+1}$

x	0	1	2	3	4
y	2	4	8	16	32

(3)

2.2.2



(3)

Question 3

3.1 A strategy that is commonly used for that problem solving is to draw table

Age	Now	5 years ago
Mary	x	$x-5$
Mary's mother	$3x$	$3x-5$

Set up an equation for five years ago

4 x Mary's age = mother's age

4 x $(x-5) = 3x-5$

Solving the equation to find the value of x

$4x-20 = 3x-5$

$4x-3x = -5+20$

$x = 15$

Mary is currently 15 years old (4)

3.2

3.2.1 $perimeter = 2 \times length + 2 \times breadth$

P $= 2(3x) + 2(x-2)$

$= 6x + 2x - 4$ (1)

$= 8x - 4$

3.2.2 $68 = 8x - 4$

$72 = 8x$

$x = 9$

The length must be $3 \times 9 = 27cm$ (1)

3.2.3 $A = length \times breadth$

$= 3x \times (x - 2)$

$= 3x^2 - 6x$ (1)

3.2.4 9:2 (1)

Question 4

4.1.1 $(3x)^2 + (4x)^2 = 15^2$

$9x^2 + 16x^2 = 225$

$25x^2 = 225$ (3)

$\div by\ 25$ $x^2 = 9$

$x = 3 \text{ or } x = -3$

4.1.2 $Height = 2x^2 - x + 3$

$= 2(3)^2 - (3) + 3$

$= 18 - 3 + 3$ (3)

$Height = 18\ cm$

4.2.

4.2.1 $(x + 5)^2 = (x + 3)^2 + (x + 4)^2$

$x^2 + 10x + 25 = x^2 + 6x + 9 + x^2 + 8x + 16$

$x^2 + 14x + 25 = 10x + 25$

$x^2 + 4x = 0$ (3)

$x(x - 4) = 0$

$x = 0 \text{ or } x = -4$

4.2.2 $Area = \frac{1}{2}(4)(3)$

$A = 6mm^2$

$A = 2 \times 6mm^2$

$A = l \times b$

$A = 12 \times 4$

$A = 48mm^2$

$A = l \times b$

$A = 5 \times 12$

$A = 60mm^2$

$Total\ surface\ area = 12 + 48 + 60 + 36$ (3)

$Total\ surface\ area = 156mm^2$

APPENDIX C: Focus group interview questions (*see code switching)*

1. If you look at the question how you can find value of x ? (Sizomthola kanjani $\boldsymbol{x}$ $(\boldsymbol{uyilenamba\ esingayazi})$? (Refer to 1.1 Appendix A)

2. How can you solve for in x algebraic equation with fractions*? **Sizoyithola kanjani I value ka x (yilenamba esingayazi?)*** (Refer to 1.2 Appendix A)

3. How can you find set of ordered pairs? How can you find set of ordered pairs? $(\boldsymbol{sizozithola\ \ kanjani\ izinamba\ ezinobudlelwano\ \ phakathi\ ka\ x\ no\ y})$? (Refer to 2.1 Appendix A)

4. How can you determine the equation in the graph that you are given? ***Sizoyithola kanjani equation kule graph (umdwebo esiwunikeziwe)?*** (See Appendix A 2.1.2)

5. If you are given the equation how can you draw the table using given $x\ values$?

(Sizolidweba kanjani tabular sisebenzisa izinamba esizinikiwe zika x siligcwalise futhi?)

(See Appendix A 2.1.3)

6. How can you solve for x in this word-problem (see Appendix A 3.1). ***Sizoyithola kanjani le value esingayazi ewu x kule word-problem?***

7. How can you write an expression for it perimeter? **Sizoyithola kanjani ipherimetha?** (See Appendix A 3.2 a)

8. How can you find length if a perimeter given is 68? ***Sizobuthola kanjani bude bendawo esiyinikiwe*** (see Appendix A 3.2 b)

9 How can you find area? ***Sizoyithola kanjani indawo***? (See Appendix A 3.2 c)

10 How can you find ratio? ***Sizoyithola kanjani ratio***? (See Appendix A 3.2 d)

11. How can you find equation***? Sizoyithola kanjani equation? (***See Appendix A 4.1.1)

12 How can you find height? ***Sizoyithola kanjani height?*** (See Appendix A 4.1.2

APPENDIX D: Focus group interview expected responses

1. There is no like expression. Learners need to multiply each term by those denominators, then multiply by removing brackets then simplify (Making equation to be simple) then go for solution.

2. The learner expected to take Lowest Common Denominator on the left-hand side of the equation then expand, there must be an expansion on the right-hand side to remove brackets. The cancellation rule applied after expansion and then simplify or make equation to be simple.

Lastly go for an answer.

3. Write each x values with the corresponding y values. Substitute with x values in an equation then draw and complete table.

4. The learner expected to observe the graph, observing $x\ values\ and\ y\ values$ then use any point in the graph the equation was $y = \frac{x^2}{2}$

6. There is a need for drawing table representing information on a table , set up an equation for five years ago using information in the table then solve an equation and find the known value that was represented by the variable.(see Appendix B 3.1)

7. Learners must look at what kind of figure was given, in which was given a rectangle. Learners must know properties of rectangles and use formula for finding perimeter$p = 2(l \times b)$

(See Appendix B 3.2 a)

8. The equation that was the product of a perimeter could be used as it is to substitute the value of p as the question gave perimeter of 68. (See Appendix B 3.2 b)

9. Learners must look at what kind of figure was given .Here is rectangle; learners must know properties of rectangle. The formula for finding arear to be used: $A = l \times b$

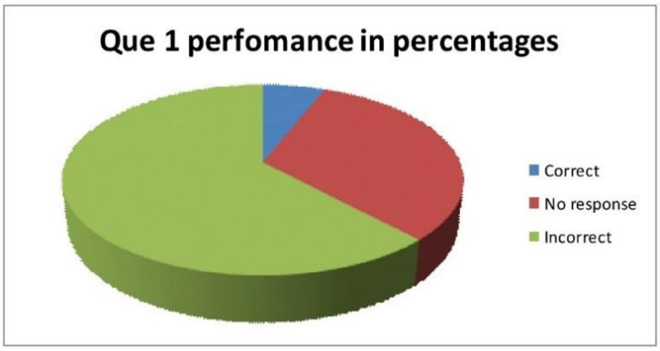
10 The learner expected to substitute x value of 9 in the equation. (See Appendix B 3.2 d)

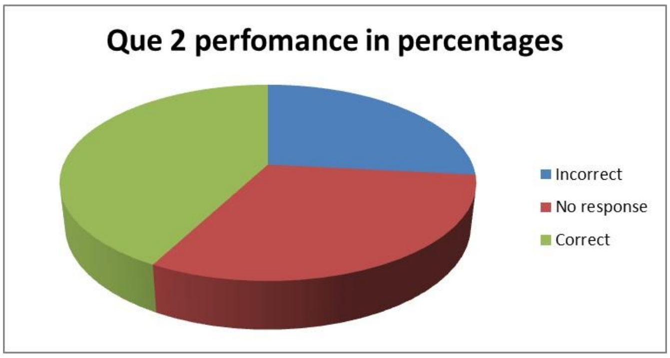
11 The figure is a rectangular prism so the learner must find the value of x by Pythagoras theorem where $r = 15$, $x = 3x$, $y = 4x$ $x^2 + y^2 = r^2$. (See 4.1.1 in Appendix B)

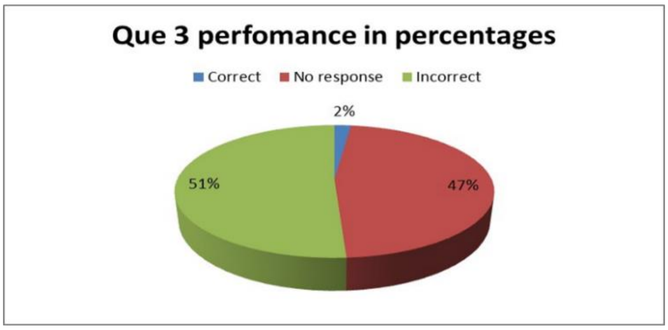
12 The height is given to the equation of $h = 2x^2 - x + 3$ then substitute with 3 from the product of 4.1.1 then the height was 18cm (see Appendix B 4.1.2)

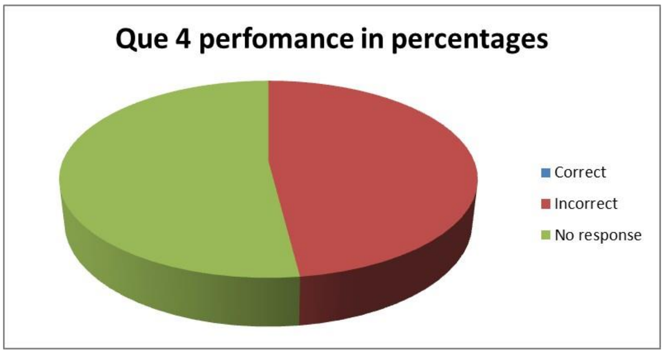
APPENDIX E: Qualitative data presentation in the graph (pie chart)

The pie chart represents the data in percentages in a qualitative data which focused on group interview as tool number 2 of collecting a data in this study. Write a paragraph explaining what value they carry or report on the further in relation to research questions.









APPENDIX F: A LETTER REQUEST FOR PERMISSION TO CONDUCT RESEARCH

University of Zululand
P.O Box x1001
Kwa-Dlangezwa
3886
24 July 2017

The Director
KwaZulu- Natal Department of Education
Provincial Office
Private x 9137
Pietermaritzburg
3200

Dear Sir/ Madam

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN RURAL SCHOOLS

My name is Philile Nobuhle Mathaba and I am studying towards a Master's degree in the Department of Mathematics, Science and Technology Education at the University of Zululand.
I would like to conduct research in your schools for the purpose of fulfilling the requirement of a full dissertation in Mathematics Education. My research topic is: Errors and Misconceptions related in Learning of Algebra in the Senior Phase. UMlalazi schools in Kwa-Zulu Natal province.
The research will conducted under supervision of Prof.A.Bayaga from the University of Zululand. I am hereby seeking your permission to approach grade nine learners in your schools in order to participate in this research. Upon completion of the research, I undertake to provide the Department of Education with a bound copy of the full research report. If you require any further information, please do not hesitate to contact me on 0734171424 email biyelapn@gmail.com.

Thank you.

Yours sincerely
Philile Nobuhle Mathaba

APPENDIX G: APPLICATION FORM TO CONDUCT RESEARCH IN KZN-DOE

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Application for Permission to Conduct Research in KwaZulu Natal Department of Education Institutions

1. Applicants Details

Title: Prof / Dr / Rev / Mr / Mrs / Miss / Ms Surname: ______________________________

Name(s) Of Applicant(s): ____________________________ Email: _____________________

Tel No: _______________ Fax: _________________ Cell: __________________

Postal Address: ____

____

2. **Proposed Research Title:** ____

____

____

3. Have you applied for permission to conduct this research or any other research within the KZNDoE institutions?	**Yes** | **No**
If "yes", please state reference Number: _______________	

4. Is the proposed research part of a tertiary qualification?	**Yes** | **No**
If "yes" **Name of tertiary institution:** ____ **Faculty and or School:** ____ **Qualification:** ____ **Name of Supervisor:** _______________________ **Supervisors Signature**____________________	

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Physical Address: 247 Burger Street • Anton Lembede Building • Pietermaritzburg • 3201
Tel.: +27 33 392 1041 • **Fax.:** +27 033 392 1203• **Email:** Phindile.Duma@kzndoe.gov.za •**Web:**www.kzneducation.gov.za
Facebook: KZNDOE….Twitter: @DBE_KZN….Instagram: kzn_education….Youtube:kzndoe

If "no", state purpose of research: ____

____

5. Briefly state the Research Background____

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6. What is the main research question(s) : ____

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7. **Methodology including sampling procedures and the people to be included in the sample:**

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8. **What contribution will the proposed study make to the education, health, safety, welfare of the learners and to the education system as a whole?:** ______________________

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KZN Department of Education Schools or Institutions from which sample will be drawn – If the list is long please attach at the end of the form		

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9. **Research data collection instruments**: *(Note: a list and only a brief description is required here - the actual instruments must be attached)*: ____

____
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10. Procedure for obtaining consent of participants and where appropriate parents or guardians:

____
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11. Procedure to maintain confidentiality (if applicable): ______________________________

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12. Questions or issues with the potential to be intrusive, upsetting or incriminating to participants (if applicable): ____

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13. Additional support available to participants in the event of disturbance resulting from intrusive questions or issues (if applicable): ____

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14. Research Timelines : ____

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Facebook: KZNDOE....Twitter: @DBE_KZN....Instagram: kzn_education....Youtube:kzndoe

15. Declaration
I hereby agree to comply with the relevant ethical conduct to ensure that participants' privacy and the confidentiality of records and other critical information.
I ____ declare that the above information is true and correct
_________________________ **Signature of Applicant** __________________ **Date**

16. Agreement to provide and to grant the KwaZulu Natal Department of Education the right to publish a summary of the report.
I/We agree to provide the KwaZulu Natal Department of Education with a copy of any report or dissertation written on the basis of information gained through the research activities described in this application.
I/We grant the KwaZulu Natal Department of Education the right to publish an edited summary of this report or dissertation using the print or electronic media.
_________________________ **Signature of Applicant(s)** __________________ **Date**

Return a completed form to:
Connie Kehologile – Tel: 033 392 1004/41
Office of the HOD; KwaZulu Natal Department of Education

Hand Delivered:
Office 318; 247 Burger Street; Anton Lembede House; Pietermaritzburg; 3201
Or
Ordinary Mail
Private Bag X9137; Pietermaritzburg; 3200
Or
Email
kehologile.connie@kzndoe.gov.za / Phindile.Duma@kzndoe.gov.za
Or
Fax
033 392 1203

..Championing Quality Education - Creating and Securing a Brighter Future

APPENDIX H: PERMISSION TO CONDUCT RESEARCH IN KZN-DOE

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Enquiries: Phindile Duma	Tel: 033 392 1041	Ref.:2/4/8/1391

Ms PN Mathaba
PO Box 2032
Mtunzini
3867

Dear Ms Mathaba

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"ERRORS AND MISCONCEPTIONS RELATED IN LEARNING OF ALGEBRA IN THE SENIOR PHASE",** in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 01 November 2017 to 09 July 2020.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma at the contact numbers below
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

(List of Schools Attached)

Dr. EV Nzama
Head of Department: Education
Date: 06 November 2017

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KWAZULU-NATAL DEPARTMENT OF EDUCATION
Postal Address: Private Bag X9137 • Pietermaritzburg • 3200 • Republic of South Africa
Physical Address: 247 Burger Street • Anton Lembede Building • Pietermaritzburg • 3201
Tel.: +27 33 392 1004/41 • **Fax.:** +27 033 392 1203• **Email:**Phindile.Duma@kzndoe.gov.za •**Web:**www.kzneducation.gov.za
Facebook: KZNDOE....Twitter: @DBE_KZN....Instagram: kzn_education....Youtube:kzndoe

APPENDIX I: REQUEST LETTER TO PRINCIPALS TO CONDUCT RESEARCH

University of Zululand
P.O. Box x1001
Kwa-dlangezwa
3886
24 July 2017

The Principal
(Address)

Dear Sir/ Madam

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN YOUR SCHOOL

My name is PHILILE NOBUHLE MATHABA and I am currently studying towards a Master’s degree in the Department of Mathematics, Science and Technology Education at the University of Zululand. I would like to conduct research in your school for the purpose of fulfilling the requirement of a full dissertation in Mathematics Education. My research topic is: Errors and Misconception related in learning of Algebra in secondary schools. UMlalazi and Mtunzini schools in KwaZulu-Natal province.
The research will conducted under supervision of Prof.A.Bayaga from the University of Zululand. I am hereby seeking your permission to approach grade 9 Mathematics learners in your schools in order to participate in this research.
Upon completion of the research, I undertake to provide the Department of Basic Education with a copy of the full research report upon completion. Should you require any further information, feel free to contact me on this number 0734171424 email:biyelapn@gmail.com.

Thank you.

Yours sincerely
Philile Nobuhle Mathaba

APPENDIX J: - INFORMED CONSENT DECLARATION

PARENT AND GUARDIAN'S INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Parent or Guardian)

Project Title: **Errors and Misconceptions related to Learning Algebra in the Senior Phase**

PHILILE NOBUHLE MATHABA from the Department of Education, University of Zululand has requested my permission to allow my child/ ward to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to investigate the errors and misconceptions learners display in Algebra, identify mistakes and misconceptions learners have in response to Algebra expressions, as well as to explain how learner Algebra errors link with their misconceptions**.**
2. The University of Zululand has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate.
3. By participating in this research project my child/ward will be contributing towards the benefits on Algebra expression and equations in grade nine learners.
4. My child/ward will participate in the project by writing a test and answering interview questions.
5. My child's/ward's participation is entirely voluntary and if my child/ward is older than seven (7) years, s/he must also agree to participate.

6. Should I or my child/ward at any stage wish to withdraw my child/ward from participating further, we may do so without any negative consequences.

7. My child/ward may be asked to withdraw from the research before it has finished if the researcher or any other appropriate person feels it is in my child's/ward's best interests, or if my child/ward does not follow instructions.

8. Neither my child/ward nor I will be compensated for participating in the research.

9. There may be no risks associated with my child's/ward's participation in the project. Low risks are anticipated.

10. The researcher intends publishing the research results in the form of …………………………………….. However, confidentiality and anonymity of records will be maintained and that my or my child's/ward's name and identity will not be revealed to anyone who has not been involved in the conduct of the research.

11. I will receive feedback in the form of email biyelapn@gmail.com regarding the results obtained during the study.

12. Any further questions that I might have concerning the research or my participation will be answered by Prof .A. Bayaga.

13. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies that I or my child/ward may have.

14. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

I, Have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of my child/ward during the research.

I have not been pressurised in any way to let my child/ward take part. By signing below, I voluntarily agree that my child/ward ……………………………………………………………… (**Insert name of child/ward**), who is …………….. Years old, may participate in the above-mentioned research project.

…………………………………….

…

………………………………….

Parent/Guardian’s signature **Date**

APPENDIX K: PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Participant)

Project Title: **Errors and Misconceptions related to Learning Algebra in the Senior Phase**

From the Department of Education, University of IsiZululand has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. **The purpose of the research project is to investigate the errors and misconceptions learners display in Algebra, identify mistakes and misconceptions learners have in response to Algebra expressions, as well as to explain how learner Algebra errors link with their misconceptions.**

2. The University of Zululand has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate.

3. By participating in this research project I will be contributing towards benefits on Algebra expression and equations in grade nine learners.

4. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.

5. I will not be compensated for participating in the research, but my out-of-pocket expenses will be reimbursed.

6. There may be low risks associated with my participation in the project. I am aware that.

a. Low risks are anticipated.

7. The researcher intends publishing the research results in the form of …………………………………….. However, confidentiality and anonymity of records will be maintained and that my name and identity will not be revealed to anyone who has not been involved in the conduct of the research.

8. I will receive feedback in the form of emails regarding the results obtained during the study.

10. Any further questions that I might have concerning the research or my participation will be answered by Prof .A Bayaga (Supervisor)

11. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.

12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

I, have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

………………………………….

…

……………………………….

Participant's signature **Date**

APPENDIX L: PARTICIPANT INFORMED CONSENT FOR PARENTS (Zulu)

IFOMU LOKUZIBOPHEZELA

(obambe iqhaza)

Isihloko socwaningo:

.... (**Igama lomcwaningi/lomuntu oxhumanise izinsiza zocwaningo**) ovela ku Mnyango ……......................................, inyuvesi yakwaIsiZulu ube nesicelo semvume yokuzibandakanya kulolucwaningo olulotshwe ngenhla.

Imvelaphi kanye nenhloso yalolucwaningo, nalolu lwazi nophawu lokwamukela ukuzibophezela ngichazeliwe ngalo ngolimi engilwaziyo.

Ngiyakuqonda ukuthi:

1. Inhloso yalolucwaningo uku ……………………………….
2. Inyuvesi yakwaIsiZulu inikezele ngemvume kubenzi balolu cwaningo ukuba benze loluhlelo futhi ngiyibonile leyomvume/ngingacela ukubona isitifiketi semvume.
3. Ngokubamba iqhaza kulolucwaningo ngizonikezela iqhaza ngoku ……………………………… **(chaza ubungako obulindelekile noma inzuzo emphakathini noma abantu abangaphumelela ngalolucwaningo).**
4. Ngizobamba iqhaza kulolucwaningo ngoku ……………………………… **(chaza imininingwane ephelele yokuthi ozimbandakanyile uzobe enzani).**
5. Ekuzibandakanyeni kwami angizukubheka nzuzo futhi akukho lapho engizotholakala ngihoxa ocwaningweni, umakwenzeka ngeke kube nemiphumela emibi ocwaningeni.
6. Mina angizukunxephezelwa ngokuzibandakanya kwami kulolucwaningo, kodwa izindleko ephume kwelami iphakethe zizokhokhelwa. **(Uma kukhona isinxephezelo nikeza imininingwane)**.
7. Kuzoba nezimo ezibucayi ekuzibandakanyeni kwami kulolucwaningo , ngiyakuqonda ukuthi: Azikho izimo ezibucayi ezingaba khona.
8. Umphequluli uzoshicilela imiphumela yalolucwaningo ngohlelo lokubhala kwphepha.Nokho, ubhalomfihlo, nofihlo-gama lwemininingwane izobe igciniwe nokuthi igama lami nobutho kwami angeke kubonakaliswe kumona yimuphi umuntu

obengayona inhlangano yocwaningo.

9. Angeke ngiyamukele imiphumela/ngizoyamukela imiphumela engaloluhlelo……………….. emayelana nemiphumela etholakale ngesikhathi sesifundo.

10. Eminye imibuzo ephathelene nalolucwaningo noma mayelana nokuzibandakanya kwami ingaphendulwa ngu …………………………………… **(bhala igama nemininingwane yokuxhumana)**

11. Ngokusayina lamafomu angiqubuli ubuthi noma amalungelo kwezomthetho

12. Ikhophi enolwazi oluphelele nophawu lokwamukela ukuzibophezela kwami ngizonikezwa, bese okungungqo kuyagcinwa.

Mina ……………………………….ngilufundile loku okubhalwe ngenhla/ ngiyavuma ukuthi lolulwazi olungenhla ngichazelwe ngolimi lwami engiluqondayo futhi ngiyakuqonda okuqukethwe nokubhaliwe. Ngiyibuze yonke imibuzo engifunayo ukuyibuza, futhi yaphendulwa ngendlela engenelisayo. Ngiyayiqonda kahle ukuba kulindelekile ini kimi kulolucwaningo.
Angiphoqwanga nakancane ukubamba iqhaza kulokhu kulolucwaningo

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isishicilelo kobambe iqhaza **usuku**

UKUZIBOPHEZELA KOMCWANINGI

Mina
………………………………...ngiyavuma ukuthi

- Ngichazile ulwazi olukuleli bhuku ku

  ……………………………………………………………….
- Ngicelile ukuthi kubuzwe imibuzo uma kukhona la kungaqonakali khona ngizoyiphendula ngobuqotho
- Nginelisekile ukuthi u-------------------------------uzwile indlela lolucwaningo oluzosebenza ngayo, lokhu okumenze wathatha isinqumo sokuthi alibambe yini iqhaza noma cha
- Ingxoxo yennziwa ngesiIsiZulu
- Ngimsebenzisile noma/ angimsebenzisanga utolika

-------------------------------------- **Isishicilelo somcwaningi** **usuku**

APPENDIX M: INTERVIEW SHEET/SCHEDULE

INTERVIEW INFORMATION SHEET

Purpose of the research: To investigate the errors and misconceptions learners display in Algebra, identify mistakes and misconceptions learners have in response to Algebra expressions, as well as to explain how learner Algebra errors link with their misconceptions.

What you will do in this research: Each learner will be asked six questions. All of them will be about errors and misconception of algebraic equations and expressions .With your permission, I will tape record the interviews, I will also take notes with pen and paper. You will not be asked to state your name on the recording.

Time required: The interview will take approximately 20 mins.

Risks: No risks are anticipated.

Benefits: This is a chance for you to tell your story about your experiences concerning **Algebra expressions and equations**.

Compensation: You will receive **R20** in cash at the end of the interview.

Confidentiality: Your responses to interview questions will be kept confidential. At no time will your actual identity be revealed. You will be assigned a random numerical code. Anyone who helps me transcribe responses will only know you by this code. The recording will be destroyed [OR erased] as soon as it has been transcribed, OR when my final paper has been graded, OR when my dissertation has been accepted. The transcript, without your name, will be kept until the research is complete.

The key code linking your name with your number will be kept in a locked file cabinet in a locked office, and no one else will have access to it. It will be destroyed [explain when]. The data you give me will be used for [explain what, i.e., an article I am currently writing] and may be used as the basis for articles or presentations in the future. I won't use your name or information that would identify you in any publications or presentations.

Participation and withdrawal: Your participation in this study is completely voluntary, and you may refuse to participate or withdraw from the study without penalty or loss of benefits to which you may otherwise be entitled. You will receive payment based on the proportion of the study you completed. You may withdraw by informing the experimenter that you no longer wish to participate (no questions will be asked). You may skip any question during the interview, but continue to participate in the rest of the study.

To contact the researcher: If you have questions or concerns about this research, please contact: Philile Nobuhle Mathaba, University of IsiZululand, kwa-Dlangezwa P.O box x1001 1 MAIN ROAD,Vulindela, kwa-Dlangezwa 3886, 0734171424, biyelapn@gmail.com. You may also contact the faculty member supervising this work: Prof.A.Bayaga, email: bayagaA@uniisiZulu.ac.za.

Whom to contact about your rights in this research, for questions, concerns, suggestions, or complaints that are not being addressed by the researcher, or research-related harm: University of Zululand Research Ethics Committee [UZREC], Research & Innovation Office: 035 902 6887 or the Researcher's department/supervisor.

APPENDIX N: CHILD PARTICIPANT'S CONSENT FORM

INFORMED CONSENT DECLARATION

(Child participant)

Project Title: Errors and Misconceptions to Learning Algebra in the Senior Phase

Researcher's name: PHILILE NOBUHLE MATHABA

Name of participant: ……………………………………..

1. Has the researcher explained what s/he will be doing and wants you to do?

YES	NO

2. Has the researcher explained why s/he wants you to take part?

YES	NO

3. Do you understand what the research wants to do

| YES | NO |

4. Do you know if anything good or bad can happen to you during the research?

| YES | NO |

5. Do you know that your name and what you say will be kept a secret from other people?

| YES | NO |

6. Did you ask the researcher any questions about the research?

| YES | NO |

7. Has the researcher answered all your questions?

YES NO

8. Do you understand that you can refuse to participate if you do not want to take part and that nothing will happen to you if you refuse?

YES NO

9. Do you understand that you may pull out of the study at any time if you no longer want to continue?

YES NO

10. Do you know who to talk to if you are worried or have any other questions to ask?

YES NO

11. Has anyone forced or put pressure on you to take part in this research?

YES NO

12. Are you willing to take part in the research?

YES NO

______________________________ ______________________

Signature of Child **Date**

APPENDIX 0: CLEARANCE CERTIFICATE

UNIVERSITY OF ZULULAND
RESEARCH ETHICS COMMITTEE
(Reg No: UZREC 171110-030)

RESEARCH & INNOVATION
Website: http://www.unizulu.ac.za
Private Bag X1001
KwaDlangezwa 3886
Tel: 035 902 6887
Fax: 035 902 6222
Email: MangeleS@unizulu.ac.za

ETHICAL CLEARANCE CERTIFICATE

Certificate Number	UZREC 171110-030 PGM 2017/391							
Project Title	Errors and misconceptions related to learning Algebra in the senior phase							
Principal Researcher/ Investigator	PN Mathaba							
Supervisor and Co-supervisor	Prof A Bayaga							
Department	MSTE							
Faculty	Education							
Type of Risk	Medium risk – Data collection from people							
Nature of Project	Honours/4th Year		Master's	x	Doctoral		Departmental	

The University of Zululand's Research Ethics Committee (UZREC) hereby gives ethical approval in respect of the undertakings contained in the above-mentioned project. The Researcher may therefore commence with data collection as from the date of this Certificate, using the certificate number indicated above.

Special conditions:

(1) This certificate is valid for 2 years from the date of issue.
(2) Principal researcher must provide an annual report to the UZREC in the prescribed format [due date-01 July 2018]
(3) Principal researcher must submit a report at the end of project in respect of ethical compliance.
(4) The UZREC must be informed immediately of any material change in the conditions or undertakings mentioned in the documents that were presented to the meeting.

The UZREC wishes the researcher well in conducting research.

Gde Wet

Professor Gideon De Wet
Chairperson: University Research Ethics Committee
Deputy Vice-Chancellor: Research & Innovation
10 August 2017

CHAIRPERSON
UNIVERSITY OF ZULULAND RESEARCH
ETHICS COMMITTEE (UZREC)
REG NO: UZREC 171110-30

10 -08- 2017

RESEARCH & INNOVATION OFFICE

APPENDIX P: Editor's letters ***Letter A – Editor 1 Nkhensani Maluleka***

Professional EDITORS Guild

Membership No: MAL005

EDITORIAL CERTIFICATE

This document certifies that the Dissertation listed below was edited for proper English language, grammar, punctuation, spelling, and overall style by:

Ms. N Maluleke

MalulekeN@unizulu.ac.za
nmaluleke@gmail.com

Title of the Manuscript:
ERRORS AND MISCONCEPTIONS RELATED TO LEARNING ALGEBRA IN THE SENIOR PHASE –GRADE 9
Author

PHILILE NOBUHLE MATHABA

Editor's Signature	**Date**
	2019. 01. 15

Disclaimer: Track Changes and Comments to be effected by the Client

Letter B: Editor 2: Alexa Barmby

Alexa Barnby

Language Specialist

Editing, copywriting, indexing, formatting, translation

BA Hons Translation Studies; APEd (SATI) Accredited Professional Text Editor, SATI

Mobile: 071 872 1334

Tel: 012 361 6347 alexabarnby@gmail.com

6 April 2019

To whom it may concern

This is to certify that I, Alexa Kirsten Barnby, an English editor accredited by the South African Translators' Institute, have edited the master's dissertation titled "Errors and misconceptions related to learning Algebra in the Senior Phase – Grade 9" by Philile Nobuhle Mathaba.

The onus is, however, on the author to make the changes and address the comments made.

AKBarnby