UNIVERSITY OF ZULULAND

THE EFFECTS OF MANIPULATIVE OBJECTS AS SCAFFOLDERS IN THE CONTEXT OF LEARNING GRADE 11 GEOMETRY

Submitted to the Faculty of Education, in the field of Mathematics, Science and Technology Education (MSTE)

In fulfilment of the requirements for the degree of Master of Education (Mathematics Education)

by

Mbekezeli Absolom Jiyane

Supervisor: Prof. Sibaya D. C.

Co-Supervisor: Mr Chibisa A

January 2020

DECLARATION

I, MBEKEZELI ABSOLOM JIYANE, hereby declare that this dissertation, entitled '**THE EFFECTS OF MANIPULATIVE OBJECTS AS SCAFFOLDERS IN THE CONTEX OF LEARNING GRADE 11 GEOMETRY'** is my own work and that all resources that I have used or quoted have been indicated and acknowledged using complete references. It has been submitted for the degree of Master of Education at the University of Zululand. I have not submitted it to any other university for any degree or examination.

…………………………………………. **Date: 15 JANUARY 2020**

MBEKEZELI ABSOLOM JIYANE

Student number: 201062206

DEDICATION

I dedicate this research to my parents, Ntombi Philisiwe Jiyane (MaNtuli) and late Musa Ozious Jiyane (Mthunzi).

ACKNOWLEDGEMENTS

I appreciate the support that my supervisors, Prof. D.C Sibaya and Mr A Chibisa have given to me. The research journey was not an easy one but through the continuous guidance and motivation that I got from my supervisors, I was able to make it to the finish line. My gratitude also goes to the Faculty of Education at UNIZULU for arranging the workshops which supported postgrad students on the research journey.

My profound thanks go to mama, N.P Jiyane (MaNtuli) and Thulile Gasela, for continuously supporting me during the hard times of the research journey. I acknowledge Gideon Jiyane and Themba C. Jiyane.

I would also like to extend my appreciation to the KZN DBE HOD and Director of Education at King Cetshwayo District for allowing me to conduct this study. Special thanks to Mr S. H Zwane (Ntshosho), H Zulu (Mageba), Khumalo M.S (Khumza), S.K Fihlela (Qabane), Sabelo (Pini) Mbuyazi and learners who volunteered to participate in this study; it would not have been possible to complete this degree without their support.

Finally, I thank the Almighty God, who made it possible for me to start the research journey and finish it.

ABSTRACT

Although the poor performance of learners in geometry has been an issue of concern for mathematics researchers and teachers for decades, a solution to this problem has not yet been found. The majority of South African learners in the Further Education and Training (FET) phase in South Africa are unable to perform at the expected levels of Van Hiele's model geometry thinking, namely, levels 3 and 4. Teaching strategies which promote memorisation as a type of learning, for example the lecture method, tend to hinder learners in reaching the expected levels of the model. Thus, the introduction of learning aids such as geoboards may help both in promoting active learning and in developing the relevant skills which learners require if they are to reach levels 3 (abstraction) and 4 (deduction). Thus, this study investigated the effects of manipulative objects as scaffolders towards the learning of selected geometry theorems in grade 11.

This study was conducted on 24 eleventh grade learners in one of KwaZulu-Natal public schools found in King Cetshwayo District. Van Hiele's theory was adopted to observe, examine and analyse the effects of manipulative objects on problem-solving, learners' engagement time and the extent to which these objects can scaffold geometry learning in reaching levels 3 and 4 of the theory. Level 3 involves the application of geometry theorems while their proofs are done in level 4. Written tests, observations and semi-structured interviews were used to answer the following research questions: Does the incorporation of manipulative objects enable learners to engage in solving geometry problems? What effects do manipulative objects have on learners' engagement time in geometry learning at grade 11? To what extent are manipulative objects able to scaffold learners in reaching level 3 and level 4 of Van Hiele's model of geometry thinking?

During this investigation, learners received instruction with manipulative objects. This study employed a mixed-methods sequential explanatory design. Quantitative data collected were analysed using Microsoft Excel while thematic analysis was used in the analysis of qualitative data.

Findings of this study revealed that manipulative objects enhanced learners' problem-solving skills in geometry such that learners are able to perform at level 3 of Van Hiele's model. The findings also revealed that spending least amount of time on verbal learning actions like asking and answering questions in the class may have negative effects on learners' verbal communication as the majority of learners correctly answered level 3 questions but were unable to explain what they had written. Learners had difficulties with geometry proofs prior to and after the treatment which indicated that manipulative objects were unable to help learners reach level 4.

These findings implied that to improve learners' performance in geometry and help them reach level 3, manipulative objects should be used by mathematics teachers. However, to help learners with geometry proof, this study recommends that further investigation may focus on other forms of manipulatives as manipulative objects were unable to help learners with geometry proofs i.e. virtual manipulatives or manipulative objects which are different from the ones used in this study.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

1.1 Introduction

Geometry is a branch of mathematics that focuses on an axiomatic system to construct representations of spatial objects, relationships and transformations (Clements, 2003). It is one of the branches of mathematics that contains many concepts some of which are found in other branches of mathematics (Rofii, Sunardi, & Irvan, 2018). Apart from the field of mathematics, geometry occupies a special position even in other fields like Science, Geography, Art, Design and Technology (Armah, Cofie, & Okpoti, 2017).Thus geometry is considered an important branch in the mathematics curriculum for the Further Education and Training (FET) phase in South Africa.

In South African education context, the term “geometry” refers to Euclidean geometry. Euclidean geometry is mainly about plain geometry which focuses more on the geometry of line and angles (Larvor, 2019). This postulates-based type of geometry according to Maričić and Stamatović (2018) is used to help learners in the development of logical, critical and analytical thinking. However, all these thinking abilities, are in the South African context, developed through studying circles, straight lines and triangles.

Despite its importance, FET learners still perform poorly in geometry related questions in the grade12 final examination. This is affirmed by findings from a study conducted by Luneta (2015), which indicate that learners are not performing at the expected standard in FET phase. Learners in FET are expected to perform at levels 3 and level 4 of Van Hieles model of geometry thinking, yet the majority perform at levels 1 and 2 (Alex & Mammen, 2016a). The National Senior Certificate (NSC) results for the past five years also affirm this poor performance (Department of Education, 2014, 2015, 2016, 2017, 2018).

The assessment of geometry in grade 12 does not only include the theorems learnt in the grade, but also includes circle geometry theorems which are taught in grade 11 as they are examinable in grade 12 (Department of Education, 2011). As a result, learners may perform poorly in geometry at grade 12 if circle geometry theorems were not well understood in grade 11. Hence, improving learners’ understanding of circle geometry may be one way to improve geometry performance in grade 12. Further, the knowledge of circle geometry theorems is also assessed in analytical geometry as according to Pierce (2014) as cited in Naidoo and Kapofu (2020) analytical geometry involves combining geometry with the coordinate system while using algebraic notations. This may mean that improving learners' understanding of circle geometry will not only improve their performance in geometry but also in analytical geometry. However, to improve learners' performance in circle geometry requires a special kind of learning.

The literature reviewed revealed that manipulative objects may help learners understand geometry (Balinha & Mamede, 2018; Jamhari & Wongkia, 2018). In addition, manipulative objects are able to make learning geometry fun and interesting to learners (Naidoo & Kapofu, 2020; Sibiya, 2019). Thus, this study investigated if learners' thinking and achievement in geometry (circle geometry) could be enhanced through the use of manipulative objects called geoboards. To achieve this, action participatory research with a sequential mixed method explanatory design was conducted in grade 11.

1.2. Literature review

1.2.1 Manipulative objects

There are different types of manipulative objects as they are used differently by different scholars. Some studies use physical manipulatives to refer to manipulative objects, for example, (Baki, Kosa, & Guven, 2011; Rouinfar, Madsen, Hoang, Puntambekar, & Rebello, 2012; West, 2011) while others used the concrete materials or models to refer to manipulative objects i.e.(Bayram, 2004; Clements & McMillen, 1996; Hall, 1998). Bayram (2004) defined concrete models as materials that can be moved around or manipulated by students. In this study, physical manipulatives, concrete models and manipulative materials are treated the same as manipulative objects.

The use of manipulative objects is of great value in the world of mathematics. According to Yeatts (1991), manipulative objects provide learners with concrete representations of abstract mathematical ideas and manipulative objects may be used in all branches of mathematics. For example, an abstract mathematical idea shown in the textbook may be concretely represented through manipulative objects. In this regard, complicated mathematical ideas may be visualised through the use of manipulative objects.

Where there is a little or no connection between concrete and abstract mathematical ideas, manipulative objects may mediate this disconnection (Kamina & Iyer, 2009). As noted in Bruner (1966), that knowledge can either be represented concretely, pictorially and abstractly. Thus, according to Kamina and Iyer (2009), manipulative objects may help learners progress from concrete to pictorial and abstract representation.

Concrete manipulatives are also useful in cases where a mathematical concept was wrongly understood or was never understood (Yeatts, 1991). In this way, concrete manipulatives allow learners who did not understand means to rebuild their conceptual foundation. Concrete manipulatives also play an important role in providing learners with an opportunity to experiment, thus enhancing their problem-solving skills (Bayram, 2004). However, some concrete manipulatives which are not properly designed can hinder learners' learning, hence, there must be a proper consideration to concrete learning aids to make sure that they do not prevent learners from constructing their mathematical knowledge (Hall, 1998).

In mathematics, different manipulative objects are used with the different topics. Some examples of how manipulative objects are used in mathematics include: algebra tiles which are used to enhance learners' understanding of the addition and subtraction of polynomials in algebra (Castro, 2017; Padmore, 2017); base ten blocks which are used to help learners with practical exploration of numbers (Bouck, Satsangi, Doughty, & Courtney, 2014); fraction pieces which can be used to explore fractions and geoboards which are used to explore two dimensional geometric shapes, area and perimeter, angles (Johnson, 2018). However, this study used geoboards since its focus was on circle geometry theorems which are mainly about angles, lines and two dimensional geometric shapes.

In geometry learning, activities involving manipulative objects like geoboards give learners more time to engage in both mental and physical learning, by practising spatial reasoning through visualisation (Bayram, 2004). The development of spatial reasoning is important in geometry as it enhances learners problem-solving skills (In'am, 2016). However, using geoboards with large groups may not always yield good results because it may not be easy to manage such classes (Mildenhall, Swan, Northcote, & Marshall, 2008).

According to Carroll (1992), geoboards may be used to engage primary learners in an investigational and problem-solving type of learning in geometry. In line with this, Balinha and Mamede (2018) found that the concepts of area and perimeter are easily understood by primary school learners when taught with geoboards. They further noted that geoboards may provide learners with spatial visualisation of geometric shapes and their properties. Geoboards in this respect, enhance learners' problem-solving abilities.

Geoboards may be used in the learning of different mathematical topics. Scandrett (2008) recommended that geoboards be used together with isometric dots to facilitate harmonious progression from concrete to abstract mathematical representations. For example, when learners are learning about translation and reflection in transformation geometry, the combination of isometric dots and geoboards may enable learners to transfer what has been represented on the geoboard to isometric dots drawing. In this way, geoboards facilitate representations of mathematical ideas from concrete to pictorial.

In spite of the pre-mentioned benefits of using geoboards in geometry learning some of which include promoting active learning and enhancement of visualisation and problem-solving abilities (Furner & Marinas, 2011; Scandrett, 2008; Sibiya, 2019), geoboards were also proposed considering the South African educational situation in terms of technological developments. As noted in Lim, Zhao, Tondeur, Chai, and Tsai (2013) that the implementation of computer-based learning support materials in many developing country such as South Africa is still a problem since some schools still do not have computers to implement software-based kind of interventions let alone the issue of internet connection. Hence, the use of geoboards may particularly be of great importance in South African rural schools as according to Mushipe (2016) they are the ones who are mostly affected by the unavailability of prerequisites for computer-based learning support materials.

The proposed study was in support of the assertion made by Scandrett (2008) that the use of computer-based learning support materials has made teachers forget the potential that geoboards have to mathematical learning. She argues that geoboards should be used in conjunction with computer-based learning support materials as they are meant to complement and not replace each other's use. Computer-based learning support materials provide learners with an onscreen manipulation of geometry objects while geoboards help learners to concretely manipulate geometry objects while acquiring what she calls hand-on experience. As a result, she believes that geoboards must be used throughout the grades.

1.2.2. Problem-solving

Polya (1962) categorised mathematical problems into two types namely: "*to find*" and "*to prove*" types of problems. According to Polya, being able to classify a given problem enables learners to devise an appropriate solving approach. Each type of problem has got different cognitive demands with different approaches to finding solutions (Polya, 1962). On the one hand, the "*to find*" type of problems are those, for example, where learners are expected to apply the knowledge of geometry theorems in finding the size of missing angles. These types of problem fall in level 3 of Van Hiele's model of geometry thinking (Alex & Mammen, 2016a). On the other hand, the "*to prove*" type of problems may include performing a proof of certain geometry theorem and such questions belong to level 4 of Van Hiele's model of geometry thinking. The geometry questions in the eleventh grade are mostly characterised by these types of questions. Thus, learners need to understand the problem first before attempting to find a solution as failing to understand the problem may hinder them from solving the problem (Saleh, Prahmana, & Isa, 2018).

In the learning of geometry particularly, solving some problems may help learners to solve other problems and according to Polya (1962, p. 36), such problems are called auxiliary problems "means to an end". This includes finding the size of the third angle in a triangle given the size of the second angle and that the triangle is a right-angled one. To this end, being able to see 90^0 as one of the interior angles of a triangle may serve as an auxiliary problem towards finding the size of the third angle, while the opposite may hinder progress.

Understanding the problem involves critically analysing the given information which helps in devising appropriate ways in which given information can be used to find the required information (Saleh et al., 2018). According to Carifio (2015), devising appropriate ways of solving a problem involves arranging plans, carrying out the plans and looking back. It is believed that these steps may enable learners to successfully solve mathematical problems.

Mathematics, according to South African curriculum CAPS, ought to develop in learners' minds abilities that enable them to think critically, creatively, logically, analytically, and systematically. According to Maričić and Stamatović (2018), logical, critical and analytical thinking is mainly developed through the learning of geometry. However without problem-solving abilities, all these skills may be difficult to develop (Maulyda, Hidayati, Rosyidah, & Nurmawanti, 2019). As a result of this, developing problem-solving abilities is of paramount importance in mathematical learning in South Africa.

For learners to be able to solve any mathematical problem, Polya (1962) emphasized the importance of problem-solving which is what he calls a mental action that enables learners to solve mathematical problems. Poly's mathematical problem solving is characterised by mental actions which include mobilization, organization, isolation and combination (Carifio, 2015). According to Polya (1962), these activities complement each other i.e. mobilization complements organization and isolation complements combination. The mental actions are necessary for mathematical problem-solving which is an essential part for learners to solve mathematical problems. Furthermore, each of the mental actions is based on the intermediate mental actions namely: relevancy, proximity, and quality (Polya, 1962).

Mobilization and Organization

Mobilization involves understanding a problem and recall relevant facts which will be helpful in further analysis of the problem and this action may result in a better understanding of a problem (Polya, 1962). It is after recalling relevant facts about a problem that such facts are linked to a problem and linking facts to the problem is what Polya calls organization. Hence, learning support materials which allow learners to recall and link relevant facts to the given problem may be needed to enhance these mental actions.

Isolation and Combination

Isolation is according to Polya a process of singling out an element in a group of elements while combination is the opposite which is an assembling process to form a meaningful whole (Polya, 1962). A mental ability to break down a geometry rider to its constituent theorems and assemble it may illustrate isolation and combination processes. Polya believes that true problem solving is characterised by reaching all mental actions.

Rofii et al. (2018) argue that the development of problem-solving in geometry particularly may enable learners to critically analyse and reason logically to make informed mathematical decisions. To make an informed mathematical decision in this respect is to find a correct solution to the given mathematical problem. In the same sense, Karimah, Kusmayadi, and Pramudya (2018) believe that learning based on problem-solving connects what is learnt inside the classroom with what is happening outside the classroom thus preparing learners for informed decision making in real life.

Problem-solving abilities may be provoked through the activities given to learners (Polya, 1962). Polya believes that activities with no routine algorithmic way of finding a solution to the given mathematical problem provoke learners' problem-solving abilities. This includes activities derived from or linked to real-life context (Schoenfeld, 1987). However, the development and use of such activities in the classroom is mainly dependent on teachers' willingness to employ different instructional methods which are in line with problem-solving as according to Maulyda et al. (2019), it is difficult for problem-solving abilities to be developed in learners' minds if teachers do not provide such opportunities. To this end, it seems that teachers are the ones who are responsible for the lack of problems-solving abilities in mathematical learning by not giving activities which allow learners to be engaged in problem-solving.

Teachers must also be good at problem-solving for them to use instructional methods which allow learners to be engaged in mathematical problem-solving (Maulyda et al., 2019). This means that teachers themselves should have problem-solving abilities as they cannot be expected to impart such abilities to learners if they do not have those abilities. It is thus, after having problem-solving abilities that teachers will be able to use instructional methods which promote problem-solving.

Research evidence reveals that instructional methods based on learning support materials promote problem-solving in mathematical learning (Chimuka & Ugorji, 2016; Özsoy & Ataman, 2017). This includes learning support materials like manipulatives.

Making use of both physical and virtual manipulatives in mathematical learning may enhance learners' problem-solving abilities (Chimuka & Ugorji, 2016; Karatas & Baki, 2017). In an attempt to enhance and provoke learners' problem-solving abilities, this study proposed the introduction of manipulative objects in learning geometry as according to Rofii et al. (2018), activities that are based on geometry may provoke learners' problem-solving abilities.

Polya (1962) emphasized the principle of action learning to enhance and provoke mental actions needed for mathematical problem-solving. Action learning according to Zuber-Skenitt (1993) is mainly about learning by doing which may either include written activities or practical exploration of mathematical ideas. In the learning of geometry, in particular, research evidence shows that manipulative objects promote action learning which allows problem-solving abilities to be provoked and enhanced in learners' minds (Balinha & Mamede, 2018; Jojo, 2017). It is not clear though as to how in terms of mobilization, organization, isolation and combination do manipulative objects enhance learners' problem-solving abilities.

According to Yeatts (1991), manipulative objects are essential tools for enhancing learners' problem-solving abilities since they promote active and interactive learning. This allows learners to learn from their actions as they interact with the subject matter through the use of manipulative objects. In doing so, Jojo (2017) argues that problem-solving abilities are enhanced in learners' minds.

Bakker, Smit, and Wegerif (2015) argued that manipulative objects may help to foster learners' creative thinking and problem-solving, such that learners pose and solve problems on their own, and through their feedback from their actions, learners evaluate their solutions. This implies that manipulative objects may have a positive impact on learners' problem-solving.

Manipulative objects provide learners with hands-on experience, which is a key component in allowing students to better understand how numerical symbols and abstract equations operate at a tangible level (Anstrom, 2006). According to Wisniewski and Smith (2002), hands-on mathematical activities are important in assisting learners with disabilities, since these activities allow different learning styles to be accommodated and thus enhance their problem-solving skills. Similarly, Saleh et al. (2018) assert that knowledge acquired through experience stays longer in learners' minds compared to the knowledge acquired through listening or reading. Hence, this study proposed the introduction of manipulative objects in enhancing learners' problem-solving abilities to improve performance in geometry.

1.2.3. Engagement

Manipulative objects may enhance learners' engagement in mathematical learning according to Cockett and Kilgour (2015), however, the period engagement is an issue in question. Greenwood (1991) came up with what he called engagement time which he defined as the amount of time which learners spent being engaged in meaningful learning. Engagement time may have different effects on learning. Zakszeski, Hojnoski, and Wood (2017) noted that learners' engagement time may affect their achievement such that, learners may not perform well if they spent a short period of time being engaged in meaningful learning.

Studies have been conducted on the effects of manipulative objects on learners' engagement, for example, Sibiya (2019), Sibiya and Mudaly (2018), Balinha and Mamede (2018) and Jamhari and Wongkia (2018). These studies generally report that manipulative objects positively affect learners' engagement, however, it seems that less has been reported about the effects of manipulative objects on learners' engagement time, particularly in geometry. Some studies, for example, (Greenwood, 1991; Spanjers, Burns, & Wagner, 2008; Zakszeski et al., 2017) have investigated learners' engagement time in mathematics but they have not specifically focussed on geometry. Thus, this study focused on the effects that manipulative objects have on learners' engagement time when learning geometry.

1.2.4. Scaffolding

Scaffolds are frameworks used in engineering and construction, which give support to walls in the building process until they get strong enough to hold on their own. Scaffolding was first introduced in the process of teaching and learning when mothers were helping their children to learn the language and play games (Bruner, 1975, 1976). In the formal process of teaching and learning, the term scaffolding refers to the support given by teachers to learners, up until learners reach the required level of mastery, and, as learners reach mastery, the support is slowly withdrawn (Ozmantar & Roper, 2004).

In the context of teaching and learning, scaffolding is an important tool for meeting the challenging needs of heterogeneous groups of learners in inclusive classrooms as it is especially useful when supporting low achievers (Pfister, Opitz, & Pauli, 2015). Of paramount important to mathematical education particularly, is to provide scaffolding support which accommodate different learning style as such according to Solvie and Kloek (2007) may help learners understand the subject matter.

Educational scaffolding is mainly based on the instructional supports given to learners so that they reach their full potential. This instructional support may either be teachers' explanations which involves defining terms or teachers' use of manipulatives which specially designed to help learners master what they ought to master (Ferguson & McDonough, 2010).

Vygotsky (1981) developed the concept of the Zone of Proximal Development (ZPD) as a way of measuring an individual's intelligence. According to Burns and de Silva Joyce (2005), Vygotsky's key principles with regards to learning experiences are based on the interaction with others. In the interaction process, language is carefully considered, since this involves learning as being mediated by knowledgeable people supporting less knowledgeable people. This involves knowledgeable people coming down to the level of less knowledgeable people as according to Usman, Yew, and Salleh (2018) learning success may not be reached if the language used by knowledgeable people is beyond the level at which less knowledgeable people are operating. In this respect, it is believed that learners can achieve more when they are supported than when they are alone. Finally, after learners have acquired the relevant skills, the support will be gradually withdrawn.

Different learning theories used in mathematical education view learning as being characterised by levels or stages i.e. Van Hiele's model and Bruner's registers of learning which are Concrete, Pictorial and Abstract (CPA). However, each theory emphasizes the use of scaffolding in helping learners progress from one level to the next. Van Hiele's model asserts that teachers' explanations together with the use of manipulative objects in learning geometry may help in providing learners with support which will enable them to progress through levels (Crowley, 1987). In line with this, Bruner (1966) also emphasized the use of manipulative objects in helping learners to move from concrete to pictorial and pictorial to abstract knowledge representations. The use of manipulative objects according to Ferguson and McDonough (2010) is covered under teachers' use of manipulatives type of scaffolding. In this respect, the scaffolding support from manipulative objects can act as a bridge in helping learners to progress from one level or register to another (Maulyda et al., 2019).

Alex and Mammen (2016a) noted that South African learners in grade 11 are unable to reach levels 3 and 4 of Van Hiele's model of geometry thinking. According to Scandrett (2008), teachers' use of manipulatives type of scaffolding like the introduction geoboards may, however, be used to support learners' geometry learning such that they can reach mastery level. There is much literature about scaffolding in the general field of mathematics education, but less has been investigated about scaffolding in geometry, which is one of the worst-performing branches of mathematics in high schools in South Africa. The interconnectedness of scaffolding and the use of manipulative objects in geometry is generally ignored.

1.2.5. Challenges in learning geometry

The general source of errors and difficulties in learning geometry emanate from the lack of background knowledge, reasoning and basic operation mistakes (Özerem, 2012). Meanwhile, South African learners, in particular, were found to lack conceptual understanding of the basics needed in geometry, and consequently learners perform poorly in geometry related question (Luneta, 2015). According to the NSC report, some South African grade 12 learners leave blank spaces in the examination instead of answering geometry related questions (Department of Education, 2018).

Like any other mathematical topic, learning of geometry requires a sufficient understanding of basics like symbolic representations and language (Chilton, 2014). How language is used in mathematical learning may affect learners' reasoning, argumentations and proofs (Planas, Morgan, & Schütte, 2018). For example, it may be difficult to understand and apply the theorem which involves angles and line segments if learners do not understand their symbolic representations and use. In this respect, understanding of symbols and language should be taken into account to help learners understand the basics needed in geometry,

Lack of geometry vocabulary may, on the one hand, result in the inability to define geometry terms which may hinder learners from understating and properly using geometry language and symbols (Makhubele, 2014). For example, the definitions of terms like tangent, chord and alternate segment are key components in understanding the theorem which says: The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. On the other hand, understanding terms like tangents and chords may require a specialized type of learning which enables learners to visualise such terms like geoboards (Sibiya & Mudaly, 2018).

Geometry proofs

Poor performance on geometry proofs may be due to many factors such as understanding the proof itself. One may not be expected to perform geometry proofs without understanding the language used in the proof. As stated in Lin and Yang (2007), a lack of understanding in the language used in geometry proofs may hinder one from understanding the entire proof; as a result, learners fail to infer knowledge gained from other contexts.

In Van Hiele's levels, learners may not progress from one level to the next if the language used by either a teacher or text written in the proof is unfamiliar to learners (Usman, Yew, & Salleh, 2018). Even the question, in the examination may be difficult for a learner to answer, not because they do not know, but because of the language used if it is not familiar to them. Thus, it may not be enough to conclude from the end product of what learners have written that they are poorly performing in geometry proofs without taking language used into account.

The role of language in geometry proofs is essential such that one may not be able to even interpret and evaluate a written proof (Mejia-Ramos, Fuller, Weber, Rhoads, &

Samkoff, 2012). In this regard, learners may fail to see how each element of a proof is related to another. For example, proving a certain geometry theorem may involve more than one theorem being connected to support the proof in question. Hence, if there is a language lack, a learner may not see how each theorem is connected to the proof in question. Hence, failing to interpret may also cause a student to fail to validate whether or not the proofs are correct (Yang & Lin, 2008).

Constructions form part of the validation of geometry proofs. Despite drawings of the construction, Janičić (2010) argues that a learner may encounter difficulties in explaining how they have done their construction if the language of construction is poorly understood. In this way, it does not matter whether the construction was done on the computer or using the pen and paper, the issue of language remains crucial. Hence, the lack of understanding of construction language may hinder learners from performing geometry proofs.

The role of language in geometry is important. Now that technology has impacted how teaching and learning is taking place, more studies i.e. (Chimuka & Ugorji, 2016; Dockendorff & Solar, 2018; Olsson, 2017) have been done on using technology to improve learners' understanding of proofs. However, such interventions may not work if learners have not understood the language. This is because the learning tools also use the language of geometry, hence, without a proper understanding of language in geometry proofs, learning progress may be hindered.

It is difficult to judge whether one understood what they ought to understand in mathematics without considering the impact of language (Riccomini, Smith, Hughes, & Fries, 2015). Even something that one knows may be difficult for them if expressed in the language they do not understand. For example, expressing 1+1 in Chinese language may hinder a grade 11 learner from getting an answer if they do not understand the Chinese language. The same may apply regarding geometry proof; learners may struggle with answering a question based on the proofs if the language used in the question is not familiar to them (Luneta, 2015).

A study by Yang and Lin (2008) has reported about a model of assisting learners to understand geometry proof which they call a model of reading comprehension of geometry proof. In their model, they highlighted the importance of language in the process of understanding geometry proof. They believe that understanding geometry proof may require one to understand the meaning, logical status and logical chaining of its statements. To expand the understanding of the model, Mejia-Ramos et al. (2012) conducted a study focussing on how each of the three components of the model can be assessed in the number theory where they also found that it is difficult to reach success in learning without a proper understanding of language.

1.2.6 Theoretical framework

1.2.6.1 Van Hiele's theory

Van Hiele's theory is a theory used in geometry (Mason, 2009). The theory was originally developed in 1957 by mathematics teachers, Dina Van Hiele-Geldof and Pierre Van Hiele. It explains how learners learn geometry and associates the learning of geometry with levels (Alex & Mammen, 2016). The theory is based on certain assumptions, namely, the level of difficulty in the geometry content increases as the levels of the theory progress; the same language is not used in the various levels and learners may not skip levels (Van Hiele, 1984). In other words, the language used in level 1 is simpler than the language used in the higher levels, i.e. levels 2, 3, 4 and 5. Thus, the language used by teachers when teaching should be at the level of learners because, if not, the learners may not progress to the next level. To this end, the language used by a teacher may either hinder or help the learners' progress.

Van Hiele believed that the learning of geometry should be associated with the levels of geometrical thinking, namely, levels 1, 2, 3, 4, and 5 (Clements, 2003).

Level 1 – the visualisation stage

Learners recognise the figure holistically. This means that the geometrical figure is defined according to its appearance (Van Hiele, 1959).

Level 2 – Analysis

At this level, learners may begin to define a figure using its class properties, for example recognising a square and a rhombus as figures that belong to the same class, namely, a parallelogram (Van Hiele, 1984).

Level 3 – Abstraction

At this level, a complete definition of figures according to their properties is done, thus allowing learners to formulate informal types of arguments (Van Hiele, 1984). Learners at this level may be able to define a figure in a more specific way, for example defining a square as a rectangle with all the angles and sides equal. Many South Africa learners are unable to perform at this level (Alex & Mammen, 2016).

Level 4 – Deduction

Learners at this stage use formal arguments to prove their conjectures and without memorisation (Van Hiele, 1984). This may include performing proofs and their converses – something which has been a problem for many learners in both primary school and high school (Alex & Mammen, 2016; Jamhari & Wongkia, 2018).

Level 5 – Rigour

According to Van Hiele (1959), this type of geometry includes non-Euclidean geometry content and, thus, it was not included in this study.

This study focuses more on levels 3 and level 4 of Van Hiele's geometry thinking. This is motivated by the observation that learners at FET are unable to reach these levels (Alex & Mammen, 2016a). Thus, this study investigates the effects of manipulative objects in helping learners reach levels 3 and 4 of the theory.

In addition to levels, progressing from one level to the next involves five phases namely: information, guided orientation, explication, free orientation and integration (Fuys, Geddes, & Tischler, 1988). The phases are lesson specific which means that these are things that teachers need to consider when planning geometry lessons so that learners will be able to progress from one level to the next.

Van Hiele's levels have been associated with the concrete, pictorial and abstract by (Bayram, 2004). In this respect, the levels of Van Hiele's were categorised as follows: level 0 - concrete, levels 1 and 2 - visual geometric structures, and levels 3 and 4 abstract structures, which refers to the idea of concrete, pictorial, then abstract approach.

Instructional methods which are based on Van Hiele's theory yield positive results (Alex & Mammen, 2016b; Erdogan, Akkaya, & Celebi Akkaya, 2009). The theory helps teachers to understand their learners' operational levels which in turn help them to give appropriate support (Luneta, 2015). In addition to learners' operational levels, the theory can also help teachers to be aware of the language they use to teach learners as using the language which is beyond learners' operational levels may hinder learning progress. Importantly, Van Hiele's theory was specifically developed for geometry, hence an appropriate theory to guide and interpret the results of this study.

The concrete, pictorial, then abstract approach is named differently by different scholars: Dorier, Gutiérrez, and Strässer (2003, p. 2) called concrete, pictorial, and abstract registers of representation, which are categorised as: material representation (in paper, cardboard, wood, plaster, etc.), a drawing (made either with pencils on a sheet of paper, or on a computer screen, with use of a geometric software, etc.), and a discursive representation (a description with words using a mixture of natural and formal languages). Learners should be able to move from one register to the other and also be able to move backwards and forwards.

Learners learn geometry in different ways. According to Bossé and Adu-Gyamfi (2011), students learn best when their learning is associated with communication, technology, multiple representation, and collaboration, as well as real-world examples. Furthermore, manipulative objects may be used to enhance learners' multiple representations (Balinha & Mamede, 2018; Cockett & Kilgour, 2015; Sibiya, 2019). Using Van Hiele's theory, this study utilised manipulative objects (circle geometry geoboards) to support learners in moving from one level to the next level in learning selected circle geometry theorems. This study investigated the effects of manipulative objects at the FET phase with a specific reference to grade 11. Since the study was conducted with grade 11 learners, it focussed on levels 3 and 4, as according to Alex and Mammen (2016a), geometry done in grade

11 ranges between levels 3 and 4 of Van Hiele's model. They further stated that, grade 11 learners in South Africa are not able to reach levels 3 and 4. Thus, this study investigated the effects of manipulative objects in helping learners reach levels 3 and 4.

1.3. Problem statement

In spite of interventions made to help learners to understand geometry, majority of South African learners at the FET phase are still unable to reach levels 3 and 4 of Van Hiele's model in geometry even though it is expected of them (Luneta, 2015; Makhubele, 2014; Ramlan, 2016). There has been an ongoing concern regarding learners' inability to properly apply and prove geometric theorems as observed in the NSC diagnostic reports which are released on yearly basis based on how grade 12 learners have answered certain questions on their end of the year examinations (Department of Education, 2014, 2015, 2016, 2017, 2018). Poor reasoning and leaving certain questions in the geometry section unanswered particularly those in the Van Hiele's levels 3 and 4 is common in all the reports. This indicates lack of problem-solving abilities which are of paramount importance for learners to progress through the levels of Van Hielie's model of geometry thinking (Sulistiowati, Herman, & Jupri, 2019). This lack may however be due to the instructional methods used to teach the topic, that is, spending large amount of learning time on route learning, which does not allow learners to be actively engaged in their learning while promoting problem-solving.

Given the benefits of using geoboards some of which include the enhancement of problem-solving and visualisation abilities (Balinha & Mamede, 2018; Carroll, 1992; Furner & Marinas, 2011; Sibiya & Mudaly, 2018), this study sought to investigate the effects of these objects in the improvement of learners' performance in geometry. An improvement in geometry performance may yield improved performance in mathematics, since the major cause of poor performance in mathematics stems from geometry (Cassim, 2006).

1.4. Aim

This study aimed to investigate the effects of manipulative objects (geoboards) as scaffolders in the context of learning grade 11 geometry.

1.5. Objectives of the study

The specific objectives of the study were to:

1. Engage grade 11 learners in solving geometry problems through the use of manipulative objects.
2. Explore the effect of manipulative objects on learners' engagement time in learning of circle geometry at the eleventh grade.
3. Investigate the extent to which manipulative objects are able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking.

1.6. Research questions

The research questions for this study were as follows:

1. Does the incorporation of manipulative objects enable learners to engage in solving geometry problems?
2. What effects do manipulative objects have on learners' engagement time in geometry learning at grade 11?
3. To what extent are manipulative objects able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking?

1.7. Intended contribution of knowledge

The researcher hopes that the use of manipulative objects may help learners to better understand Euclidean geometry. In this regard, this study may inform different stakeholders at the macro level, i.e., when curriculum is developed at national level, as well as at the implementation stage which is at school level (meso level) and classroom level (micro level) about the use of manipulative objects in the learning of geometry in the FET phase.

1.8. Research methodology

1.8.1. Research design

This study is a participatory action research. Participatory action research is research which seeks answers to questions facing the researchers in their daily practice (Delport, De Vos, Fouche, & Strydom, 2005). Participatory action research is based on the notion of action learning (Zuber-Skenitt, 1993).

This study employed a mixed-method sequential explanatory design. It involved collecting and analysing data from both quantitative and qualitative with respect to the priority, sequence and stage of integration (Subedi, 2016).

Priority

This study employed three methods of data collection, namely: test, observations and semi-structured interviews. Two out of three methods which are tests and observations were used to collected quantitative data. Although the research questions of this study are overarching as advocated in Kroll and Neri (2009), the major part of them is addressed quantitatively, thus the study prioritised more quantitative over qualitative data.

Sequence

The mixed-methods sequential explanatory design sequence implies that the collection and analysis of data have to be done quantitatively first followed by qualitative later (Ivankova, Creswell, & Stick, 2006). As a result, interviews conducted later helped to explain and verify what was observed during the observations and tests conducted earlier and providing more in-depth learners' views about the effect of manipulative objects in the learning of geometry.

Integration

The findings from quantitative and qualitative measures were integrated at the interpretational stage of the study. As noted in Ivankova et al. (2006), the integration or mixing of quantitative and qualitative may happen in the beginning stages of the study at a point where the research questions from both quantitative and qualitative parts are formulated.

1.8.2. Population

Since this study is a participatory action research, the sample comprised grade 11 learners that were enrolled in the mathematics and science stream at a school where the research was conducted. Learners were told in the beginning that their participation was voluntary; if they did not feel like participating, they are free to do so. This study focussed on circle geometry theorems. The researcher chose the grade 11 class because more circle geometry theorems are introduced in this grade. In this study, physical manipulatives were used to assist learners to better understand circle geometry theorems.

1.8.3. Research instruments

Written tests together with observation sheets were used to collect quantitative data, while qualitative data was collected through semi-structured interviews. Written tests in this respect were used to answer research questions 1 and 3. This means that the effects of manipulative objects on geometry problem-solving and Van Hiele's levels were investigated using written tests. Written tests were Pre-and Post-tests. In order to find out if there was an improvement on learners' performance after the introduction of manipulative objects, learners wrote a pre-test, then a post-test was written after the introduction of manipulative objects to find out if there was any improvement. Thus, it is for this reason that this study employed pre-and post-tests.

Observation sheets were used to answer research question 2: What effects do manipulative objects have on learners' engagement time in geometry learning at grade 11? Special observation sheets called interval time sampling were used. These sheets enabled the researcher to observe learners' actions in every 15-seconds time interval. This was done with an aid of videotaping camera. A group of eight learners were videotaped during the intervention, later the video clips were then watched by observers so as to code learners' actions in every 15-seconds time interval. Thus, to estimate learners' engagement time, the researcher counted the number of 15-second intervals learners spent being engaged. Observations are good data collection methods, particularly when observing how individuals behave under certain situations (De Wet, Monteith, Venter, & Steyn, 1981). In this study, observation sheets were used to collect data about how learners react to the subject matter when learning geometry through manipulative objects.

Semi-structured interviews were used to get learners' views about learning geometry with the aid of manipulative objects. Semi-structured interviews allow the researcher to probe and explain to interviewees where certain questions are not properly understood (Braynard, Hanekom, & Brynard, 2015). Since participants in this study were learners, it was possible that they might not interpret questions found in the interview schedule in a correct manner, thus employing semi-structured interviews was appropriate for probing and explaining where necessary in this study.

1.8.4. Data analysis

Microsoft Excel was used to analyse quantitative data collected using written tests and observation sheets. In this regard, Microsoft frequency tables and the pie charts were used to present quantitative data. On the other hand, the thematic analysis was used to analyse qualitative data collected from semi-structured interviews.

1.9. Ethical and safety issues

The researcher read and understood the University of Zululand’s Policy and Procedures on Research Ethics and the University’s Policy and Procedures on Managing and Preventing Acts of Plagiarism. Hence, to ensure that the present study would not violate human rights, the following were considered before the data were collected.

The researcher first got permission to collect from the KwaZulu-Natal Department of Education, thereafter the ethical clearance was obtained from the University of Zululand. After this, permission to conduct the study was obtained from the principal of King Cetshwayo High School, where the research was conducted. Since participants of the study were learners, the data collection process did not take place without the parents’ permission, allowing their children to participate in the study. Participants were told that their participation was voluntary and that they have a right to withdraw should they feel uncomfortable. Anonymity was considered and maintained.

1.10. Operational definition of terms

Manipulative objects

Manipulative objects, as described by Yeatts (1991, p. 3), are "objects or things that appeal to several of the senses and can be touched, moved and handled by learners". However, manipulative objects in this study are tangible wooden made circle geometric geoboards which are used to help learners to better visualise more complicated or abstract structures (geometry riders in this case).

Geometry in this study, refers to circle geometry which is studied in grade 11, as stated in the FET mathematics curriculum policy document (Department of Education, 2011, p. 14).

Arc - a portion of the circumference of a circle.

Chord - a straight line joining the ends of an arc.

Circumference - the perimeter or boundary line of a circle.

Radius (r**)** - any straight line from the centre of the circle to a point on the circumference.

Diameter - a special chord that passes through the centre of the circle. A diameter is a straight line segment from one point on the circumference to another point on the circumference that passes through the centre of the circle.

Segment - part of the circle that is cut off by a chord. A chord divides a circle into two segments.

Tangent - a straight line that makes contact with a circle at only one point on the circumference.

Figure 1.1 together with all definitions linked to it were taken from grade 11 Siyavula mathematics textbook for learners.

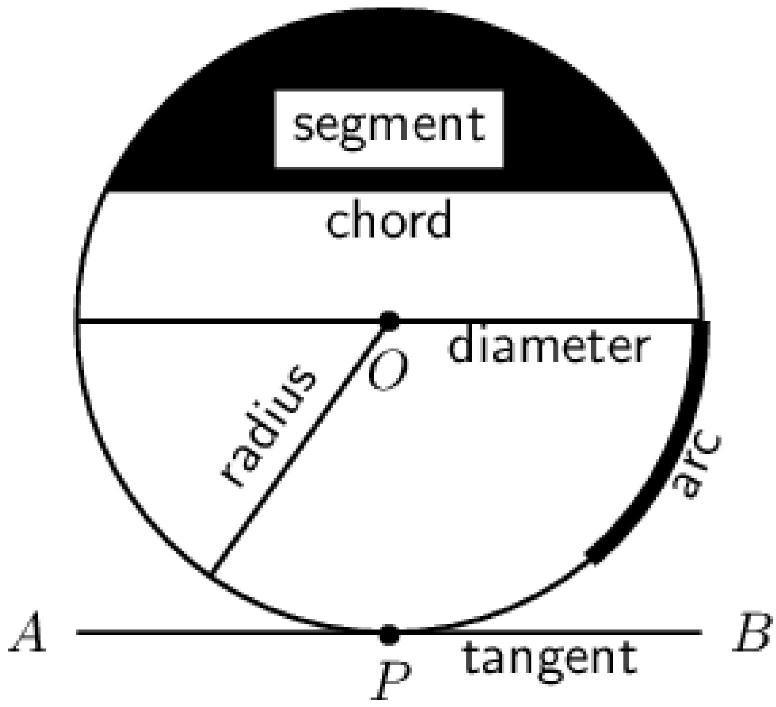


Figure1.1: Parts of the circle

Scaffolding

According to Ferguson and McDonough (2010), scaffolding is educational support which can come from either a person or manipulatives given to people in need to help them reach their full potential. This support is withdrawn as soon as people supported reach their full potential. The manipulatives-based scaffolding is the one used in this study. This study uses a special type of manipulatives called geoboards to help learners in the learning of geometry to reach their level of mastery.

Engagement

Kahu and Nelson (2018) defied engagement as one's psychosocial state which can either be their behavioural, emotional and cognitive connection to their learning. Behavioural engagement includes learners' classroom participation, on-task behaviour, and academically-oriented extracurricular activities; emotional activities such as anxiety, anger and enjoyment are catered under emotional engagement and cognitive engagement includes factors of learning such as thinking and problem-solving strategies (Kong, Wong, & Lam, 2003).

However, the scope of this study is only limited to learners' behavioural engagement which covers learners' classroom participation and on-task behaviours of both passive and active engagements. To this end, learners were regarded as engaged if they did any of the following observable learning actions: Listening attentively; Asking a question; Answering a question; Using geoboard; Doing exercise; Seeking for assistance and Assisting other learners.

Disengaged

The opposite of engagement is disengagement, which means that absence of all the observable learning actions mentioned under engagement results in disengagement. In this study, disengagement refers to all non-lesson related observable actions which include playing, laughing and making noise during the teaching and learning of geometry.

Engagement time

Carroll (1989) defined engagement as the potion of time allocated for learning which was spent by learners acquiring necessary skills intended for a particular topic. This includes time spent on actions related to what the learning was about. The engagement time in this study refers to the amount of time that learners spent being engaged in meaningful learning of geometry which includes all the observable learning actions listed under engagement.

1.11. Organisation of the study

Chapter 1

In this chapter, a detailed plan (the steps and their sequence) of how the whole study will be conducted is discussed. This chapter covers the introduction, background, problem statement, objectives, research questions and research methodology of the present study.

Chapter 2

In this chapter, relevant literature is reviewed. Studies based on the effect of manipulatives in the learning of geometry were consulted.

Chapter 3

This chapter presents the research design, methods of data collection, instruments of collecting data and how the observation sheets were administered.

Chapter 4

In this chapter, the results are presented and interpreted by means of graphs, table and qualitative statements.

Chapter 5

In this chapter, a discussion of findings, conclusions and recommendations are presented.

1.12. Summary

Having considered the poor performance of learners in geometry at FET phase, this study set out to investigate the effects of manipulative objects as scaffolders in the context of learning grade 11 geometry. Mixed-method sequential explanatory design in conjunction with written tests, observations and semi-structured interviews as data collection methods were used to answer the following research question:

1. Does the incorporation of manipulative objects enable learners to engage in solving geometry problems?
2. What effect do manipulative objects have on learners' engagement time in geometry learning at grade 11?
3. To what extent are manipulative objects able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking?

The next chapter presents the review of the literature and theoretical framework related to the effects of manipulative objects as scaffolders towards the learning of geometry.

CHAPTER 2: LITERATURE REVIEW AND THEORICAL FRAMEWORK

2.1.1. Introduction

This chapter reviews the literature on the effects of manipulative objects as scaffolders towards learning of geometry. The researcher was able to do a critical analysis and synthesis of what other researchers have reported about the effects of manipulative objects in the learning of geometry at local, national and international level. The researcher was also able to adapt other researchers' methods and instruments of data collection to this study. The review is organised according to the objectives of the study.

2.1.2 Problem-solving in geometry with the use of manipulative objects

Mathematics is a field of study which covers a wide range of topics. Mathematics as per Curriculum and Assessment Policy Statement (CAPS) for Further Education and Training (FET) phase covers ten main topics namely: functions; number patterns, sequences, series; finance, growth and decay; algebra; differential calculus; probability; Euclidean geometry and measurement; analytical geometry; trigonometry and statistics. In this way, each topic contributes towards the acquisition of problem-solving skills (Department of Education, 2011). Problem-solving is essential in mathematics as it relates mathematics to the real world, thus when learners' problem-solving skills are properly developed they may not have difficulties in solving real-life problems (Widodo & Ikhwanudin, 2018). However, problem-solving needs logic and creativity, and these abilities are found through geometry learning (In'am, 2016). Further, Rofii et al. (2018), view geometry as the branch of mathematics which supplies other branches with different approaches towards problem-solving. Thus, this study focuses on geometry while engaging learners in mathematical problem solving through the use of manipulative objects.

Problem-solving is a mental activity that is used as an approach to solve different mathematical problems (In'am, 2016). This means that solving problems may not be achieved without adequate problem solving. Furthermore, approaches to problem-solving include planning, understanding the problem, devising the strategy of solving, implementing and evaluating (Rofii et al., 2018). Although problem-solving happens in one's mind, however, its results may be seen in one's actions. For example, failing to solve a given geometry rider may indicate poor problem-solving skills.

According to Rofii et al. (2018), geometry is considered as a source of problem-solving approaches for mathematics, but what happens if a learner fails to understand geometry? Greeno (2017) argues that problem-solving may not be reached in geometry without understanding. Thus, failing to understand geometry may have a negative impact on mathematical problem solving (In'am, 2016; Rofii et al., 2018). This implies that problem-solving within geometry may be needed.

Problem-solving as per Department of Education (2011), CAPS for mathematics, forms the last level of Bloom's taxonomy. The majority of learners at FET phase are unable to reach the last level of Bloom's taxonomy in geometry (Luneta, 2015). Thus, strategies of engaging learners in geometry problem-solving may come into place. This implies that learning methods that promote problem-solving must be prioritised as according to Greeno (2017) such learning methods yield understanding. Balinha and Mamede (2018) view manipulative objects as learning materials that may be used to enhance learners' problem-solving skills in geometry.

Manipulative objects and problem-solving

There are different types of manipulative objects with different names and functions. Different topics of geometry use different manipulative objects to enhance learners' problem-solving skills and understanding. For example, Sibiya (2019), in his study used protractors to measure angles on manipulative objects called geoboards to enhance learners' understanding of circle geometry theorems in grade 11. This study also makes use of geoboards, however, instead using protractors, star and round self-adhesive labels were used to denote equal angles as according to Wells (1970), the use of geoboards may vary depending on how one wants to use them. Geoboards were also used by Balinha and Mamede (2018) to enhance learners' understanding and problem-solving skills in the learning of parallelograms which is a topic of geometry done in primary schools. Also, Jamhari and Wongkia (2018) used manipulative objects called CircleBoard-Pro to enhance learners' understanding of circle geometry theorems in secondary school. However, like Sibiya (2019) and Balinha and Mamede (2018), this study employs manipulative objects called geoboards to investigate their effects in problem-solving when learning geometry in grade 11.

Manipulative objects are powerful tools in assisting learners to successfully solve challenging problems in geometry (Balinha & Mamede, 2018; Doias, 2013). They provide learners with relevant support needed to solve challenging problems. According to Karatas and Baki (2017), problem-solving in geometry involves analysing, interpreting, reasoning, predicting, evaluating and reflecting on the given information in order to make informed decisions. In addition, when problem-solving is properly developed in learners' minds, it becomes easy for them to analyse given information and remember relevant facts which will help in finding the solution to the given problem (Epelboim & Suppes, 2001). This involves separating facts that are not relevant in solving a particular geometry problem. Incomplete reasoning, a mismatch between a statement and a reason and inclusion of irrelevant facts are indicators of weak problem-solving in geometry (Greeno, 2017).

Manipulative objects are seen as important sources of visualisation skills when learning geometry (Karakus & Aydin, 2017). Learners need visualisation skills when working with more abstract geometry symbols, notations and diagrams. Thus engaging learners in geometry problem-solving requires visualisation and spatial skills to be properly developed in their minds (Arıcı & Aslan-Tutak, 2015). Thus, visualisation and spatial skills help learners to mentally manipulate complex geometry objects so as to understand. Research evidence found in Balinha and Mamede (2018) reveals that visualisation and problem-solving skills are properly developed when incorporating manipulative objects in geometry. Manipulative objects in this manner lay a strong foundation that helps learners understand complex geometry diagrams. Laski, Jor'dan, Daoust, and Murray (2015) suggest the incorporation of manipulative objects when working with complex geometry tasks because manipulative objects are capable of helping learners understand abstract content. This means that learners are able to reach a state of understanding when manipulative objects support their geometry learning.

The first step towards problem-solving in geometry is understanding (Greeno, 2017). In this sense, it is not possible for learners to critically analyse, reason logically and evaluate if they do not understand. Lambdin (2003) argued that when learners understand, their problem-solving skills are developed; likewise, when they learn in a manner that promotes problem-solving, they are likely to understand. Hence, one way of promoting problem-solving in geometry is to make learners understand. Understanding is an important factor in applying problem-solving procedures which give strong support in solving geometry proofs (Greeno, 2017). For example, it is difficult for a learner to apply a certain theorem when they do not know how and when it is applied. Thus, learning abstract geometry facts requires problem-solving kind of learning for learners to understand. Hence the use of manipulative objects may be recommended since they are able to engage learners in geometry problem- solving which leads to understanding (Balinha & Mamede, 2018; Doias, 2013; Jamhari & Wongkia, 2018).

The relationship between manipulative objects and problem-solving in geometry is vital. For example, in cases where learners are required to solve non-routine geometry problems, Rahman and Ahmar (2016), believe that exposing learners to problem-solving can assist them in achieving a learning goal. Problem-solving may help learners to have a proper plan, strategy, implementation and evaluation of a given geometry problem (Rofii et al., 2018). If problem-solving skills are not in place, learners may struggle in solving abstract geometry problems. Jojo (2017) on the other hand, believes that manipulative objects may be used to enhance learners' problem-solving skills in geometry.

Manipulative objects are able to assist learners to move from concrete to abstract geometry content (Balinha & Mamede, 2018). Moving from concrete to abstract in geometry is difficult if problem-solving skills are not developed in learners' minds (Rahman & Ahmar, 2016). Problem-solving in this regard, helps learners to have a deeper understanding of what learning was about. Furthermore, Bayram (2004) believes manipulative objects can assist learners to progress from a concrete level to an abstract level. Saleh et al. (2018) assert that manipulative objects are like bridges through which learners use to progress from concrete to pictorial and abstract while allowing learners to construct knowledge on their own.

Manipulative objects called geoboards may positively affect the learning of geometry. The findings from Sibiya and Mudaly (2018) indicated that geoboards may improve learners' understanding of circle geometry theorems. This was noted from the interviews conducted on how geoboards affected learners' learning of geometry theorems. Many learners reported that geoboards allowed them to learn geometry practically. Sibiya and Mudaly (2018) noticed that geoboards are able to develop learners' reasoning skills and geometry terminology. Furthermore, Sibiya (2019) noted that geoboards also improve learners' motivation and confidence in geometry.

Engaging learners in problem-solving mainly depends on the learning method used. Most studies e.g. (Balinha & Mamede, 2018; Jamhari & Wongkia, 2018; Larkin, 2016) indicate a positive correlation between learning methods which were aided with manipulative objects and problem-solving. In addition, Graham and Chick (2015) have advocated that manipulative objects are also able to improve learners' performance in geometry. In this way, manipulative objects are viewed as learning tools that promote problem-solving while improving learners' performance in geometry.

Some of the learning methods do not promote problem-solving. Makhubele (2014) believes that learning methods that are not promoting problem-solving are the main causes of errors and misconceptions which results in the poor performance by learners in geometry. Thus, the traditional way of learning geometry which is a lecture method leads to no understanding, nor problem solving rather in learners memorising what they were told (Luneta, 2015). Moreover, Makhubele (2014), also reports that learning methods that are not promoting problem-solving lead to weak procedural and conceptual understanding in geometry.

Manipulative objects have the potential of setting a conducive learning environment for problem-solving in geometry to take place (Cockett & Kilgour, 2015). Manipulative objects support action learning, where learners learn by doing. According to (Jojo, 2017), action learning is what is needed the most to promote problem-solving in geometry. In other words, the learning of geometry happens better when the classroom environment is conducive for learning by doing. Moreover, Karatas and Baki (2017) view the feasibility of problem-solving in learning geometry as mainly dependent on the learning environment of learning. In this regard, the learning environment has to be conducive for the learning of geometry. According to Cockett and Kilgour (2015), manipulative objects are able to set the learning environment such that they keep learners focussed on the learning of geometry for a long period of time.

It is through manipulative objects that learners become actively involved in a hands-on manner of exploring complex ideas, concepts and procedures in geometry (Cockett & Kilgour, 2015; Menghini, 2015). Manipulative objects in this way enable learners to be actively engaged in their learning. Menghini (2015) believes that engaging learners in geometry problem solving properly happens when learners learn through practicals or experiments. Moreover, problem-solving in geometry is a key component in the mental engagement of learners in their learning (Özsoy & Ataman, 2017). Thus, it may be possible to engage learners in geometry problem solving through manipulative objects because manipulative objects promote practical learning. As advocated in Piaget and Cook (1952), action learning should be encouraged at all stages of learning because it allows learners to make mistakes and learn from their mistakes.

2.1.3. Engagement time in geometry through manipulative objects

Engagement

Learners' engagement can be defined as "students' psychological investment in and effort directed toward learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote" (Lamborn, Newmann, & Wehlage, 1992, p. 5). In this way, learners' engagement is regarded as mental and physical actions that learners undergo in order to reach understanding. According to Kong et al. (2003), there are three types of learners' engagement namely: cognitive, emotional and behavioural. Cognitive engagement is associated with learning factors including thinking and problem-solving strategies. Emotional engagement is associated with attitude and sense of belonging towards schooling while behavioural engagement is associated with learners' classroom actions including on-task and off-task behaviours. However, learners' actions may not be separated from what they think as they (learners' actions) reflect their (learners') thoughts. This study focuses on a behavioural engagement; however, behavioural actions that are done by learners are assumed to be reflecting their mental actions. Thus, to work out how much time learners spent being engaged, this study considered both passive and active behavioural engagement actions.

Engagement in mathematical learning is one of the most important things to consider in helping learners understand abstract concepts (Sinatra, Heddy, & Lombardi, 2015). This means that the more learners become engaged in their learning, the more they understand what is being taught. According to Fredricks and McColskey (2012), learners' engagement is a key component in addressing issues of low learners' achievement in mathematics. Thus, low achievement is associated with no or less learners' engagement.

Keeping learners engaged in mathematical learning is a problem faced by educators in their daily practices (Toussaint & Brown, 2018). On the other hand, a lack of learners' engagement in mathematical learning may not only affect their learning, but also educational success (Bobis, Way, Anderson, & Martin, 2016). For example, lack of engagement may lead to poor performance in mathematics which may negatively affect learners' carriers which are mathematical related. Thus, learners need to be taught in a way that promotes engagement as according to Fredricks and McColskey (2012), such an approach may enhance learners' achievement in mathematics.

Different strategies of learning are used to promote learners' engagement in learning. For example, Sibiya (2019) recommended the use of geoboards to promote learners engagement in geometry. His recommendations were based on the findings of his study which revealed that geoboards promote active learning which enhances learners' motivation and confidence in geometry learning. His findings supported the idea by Toussaint and Brown (2018) who stated that motivation is a key component to keep learners engaged in learning. However, choosing a learning method that promotes learners' engagement like geoboards depends more on the teachers' beliefs, i.e. some teachers may view the incorporation of geoboards as a waste of time, thus, teachers' beliefs may hinder or promote learners' engagement (Bobis et al., 2016).

Engagement time

Another dimension of studying learners' engagement in learning is to study the amount of time spent being engaged. Some studies, for example, (Balinha & Mamede, 2018; Sibiya, 2019) have explored the effects of manipulative objects on learners' engagement in geometry; however, they have not explored the period of engagement. Kong et al. (2003) in their study specified the period of engagement; however, they did not use manipulative objects. To my reading, it appears that literature lacks a discussion on how manipulative objects affect the period of engagement in learning geometry. This study focuses on the effects of manipulative objects on learners' engagement time when learning geometry.

Learners' engagement depends more on the amount of time available for teaching and learning (Caldwell, Huitt, & Graeber, 1982). Thus, learners' engagement is a fraction of time which learners spent being engaged over the total time which was available for teaching and learning. In this respect, when observing learners' engagement, one needs to consider the amount of time spent being engaged in task performance. For example, the amount of time a particular learner spent on listening, asking or answering a question and or using geoboard if geoboards are learning tools on that particular day. Hence, the focus in this study is on learners' engagement time which is the amount of time which was spent by a particular learner in performing a task. Thus, this objective of this study investigates the link between manipulative objects (geoboards) use in geometry learning and learners' engagement time (time on task).

Focusing on engagement time helps in finding out what fraction of available time for learning learners spent being engaged in meaningful learning. Carroll (1989), in his model time, defines engagement time as time learners spent being engaged in meaningful learning when performing a task. Meaningful engagement in this regard means performing activities that are in line with the learning goal like attentive listening, discussion about the task and answering the questions asked. However, it is unlikely that the time allocated for learning geometry content at a particular moment will be the same as learners' engagement time (Rosen, Cheever, & Carrier, 2015).

Time spent on learning is an important factor in learners' performance in mathematics (Caldwell et al., 1982). Each section of mathematics is allocated with a number of hours, days or weeks of teaching and learning. Euclidean geometry, for example, three weeks are allocated on it in grade 11 as per CAPS (Department of Education, 2011). It is assumed that learners would have mastered all the learning content needed in this three-week period. However, such may be possible, if in all the lessons learners would have been engaged in meaningful geometry learning. That is, learners become engaged for every moment of the amount of time allocated for learning a particular geometry task. However, according to Rosen et al. (2015), learners hardly spent 100% of the time allocated for a particular task being engaged in meaningful learning. Rosen et al. (2015)

believe that learners may spend part of the time allocated for learning on multitasking and other disturbances like socialising, daydreaming, or doodling. Thus, methods of learning that promote engagement may be recommended to enhance learners' engagement time so as to reduce the amount of time learners spent on disturbances.

Improving learners' engagement time demands more effort on measures that keep learners engaged and motivated in their learning. Maximising learners' engagement time requires engagement on task to be also maximised (Fyfe, McNeil, & Borjas, 2015; Perry & Steck, 2015). According to Cockett and Kilgour (2015), incorporating manipulative objects may positively affect learners' on-task engagement. Cockett and Kilgour argue that manipulative objects when learning abstract content are able to motivate learners by keeping them focussed and engaged for a long period of time as noted in Toussaint and Brown (2018) that keeping learners motivated may lead to high engagement. Thus, learning material that motivates learners is important in geometry because lack of motivation is a serious cause of disengagement which negatively affects learners' engagement time (Hopstaken, Van Der Linden, Bakker, & Kompier, 2015). Manipulative objects may enhance learners' motivation and understanding of geometry (Sibiya, 2019). This implies that manipulative objects may also enhance learners' engagement time.

The amount of engagement time has the power to motivate learners which may positively affect their achievement in mathematics (Carroll, 1989). In this regard, if learners spend less time being engaged they are likely to be demotivated, and demotivation according to Bobis et al. (2016), may cause low learners' achievement in mathematics. However, research has shown that manipulative objects may increase learners' engagement, motivation and achievement (Balinha & Mamede, 2018; Cockett & Kilgour, 2015). Thus, manipulative objects may positively affect learners' engagement time as they are capable of increasing learners' engagement, motivation and achievement in mathematics.

Manipulative objects may positively affect learners' engagement time and the time it takes them to finish a given task. Goodman, Seymour, and Anderson (2016) observed that a scaffolding from manipulative objects may help learners to finish a given task in a short space of time. According to Perry and Steck (2015), learners' engagement is what is needed the most to maximise learners' understanding when learning geometry. In this way, manipulative objects improve learners' understanding of a learning task because the more learners understand, the shorter it takes for them to finish a given task. Goodman et al. (2016) also noted that manipulative objects improve learners' engagement as they help learners to visualise geometric shapes. Instructions which promote high learners' engagement are also likely to promote high learners' academic achievement (Harbour, Evanovich, Sweigart, & Hughes, 2015). Thus, prioritising engagement yields improved results.

Manipulative objects close the gap between the expected finish time and the actual finish time when learners are doing a particular task by keeping learners engaged for a long period of time (Fyfe et al., 2015). It is possible that learners finish a 30-minute long task in 20 minutes, hence, working out an expected finish time for learners when doing a task is never easy (Johns, Crowley, & Guetzloe, 2017). Thus, in such cases where learners finish before the expected time, manipulative objects may help in keeping them engaged even after finishing a given task. This may be possible because learners may use the remaining time to test and verify their conjectures with the use of manipulative objects (Fyfe et al., 2015). For example, when using geoboards, learners may use the remaining time by representing one figure or theorem in many different forms on their boards.

Literature generally reveals less about the relationship between manipulative objects and engagement time. There seems to be a dearth of literature on whether or not the learners' level of engagement affects their performance. According to Bobis et al. (2016), engagement time spent by learners when learning geometry may affect their performance. There is also limited literature on the effects that manipulative objects have with regards to learners' engagement time in geometry. Smith and Stephanie (2015) noted that there is a relationship between manipulative objects and learners' active engagement in geometry learning. Active engagement in learning yields high learners' engagement and improves learners' performance in geometry (Johns et al., 2017; Norris, Shelton, Dunsmuir, Duke-Williams, & Stamatakis, 2015). Thus, manipulative objects may positively affect learners' engagement time as they promote active learning. However, the extent to which manipulative objects are able to positively affect learners' engagement time is not clear. For example, the magnitude engagement time that is estimated when learners learn geometry with the aid of manipulative objects is not specified in literature. Thus, this study hopes to estimate the amount of time spent by learners being engaged in their learning of geometry.

Time and behavioural engagement are closely related. Behavioural engagement refers to learners' actions which can be seen through direct observations (Fredricks & McColskey, 2012). For example, behavioural engagement may include listening, making use of learning material and assisting other learners. Some behaviours happen for a short period, hence it is important to mention how long did a certain behavioural engagement last (Kong et al., 2003). Kong et al noted that learners spent most of their time on listening and preforming exercises given by the teacher to them. Thus, observing the occurrence of behavioural engagement in a short period helps in working out how long did a certain behaviour last. For example, Kong et al. (2003) noted that some learners can spend up to half an hour not being actively engaged in meaningful learning.

Judging whether learners are engaged or not may be problematic, as noted in Jackson (1992) that some learners may appear engaged in meaningful learning while they are not and others may appear disengaged whereas they are engaged. This is based on the assumption that learners' actions are reflections of their thoughts. Thus, the incorporation of manipulative objects may help in maintaining a connection between what was observed and what learners really do by increasing the time learners spent being actively engaged in meaningful learning of geometry (Tabach & Nachlieli, 2015).

Keeping learners engaged and motivated for the entire learning period is a problem faced by many educators and researchers (Barkley & Major, 2020). Learning geometry through problem-based learning requires engagement techniques which maintain the balance between the task's demands and learners' engagement time as learners need sufficient engagement time for them to understand a given problem (Erickson, 1999). According to Rofii et al. (2018), understanding the problem is one of the key components needed to enhance learners' problem-solving skills in geometry. Thus, if a learner does not understand the problem he or she may not know what the problem demands and may fail to devise a proper strategy to solve a problem.

Thus, the amount of engagement time has the power to motivate or demotivate learners which may either positively or negatively affect their achievement in mathematics (Carroll, 1989). In this regard, if learners are not engaged they are likely to be demotivated and demotivation may cause low learners' achievement in mathematics. However, manipulative objects may increase learners' engagement, motivation and achievement (Balinha & Mamede, 2018; Cockett & Kilgour, 2015). Thus, manipulative objects may positively affect learners' engagement time as they are capable of increasing learners' engagement, motivation and achievement in mathematics.

Learners may need more engagement time to understand abstract geometry content. According to Harbour et al. (2015), the more the engagement time they have, the more learners understand. Further, Johns et al. (2017) emphasise that to reach a high degree of success in geometry learning, appropriate learning materials are needed to maximise learners' engagement time. However, it is not specified as to what type of materials are appropriate to promote a high degree of success in learning abstract content. However, Larbi and Mavis (2016), specified that the visualisation that manipulative objects provide to learners may reduce the abstract nature of content in geometry thus positively affecting learners' engagement time.

Observations are commonly used to investigate learners' engagement time. Kovanovic, Gašević, Dawson, Joksimovic, and Baker (2016) noted that there is a relationship between observational methods of data collection and time-related studies. Special types of observations like momentary time and partial-interval sampling observations may be used to estimate engagement time (Fedewa, Davis, & Ahn, 2015). These kinds of observational methods enable researchers to observe a specific behaviour on a given amount of time, e.g. learners' engagement time in every 5-second (5-s) time interval. However, to observe the occurrence of behaviours in a short period like 3-s, 5-s and 15-s may be difficult without the use of technology like videotaping cameras. Many engagement time studies e.g. (Jamhari & Wongkia, 2018; Kong et al., 2003; Pfister et al., 2015; Storms, 1973) have used videotaping cameras to observe the occurrences of targeted behaviours in short periods of time. Hence, this study used interval time sampling to code targeted learners' behaviours with an aid of videotaping camera to investigate the effects of manipulative objects on learners' engagement time in the learning of geometry.

Partial-interval sampling is one of the measures that are used to provide an accurate estimation of learners' engagement time. Partial-interval sampling divide duration of learners' engagement into small time intervals and code the engagement and disengagement on a particular task (Droit-Volet & Wearden, 2015). Through this, learners are able to be observed in a short period, hence producing more accurate observations. According to Zakszeski et al. (2017), breaking duration into small intervals yields more accurate engagement estimation. Although it is assumed that one's behavioural engagement is a reflection of their mental and emotional engagement, partial-interval sampling does not give an exact picture of one's mental and emotional part of engagement (Fredricks & McColskey, 2012).

Breaking down learning duration into small intervals helps in observing learners' behaviour in a short period. Calderwood, Ackerman, and Conklin (2014) observed that, in a 3-hour long period, learners spend about 6 seconds on average in every 25 minutes not being actively engaged in learning. Moreover, an observational study of 263 participants on learners' learning behaviour indicates that about 5 minutes out of a 15-minute period is spent by learners not being actively engaged in their learning (Rosen, Carrier, & Cheever, 2013). Hence, putting more effort into maximising learners' engagement time will benefit learners.

2.1.4. Scaffolding geometry learning through manipulative objects

Manipulative objects in this study are investigated if they can be used to help learners reach levels 3 and level 4 of Van Hieles model of geometry thinking. This part of the study investigates the extent to which manipulative objects can scaffold the geometry learning in reaching levels 3 and 4. This study is of the view that learners in Further Education and Training (FET) phase are not performing at the expected levels and those are levels 3 and 4 of the model (Alex & Mammen, 2014; Luneta, 2015; Makhubele, 2014). In other words, they are investigated if they can scaffold geometry learning such that learners are able to reach levels 3 and level 4. Thus, manipulative objects are viewed as scaffolders in this respect.

Scaffolding is a kind of support that knowledgeable people (teachers or other learners) provide to less knowledgeable people, learners in particular (Radford, Bosanquet, Webster, & Blatchford, 2015). This kind of support called scaffolding is systematically provided to learners until they reach their full potential in performing a task which is beyond their ability at that point (Chi, Siler, Jeong, Yamauchi, & Hausmann, 2001). In geometry, learning based on scaffolding is characterised by comparison, explanations, justifications, agreement and validation learners' conjectures (Brown & Redmond, 2016).

Ferguson and McDonough (2010) noted that there are two types of scaffolding, where the first one is, one-to-one discussions between teacher and learner and the second is the teachers' use of manipulatives. In this respect, the one-to-one discussion allows less knowledgeable learners to ask questions where they do not understand. On the other hand, teachers' use of manipulatives refers to support that is not only coming from the teachers but also from manipulatives use (Ferguson & McDonough, 2010). In this way, the use of manipulative objects comes into place. Manipulative objects supplement the teacher's explanations, in cases where what was explained by the teachers was partially understood by the learners.

Scaffolding from manipulative objects may become useful in cases where learners struggle to understand abstract geometry content. Manipulative objects create an environment that enables learners to think abstractly by engaging learners both mentally and physically (Arıcı & Aslan-Tutak, 2015). This happens through the visualisation that manipulative objects provide. Furthermore, manipulative objects are also capable of promoting learners' understanding and enhancing their geometry reasoning (Arıcı & Aslan-Tutak, 2015).

Manipulative objects may help in scaffolding geometry representation of figures from Concrete to Pictorial and Abstract (CPA). Bruner (1966) came up with enactive, iconic and symbolic approach to learning commonly known as concrete, pictorial and abstract (CPA) approach. Bruner believes that any kind of knowledge can be represented in concrete, pictorial or abstract. Bruner's approach is commonly used in geometry since geometry works more with figures which can either be represented in concrete, pictorial or abstract. However, moving from concrete to pictorial and pictorial to abstract is never easy (Putri, 2015). It demands a specialised kind of learning not rote learning. Bouck, Satsangi, and Park (2017) asserted that the incorporation of manipulative objects in the learning of geometry may help learners to progress from one stage of CPA to another.

Balinha and Mamede (2018) noted that learning geometry involving rectangles and triangles with an aid of manipulative objects like geoboards develops learners' visualisation skills in geometry. In this regard, manipulative objects act as scaffolders in helping learners visualise sides and angles of shapes. According to Gürsoy and Özkar

(2015), visualisation is what learners need in order to understand abstract geometry content. The same conclusion regarding visualisation was made by Habegger and Emen (1994) that without visualisation it is difficult for learners to progress from one level of Van Hiele's model to another. Apart from visualisation, manipulative objects called geoboards are also capable of improving learners' achievement in geometry (Keraf, 2017). This is because geoboards allow learners to represent one geometry shape in many different ways and that may help in increasing their level of understanding.

Geometry reasoning and proofs are not easily understood by learners. Learners in senior and FET phases are still struggling with regards to geometry proofs (Jamhari & Wongkia, 2018; Luneta, 2015). In other words, learners are unable to reach levels 3 and 4 of Van Hiele's model as formal geometry reasoning and proofs are performed in levels 3 and 4 of Van Hiele's model (Hock, Tarmizi, Yunus, & Ayub, 2015). However, manipulative objects may scaffold geometry learning such that learners are able to confront and solve challenging geometry problems (van de Pol, Volman, Oort, & Beishuizen, 2015). In this way, manipulative objects may be used to assist learners to reach levels 3 and 4.

In the learning of geometry related to the circle in grade 8, it was noticed that manipulative objects called CircleBoard-Pro improve learners' understanding of angles although it was noted that learners are still unable to perform geometry proofs by themselves (Jamhari & Wongkia, 2018). Performing geometry proofs according to Van Hiele's model, takes place in level 4. The inability of learners to perform geometry proof was noticed through the pre-test and post-test together with semi-structured interviews conducted in Jamhari and Wongkia (2018). This study also uses pre-post-test and semi-structured interviews to investigate the effects that manipulative objects have in scaffolding learners' geometry learning such that they are able to reach levels 3 and 4 in the eleventh grade.

Manipulative objects overcome the limitations of computer-related manipulatives (Larkin, 2016). For example, some computer-related manipulatives neglect the concrete stage of Concrete, Pictorial and Abstract (CPA) approach at times (Larkin, 2016). In this regard, using manipulative objects as scaffolders to geometry learning, gives learners the freedom to manipulate geometry figures concretely. Moreover, it is through manipulative objects that learners get what Zheng, Yu, Wang, Zhong, and Xu (2017) call tangible experience. Tangible experience is what learners need in order to understand abstract geometry ideas (Doias, 2013). Manipulative objects also help learners to explore geometry ideas and reflect on their action (Catala, Sylla, Theune, Brooks, & Read, 2018). Hence, scaffolding based on manipulative objects may work on learners' advantage when learning geometry.

According to Larkin (2016), when representing knowledge in terms of concrete, pictorial and abstract, the age of learners matters. In this regard, a concrete representation that involves using manipulative objects is more effective when scaffolding primary and senior primary school learners while pictorial and abstract representations are viewed to be more suitable for older learners (Larkin, 2016). To this end, it seems that as a grade level increases, the use of manipulative objects decreases. Based on my reading it appears as though manipulative objects are less used with high school geometry.

Scaffolding support from manipulative objects is able to assist learners to progress from concrete to abstract (Putri, 2015). Findings from Bayram (2004) also indicate that manipulative objects are able to provide scaffolding support which learners need to progress from one level to the next when working with grade 8 learners. According to Balinha and Mamede (2018), scaffolding support from geoboards is able to develop primary learners' problem-solving skills, geometric vocabulary and help them reach upper levels of Van Hiele's model. However, this study seeks to investigate whether manipulative objects are also able to assist learners to progress from levels 3 to level 4 of Van Hieles model in grade 11. This is informed by the findings from Luneta (2015) which indicate the inability of learners to reach levels 3 and 4 of van Hiele's model as they are expected to. In this context, manipulative objects are being investigated if they can help learners reach levels 3 and level 4 of Van Hiele's model of geometry thinking.

2.1.5. Summary

The reviewed literature with regards to the use of manipulative objects in the learning of geometry confirms what Scott (1983) noted, that manipulative objects are commonly used in the lower grades i.e. (Balinha & Mamede, 2018; Bayram, 2004; Jamhari & Wongkia, 2018).

Several studies have been conducted regarding the use of manipulative objects in geometry. Among them are Balinha and Mamede (2018), Jamhari and Wongkia (2018), Sibiya and Mudaly (2018) and Sibiya (2019). These studies are generally focusing on the use of manipulative objects in enhancing learners' understanding of geometry. Based on my reading it appears as though not much has been reported about the effects that manipulative objects have on the following: problem-solving in learning circle geometry at the eleventh grade; learners' engagement time and levels of Van Hiele's model of geometry thinking in grade 11.

2.2 Theoretical Framework

The importance of theory in research is that it explains and predicts the relationships which dependent and independent variables might have (Creswell, 2014). In this regard, theory serves as the cornerstone of a research project, as Silverman (2013) believes that it is impossible to conduct a research project without theory. Van Hiele's model of geometry thinking is the theory that serves as a cornerstone of this study. The learning progress in this regard is viewed in terms of Van Hiele's model.

2.2.1 Van Hiele's theory

The reviewed literature reveals that despite the importance of geometry, many learners in South African do not understand it (Luneta, 2015; Makhubele, 2014). The issue of geometry being seen as a difficult branch of mathematics is not a new issue. It became a reason for the development of Van Hieles theory in the 1980s by Dina van Hiele-Geldo and Pier van Hiele who are the founders of the theory (Burger & Shaughnessy, 1986).

Van Hiele's (VH) model and Bruner's registers of knowing as called Concrete, Pictorial and Abstract (CPA) define success in geometry learning as progress from one stage to the next. In VH, success is viewed as progress from one level to the next (Crowley, 1987). In CPA, success in learning is progress from one register to the next (Bruner, 1966). However, visualisation is an important aspect of learning progress in both VH and CPA (Balinha & Mamede, 2018; Bouck et al., 2017). Habegger (1994) views visualisation as a basic skill that every learner needs to master in order to move from the first level to the last level. On CPA, without visualisation, it is difficult for learners to progress from one register to the next i.e. concrete to pictorial and pictorial to abstract (Carden & Cline, 2015). This means that it is likely to reach success in geometry learning when maximising visualisation in both VH and CPA. Bayram (2004, p. 17) linked the two VH and CPA as follows: "level 0 - concrete, levels 1-2 – visual geometric structures (pictorial), and levels 3-4 abstract structures". Manipulative objects as recommend by Bayram (2004) seem to be linking VH and CPA. At the concrete stage of CPA and direct orientation (phase 2) of VH learners visualise geometry figures through manipulative objects.

The South African curriculum is designed such that, Van Hiele's model and Bloom's Taxonomy (BT) give structure to geometry learnt in primary and secondary schools (Alex

& Mammen, 2016b; Department of Education, 2011). There is a connection between VH model and BT in terms of how these theories define success in learning. Both theories, VH and BT, view geometry learning as being based on hierarchical levels (Žilková, Guncaga, & Kopácová, 2015). The hierarchy of the levels in both theories is such that the level of complexity of knowledge increases as levels increase. That is, the content found in level 2 is more complex compared to the content found in level 1. The same happens with levels 3 and 4, the content matter found in level 3 is more complex than that of level 2, but it is less complex than the content matter found in level 4. Although VH model has five levels in total, however, according to Alex and Mammen (2016b), primary and high school geometry content goes up to level four. Also, with BT, there are six levels in total; however, for primary and high school geometry, level 6 is not considered as per CAPS document for mathematics (Department of Education, 2011). Walsh (2015) categorised levels in both BT and VH as follow: Level 1- Visualisation (VH) - knowledge and understanding(BT); Level 2 - Analysis (VH) -Routine procedures (BT); Level 3 - Abstraction(VH) - Complex procedures (BT) and Level 4 - Deduction(VH) - Problem Solving (BT). In this regard, both B T and V H model have levels 1 to 4 when considering the geometry learned at primary and high school. However, this study used VH level as according to Burger and Shaughnessy (1986) VH levels are closely related to geometry as the main focus of the study.

According to Van Hiele (1959), progressing from one level to the next does not depend on the age of a learner, but it depends more on the content and methods used. Instructional models and learning materials that are in line with problem-solving may help learners to progress from one level to the next (Balinha & Mamede, 2018). In this way, memorisation based learning is discouraged because it leads to no understanding and does not help learners to progress from one level to the next, but rather brings them back to the previous level since learners cannot skip levels (Van Hiele, 1984). This gives an illustration of why for example, South African learners in grade 11 and 12 are performing at levels 1 and 2 while they are expected to perform at levels 3 and 4 (Luneta, 2015; Makhubele, 2014). Makhubele (2014) believes that one of the possible reasons for this is the memorisation kind of learning which is mainly caused by employing the traditional (lecture method) way of teaching geometry. Karakuş and Peker (2015) view manipulative objects as learning tools that help in assisting learners to progress from one level to the next.

The levels of the model build on each other in a hierarchical manner (Alex & Mammen, 2016a). In other words, there is no level 2 without level 1 and level 3 cannot be reached without level 2 knowledge. Knowledge obtained in the previous level forms the foundation of what the next level is about (Van Hiele, 1959). That is, a learner will not be able to understand the properties of a square (level 2) if he or she has not seen the square before (level 1). This is possible because, the level of complication increases as the levels of the model go higher (Van Hiele, 1984).

It is also possible that learners spend more time on one level, at that moment learning progress may seem to stop, but it continues after sometime (Van Hiele, 1959). It is for this reason that the Van Hiele's model of geometry thinking views learning progress as mainly dependent on methods and types of materials available to help learners construct knowledge (Yazdani, 2008). Although teachers are seen as best instructional materials, learning can also be enhanced through the other materials likes manipulatives to accommodate different learning abilities (Beltran, Regala, & Bog-ot, 2018). Such integration of other material helps to scaffold learning progress from one level to the next hence reduces time spent on each level of the model.

There are two numbering systems used for levels of VH model. There is the one that starts from level 0 to level 4 and the other one which starts from level 1 to level 5. The numbering system which starts from level 1 to level 5 is an American way of numbering levels of VH model (Mason, 2009). This study employs an American way of naming levels which starts from level 1 to level 5.

Progressing from one level to the next involves five phases, namely, information, guided orientation, explication, free orientation and integration (Fuys, Geddes, & Tischler, 1988). These phases are important so the teacher should take them into account when planning geometry lessons to ensure that learners will be able to progress from one level to the next (Armah, Cofie, & Okpoti, 2018).

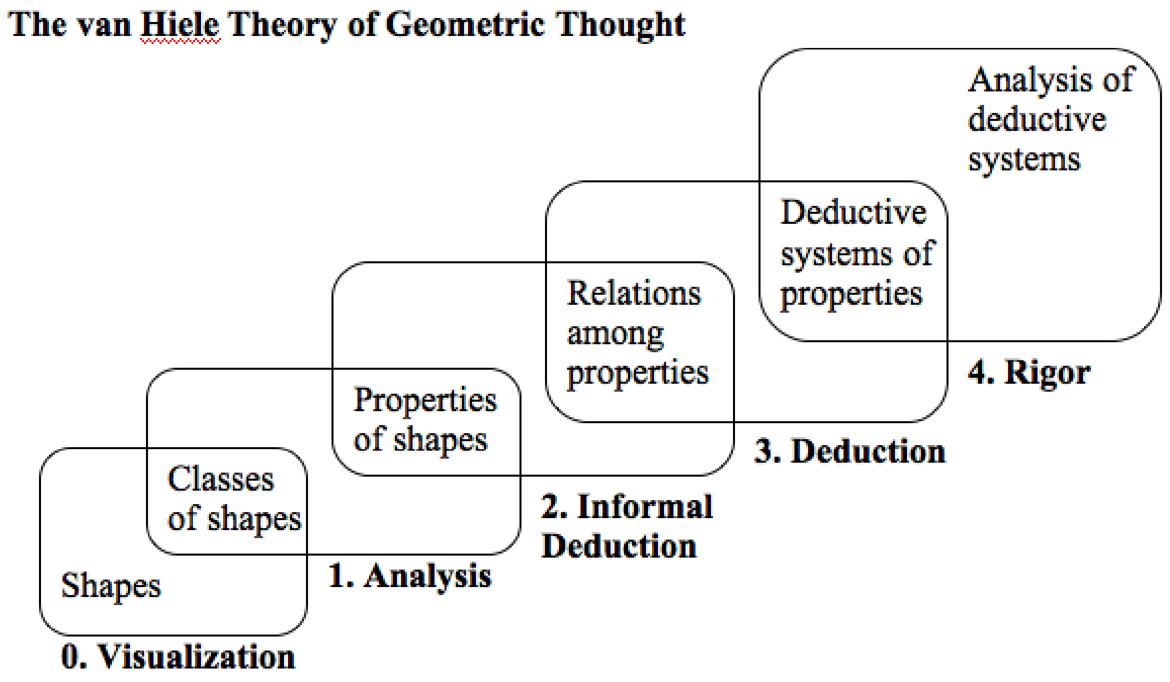


Figure 2.1: Van Hiele's Geometric Thought (Walle, 2004)

Level 1- Visualisation

At level 1, learners are expected to recognise figures holistically (Mason, 2009). This means that the properties of figures are not as important as at levels 2, 3, 4 and 5. Through the information phase level, learners are required to describe objects using their perceptions (Naidoo & Goba, 2011). Learners describe objects according to how they appear in their mind, thus, manipulative objects like geoboards may be used on the guided orientation phase of this level to help learners visualise geometry figures (Balinha & Mamede, 2018). Learners must visualise geometry ideas at this point because without visualisation it is less possible for learners to reach the analysis, abstraction, deduction and rigor level (Habegger, 1994). In this regard, learners still need visualisation skills even in levels 2, 3 and 4. Sinclair, Bussi, de Villiers, Jones, Kortenkamp, Leung, and Owens (2017) highlighted that visualisation must happen mentally and physically. This means that practical work helps in the development of learners' mental visualisation. Thus, manipulative objects may be used to enhance learners' mental visualisation skills (Balinha & Mamede, 2018). The research may help learners at the explication phase, where learners' prior knowledge is guided by a teacher through discussions towards getting a new understanding of the problem being studied (Crowley, 1987). In this respect, learners might be asked: what is a triangle? How does it look like? Asking questions of this nature helps in shaping learners' prior knowledge towards taking a new direction (Van Hiele, 1959).

Level 2 - Analysis

At level 2, learners recognise geometry figures by their general properties (Pusey, 2003). This includes describing a triangle as any closed three-sided figure. When describing a geometry figure, a learner might write anything that he or she knows about that figure, even the unnecessary properties may be included (Mason, 2009). For this, the manner in how figures are defined by learners at this level lacks logic. This level requires more attention to be given to assisting learners in shaping their knowledge of how they describe figures at guided orientation and explication phases. Research findings from Makhubele (2014) and Luneta (2015) report that many grade 11 and 12 learners are still performing at level 2. This indicates the lack of visualisation and problem-solving skills which are important aspects in making learners understand geometry (Arıcı & Aslan-Tutak, 2015).

Level 3 - Abstraction

According to Clements (2003), at level 3, learners use their logical arguments to justify their statements in a geometric domain at more of an abstract level. This may include applying the knowledge of theorems to find the size of the missing angles. However, the level of abstraction at this level is limited to a familiar context (Walsh, 2015). The majority of learners are unable to reach this level (Ramlan, 2016). At this level, formal definitions of geometry figures take place. When given a figure, learners are able to recognise an interrelationship between geometry figures that belong to the same class (Mason, 2009). The assistance of manipulative objects in a guided orientation phase may also be useful in helping learners reach explication, free orientation, and integration phase so as to progress to level 4 (Crowley, 1987).

Level 4 - Deduction

At level 4, learners make conjectures and use logical arguments and formal proofs to verify their conjectures (Crowley, 1987). Furthermore, learners are able to construct proofs of theorems and their converse without memorisation at this level. The learning success at this point is characterised by the use of the appropriate geometry language in the explication phase and the ability to solve problems using many different ways in the free orientation phase (Mason, 2009). The summary phase (integration) is only reached after the explication and free orientation phase has been successfully achieved by learners in learning geometry (Van Hiele, 1959). Learners in grade 11 and 12 are expected to understand Euclidean geometry up to level 4, however, very few learners according to Luneta (2015) are able to reach this level.

Level 5 - Rigor

Learners are expected to establish theorems at this stage (Alex & Mammen, 2016b). According to Crowley (1987), geometry thinking that is developed at this level is not part of secondary school geometry. It is done at more advanced levels of schooling and not primary and high school levels. Hence, this level will not be considered in this study since the study's focus is on high school geometry.

The language of teaching and learning geometry is very important in the development of learners' geometry proficiency (Riccomini, Smith, Hughes, & Fries, 2015). The language barrier to learning makes it possible for learners to think that they understood exactly what the teacher was expecting them to understand, whereas they understood something else which is not the same as the one that was expected from them (Van Hiele, 1984). the model emphasises the importance of having the same language spoken by teachers and learners; learning content and learners; instructional materials and learners as each level of the model uses its own language (Mason, 2009). The language used by a teacher, content or instructional materials has the potential of hindering the learning if these are not at the same level as that of a learner (Van Hiele, 1984). Thus, the incorporation of manipulative objects may help to supplement what is said by a teacher because teachers explain content to learners according to how they understand it, which may be a difficult way of understanding to learners (Sinclair & Bruce, 2015).

2.2.2 Summary

The primary aim of VH model was to equip teachers with knowledge about how learners learn geometry so as to give an appropriate support to help learners succeed (Erdogan, Akkaya, & Celebi Akkaya, 2009). The first step towards helping learners with geometry is to identify the levels at which they are operating at (Luneta, 2015). Luneta found the analysis of grade 12 learners' misconception in transformation geometry to be best supported by VH model. Further, instructions which are based on the model enable learners to progress from one level to next, for example, a study by Bayram (2004) revealed that introduction of concrete objects together the HV model may help learners to solve abstract geometry problem. Furthermore, Alex and Mammen (2016b) noted a significant improvement in the performance from learners who were taught using VH based instruction compared to those who were taught using traditional(lecture) method. In the light of these findings, this study used Van Hiele's model as a theoretical framework. The VH levels were chosen because they describe how learners learn geometry. The theory specifies different age levels at which learners understand geometry. The theory served as guide on how learners solve geometry problems using geoboards while learning of geometry. On the basis of research instruments, the reviewed literature revealed that interviews and written tests may be used to measure learners' level of understanding in geometry i.e. (Jamhari & Wongkia, 2018; Walsh, 2015) while observations were found to be common in investigating learners' engagement time(Lane & Harris, 2015; Zakszeski et al., 2017). Hence, all three methods (tests, observations and interviews) were used in this study. A detailed description of research methodology is presented in the next chapter.

CHAPTER 3: METHODOLOGY

3.1 Introduction

This chapter explains how the process of data collection was done in this study. This is the second step, following the planning step where researchers demonstrate means of how to plan to collect and analyse the data (Biyane, 2007). Accordingly, in the second step, a detailed description of how the data were collected and analysed is provided. This is not limited to the research paradigm, design, participants, instruments and the procedure of the data collection.

The main aim in this study was to investigate the effect of manipulative objects as scaffolding in context of learning grade 11 geometry. Hence, to achieve the aim of this study, the following research questions were formulated:

1. Does the incorporation of manipulative objects enable learners to engage in solving geometry problems?
2. What effect do manipulative objects have on learners' engagement time in geometry learning at grade 11?
3. To what extent are manipulative objects able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking?

3.2. Research Paradigm

Research paradigm, according to Yvonne Feilzer (2010, p. 7), is "an organizing structure, a deeper philosophical position relating to the nature of social phenomena and social structures". Mackenzie and Knipe (2006) state that a paradigm serves as a viewpoint of how a phenomenon was studied. This affects how the data are collected, analysed and interpreted. For example, if a researcher uses positivist paradigm, they are likely to collect quantitative data while qualitative data may be collected from using interpretivism paradigm (Dean, 2018). She further states that the difference between positivist and interpretivism paradigm depends on researcher's underlying assumptions about reality, for example, positivists believe that there is one reality while interpretivists believe that reality is multiple and socially constructed. Although the nature in which positivism and interpretivism paradigm operate is different, they may overlap in methodological strategies (Lincoln, Lynham, & Guba, 2011). This study, however, adopted both positivist and interpretivism paradigms as highlighted in Mackenzie and Knipe (2006) that two paradigms may be used in one study particularly if a study employs a mixed method. This enabled the research to collect quantitative and qualitative data through the mixed-methods sequential explanatory design which was employed in this study.

3.2.1. Positivist paradigm

Wilson (2001) explains that positivism is based on the view that there is one reality and it is upon human beings to discover and interpret it. Furthermore, conducting research is a way of discovering and interpreting this one reality (Unluer, 2012). Positivist paradigm is based more on objectivism (Mackenzie & Knipe, 2006). Objectivism according to Unluer (2012) means that judgements made when collecting and analysing data are not affected by researchers' personal feelings, beliefs and values. Mackenzie and Knipe (2006) further state that researching positivism helps in testing theories through observations and measurements to predict and generalise collected data. Hence, causes and effects, measuring and quantifying the studied phenomena are special features of the positivist paradigm Flick (1998) as cited in (Dean, 2018).

The positivist paradigm is mostly used in quantitative studies because of its subjectivity. Unluer (2012) stated that positivism research methods may include those in line with quantitative studies such as standardised tests, closed-ended questionnaires and descriptions of phenomena using standardised observation tools. However, this study only used standardised tests in testing the effects of manipulative objects in geometry learning in grade11. As emphasized in Mackenzie and Knipe (2006), subjectivism was maintained when analysing data collected from standardised tests. Employing this paradigm helped the researcher to use standardised tests in investigating the effects of manipulative objects in the learning of geometry.

Standardised tests were considered because part of the study investigated the effects of manipulative objects have in geometry problem-solving and the extent to which manipulative objects can help learners in reaching levels 3 and 4 of the VH theory. Thus the researcher believed that standardised tests would give a clear indication of whether there is an improvement in what learners knew or not. The same test was used for pre-test and post-test. As asserted by Doias (2013) using the same test in the pre-test and post-test helps in ensuring the consistency of the results. The standardised test in this study referred to a common test for Mathematics paper 2 which was written by all grade 11 learners in KwaZulu-Natal province for the June examination. The test was set by the provincial department of education's experts in mathematics. It was set such that it has six questions in total. Different mathematical topics were spread across the paper such that questions 1 and 2 assess analytical geometry; questions 3 and 4 assess trigonometry and questions 5 and 6 assess geometry (Euclidean geometry). However, this study's focus is on geometry, thus only questions 5 and 6 were considered. In this regard, the test which was used as a pre-test and post-test consisted of questions 5 and 6. Pre-test and post-test were used to answer research questions 1 and 3 of this study.

3.2.2. Interpretivism paradigm

The interpretivism paradigm is primarily based on the subjectivity which means that the collection, analysis and interpretation of data may be directly affected by researchers' own values and beliefs (Ryan, 2018). In the same way, Pham (2018, p. 3) believes that researchers' experiences which are based on their history and culture may result in one phenomenon to have multiple interpretations rather than having truth which can be measured. As opposed to generalisation, interpretivism paradigm focuses more on an in-depth understanding of a particular phenomenon (Dean, 2018).

The interpretivism paradigm also allows researchers to collect data about how people behave under a certain condition and such may be done based on their own meanings while meanings are generated from social interactions (Ryan, 2018, p. 9). Methodologically, the interpretivism paradigm adopts those tools, techniques and methods which are in line with qualitative research as they enable researchers to interpret and understand texts and have a deeper meaning (Dean, 2018). In line with this, observations and semi-structured interviews underpinned the interpretivism paradigm in this study.

One principle of interpretivism is that it is based on people's experiences including their behaviours and attempt to understand such behaviours (Ryan, 2018). On the basis of understanding people's behaviours, this study conducted observations using a videotape camera. The aim was to observed learners' behaviours during teaching and learning which result in either engagement or disengagement. As noted by Chafouleas, Sanetti, Jaffery, and Fallon (2012), observing learners using videotape cameras may help in capturing learners' actions throughout the lesson and allow observers to pause and replay in order to correctly code learners' actions.

An observation sheet called interval time sampling was used to code learners' behaviours in every 15-second interval (see Appendix A). Interval time sampling is used to systematically observe the occurrence or the non-occurrence of targeted observable behaviours in an equal time interval (Zakszeski et al., 2017). Although the observations conducted in this study are quantitative, however, their analysis makes them underpin the interpretivism paradigm. In this respect, deciding on the occurrence and the non-occurrence of certain behaviours depended on the trained observers' judgements as according to Jackson (1992) a learner may appear engaged when observed while they are not. Hence, observers' judgements with this respect, was based on their knowledge of observable behaviours and their thinking as according to Pham (2018), observers' judgements may not be taken away from their beliefs, values and history. Using this type of observation enabled the researcher to investigate the effects of manipulative objects on learners' engagement time (research question 2).

One advantage of using semi-structured interviews under the interpretivism paradigm is that they allow researchers to probe while socially constructing the reality (Pham, 2018). Also in this study, semi-structured interviews as they underpin interpretivism enabled the researcher to probe while getting learners' views about the introduction of manipulative objects on the learning of geometry.

Semi-structured interviews were used to verify what was observed through the tests and videotape camera. Ryan (2018) assets that conducting semi-structured interviews enabled the researcher to understand the problem under investigation from learners' point of view. The conduction of interviews after observations is important in attaching and verifying meaning to what was observed (Baker, 2006). In this way, some learners may be observed as engaged whereas they are not while others may be observed as disengaged whereas they are engaged, thus, employing interviews helps in verifying such behaviours. Semi-structured interviews enabled the researcher to verify what was observed and to get learners' views about learning geometry with the assistance of geoboards in this study. Eight learners were randomly selected for both observation and semi-structured interviews.

Observing and interviewing the same learners helps to attach meaning to the observed actions Kong et al. (2003). For example, if a learner was observed to be disengaged, it is likely that he/she does not correctly explain the procedure of how they have answered a certain question on a test when they are interviewed. Hence, using interpretivism paradigm with observations and semi-structured interviews helped the researcher to gain a fuller understanding of the meanings, reasons and insights of behavioural actions which happen during learning of geometry.

3.2.3. Participatory action research

This study is a participatory action research. Participatory action research seeks answers to questions faced by researchers in their daily practice (Delport et al., 2005). Problems faced by a teacher when teaching learners in a certain school is one example of illustrating participatory action research (Imenda & Muyangwa, 2006). In this regard, the researcher becomes an insider researcher. Unluer (2012) explains that insider researchers are those who collect data from the group in which they belong. In this respect, learners were taught geometry by the researcher. The researcher in this study is investigating the effects of manipulative objects in the learning of geometry at the school where he teaches.

3.3. Research design

Maxwell (2012, p. 4) defined research design as "the underlying structure and interconnection of the components of the study and the implications of each component for the others". Maxwell further emphasized the importance of working together of components of a study in a harmonious way so as to promote efficient and successful functioning.

This study employed a mixed-methods sequential explanatory design. This design involves collecting and analysing data from both quantitative and qualitative with respect to the priority, sequence and stage of integration (Subedi, 2016).

3.3.1. Priority

Based on the research questions and methods used to collect data in this study, the quantitative part of the study was more prioritised over the qualitative part. This study employed three methods of data collection, namely: tests, observations and semi-structured interviews. Two out of three methods which are tests and observations were used to collect quantitative data. Although the research questions of this are overarching as advocated in Kroll and Neri (2009), the major part of them is addressed quantitatively, thus the study prioritised more the quantitative over the qualitative part.

3.3.2. Sequence

The mixed-methods sequential explanatory design sequence implies that the collection and analysis of data have to be done quantitatively first followed by qualitative later (Ivankova et al., 2006). In this regard, the qualitative part builds upon the quantitative part of data collection and analysis. For example, this study used observations and standardised test to collect data quantitatively and semi-structured interviews were conducted later to collect qualitative data. As advocated in Ivankova et al. (2006), the interviews conducted later helped to explain and verify what was observed during the observations and tests conducted earlier and provided an in-depth learners' views about the effect of manipulative objects in the learning of geometry.

3.3.3. Integration

The findings from quantitative and qualitative measures were integrated at the interpretational stage of the study. As noted in Ivankova et al. (2006), the integration or mixing of quantitative and qualitative may happen in the beginning stages of the study at a point where the research questions from both quantitative and qualitative parts are formulated. In this way, findings from both quantitative and qualitative measures are merged to form one conclusion at the interpretational stage. However, according to Kroll and Neri (2009), it is possible not to reach a state of common conclusion from both quantitative and qualitative parts.

3.4. Research Methods

This participatory action research employs a mixed method. The mixed method is whereby the language, approaches and techniques used in quantitative and qualitative are combined to form a single study (Johnson & Onwuegbuzie, 2004). Investigating the problem from both qualitative and quantitative point of view broadens the researcher's understanding (Doias, 2013). Hence, employing both quantitative and qualitative research methods in this study helped the researcher to investigate the effect of manipulative objects in the learning of geometry from the broader sphere.

As mentioned before in 3.3, the data collection process in this study was first done quantitatively and then the qualitative part came second. This study employed tests and recording sheets to collect data quantitatively and semi-structured interviews were also used to collect qualitative data. Creswell (2014) asserts that, for a study to employ mixed-method properly, findings from both respects must be combined to form one conclusion. Thus, data collected from tests, recording sheets and semi-structured interviews were combined at the integration to verify if one conclusion is reached from both quantitative and qualitative data collection instruments.

3.5 Sampling design

The study used a convenience form of sampling. Convenience sampling is a nonprobability sampling where the sample is made up of participants who are easily accessible to the researcher based on their availability at a given time, the willingness to participate or resources (Etikan, Musa, & Alkassim, 2016). In further explaining convenience sampling Dörnyei (2007) cited an example whereby a study is done with learners from the researcher's own institution. Thus, in this study, the researcher collected data in the school where he works because of being limited by available resources and time. That is, each learner had their own manipulative object, thus it was going to be costly to involve more than one school and make these objects available for each learner and more time was also going to be needed.

The school where the research took place (research site) is a rural school found in King Cetshwayo in the province of KwaZulu-Natal in South Africa. The school comprised both male and female learners and is a government school which is offering grade 8 up to grade12. Each grade at Further Education and Training (FET) (garde10-12) phase had three streams: mathematics and science; commerce and general. Mathematics at FET phase in the school is only done by learners who are enrolled in mathematics and science stream. Thus, the sample in this study consisted of only grade11 learners who enrolled in the mathematics and science stream in the school. The grade 11 mathematics and science group were chosen because they were easily accessible to the researcher in terms of availability, specialisation (geometry) and resources. In this regard, the whole class of 32 grade 11 learners who are enrolled in the mathematics and science stream at the school in 2019 was expected to participate in the study. However, out of 32 learners, only 24 volunteered to participate in the study. Thus, all learners who volunteered to participate were considered in the study.

Further sampling for videotaped observations and semi-structured interviews was done. In this regard, random sampling was used to select eight learners who were going to be observed through a videotape camera and interviewed. Learners who were observed were also interviewed at the end of the intervention. The order used in observing learners from videotape camera was also followed when interviewing them.

3.6. Research instruments

As mentioned earlier, this study employed pre-test, post-test and observations to collect quantitative data. Semi-structured interviews were used to collect qualitative data about how manipulative objects affect geometry learning. Hence a detailed description of how each instrument was used to collect data is explained.

3.6.1. Description of manipulative objects

This study investigates the effects of learning circle geometry with the assistance of manipulative objects as shown in Figures 3.0. Manipulative objects in this study involve geoboards which are specially designed by a researcher to illustrate circle geometry theorems. To design these objects, the researcher needed the following:

- square soft boards (30cm×30cm)
- Nails (13×1.20mm panel pins)
- Paint (green and brown)
- Star and round self-adhesive labels
- Elastic rubber bands of different colours.
- Ruler
- Pencil
- Protractor

The instructions include:

- A circle with a radius of 9.5cm was drawn in the middle of the board using compass and pencil.
- The nail was inserted at the centre of the drawn circle.
- Using the ruler, compass and the pencil, points were marked at the circumference of the drawn circle which were each 5cm apart.
- The line segments which were tangents to the circle were drawn using the ruler, pencil and protractor such that the tangents are perpendicular to the radius or diameter.
- Nails were inserted at each point marked at the circumference of the circle.
- Nails were inserted to each end of the drawn tangent.

- The circular part of the board with an area of about 284 cm^2 was painted green and the remaining area of 616 cm^2 of square board was painted brown which made drawing lines to be invisible See figure 3.0.

Elastic rubber bands were used to represent lines between points, and nails were used to denote points on and outside the circle on the wooden square board. For example, to illustrate a chord, one needed to hook elastic rubber band on two nails at the circumference. More rubber bands were used to create more complicated figures like triangles and quadrilaterals on the circle by hooking rubber bands on different nails on the circle. In a case where rubber bands were used to simulate a certain theorem, star and round self-adhesive labels were used to denote a set of equal angles (see Figure 3.5).

In ensuring that manipulative objects serve what they ought to serve, the researcher considered guidelines as suggested by Cope (2015). The guidelines helped the researcher in connecting the abstract knowledge of circle geometry theorems found in textbooks and their concrete representations on manipulative objects. The summary of the guidelines as stipulated by Jamhari and Wongkia (2018, p. 2) is as follows:

- Manipulatives must have parts or things which can be moved to portray a mathematical process or concept.
- The direct correlation between the process portrayed by the manipulative and the process conducted using pencil and paper must exist.
- Manipulatives must correctly illustrate the actual mathematical process.
- Learning does not derive from a manipulative itself but from students' physical actions on manipulatives.
- The use of manipulative requires explicit instruction.

These objects were specifically used to help learners with visualisation, engagement and problem-solving in this study. According to Balinha and Mamede (2018), geoboards, when correctly used, may enhance learners' visualisation and problem-solving skills in geometry. These objects were designed such that they help learners to visualise equal angles as opposed to measuring. In this respect, shapes and colours were used to denote equal angles and lines, respectively. Star and round self-adhesive labels were used to denote equal angles when constructing theorems on manipulative objects while coloured elastic rubber bands and spaces between the nails on the conference of the circle were used to denote equal lines (see Figure 3.0 and 3.0 a). In Figure 3.0 a, round self-adhesive labels denoted equal angles while equal spaces between the nails as indicated by yellow elastic rubber bands denoted equal chords. All the geometry figures used in this chapter were taken from Siyavula learners' textbook which was used in conjunction with manipulative objects during the intervention period.

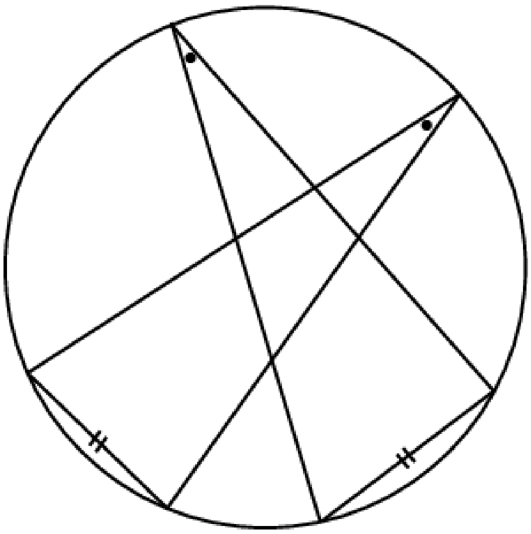


Figure 3.0 : Equal chords subtend equal angles at the circumference

Figure 3.0 a: Equal chords subtend equal angles at the circumference

3.6.2. Description of pre-test and post-test

The effects of manipulative objects towards geometry problem solving and learners' performance on Van Hiele's levels were assessed through written tests. Written tests allow learners' problem-solving, conceptual and procedural understanding to be expressed in writing (Evbuomwan, 2013). As mentioned earlier, pre-test and post-test were the same in this study. As advocated by Doias (2013), administering the same test for pre-test and post-test helps in ensuring the reliability between the tests. Written tests were used to investigate if manipulative objects affect learners' academic achievement in geometry. Written tests enabled the researcher to assess the effects of manipulative objects on learners' problem-solving when learning geometry (research question 1). The tests also helped in investigating whether manipulative objects are able to help learners reach levels 3 and 4 of Van Hiele's model (research question 3).

The intervention took place at the time when learners had already been exposed to the knowledge of geometry. This was because according to the 2019 KwaZulu-Natal Annual Teaching Plan for mathematics in grade 11, geometry was supposed to be taught in term one. However, the instruction which was used in the first term did not incorporate manipulative objects. The diagrams from textbooks were the only tools used to help learners with visualisation. The geometry part in the KwaZulu-Natal (KZN) Department of Education's common test for the second paper of mathematics June examination was used as a pre-test and post-test in this study. This paper assesses what has been learnt in both term one and two. The whole mathematics paper had a duration of two hours with one hundred as a total mark (see Appendix C). As explained in 3.2, the mathematics June examination had six questions in total; however, only questions 5 and 6 were assessing geometry in this paper. Thus, learners' performance in the pre-test and post-test considered in this study is based on questions 5 and 6 of the test, not the whole paper because this study's focus is on geometry.

Learners' pre-test scores were extracted from their performance of the second paper of mathematics June examination. In each answer book, learners' scores for questions 5 and 6 were recorded aside for pre-test learners' performances. Hence, considering learners' pre-test performance helped in finding learners' level of understanding in geometry before the intervention so as to compare them with scores obtained by learners in the post-test, thus evaluating if manipulative objects had assisted learners in reaching levels 3 and 4 of Van Hiele's model of geometry thinking. All question papers were collected soon after learners had completed their pre-test.

The post-test was administered on the last day of the intervention which was the fifth day. Unlike the pre-test, in the post-test, the twenty-four learners who volunteered wrote only questions 5 and 6 and the duration was forty-five minutes with a total mark of thirty-four (34) which was only questions 5 and 6. The test had nine sub-questions with different mark allocations, depending on the level at which the question was set. Questions 5 had 6 sub-questions and three sub-questions were found in question 6.

Both questions 5 and 6 were assessing learners' knowledge of the following theorems: the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre; the angle subtended by the diameter at the circumference of the circle is right angle; angles subtended by a chord (arch) of the circle, on the same side of the chord, are equal; the opposite angles of a cyclic quadrilateral are supplementary and the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. However, the levels at which the theorems were assessed were different. According to Alex and Mammen (2016b), geometry done in grade 11 ranges between level 3 and level 4 of Van Hiele's model, hence geometry set in the June examination was set such that it ranged between levels 3 and 4.

Question 5 was more about the application of the theorems. Learners were required to use the knowledge of theorems in finding the missing angles. In Van Hiele's level, all questions in question 5 were set at level 3 as emphasised by Clements (2003) that learners in level 3 used logical reasoning to justify their statements. In the context of the June examination, learners were expected to use logical reasoning of the theorems assessed in order to find missing angles. On the other hand, question 6 was more about geometry proof. Learners were required to prove the theorem which says the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre in 6.1. In 6.2, learners were required to use the given information in order to prove the cyclic quadrilateral and line parallel. These questions qualify to be at level 4 of Van Hiele's model as according to Crowley (1987) geometry proofs are only done at level 4 of Van Hiele's model. The data collected through pre-test and post-test were used to answer research questions 1 and 3.

3.6.3. Description of classroom observations

To observe as used in research is to carefully collect the data about how behaviours happen, especially in an attempt to understand more about such behaviour (Ciesielska, Boström, & Öhlander, 2018). This may involve using either direct or indirect observations. Direct observations according to Ryan, Daum, Bauman, Grisez, Mattimore, Nalodka, and McCormick (1995) require one to go to an area of observation and collect data using senses which include watching how certain behaviours happen. Indirect observations, as opposed to direct observations, involve using devices to collect data about how certain behaviours take place (Ciesielska et al., 2018). Using videotape cameras to watch how behaviours happen and analyse the video clips later and using videotape cameras in conducting indirect observation are common in social sciences i.e.(Ciesielska et al., 2018; Dignath & Büttner, 2018). Videotaped observations have benefits of being paused and be watched again (Storms, 1973).

All observations of learners' behaviours were conducted with an aid of a videotape camera in this study. One videotape camera was placed in the front of the classroom such that it captured as many learners' behaviours as possible. However, before the intervention takes place, learners' parents were informed that their children would be videotaped and volunteered learners' parents did not have a problem with their children being videotaped. However, some learners did not participate in the study because their parents did not like their children to be videotaped. As a result, 24 out of 32 volunteered to participate. Learners' parents were also told that anonymity will be highly maintained in this study.

De Wet et al. (1981) assert that observations are good when investigating how individuals or groups of people behave under specific circumstances. This includes observing how learners interact with the subject matter under certain conditions.

Observations are one of the research methods underpinning the interpretivism paradigm as they allow knowledge to be socially constructed while allowing researchers' judgements to be shaped by their experiences, beliefs and values (Ryan, 2018).

According to Baker (2006), observations of certain behaviours should be conducted in their natural world setting so as to understand their meanings. This means that the presence of a camera or an unknown person who is there to observe might influence the behaviour of participants. Briesch, Hemphill, Volpe, and Daniels (2015) suggest that the first few sessions may be used to familiarise learners with the presence of cameras in the classroom in a case where learners are not used to such kind of environment. Thus the observations for the first two sessions were not included in the analysis because at this point learners were still not used to the presence of cameras in a classroom. Hence, after two sessions of the intervention learners were observed to be familiar with the presence of cameras in the classroom and started to behave in a normal way. Hence, it is believed that observations thereafter were conducted in a natural classroom setting.

In this study, learners' engagement during the instruction was observed to estimate learners' engagement time. Thus, the term engagement was operationalised to cover both passive and active engagements. On the other hand, disengagement covers all irrelevant behaviours done by learners i.e., playing during the instructional period, out of the seat and making noise or any other actions that are not lesson-related. The observations were based on the following learning actions as shown in Appendix A:

1. Disengaged
2. Listening attentively
3. Asking a question
4. Answering a question
5. Using geoboard
6. Doing exercise
7. Seeking for assistance
8. Assisting other learners.

Each learning action was assigned with a number, i.e. 5 represented “using geoboard”, the same happened with 6 which represented “doing exercise”. In this regard, when a learner does one of the learning actions assigned numbers from 2 to 8 he or she was regarded as engaged, while 1 was assigned to disengaged.

Videotaped observations allowed learners’ behavioural engagement and disengagement to observed using partial time sampling in a thirty-minute long period. The duration of each session was fifty minutes; however, a fifty-minute long session was edited such that it became a thirty-minute long session. As mentioned in section 3.2, using interval time sampling enabled the researcher with an assistance of trained observers to systematically observe the occurrence or the non-occurrence of targeted observable behaviours in an equal time interval as asserted in Zakszeski et al. (2017). To observe and code learners’ behavioural engagement, a thirty-minute long session was broken down into equal time intervals of 15-second (15-s) so as to code learners’ behaviours in a short period of time.

When coding the occurrence of behaviours, it is important to consider a shortest possible time interval because the shorter the time interval, the more accurate the measurements are going to be (Zakszeski et al., 2017). Limited resources in this study, however, influenced the researcher to use 15-s as an appropriate time interval. In this respect, time interval less than 15-s was going to require more observers, time and money.

The first two sessions were not included in data analysis because learners during these sessions were still not familiar with the presence of the cameras and the use of manipulative objects (geoboards). Hence day three of the intervention became day one of observing learners for data analysis purposes. Also, on the last day of intervention learners were not observed as this was used for post-test and interviews. The aim was to observe the amount of time learners would spend being actively engaged (engagement time) in the learning of geometry when manipulative objects were used as scaffolders.

The numbers assigned to each learning action were used by observers to code the presence of each learning action in every 15-s time interval over one-hour long period. They then counted number the number of ones present on the observation sheets and multiplied by 15 as learners were coded after every 15-second time intervals. This made possible for observers to workout engagement time by subtracting time spent on “disengaged” from the total time of observation which was one hour.

3.6.4. Description of Semi-structured interviews

Semi-structured interviews were used to collect qualitative data. They are important in the qualitative data collection because they allow researchers to explain where questions on the interview schedule are wrongly understood and probe to get more information about the problem under investigation (Braynard et al., 2015). Semi-structured interviews were used to get learners’ views about how manipulative objects affected their learning of geometry. Furthermore, semi-structured interviews may also be used to qualitatively address aspects which were not fully addressed quantitatively (Baker, 2006). Also in this study, semi-structured interviews were considered to address the aspects which were not fully addressed in achievement test and observations. For example, the aspect of considering learners’ views about using manipulative objects when learning geometry could not be fully addressed in observations and tests, thus semi-structured interviews were used. Semi-structured interviews were also used to investigate if there is a relationship between the observed learners’ behavioural engagement and what learners think about learning geometry with the assistance of manipulative objects.

The semi-structured interview schedule (see Appendix B) was designed such that it would allow learners to express themselves about how manipulative objects affected their learning of geometry. In this respect, learners were asked which test (questions 5 and 6) between the one they wrote first (pre-test) and the second one (post-test) did they consider as a difficult one and why do they think so. The effects of manipulative objects on learners’ geometry problem-solving were also investigated in the semi-structured interview schedule. In this respect, learners were asked to explain their procedures of geometry proofs asked in questions 6.1 and 6.2 in the post-test. Learners’ responses were written down as they answered each question on the semi-structured interview schedule. The audiotape was also used in addition to pen and paper recording of learners’ responses in interviews. All the interviews conducted took place within the school premises. The interviews were conducted after observations and tests had been completed since this study employed a mixed-methods sequential explanatory design. This design allows the research to collect and analyse data quantitatively first and then qualitative follows.

The eight selected learners for observations were also interviewed. The interviews took place on the last day of the intervention. All eight learners were interviewed on the same day. The interviews were conducted soon after the learners had completed their post-test. The interview sessions ranged between 3 minutes and 10 minutes. The interview schedule had six main questions. The whole interview session lasted about fifty minutes. Multiple data analysis arising from written tests, observations and interviews were compared to find out if there were common patterns that existed regarding the effects of manipulative objects towards the learning of geometry in grade 11. As emphasised by Kong et al. (2003), interviewing the observed learners helps in finding learners' perceived learning. Some learners when observed may appear as engaged whereas they are not, while others may appear as disengaged while they are engaged (Jackson, 1992). Thus, to ascertain engagement or disengagement observed, follow-up interviews were conducted. Baker (2006) highlighted that interviewing the observed learners helps in obtaining accuracy in the findings while promoting criterion validity. The order followed when observing learners was also followed when interviewing learners. For example, the first learner to be observed was also the first one to be interviewed.

3.7. Data quality control

3.7.1 Reliability and Validity of geometry achievement test

Since the test was set by the Department of Education’s experts, thus the test was considered to be valid and reliable. Imenda and Muyangwa (2006) assert that a test is considered to be valid if it measures what it is supposed to measured, while reliability, on the other hand, refers to a consistency of a test to give similar results when administered more than once. In this way, the content which was assessed in the test and the level at which the content was assessed in the test was considered to be valid.

3.7.2. Validity of classroom observations

Supervisors provided some input in the development of the observation sheet called interval time sampling. This was done to ensure the validity of the data collected through interval time observation. The input from supervisors led to revision of interval time sampling twice before it became final (see appendix A). Observations conducted by the researchers themselves have a potential of introducing errors in the collected data thus multiple observers may be used to promote validity and reliability of data collected through observations (Baker, 2006). Hence, two university students were trained to do observations in this study. Before the experimental observation took place, both observers were trained on how to correctly observe and code learners' targeted behaviours with respect to frequency and duration.

3.7.3 Reliability of classroom observations

Video clips for the first two days of the intervention were used in the training of the observers to ensure reliability. A five-minute long video clip was observed by the observers and their scores were discussed with the trainer (the researcher). The training continued until each observer became comfortable with observing. Observers were allowed to observe a video over and over again, as according to Hintze (2005), watching the same video over and over again may be used to ensure intraobserver reliability, thus such is regarded as a test-retest kind of reliability. Intraobserver reliability in this regard refers to a consistency of one observer is observing a particular event at different times (Hintze, 2005). Intraobserver is more concerned about one observer producing the same results after watching the same video clip of the same subjects more than one time.

Each observer's observations based on the five-minute video clip were checked against the master list which was predetermined by the trainer. Determining intraobserver reliability was done by allowing each observer to watch a video clip over and over again until each observer achieved 95% against the criterion. According to Hintze (2005), observers may be regarded as competent in observing when they achieve a percentage above 80% on intraobserver reliability. Both observers achieved 95% intraobserver reliability after viewing the same video of the same subjects in three different sessions. The sessions were an hour apart. The whole training period lasted for about three hours, where the first forty-five minutes of the whole period were used to teach observers about how to correctly observe the targeted behaviours.

Observers' scores obtained in the third session of the training were used to estimate the percentage agreement index of interobserver agreement. The interobserver agreement in this way is the commonness of the occurrence or non-occurrence of certain behaviours between two observers observing the same subjects (Baker, 2006). The percentage agreement index was estimated using the formula: P% $= \frac{Number\ of\ agreements}{Number\ of\ agreements + Number\ of\ disagreements} \times 100$ as cited in (Hintze, 2005). The number of agreements in this respect refers to moments where both observers agree that targeted behaviours happened at specific moments. For example, if both observers agree that, an observed learner was "using geoboard" in the first five intervals of 15-s. The percentage agreement index was obtained to be 55% which is $\frac{11}{11+9} \times 100$ in this study. The 55% interobserver agreement was considered as valid, as according to Hintze (2005), interobserver agreement percentage should be between 20% and 80%.

3.7.4. Validity of semi-structured interviews

To ensure the reliability and validity of the data collected through semi-structured interviews in this study, the interview protocol went through verification and supervisors had their input. The verification assisted in avoiding ambiguity in the questions asked on the interview protocol and ensuring that they measure what they ought to measure as Imenda and Muyangwa (2006) advocated.

3.7.5. Reliability of semi-structured interviews

The interview schedule was also piloted using two learners who were not part of the eight sampled learners for interviews and observations. According to Imenda and Muyangwa (2006), piloting the interview schedule helps in improving the questions on the schedule so as to ensure the appropriateness of the interview schedule in measuring what it is supposed to measure. As a result of verification and pilot, some of the questions were deleted while other questions were added. For example, the interview protocol had four questions initially, however, after the verification process they were six.

3.8. Data collection procedures

3.8.1. Sampling procedure for observation

Researchers need to be careful when deciding which learners to observe while making a sample as heterogeneous as possible (Baker, 2006; Chafouleas et al., 2012). As noted by Briesch et al. (2015), some studies use random sampling when deciding which learners to observe, more especially when dealing with learners with no behavioural problems. This happens mostly on intervention-based studies where the change in behaviour is associated with the intervention's effects. In classroom-wide interventions Briesch et al. (2015) suggested that learners must be sampled since observing the whole class will take more resources and time. In this study, eight learners were randomly sampled from a group of 24 learners in a classroom. This was done to make sure that the sample was as heterogeneous as possible since the focus in this study was on the change in behaviour with respect to the intervention over a period of time. A slip of paper was placed with a name of each learner in a class, then all 32 slips were thoroughly mixed and one slip with learner's name was drawn at a time until eight slips are drawn. Without telling them, the sampled learners were put within the vicinity of a camera so as to catch every move of their behavioural engagement.

Observations of learners' engagement time from a videotape camera were coded using partial-interval recording as an observation sheet to generate quantitative data. Through the videotape camera, observers were able to use partial time recording to observe learners' behavioural engagement and disengagement in a thirty-minute long period.

The eight selected learners for observations were also interviewed. As observed by Kong et al. (2003), interviewing the observed learners helps in finding learners perceived learning of geometry with the aid of geoboards. Some learners when observed may appear as engaged whereas they are not, while others may appear as disengaged while they are engaged (Jackson, 1992). Thus, to ascertain engagement or disengagement observed, follow-up interviews were conducted. As noted by Baker (2006), interviewing the observed participants helps in obtaining accuracy in the findings while promoting criterion validity. The order followed when observing learners was also followed when interviewing learners. For example, the first learner to be observed was also the first one to be interviewed.

3.8.2. Instruction

The data collection of how manipulative objects (geoboards) affect learning of geometry in grade 11 was conducted during the school holidays. This was done to avoid disturbing normal teaching and learning. The intervention took place two weeks after learners had written their second paper mathematics June examination. Questions 5 and 6 of the second paper of mathematics were used as pre-test and post-test in this study. In this respect, intervention took place two weeks after learners had written their pre-test.

The intervention lasted for five days (one week, Monday to Friday). Thus, pre-test and post-test were separated by three weeks. Each day of the intervention had fifty (50) minutes session. However, only thirty (30) minutes of the 50-minute video clip were included in the data analysis. The 50-minute video clip was edited such that it becomes a 30-minute video clip. The observations were done using a high definition digital camera which was placed at the front corner of the classroom where the intervention was taking place.

Each learner was given a geoboard so as to demonstrate theorems on their own. The instruction used throughout the intervention was aligned with phases of Van Hiele's model of geometry thinking, namely: information, direct orientation, explicit and free orientation phase.

At the information phase, learners' prior knowledge based on the geometry of circle was obtained. Learners were asked to define terms like radius, diameter, bisect, tangent, chord, arch, centre, subtend and segment. The researcher explained each term after different responses from learners. It was also at this phase that the researcher explained to learners how manipulative objects are used.

Direct orientation phase involved a practical part of what was done in the information phase. In this respect, learners used elastic rubber bands to demonstrate a radius, diameter, tangent, chord, arch, and segment on manipulative objects. The demonstration enabled learners to practically explore different parts of the circle like the difference between minor and major arch; minor and major segment. The researcher introduced learners to geometry theorems after a thorough practical exploration of different parts of the circle and showing learners how elastic rubber bands and self-adhesive labels are used on the geoboards. Beyond this point, the researcher facilitated the learning of geometry throughout the intervention period without intervening on learners' practical explorations of theorems. In this respect, the pictures of how manipulative objects were used to explore theorems as shown in Figure 3.3a, b and c were constructed by learners themselves.

The exploration of each theorem involved using elastic rubber bands, star and round self-adhesive labels on manipulative objects as explained earlier in Section 3.6.1. The labels, however, were used to enhance learners' visualisation skills. As a result, learners' different explorations of a theorem which says the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle circumference (on the same side of the arc/chord as the centre) on their manipulative objects are shown in Figures 3.1, 3.2, 3.3. Learners put two labels at the centre and one label on the circumference to show that the angle at the centre is twice the angle at the circumference provided they are subtended by the same chord or arch. As learners were practically exploring the theorem as shown in Figure 3.1, they had to recall the knowledge of exterior angle of a triangle which is equal to the sum of the interior opposite angles. As suggested in Polya (1962), the first step in proving this theorem is to let learners explore the theorem as shown in Figure 3.1. as it is after understanding this representation that learners will be able to explore Figure 3.2 and 3.3.

Figure 3.1: Learner exploration of angle at the centre is twice the angle at the circumference

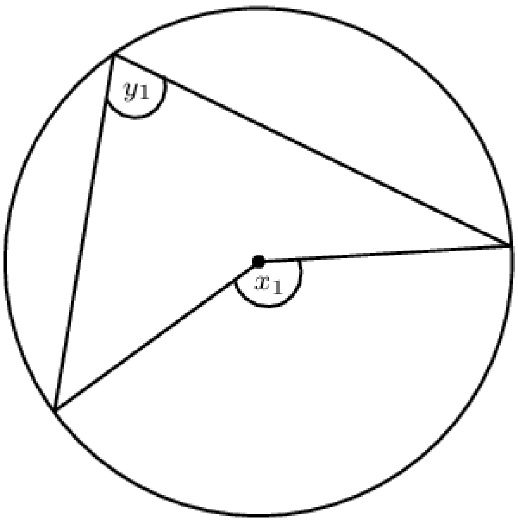


Figure 3.2: Angle at the centre is twice the angle at the circumference

To prove the theorem illustrated in Figure 3.2, a learner used the knowledge obtained in Figure 3.1. In Figure 3.2 a red elastic rubber band was used to show construction which resulted in two triangles as shown in Figure 3.2 b and 3.3a. From the construction, radii were formed and radii are equal in length, thus, angles opposite equal sides are equal. Hence, a learner used stars and rounds self-adhesive labels to denote equal angles in each triangle. With the activity done in Figure 3.1 serving as an auxiliary problem, the learner put two stars and two rounds self-adhesive labels on the exterior angle of each triangle as the exterior angle of a triangle is equal to the sum of the interior opposite angles in each triangle. A learner eventually noticed that at the centre, there are two stars and two round labels while there is one star and one round label at the circumference, hence the conclusion in Figure 3.2c and 3.3b was reached, respectively. That is, the angle subtended by an arc at the centre of a circle will always be twice the size of the angle subtended by the same arc at the circle circumference.

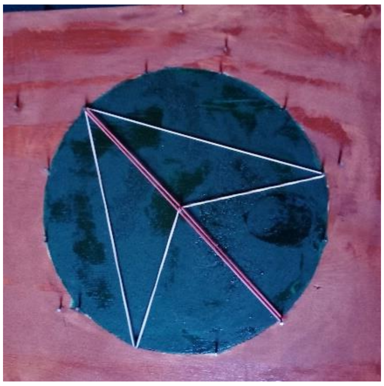
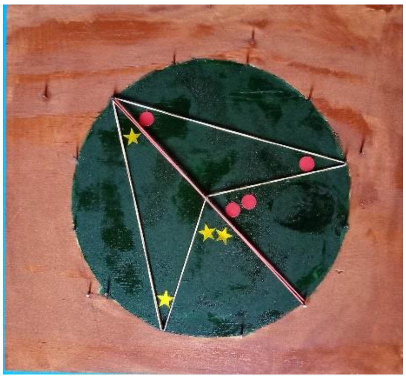




Figure 3.2a **Figure 3.2b** **Figure 3.2c**

Figure 3.2a, b and c show learner's exploration of angle at the centre

Another exploration by another learner of the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle circumference (on the same side of the arc/chord as the centre) is shown in Figure 3.3. This was done the same way as it was in Figure 3.2. The practical exploration of Figure 3.3 shown in Figure3.3a resulted in the conclusion shown in Figure 3.3b which is the same conclusion as that of Figure 3.2. These activities aimed to engage learners in a practical exploration of the theorem in question while engaging them in geometry problem-solving.

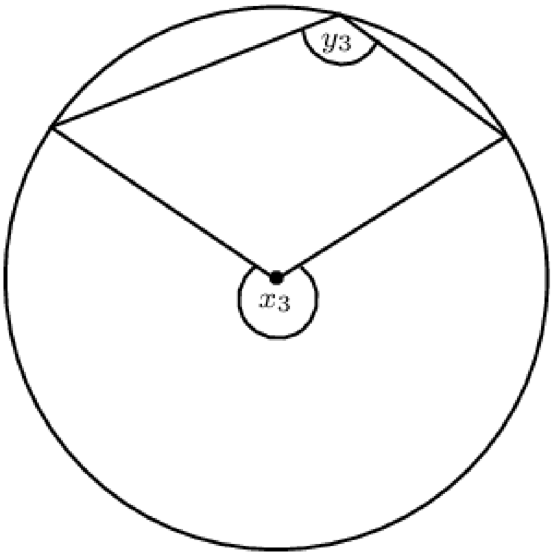


Figure 3.3: Angle at the centre is twice the angle at the circumference

Figure 3.3a **Figure 3.3b**

Figure 3.3a and b show learner's exploration of angle at the centre

Learners were encouraged to rotate manipulative objects so as to get a different orientation of their constructions as according to Naidoo and Kapofu (2020) the orientation of a figure may confuse learners when they are not used to such orientation. For example, when figures used to illustrate angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle circumference (on the same side of the arc/chord as the centre) are always facing upward, learners become confused when in the examination they find the figure turned upside down. For creativity purposes, learners were also discouraged by the researcher to copy other learners' constructions, instead, they may ask for assistance on how they have done it and do theirs differently. This allowed learners to represent one geometric idea in many different ways and such according to Polya (1962) enhances their problem-solving abilities.

Figure 3.4, shows learner's exploration of a proof of "angles subtended by a chord (arch) of the circle, on the same side of the chord, are equal". Through knowledge acquired in Figure 3.2, a learner was able to see that angles Q and P are both twice angle AOB, hence they are also equal. As shown in Figure 3.4a, a learner further complicated a figure to show their understanding by including another the third angle on their manipulative object without changing the meaning of the theorem at hand.

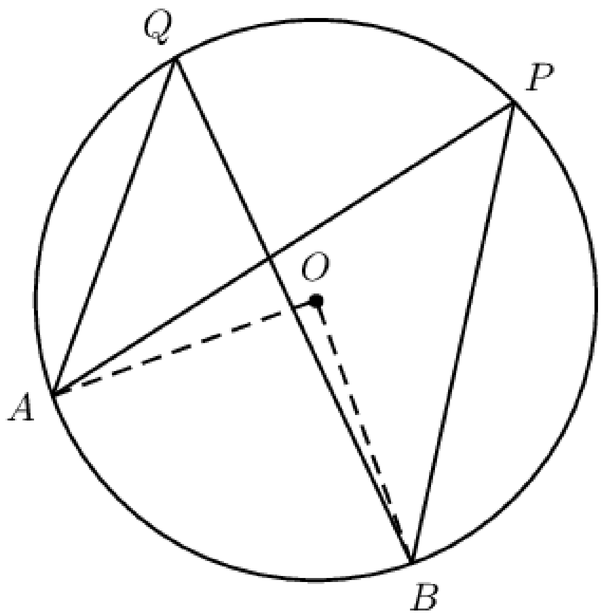


Figure 3.4: Angles on the same segment are equal

Figure 3.4a: Learner’s exploration of angles on the same segment are equal

Moreover, the evidence of learner’s understanding of the application of the tan-chord theorem which means that the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment as shown in Figure 3.5 was noted. A learner used a pair of star and round self-adhesive labels to show sets of equal angles.

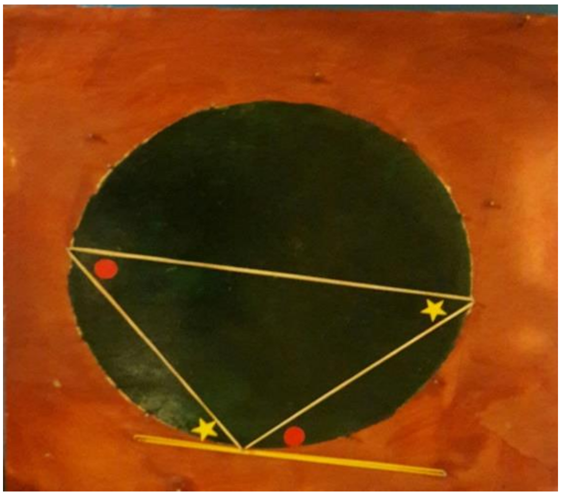


Figure 3.5: Tan-chord theorem

Also, learners’ exploration shown in Figure 3.6 was evidence that they understand the application of the theorem which says “the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle”. The researcher did not interfere with learners’ practical explorations of theorems, however, he encouraged them to share their representations with other learners to promote active learning, while encouraging as many different representations of the same theorem as possible.

Figure 3.6: Exterior angle of a circle quadrilateral

Manipulative objects were also used to prove the theorem which says that the opposite angles of a cyclic quadrilateral are supplementary. Learner first constructed Figure 3.7a then added what is shown in Figure 3.7b to form Figure 3.7c which is a combination of the two figures showing that “the angle at the centre is twice the angle at the circumference”. Angles around the centre are adding up to 360^o (revolution). This implied that two rounds plus two stars give 360^o as an answer, then divided each term by two. This resulted in one round + one star $=180^o$.

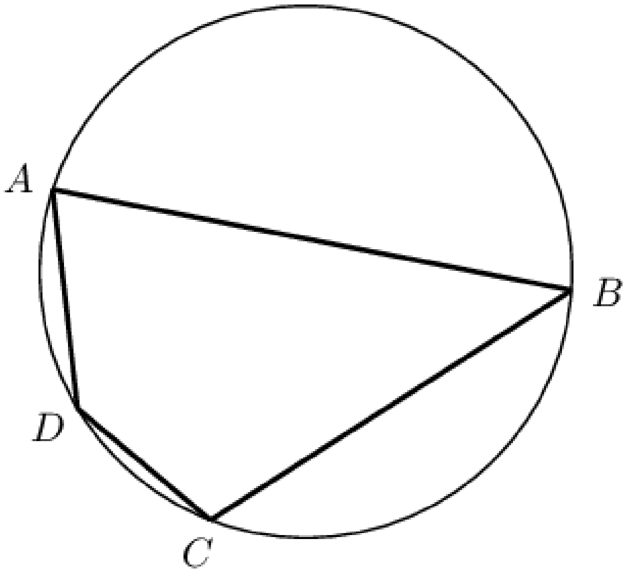
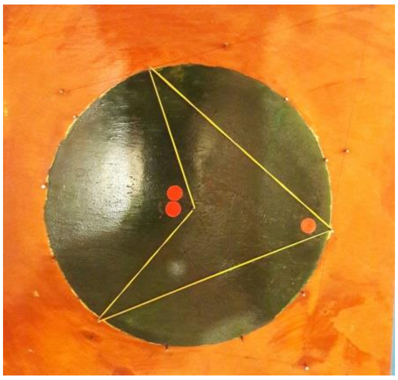


Figure 3.7: The opposite angles of a cyclic quadrilateral are supplementary



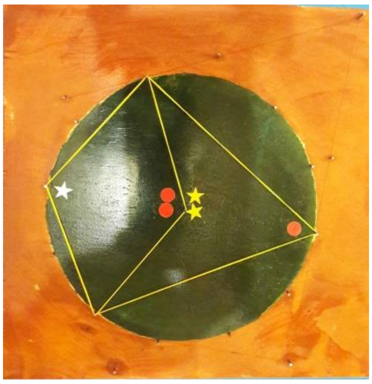


Figure 3.7a **Figure 3.7b** **Figure 3.7c**

Figure 3.7a, b and c show learner's exploration of opposite angles of a cyclic quadrilateral are supplementary

At the explicit phase, learners were encouraged to share their constructions of different theorems on manipulative objects with other learners. Sharing constructions allowed learners to discuss how they had constructed what was shown on their manipulative objects. The role of a teacher is minimal at this phase (Crowley, 1987). Thus the researcher's role at this phase was to facilitate learners' discussions.

Activity 1 was given to learners to assess the level of understanding of geometry theorems learnt. Given FH// EI and angle EIF=15°, determine the value of b in the Figure 3.8. Learners were able to see that angle EIF=IFH=15°because alternating angles are equal as lines FH// EI. They further, stated that b=IFH= 15° as they are angles on the same segment. One learner further stated that b= IFH=HEI= 15° as they are all subtended by the same arch HI.

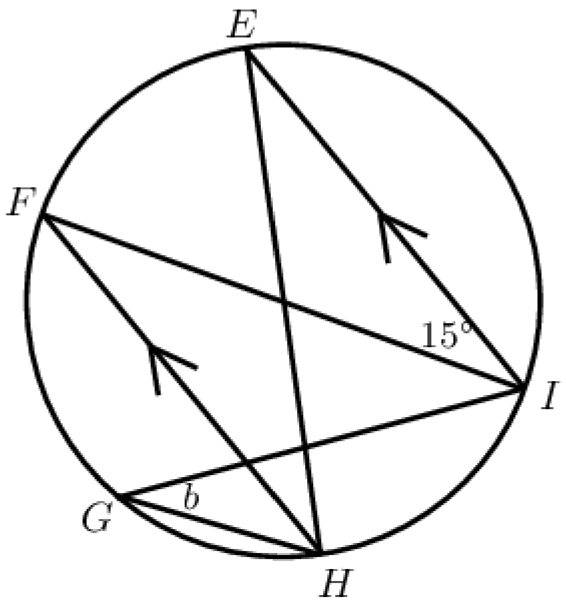


Figure 3.8: Activity 1

At the free orientation phase, learners had to solve geometry riders on Figure 3.9 with the following instruction: In Figure 3.9, O is the centre of the circle and SPT is a tangent, with OP⊥ST. Determine a, b and c, giving reasons.

One learner used the manipulative object in deconstructing and constructing a given geometry rider by adding or subtracting elastic rubber bands on a manipulative object as shown in Figures 3.9a, 3.9b and 3.9c. Breaking down a given geometry rider into constituent theorems enabled learners to see how many theorems are assessed in a particular geometry rider.

To calculate the value of "b" and "c", a learner used labels rounds and stars to denote equal angles on a figure. This enabled a learner to see that angle b=64^{o} (tan-chord theorem), while angle C=128^{o} (angle at the centre is twice the angle at the circumference). This activity was done to enhance learners' problem-solving skills in geometry which was question 1. The evidence of problem-solving with respect to isolation and combination.

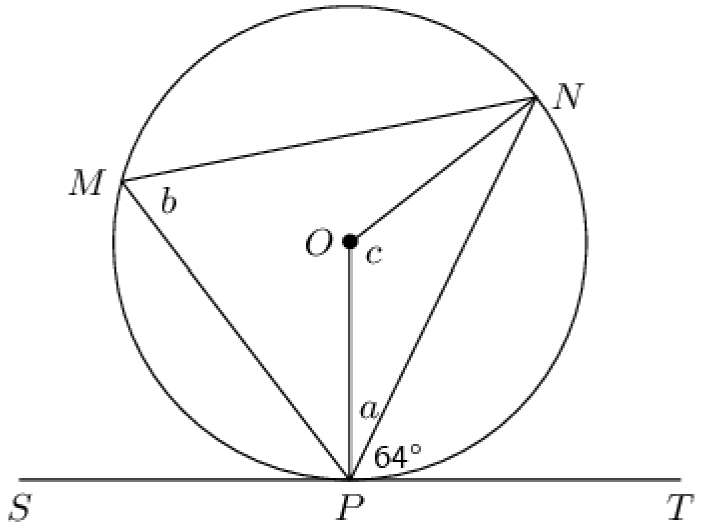


Figure 3.9: Activity 2

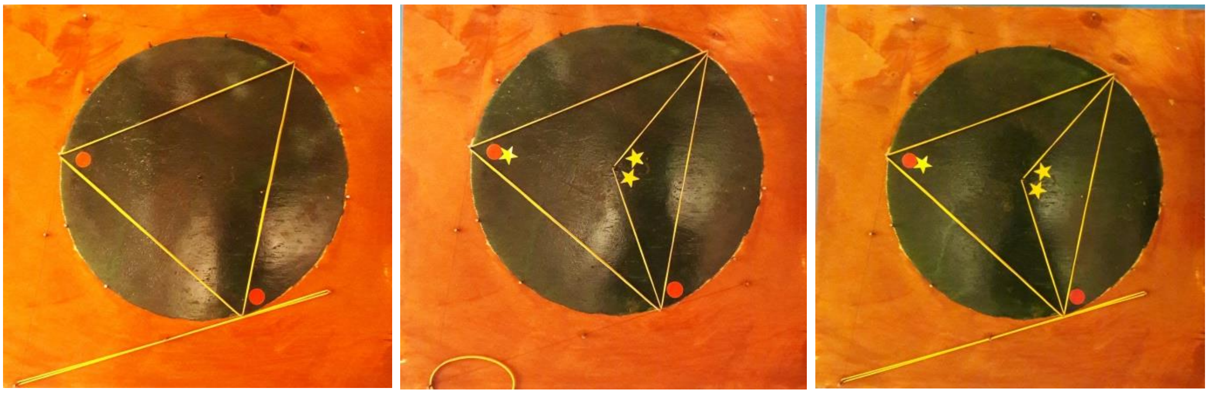


Figure 3.9 a **Figure 3.9b** **Figure 3.9c**

Figure 3.9a, b and c show learner's evidence of isolation and combination

Learners were given that AB=AC, AP∥BC and angles $A_2=B_2$. They were then required to prove that PAL is a tangent to the circle ABC. The majority of learners assumed that PAL is a tangent to the circle ABC then applied the theorem instead of proving i.e. angles A_2=ACB and A_1=ABC with the reason being tan-chord. However, although many learners seemed to be struggling with answering Activity 3 as shown in Figure 3.10, one learner decided to ignore line segment PB when using the manipulative object to help them answer the question. Figures 3.10a and b show how a learner justified using a manipulative object that indeed PAL is tangent to circle ABC. A learner constructed Figure 3.10a to show that angles ABC and ACB are equal because it was given that AB=AC, thus angles opposite equal sides are also equal. He further constructed Figure 3.10b to show that angles P_1 and ACB are equal because alternating angles are equal as AP∥BC. This led to the conclusion that PAL is tangent to circle ABC as the angle between the tangent and chord(P_1) is equal to the angle subtended by the same chord on the alternate segment angle ABC which is equal to angle ACB.

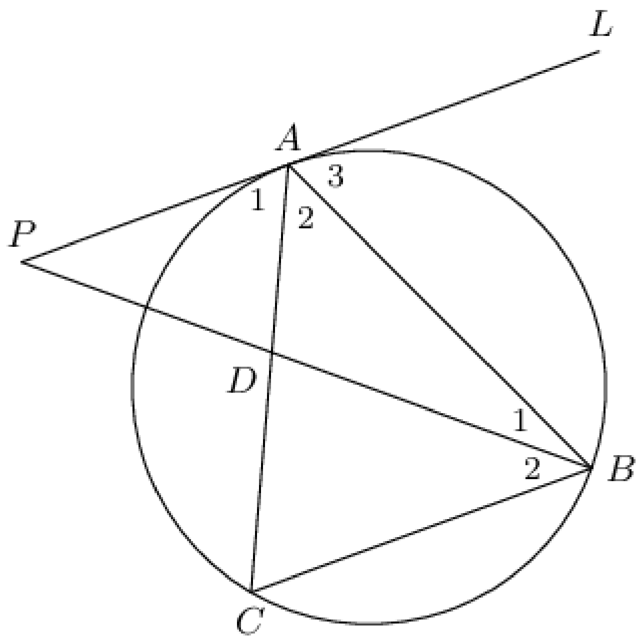
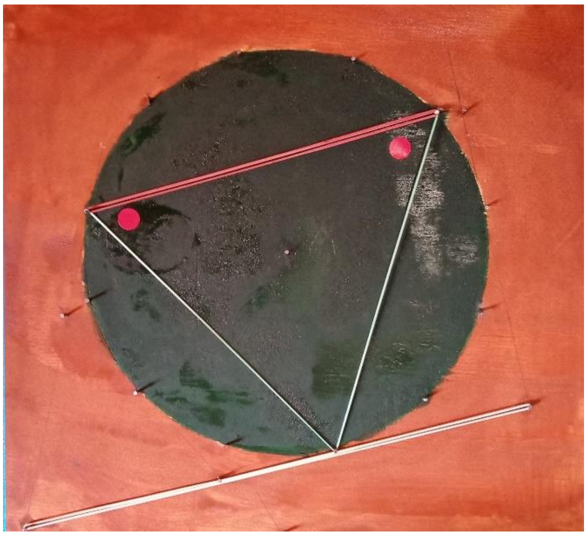


Figure 3.10: Activity 3



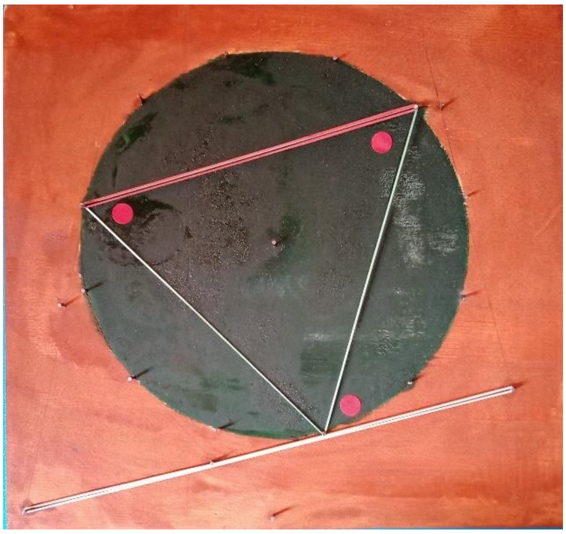


Figure 3.10a **Figure 3.10b**

Figure 3.10a and b show learner's evidence of isolation and combination

A summary of what learners understood was done in the integration phase which served as a conclusion part of each lesson. All what learners were doing was captured by a video camera which was placed in the front corner of the classroom.

This study was limited to the following selected circle geometry theorems:

- The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre).
- Angles subtended by a chord (arch) of the circle, on the same side of the chord, are equal.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle; the opposite angles of a cyclic quadrilateral are supplementary
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.

The theorems were taught in the same order as illustrated above. Accordingly, one theorem was taught in one day i.e. first theorem was taught on Monday and the last was done on Thursday. The fifth day of the intervention was used for post-test and conduction of interviews. All eight learners were interviewed on the same day. The interview sessions ranged between three minutes and ten minutes on each of the interviewed learners. In this regard, the entire interview session lasted for about fifty minutes. Learners' responses obtained from interviews were transcribed and analysed using thematic analysis. To ensure that learners' responses were taken verbatim, the interviews were recorded following the order of observation.

3.9. Ethical considerations

Ethical considerations in research mean adhering to principles, standards and norms of conduct that govern participants (Wilkins, 2017). Taking into consideration participants' ethics when conducting research helps to protect the rights of participants and such rights according to Connelly (2014) include a right to participate which must be voluntarily and a right to withdraw from the research at any time; such rights must be reflected on the informed consent where participants in the study are minors.

Since participants in the study were minors, they obtained parental consent to participate and parents were also informed that learners were going to be videotaped before the intervention commenced. To ensure that participants understood what was being asked of them as emphasized in (Connelly, 2014), the researcher used participants' home language when explaining their rights in a meeting which was conducted prior the intervention. In the meeting, participants were told about the aim of the study and its significance in both teaching and learning of geometry.

All participants were told before participating that their participation in this study was voluntary and they were not forced to participate, should they at any point feel that they did not want to continue participating, they were free to do so. Participants were told that there are no material things such as money which they are going to get for participating in this study. Also, participants were told that, data collected from them through videotaping and interviews particularly was only going to be used in this study not anywhere else and that their confidentiality was going to be maintained at the highest level. Permission obtaining from the KwaZulu-Natal Department of Education and the ethical clearance from the University of Zululand's Research Ethical Committee (UZREC) which enabled the researcher to conduct the study.

3.10 Summary

This chapter explained the research methodology followed by the researcher when investigating the effects of manipulative objects in scaffolding learners' geometry learning at grade 11. Research instruments in line with the objectives of this study were also discussed, namely: achievement test; observation sheets (interval time sampling) and interview schedule. Chapter 4 presents the analysis of data collected from achievement tests, observations and interviews.

CHAPTER 4: PRESENTATION AND ANALYSIS OF DATA

4.1. Introduction

This chapter presents the results and analysis of the data found from written tests, observations and interviews. Quantitative data were collected through written tests and observations and semi-structured interviews were used to collect qualitative data. The detailed presentation and analysis of the data collected from written tests, observations and interviews was with respect to the research questions of the study namely:

1. Does the incorporation of manipulative objects enable learners to engage in solving geometry problems?
2. What effects do manipulative objects have on learners' engagement time in geometry learning at grade 11?
3. To what extent are manipulative objects able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking?

The data collected from written tests and observations were analysed using Microsoft Excel. Meyer and Avery (2009) argued that Microsoft Excel may be used to analyse data collected from both quantitative and qualitative methods. They further argued that Microsoft Excel has an ability to handle a large amount of data and allow data collected to be displayed in different ways. Despite the ability that Microsoft Excel has, having 24 participants in this study was also a reason for the researcher to consider Microsoft Excel in analysing data. In this study, Microsoft Excel data tabulations were used in the analysis of data collected quantitatively. This means that data collected from tests and observations were analysed using Microsoft Excel. In this respect, percentages and averages shown in Tables 1-4 and Figure 2 were calculated using Microsoft Excel.

The data collected from semi-structured interviews were analysed using thematic analysis. "Thematic analysis is an interpretive process, whereby data is systematically searched to identify patterns within the data in order to provide an illuminating description of the phenomenon" (Smith & Firth, 2011, p. 3). Further, Morse and Richards (2002) stated that identifying patterns involves developing data code system which must be linked in order to form themes used to describe the problem under investigation. Also in this study, the codes were developed from learners' responses on how manipulative objects affected learner's geometry learning. Semi-structured interviews were used to get learners' views about how manipulative objects affect their geometry learning. The semi-structured interviews were conducted with the eight learners who were observed. The interviews were conducted after the tests and observations were conducted. Conducting interviews after tests and observations assisted in investigating whether there is a connection between quantitatively and qualitative data collected about how manipulative objects affect the learning of geometry. To ensure accuracy in the transcription of learners' responses in the semi-structured interviews, the researcher listened to a voice recorder a couple of times.

4.2. Engagement of learners to problem-solving through manipulative objects

The analysis and presentation of results with respect to research question 1, "Does the incorporation of manipulative objects enable learners to engage in solving geometry problems?", were done in two respects, tests and semi-structured interviews. This means that the data collected from written tests and semi-structured interviews were used to answer research question 1.

4.2.1 Test

The effects of manipulative objects on learners' problem solving were measured using written tests which were pre-test and post-test. As mentioned in 3.6.2, pre-test and post-test were the same. The pre-test was administered at the beginning of the intervention and the average score for 24 learners was 6.3% as shown in Table 1. Table 1 presents pre- and post-test scores for 24 learners.

Table 1: Pre- and post-test learners' score percentages on geometry achievement test (N=24)

Learner	PRE-TEST %	POST-TEST %
A	6	29
B	12	29
C	15	41
D	0	26
E	9	62
F	0	6
G	0	41
H	0	18
I	6	35
J	18	38
K	0	32
L	9	41
M	9	44
N	9	24
O	15	47
P	9	29
Q	3	21
R	0	59
S	9	41
T	0	26
U	0	18
V	18	21
W	0	31
X	0	44
Average	**6,3**	**34.0**

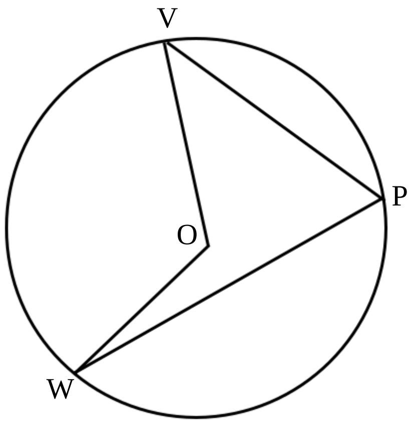
An improvement on the average scores from 6.3% on the pre-test to 34.0% on the post-test as shown in Table 1 may be associated with the introduction of manipulative objects which may have enhanced geometry problem-solving abilities in learners' minds.

4.2.2 Semi-structured interviews

Problem-solving is one of the themes which were identified from learners' responses when they were interviewed. Learners' responses from interviews showed that manipulative objects developed their problem-solving abilities in geometry. Learners indicated that manipulative objects helped them to explore one theorem in many different ways. They also stated that they were able to identify and separate each theorem in a given rider using manipulative objects. This was evidence of the enhancement of problem-solving abilities in terms of isolation and combination which could also be seen on how learners responded on written activities which were given during the intervention period and post-test. Learners' responses include the following, "Manipulative objects helped me to easily identify and separate each theorem when they are combined in a figure", "I was able to make use of elastics break down a figure with many theorems on it, so that I see how many theorems are there", "…construct theorems on my own, in my own way".

It was also found that in some aspects, problem-solving abilities were not fully developed. This was noticed from learners' inability to explain how they have answered questions 6.1, 6.2.1 and 6.2.2 on the post-test. This happened mainly on "to prove" type of questions. In these questions, learners were required to perform geometry proofs.

6.1 In the diagram, O is the centre of the circle. VP and WP are chords, and VO and WO have been drawn.



Use the diagram on the DIAGRAM SHEET to prove the theorem which states that an angle that an arc subtends at the centre of a circle is twice the size of the angle subtended by the same arc at the circle i.e. $VOW = 2P$.

The following illustrates learners' incomplete explanations of their geometry proofs as required in 6.1.

Interviewer: Explain with the aid of geoboard how you have answered 6.1?

Learner B: "Join OP produced to K. Then I said let P1=V OP=OP because of radii. I do the same to the second triangle".

Interviewer: But in the question you were asked to prove that angle VCW=2×angle P, do you think that the proof is complete?

Learner B: "eish eish eish".

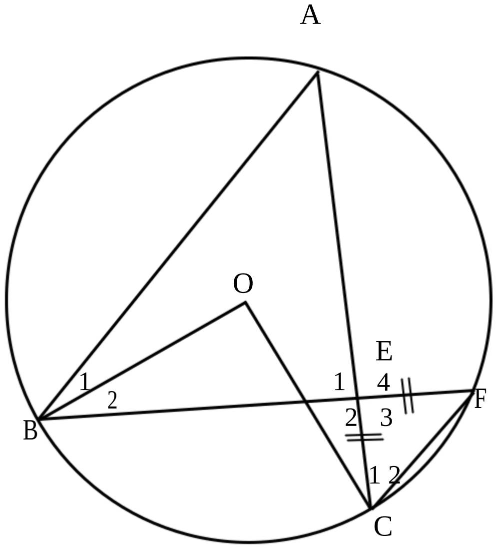
Learner E: "I constructed line, angles P1, P2 and O1 and O2 were formed. P1 is opposite to V they both equal to x; P2 opposite to W they are equal to y".

Interviewer: Why is p1=V?

Learner E: "Because, angle, angle P is, it is better to write it down than explaining".

Similar results were also found in question 6.2. Learners were also unable to explain how they have written their proofs.

6.2 In the diagram, O is the centre of the circle. A, B, C and F are points on the circumference. AC and BF intersect in E and EF = FC.



Prove that:

6.2.1 AB || FC.

6.2.2 OBCE is a cyclic quadrilateral.

Unlike question 6.1 where learners attempted to explain their procedure with a few steps, in 6.2 some learners did not even attempt to explain their procedures as to how they had answered. Their responses included the following, "I did not answer this question". "This question was difficult to me". Also, there was less connection between what the questions demanded and what learners were saying when interviewed. Learners' responded as follows when asked Why is line AB//FC?

Learner L: "Alternating angles are equal".

Interviewer: Which are those angles?

Learner L: "Angle A and F".

Interviewer: How do you know that they are equal?

Learner L did not respond.

Learner T: "…alternating, corresponding and vertical opposite angles angle, so angle A= angle C2 they alternating".

Interviewer: Ok, but how do all these angles assist you to prove that AB//FC?

Learner T: "Because corresponding angles are equal".

Interviewer: How do you know?

Learner T did not respond.

Interviewer: Did you answer question 6.2.2 in the second test, if yes, how?

Learner G: "Yes, I answered it but I did not complete answering it".

Interviewer: Can you please explain what you have written.

Learner G: "No, I don't remember".

Learner O: "Yes, but I was not sure".

Interviewer: Do you still remember what you wrote in 6.2.2?

Learner O: "No".

Learner Q: "Yes, I tried."

Interviewer: Can you please explain how you have tried answering.

Learner Q: "E=C2 they are alternating angles".

Interviewer: Ok, please explain further.

Learner Q: "I only wrote this".

4.3. The effects of manipulative objects on engagement time

Observations were used to answer research question 2 which says "What effect do manipulative objects have on learners' engagement time in geometry learning at grade 11? Learners were observed using a videotape camera in order to investigate the amount of time learners spend being engaged when learning geometry with the aid of manipulative objects during the intervention. The video clips shot were then watched by observers in order to code learners' actions in every 15-second interval over one-hour long period for each learner observed. This means that observers spent eight hours in total observing the presence of observable behaviours in every 15-second interval in each learner as there were 8 learners observed.

4.3.1. Observations

Microsoft Excel data tabulations of frequency distribution together with percent distributions were used to analyse data collected from observations. A frequency distribution was used to locate the number of 15-seconds spent by a certain learner in each learning action as shown in Table 3. The percent distribution displayed the proportion of time spent by the observed learners on each learning action as shown in Figure 2.

Observations were used in this study to address research question 2. In this respect, observations were conducted to investigate how manipulative objects affect learners' engagement time in geometry learning. To answer this question, eight learners were randomly sampled and their actions were observed in each and every 15-seconds time interval over an hour-long period which was made by two thirty-minute long sessions. As stated in Jackson (1992) that deciding whether one is engaged or not may be problematic as some learners may appear engaged in meaningful learning while they are not and others may appear disengaged whereas they are engaged. For this study, learners' actions were assumed to be true reflections of their thoughts.

Based on the interpretivist paradigm, knowledge is socially constructed and acquired through human interaction with objects, phenomena, experience and environment (Dean, 2018; Pham, 2018). The analysis of data collected from observations on how learners learn geometry was based on the assumptions of the interpretive paradigm. That is, the definition of engagement was limited to the targeted behaviours. Whether or not a certain learner is engaged at a specific period, depended mainly on the judgement made by trained observers as in interpretivism the manner in which data is collected and analysed may not be separated from their thought which is shaped by their beliefs, values, history, experiences and culture (Ryan, 2018).

To this end, learners' actions over one-hour long period were coded using the following eight learning actions as shown in Appendix A: 1. Disengaged 2. Listening attentively 3. Asking a question 4. Answering a question 5. Using geoboard 6. Doing exercise 7. Seeking for assistance 8. Assisting other learners. In this regard, each learner was observed to be doing one of the eight stated actions in every 15-second during the intervention as shown in Table 3. For example, if leaner B was listening in the first ten 15-s sessions, the observer would write ten “2_s” in the corresponding spaces on the observation sheet (see Appendix A), the same would happen if the same learner was using geoboard for the next ten sessions of 15-s, the observer would put ten “5_s” in the corresponding space. The same happened with other learning actions depending on what was observed. At the end of one hour, the observer counted the number of sessions of 15-s spent by one learner on each learning activity and the summary is shown in Table 2. This means that it took eight hours to observe all eight learners.

Table 2: Engagement time (in 15-seconds intervals) in geometry learning (N=8)

	LEARNING ACTIONS										
LEARNER	1	2	3	4	5	6	7	8	**Engaged**	**Disengaged**	**TOTAL**
B	36	121	0	0	70	0	4	9	204	36	240
E	21	75	1	5	118	20	0	1	220	21	240
G	72	80	0	7	52	26	1	2	168	72	240
J	48	93	0	4	69	24	2	0	192	48	240
L	48	104	0	2	75	11	0	0	192	48	240
O	44	85	0	0	78	28	4	1	196	44	240
Q	31	103	0	0	83	22	0	1	209	31	240
T	66	87	0	8	60	0	12	7	174	66	240
Total	366	748	1	26	605	131	23	21	1555	366	1920
average	81,3	166,2	0,3	5,8	134,4	29,1	5,1	4,7	345,6	81,3	

key: 1 -Disengaged;2- listening attentively; 3- asking a question; 4 - answering a question;5- using geoboard; 6- doing exercise; 7- seeking for assistance; 8- assisting other learners

Table 2 shows the number of moments of 15-s each learner spent on each learning action over 240 15-s intervals in one hour-long period. For example, learner B has spent 36 out of 240 moments of 15-s interval being disengaged; 121 moments were spent on listening attentively; 0 moments of 15-s were spent on asking a question, answering a question and doing exercise while 70, 4 and 9 moments of 15-s were spent on using geoboard, seeking for assistance and assisting other learners, respectively.

The average moments of 15-s intervals spent by learners on each action were calculated as shown in the last row in Table 3. This means that, for example, learners spent an average of 81,3 moments of 15-s intervals being disengaged; 166,2 moments listening attentively; 0,3 moments of asking questions; 5,8 moments of answering questions; 134,4 moments using geoboards; 29,1 moments doing exercises; 5,1 moments of seeking for assistance and 4,7 moments of assisting others. This means that, for disengagement, learners spent 81.3 moments doing irrelevant behaviours such as playing during the instructional period, out of the seat and making noise or any other actions that are not lesson-related.

Table 3 shows the amount of time in minutes each learner spent doing one of the eight stated actions. In this regard, Table 4 contains a conversion of the number of 15-s each learner spent on each action into minutes. For example, the 30.3 minutes which learner B spent on listening attentively was obtained by multiplying the number of 15-s spent by learner B on listening which is 121 by 15 to get 1815 seconds and divided the answer by 60 seconds to convert it to minutes. In this respect, Table 3 is derived from the raw data presented in Table 2.

Table 3: Engagement time (in minutes) in geometry learning (N=8)

	LEARNING ACTIONS									
Learner	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Engaged (%)**	**Disengaged (%)**
B	9,0	30,3	0,0	0,0	17,5	0,0	1,0	2,3	85	15
E	5,3	18,8	0,3	1,3	29,5	5,0	0,0	0,3	92	8
G	18,0	20,0	0,0	1,8	13,0	6,5	0,3	0,5	70	30
J	12,0	23,3	0,0	1,0	17,3	6,0	0,5	0,0	80	20
L	12,0	26,0	0,0	0,5	18,8	2,8	0,0	0,0	80	20
O	11,0	21,3	0,0	0,0	19,5	7,0	1,0	0,3	82	18
Q	7,8	25,8	0,0	0,0	20,8	5,5	0,0	0,3	87	13
T	16,5	21,8	0,0	2,0	15,0	0,0	3,0	1,8	73	27
Average	**11,4**	**23,4**	**0,0**	**0,8**	**18,9**	**4,1**	**0,7**	**0,7**	**81**	**19**
%	**19,1**	**39,0**	**0,1**	**1,4**	**31,5**	**6,8**	**1,2**	**1,1**		

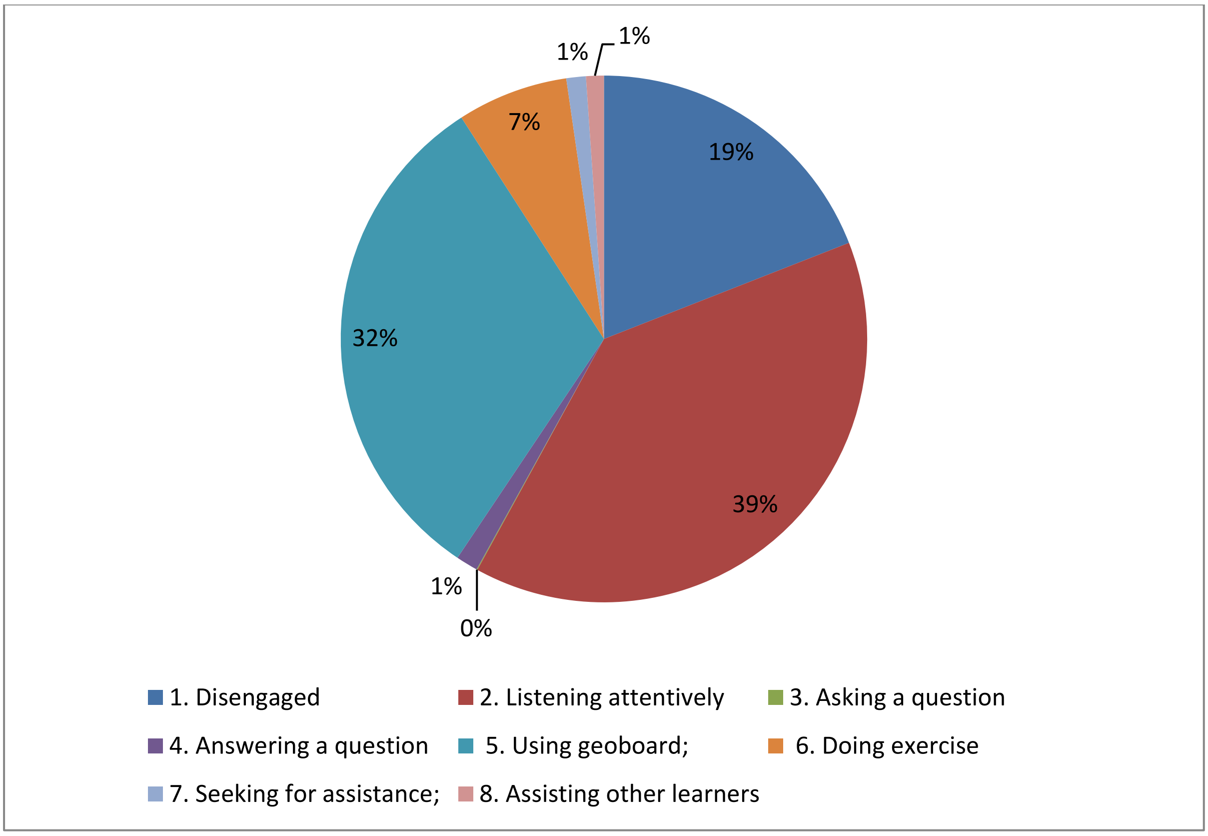
key: 1 -Disengaged;2- listening attentively; 3- asking a question; 4 - answering a question;5- using geoboard; 6- doing exercise; 7- seeking for assistance; 8- assisting other learners

As shown in Table 3, learners spent up to 81% on average in an hour-long period being engaged. In other words, learners were found to be engaged for about 48.6 out of 60 minutes when learning geometry using manipulative objects. The engagement time of 48.6 min was found by adding all learning engagement actions, that is the average amount spent on 2- listening attentively; 3- asking a question; 4 - answering a question;

5- using geoboard; 6- doing exercise; 7- seeking for assistance; 8- assisting other learners except for 1-disengagement which is shown in the first column of Table 3.

A pie chart as shown in Figure 4.1 presents the average amount of time in percentages which was spent by learners on each learning action with respect to other learning actions.

Figure 4.1: Learners' engagement time in geometry learning



As seen in Figure 4.1, learners spent most of their time listening attentively and using geoboards. Furthermore, learners' disengagement actions were also found to be the third-highest, following using geoboards and listening attentively, respectively. An average of 39% in an hour-long period was spent on listening while 32% of the time was spent on using geoboards and 19% spent on disengagement. A negligible amount of time spent on asking questions was noted, as the results indicate that it is almost 0%, unlike seeking assistance, assisting other learners and answering questions where learners spent an average of 1% on each learning action as shown in Figure 4.1.

4.4. Scaffolding learners to reach level 3 and 4 of Van Hiele's model

Written tests and semi-structured interviews were also used to answer research question 3 which says "To what extent are manipulative objects able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking?" The presentation and analyses of the results from written tests were done first followed by semi-structured interviews.

4.4.1 Tests

The tests' questions were set at different levels of Van Hiele's model of geometry thinking. However, this study's focus was on the effects of manipulative objects in assisting learners to reach levels 3 and 4 of Van Hiele's model. This was based on the findings obtained from Arıcı and Aslan-Tutak (2015) and Luneta (2015) who stated that most learners in Further Education and Training (FET) phase find it difficult to answer questions set at levels 3 and 4 of Van Hiele's model when they are being tested.

Table 4 below shows learners' performance as per Van Hiele's levels of geometry thinking. In this respect, the percentage (%) of learners who were able to reach levels 3 and 4 in the pre- and post-test are shown.

Table 4: Learners' performance per Van Hiele's Levels (N=24)

Van Hiele's level	Type of test	Number of learners achieved	%	Number of learners not achieved	%
Level 3	Pre-test	0	0	24	100
	Post-test	20	83	4	17
Level 4	Pre-test	0	0	24	100
	Post-test	6	25	18	75

As noted in Arıcı and Aslan-Tutak (2015) that most learners in Further Education and Training (FET) phase find it difficult to reach levels 3 and 4 of Van Hiele's theory, the pre-test learners' performance also affirms that, as learners were unable to reach levels 3 and 4. In the post-test however, the results indicate that 83% of the learners who wrote were able to reach level 3 with only 25% reaching level 4. Research question 3 investigated the extent to which manipulative objects can scaffold learners' geometry learning in reaching levels 3 and 4. From the post-test results, it was noted that manipulative objects are able to scaffold learners' learning of geometry up to level 3 as 75% of learners were unable to reach level 4.

4.4.2 Semi-structured interviews

Visualisation and confidence are themes that were identified from the transcription and analyses of data collected from semi-structured interviews with respect to Van Hiele's levels of geometry thinking.

4.4.3.1 Visualisation

Sinclair et al. (2017) view visualisation as a requirement that needs to be met by each learner in order to progress from one level to another. Learners' responses on whether manipulative objects supported them to understand geometry indicated that manipulative objects (geoboards) helped them to be able to visualise different orientations of geometry theorems and parts of the circle. Learners' responses when asked whether or not manipulative objects supported them to understand geometry included the following: "Yes, they supported me. I now know how to demonstrate and draw theorems. It became easy for me to understand because I was demonstrating it practically". "Yes, because I was able to visualise and construct theorems on my own, in my own way".

4.4.3.2 Confidence

Based on learners' responses it was noted that the introduction of manipulative objects had improved learners' confidence in answering questions set at levels 3 particularly. Learners were asked which they think was difficult between test one (pre-test) and test two (post-test) and why? Their responses included the following, "Test1 was difficult because I had a lack of geometry knowledge before I write test 1, but in test 2 I at least have knowledge but not that well". "Test 1 because I did not know much about how theorems are represented practically in test 1, even the proofs were difficult to me but now I have a bit of an understanding". It could be seen from these responses that improvement of learners' understanding of selected theorems had positively affected their confidence. It was also noted that learners were not adequately helped by manipulative objects regarding the proofs.

Learner E: “Test 1 was difficult because there were no manipulative objects and these objects make me understand angles and theorems easily”.

Interviewer: Don’t you think that maybe the presence of other questions like question 1, 2, 3 and 4 in the test1 affected your performance in the first test?

Learner E: “No, in test 1, it was like I knew only 25%, but in test 2 I knew about 75% of geometry”.

4.5 Summary

In this chapter, the analysis of data and presentation of results regarding the effects of manipulative objects towards the learning of geometry in grade 11 have been presented. Both quantitative and qualitative analyses were used to provide evidence on the effects of manipulative objects on geometry problem-solving. A table was used to represent quantitative data and it provided a clearer analysis of the extent to which manipulative objects affected learners’ problem-solving (see Table 1). Thematic analysis was also used to present learners’ views about how manipulative objects have affected their geometry problem-solving. A pie chart and tables were used to present the analysis of the amount of time learners spent on each learning action when learning geometry with the aid of manipulative objects. A table showing percentages of the extent to which manipulative objects can scaffold learners’ learning of geometry in reaching levels 3 and 4 of Van Hiele’s model. The next chapter presents discussion of findings, conclusions and recommendations.

CHAPTER 5: DISCUSSION OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The aim of this study was to investigate the effects of manipulative objects as scaffolders in the context of learning grade 11 geometry. This study employs a mixed-method sequential explanatory design. Written tests and observations were used to collect quantitative data and semi-structured interviews were used to collect qualitative data. This chapter presents the discussion of results obtained from employing written tests, observations and semi-structured interviews. The discussion of the findings is done with respect to research questions, literature and theoretical framework of this study. The recommendations, conclusions and limitations are also presented in this chapter.

The analyses of the results emanated from written tests, observations and semi-structured interviews concur with Van Hiele’s theoretical framework. The theory associates learning of geometry with levels where the progress is characterised by progressing from one level to the next; the instructional method used may hinder or help learners to progress through levels (Armah et al., 2018). According to Van Hiele (1959), learners may have difficulties in answering questions which are set at levels above their level as each level uses its own language. In this respect, the theory helped in analysis of learners’ responses from written tests, observations and semi-structured interviews so as to establish learners’ level of understanding prior and after the intervention.

Learners’ performance prior the introduction of manipulative objects (pre-test) showed that they have difficulties with questions set at levels 3 and 4 of the theory which means that these questions were at the levels which are above their levels. Level 3 questions, according to Van Hiele (1984), are those questions that require learners to apply the knowledge of theorems while at level 4 learners use logical arguments to prove conjectures. Similar results were found by Alex and Mammen (2016a), who in their study stated that South African learners in Further Education and Training(FET) phase are unable to reach levels 3 and 4.

Learners' performance after the treatment which is the introduction of manipulative objects affirms the notion by Van Hiele (1984) who states that the introduction of manipulative objects may help learners to progress from one level to the next. The findings of this study after the treatment indicate that learners are able to reach level 3 which enables them to answer questions related to the application of geometry theorems. This is supported by the results of the interviews, as the overall result of the interviews showed that learners' understanding of geometry increased after the introduction of manipulative objects. This finding supports the idea of Armah et al. (2018), who state that learning methods used, particularly those which do not promote rote learning like the introduction of manipulative objects may help learners progress from one level to another. Hence, this finding suggests that the introduction of manipulative objects in the learning of geometry enables learners to reach level 3 of Van Hiele's model of geometry thinking.

The finding of this study with respect to reaching level 4 which is to write geometry proofs or proving conjectures suggests that the objects used (circle geometry geoboards) and the manner in how they were used in this study may not help learners to reach level 4. Thus, with respect to these objects which were specifically designed for this study and how they were used during the instruction, this finding does not support the theory as according to Van Hiele (1984), incorporating materials like manipulative objects may help learners progress from one level to the next. However, this finding is only limited to the objects used in this study, and not all manipulative objects. Thus, manipulative objects which are not similar to the ones used in this study may yield different results. Further, similar objects as those used in this study but different exploration may yield different findings, for example, Sibiya and Mudaly (2018), used similar objects but used protractors to measure equal angles and different results were obtained as with their exploration they noted that learners are able to write proofs of selected theorems.

5.2 Summary and Discussion of the findings

The findings obtained from using tests, observations and semi-structured interviews are discussed with respect to the research questions of the present study. Pre- and post-tests and semi-structured interviews were used to answer research questions 1 and 3. Observations were used to answer research question 2. The discussion of the findings is done according to each of the following research questions:

1. Does the incorporation of manipulative objects enable learners to engage in solving geometry problems?
2. What effects do manipulative objects have on learners' engagement time in geometry learning at grade 11?
3. To what extent are manipulative objects able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking?

5.2.1 The findings in relation to research question 1

Does the incorporation of manipulative objects enable learners to engage in solving geometry problems?

This research question was addressed using written tests (pre-and post-test) and semi-structured interviews. Thus, the discussion of findings in this respect is done based on the data collected from written tests and semi-structured interviews.

5.2.1.1 Tests

The majority of South African learners at FET phase perform poorly in geometry related questions (Alex & Mammen, 2016a). Some learners leave blank space instead of answering geometry questions while others write statements without including appropriate geometry reasons (Luneta, 2015). This was firmly confirmed by learners' performance on pre-test as the majority of learners performed poorly (see Table 1). This indicated that geometry problem-solving was lacking before the treatment which is the introduction of manipulative objects. It appeared also that the teaching method used prior to the intervention did promote learners' problem-solving as poor performance was observed in geometry-related questions. However, it seemed that problem-solving abilities were enhanced after the treatment as average performance increased from 6.3 to 34.0 percent.

The findings of this study based on the pre-test learners' performance reveal that learners have difficulties with the application of geometry theorems and proofs. The majority of learners left blank spaces where they were supposed to apply and prove geometry theorems. Thus, such an inability to apply and prove geometry theorems was evident to the poor development of problem-solving skills with respect to application and proofs of geometry theorems. This finding supports the idea of Greeno (2017) who stated that the development of learners' problem-solving abilities is a key aspect in making learners understand geometry.

In the post-test, however, an improvement in the learners' performance was observed. The improvement observed was mainly based on the application of geometry theorems as the majority of learners performed well in these questions in the post-test. In terms of Polya's types of questions in problem-solving, the improvement was primarily based on "to find" type of questions. Learners when answering these questions, use the knowledge of theorems to find the size of the missing angles (Polya, 1962). The ability to solve these types of questions depends more on the mental actions which include mobilization, organization, isolation and combination (Carifio, 2015). The enhancement of problem-solving abilities was observed during and after the intervention on learners' solutions from activities and post-test, respectively. The evidence of isolation and combination could be seen during the intervention from learners' work shown in Figures 3.9a to 3.9c. In this respect, moving from Figure 3.9a to 3.9c shows the evidence of isolation while moving backwards indicates combination. Also, learners responded accordingly in questions 5.1 and 5.2 in the post-test. It seems from this finding that the introduction of manipulative objects enhanced learners' problem-solving abilities.

In question 5.1.1 of post-test (see Annexure C), learners were required to find the sizes of the missing angles given that MHN is a tangent to circle GHK at H; L is a point on GK and J a point on HK such that LJ is parallel to GH. All learners were able to see that angle K = 43˚ because of Tan-chord theorem. Further, in 5.1.2 also, all learners were able to apply the knowledge of adjacent angles on a straight line in order to find the size of angle L_2=50˚, then apply the knowledge of corresponding angles as line LJ//GH in order to find angle G which is equal to angle H_3 because of Tan-chord theorem, thus angle G= H_3 =50˚.

In Question 5.2, the majority of learners responded correctly when they were also required to find the sizes of the missing angles given that A, B, C, D and E are points on the circle having centre O; DC is produced to G; Diameter AOD bisects chord CE in F, and intersects chord BE in S; A = 32° and angle GCB = 70°. All learners were able to correctly calculate the size of missing angles except learner F. She failed to apply the theorem which says diameter always subtends right angle on the circumference which was important in finding the size of angle BED. As a result of this failure, she incorrectly calculated the size of D_1 as calculating angle BED was an auxiliary problem to D_1. In addition, the application of different theorems which results in the same solution was also observed i.e. some learners used “angle at the centre” as a reason to find angle BED while others used “diameter subtends 90^o at the circumference”.

It seems that the ability to apply the knowledge of theorems as observed in the post-test was a product of development of problem-solving skills in learners’ minds. Hence, the improvement observed may be associated with the treatment which is the introduction of manipulative objects in learning geometry as the same questions were poorly answered by learners in the pre-test which indicated learners’ level of understanding prior to the treatment.

This finding matches those observed in earlier studies, for example, (Balinha & Mamede, 2018; Jojo, 2017; Keraf, 2017; Sibiya, 2019). Balinha and Mamede (2018) observed that the incorporation of manipulative objects (geoboards) in geometry learning helps learners in the development of visualisation and problem-solving skills. Further, the improvement of the performance after the treatment supports the idea by Keraf (2017) who stated that learning based on manipulative objects may improve learners’ performance in geometry.

The findings of this study reveal that geometry proofs are still difficult for learners even after the introduction of manipulative objects. Although the improvement was observed in learners’ post-test performance regarding “to find” type of questions, however, it was also noted that majority of learners were unable to answer “to prove” type of question. Only 4 out 24 were able to correctly answer these questions. Geometry proofs were generally poorly answered by learners. For example, in question 6.2.1 (see Annexure C), learners were required to prove that line AB//FC. However, the majority of learners who attempted answering this question did not realise that angle A and F are both equal to angle O because they are subtended by the same arch, thus angle at the centre is twice the angle at the circumference. Learners instead used wrongly the properties of parallel lines, i.e. Angle A $=C_2$ because alternating angles are equal AB//FC while proving that the same line segments AB and FC are parallel. As a result, learners ended up being confused and not finishing their proofs as they failed to start their proofs in the correct way. It seemed that learners’ inability to answer question 6.2.1 became a hindrance for them to proceed to 6.2.2. This is because, the few learners who attempted answering question 6.2.1 did not proceed to 6.2.2.

The poor performance observed, indicate a lack of problem-solving and geometry understanding with respect to geometry proofs. This could also mean that, mental actions needed for problem-solving like mobilization, organization, isolation and combination were not properly developed through the introduction of manipulative objects.

Nevertheless, it was also noted particularly in Question 6.2.1 that, learners' insufficient knowledge about parallel lines had a major contribution to their inability to correctly answer the question. It seems that learners did know what they were supposed to prove as some assumed the properties of parallel lines instead of proving. That is, to assume that angle A = angle C_2 because line AB is parallel to line FC instead of proving that alternating angles, A and C_2 are equal so as to conclude that lines AB and FC are parallel. Thus, the issue of learners' prior knowledge affected learners' performance in this respect as properties of parallel lines are taught in lower grades.

This finding indicates the extent that manipulative objects have in developing geometry problem-solving skills in the learners' minds. Thus, learners' problem-solving skills with respect to geometry proofs were not fully developed by manipulative objects as learners experienced difficulties in answering geometry proofs in both pre- and post-test. This finding is in agreement with Jamhari and Wongkia (2018) findings which showed that a special type of manipulative object called CircleBoard-Pro is only able to develop learners' problem-solving such that they are able to find the size of missing angles, but unable to assist learners in performing geometry proofs.

5.2.1.2 Semi-structured interviews

The learners' performance prior to the introduction of manipulative objects indicated that geometry problem-solving was poorly developed. However, the findings of this study suggest that manipulative objects have developed learners' problem skills as the majority of the interviewed learners reported that manipulative objects helped them to understand geometry. The following are some of their responses:

"… *manipulative objects helped me because I was able to create my representation of theorems unlike when given a figure with many theorems on the paper". "Manipulative objects helped me to easily identify and separate each theorem when they are combined in a figure*".

The learners' responses provided evidence that manipulative objects developed their problem-solving skills in geometry. The results of the study in this regard were found to be inconsistent with the findings obtained in previous studies, for example (Jojo, 2017; Sibiya, 2019; Sibiya & Mudaly, 2018). Similar objects as the ones used in this study were also used in Sibiya and Mudaly (2018) although they used a conceptual framework while this study used a theoretical framework and their findings indicated that manipulative objects(geoboards) help learners to understand geometry theorems. In this study, the overall result of the interviews with respect to problem-solving showed that learners' understanding of geometry Improved after the treatment. One learner reported: *"… because I have even done theorems practically on the geoboard".* This response indeed supports the idea of Jojo (2017), who stated that the introduction of manipulative objects in geometry learning promotes active learning and develops learners' problem-solving skills in geometry.

The performance prior (pre-test) and after (post-test) the introduction of manipulative objects indicates that learners have difficulties with geometry proofs. A similar finding was found also found with semi-structured interviews. Learners' responses from semi-structured interviews showed that their problem-solving skills are poorly developed and lack of geometry understanding as they were unable to explain their procedures as to how they have written geometry proofs on the post-test. The finding indicates that learners did not have much to say about how they had written their geometry proofs. Having responses like, *"I did not answer this question"* and *"This question was difficult to me"* provided evidence about the development of learners' problem-solving skills and their understanding of geometry with this respect. This evidence indicates that learners' problem-solving with respect to geometry proofs was poorly developed which resulted in lack of understanding. This finding supports the notion by Greeno (2017), who noted that poor development problem-solving skills may lead to lack of understanding in geometry.

The majority of learners performed well in question 6.2.1 on the post-test; however, they were unable to verbally explain their procedures when they were interviewed. Learner E responded as "**…***better to write it down than explaining*", when asked to explain her procedures. This indicates learners' inability to verbally explain their procedures and poor development of geometric language. This also indicates that learners during the instruction did not receive adequate training in terms of verbally explaining their procedures using correct geometric language. This finding is supported by the amount of time learners spent on listening, answering and asking questions when they were observed (see Section 5.2.2.1). Learners spent most of their time listening (39%) while a small amount of time was spent on actions that develop verbal skills like asking questions (0.1%) and answering questions (1.4%). Thus, spending a relatively small amount of time on learning actions which develop learners' verbal skills may be associated with learners' inability to explain their procedures as observed on interviews.

5.2.2 Findings in relation to research question 2

What effects do manipulative objects have on learners' engagement time in geometry learning at grade 11?

5.2.2.1 Observations

The results of this study reveal that learners can spend up to 48.6 minutes being engaged over one-hour long period when learning geometry with the aid of manipulative objects. This indicates that the remaining part of one hour which is 11.4 minutes was spent on disengagement. This supports the idea of Rosen et al. (2015), who observed that learners hardly spent the whole time allocated for learning being engaged. On the other hand, spending more time being engaged supports what was observed by Cockett and Kilgour (2015) that manipulative objects help in keeping learners engaged for a long time. This was further highlighted in the literature that introducing manipulative objects may maximise learners' engagement in geometry (Fyfe et al., 2015; Perry & Steck, 2015). It was also noted that learners spent much of their engagement time listening which had negative effects on the time learners spent on speaking.

In contrast to earlier findings, however, no evidence of learners' engagement time in geometry was detected. Some studies focussed on engagement time in mathematics in general without specifically focusing on a particular topic of mathematics, for example (Bobis et al., 2016; Caldwell et al., 1982; Erickson, 1999). On the one hand, the reviewed literature, for example, (Balinha & Mamede, 2018; Larbi & Mavis, 2016; Sibiya, 2019; Sibiya & Mudaly, 2018), indicate that there is research evidence that manipulative objects enhance learners' engagement. On the other hand, based on researcher's reading it 'it seems that not much has been reported in the literature about the effects that manipulative objects have on the engagement time when learning geometry. Sibiya (2019) highlighted that manipulative objects have effects on learners' learning of geometry, however, he did not mention the period of engagement in his findings. Thus, the finding of this study which indicates that learners can be engaged for 48.6 minutes in an hour-long period when learning geometry with the aid of manipulative objects is the original finding of this study.

Interval time sampling was used to observe learners' behavioural engagement so as to estimate engagement time (see Annexure A). The discussion of findings as obtained from observations was based on the following learning actions:

- Non-academic action- (Disengaged)
- Listening attentively
- Asking a question
- Answering a question
- Using geoboard
- Doing an exercise
- Seeking assistance from other learners
- Assisting other learners.

5.2.2.1.1 Disengaged

These actions include all the irrelevant actions which were done by learners during the learning of geometry with an aid of manipulative objects. In other words, these actions are what learners do during the learning which disturbs them from focusing on what the learning is about. The findings of this study reveal that learners can spend up to 11.4 minutes in an hour-long period being disengaged. This means that in an hour, learners can be disengaged for up to 19.1% of the time available for learning geometry with an aid of manipulative objects. This finding confirms the association between disengagement time and poor performance as noted by Newmann (1992), that time spent being disengaged may negatively affect learners' performance. In this respect, the inability to write geometry proofs as observed in written tests may be associated with the amount of time learners spent being disengaged.

5.2.2.1.2 Listening Attentively

It was also observed that learners spend most of their engagement time listening. Learners were found to be spending almost 39% (11.4 min) of the available time for learning geometry which was one hour listening to the teacher. This indicates that learners spent most of their time being passively engaged in their learning. Thus, passive engagement of learners in their geometry learning may be associated with learners' inability to verbally explain their procedures as observed in interviews. Hence, passive engagement deprived learners' the opportunity for their verbal skills to be developed. This finding is in agreement with Kong et al. (2003) who showed that learners spend most of their time listening to the teacher when learning mathematics.

5.2.3.3 Asking a question

Few learners were found doing this action and it took the least amount of engagement time. Learners were found to be spending only 0.1% which is equivalent to 0.3 min in an hour on asking questions. Instead of asking the teacher, learners were found testing their conjectures on their manipulative objects (geoboards) and asking other learners. Spending less time on asking questions indicates that learners understood what they were learning about and such is supported by the improvement in learners' performance as observed in the post-test. However, this may also have negative effects on the development of learners' verbal skills as the results emanated from interviews indicate that learners encountered difficulties in verbally explaining their procedures.

5.2.2.1.4 Answering a question

The findings of this study indicate that learners do not spend more time answering questions when learning geometry with the aid of manipulative objects. It was found that learners could spend only 1.4% on average which is equivalent to 0.8 min on answering questions. Spending more time on listening to the teacher and less time on answering questions indicates the method of learning used which is a teacher centred approach. This indicates that learners were not encouraged to verbally answer geometry questions in the class, as a result, they were unable to verbally explain what they had written as observed during interviews. In line with this finding, Evbuomwan (2013) noted that learners are unable to properly use geometry language and that makes it difficult for them to verbally explain their geometry procedures.

5.2.2.1.5. Using geoboards

It was found that almost 32% (18.9 min) on average of the time was spent using geoboards. Using geoboards was found to be one of the learning actions which learners were spending most of their engagement time at (see Figure 4.1. Spending more time on using geoboards is directly associated with the development of problem-solving and visualisation skills observed in post- test and interviews. Learners' post-test performance improved (see Table1) and learners reported that manipulative objects (geoboards) assisted them to understand geometry when they were interviewed. This means that spending more time on using geoboards has positive effects on learners' performance in geometry. Thus, this finding concurs with what was obtained in Tabach and Nachlieli (2015), as they observed that the introduction of manipulative objects promotes understanding and engagement.

5.2.2.1.6. Doing an exercise

This finding reveals that learners spent up to 6.8% (4.1 min) on average doing written exercises. This indicates that learners were given more written exercises than oral ones. Spending more time on written exercises developed learners' problem-solving skills with respect to written tests. Learners' performance on written tests improved after the introduction of manipulative objects. This finding connects the results of the observations with those obtained on the post-test. The results of the post-test indicate that manipulative objects enhanced learners' problem-solving skills to such an extent that learners were able to reach level 3 of Van Hiele's model. Thus, the improvement in the learners' performance on the written tests is also supported by the amount of time learners spent on written exercises. Although this finding is based on geometry learning, however, it is broadly inconsistent with Kong et al. (2003) who associated the amount of time spent by learners on doing school work with the development of problem-solving in mathematics.

5.2.2.1.7. Seeking assistance from others and assisting other learners

On average, only 1.2 % (0.7min) and 1.1% (0.7 min) of one hour learners spent seeking assistance and assisting others, respectively. This finding indicates that manipulative objects allowed learners to interact with one another. Learners were found sharing their different representations of theorems with other learners through manipulative objects. Sharing representation allowed learners to explain to others as to how they have arrived at the final representation. Thus, supporting the notion by Jojo (2017) who stated that manipulative objects allow learners to be actively engaged in their learning.

5.2.3. Findings in relation to research question 3

To what extent are manipulative objects able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking?

5.2.3.1 Tests

The findings of this study with respect to learners' performance prior to the introduction of manipulative objects (pre-test) reveal that the majority of learners were unable to reach levels 3 and 4 of Van Hiele's level of geometry thinking. The inability to reach level 3 was indicated by learners' failure to apply the knowledge of geometry theorems to find missing angles while failing to write geometry proofs indicated the inability to reach level 4. This may mean that the manner in how geometry was taught to learners prior to the introduction of manipulative objects did not allow learners to reach levels 3 and 4. This finding supports previous research, for example, Alex and Mammen (2016a), noted that learners in FET phase are performing at levels 1 and 2 whereas they are expected to perform at levels 3 and 4.

The results obtained from post-test indicate that the performance of learners improved such that they were able to reach level 3 of Van Hiele's model. The ability to apply geometry theorems as shown in Section 5.2.1.1 reveals that learners are able to answer questions set at level 3. Learners' performance in the pre-test revealed that they are unable to reach level 3, thus reaching level 3 in the post-test may imply that manipulative objects helped learners to be able to reach level 3. This finding may extend the existing literature, for example, Alex and Mammen (2016a), who in their study found that learners in FET are unable to reach level 3 as this finding indicates that the scaffolding support from manipulative objects helped learners to reach level 3. This finding supports the idea of Balinha and Mamede (2018), who stated that scaffolding from manipulative objects may help learners reach higher levels of the model.

The findings of this study also indicate that the majority of learners failed to write geometry proofs as demanded by questions in the post-tests. Failing to write geometry proofs as shown in Section 5.2.1.1 indicates the inability of learners to reach level 4 of Van Hiele's model as geometry proofs are done in level 4. The inability to reach level 4 was also highlighted in the literature as Alex and Mammen (2016a) stated that South African learners' in FET phase have difficulties with geometry proofs which is a fundamental requirement for learners to perform at level 4. Generally, there is no major improvement observed as compared to pre-test performance in terms of learners reaching level 4 on the post-test. This finding may indicate the limitation that manipulative objects have in scaffolding geometry learning with respect to Van Hiele's levels of geometry thinking. Nonetheless, 25% of learners were able to answer these questions correctly. This may show the potential that manipulative objects have in scaffolding learners with respect to geometry proofs.

5.2.3.2 Semi-structured interviews

Conducting semi-structured interviews after tests and observations helped the researcher to get in-depth learners' views about how manipulative objects supported their learning of geometry. This enabled the researcher to analyse quantitative learners' scores obtained through tests and observations in a qualitative way. In this regard, three dimensions were identified from learners' responses to semi-structured interviews, namely: Visualisation, confidence and problem-solving.

5.2.3.2.1 Visualisation

Learners' performance from pre-test indicated that learners' visualisation skills were not properly developed prior to the introduction of manipulative objects in the learning of geometry. However, learners' responses based on questions of the interview schedule, it was noted that manipulative objects helped them to be able to visualise different theorems and find a connection between what they saw on paper and what they represented on the manipulative objects. Learners' visualisation may be seen from the following responses when asked whether or not manipulative objects supported them to understand geometry:

"*Yes, they supported me. I now know how to demonstrate and draw theorems. It became easy for me to understand because I was demonstrating theorems practically". "Yes, because I was able to visualise and construct theorems on my own, in my own way"*.

Learners' responses indicate that they had poor mental representations of geometry theorems prior to the introduction of manipulative objects and their visualisation skills were enhanced after the introduction of manipulative objects. The finding is inconsistent with what was obtained in Sinclair et al. (2017) who noted that learning geometry through manipulative objects enhances mental visualisation while helping learners to reach higher levels of Van Hiele's model of geometry thinking. Thus, the ability to reach level 3 as observed in the post-test performance may be associated with the development of visualisation skills which were developed by the introduction of manipulative objects

5.2.3.2.2 Confidence in answering levels 3 and 4 questions

Another finding of this study is that learners' confidence improved after the introduction of manipulative objects. Learners were asked which of the two written tests (pre-test and post-test) was difficult. There was a general consensus that the first test was difficult because at a time when the pre-test was written they did not have the knowledge that they had after being helped by manipulative objects. In this way, knowledge comes with confidence. Learner E responded as follows:

"Test one was difficult because there were no geoboards, but after using geoboards test number two became easy."

The comment from learner E indeed illustrates confidence improvement which comes from learners' improvement in understanding of geometry theorems. The results prior to the introduction of manipulative objects indicate that questions set at level 3 were poorly answered, however, after the introduction of manipulative objects, learners' confidence in answering level 3 improved as these questions were correctly answered by learners in the post-test. This indicates that manipulative objects have positively affected learners' confidence. The improvement observed supports the idea of van de Pol et al. (2015), as they noted manipulative objects enhance learners' confidence such that they are able to confront challenging geometry problems. Similar results were found by Sibiya (2019), who in his study, noted that the introduction of manipulative objects similar to the ones used in this study (geoboards) improves learners' confidence in attempting questions in the tests.

5.2.4.3 Problem-solving

The development of problem-solving abilities with respect to questions set at level 3 of Van Hiele's model was noted in this study. This finding indicates that learners performed better in answering questions set at level 3 on the post-test. Findings from semi-structured interviews with specific reference to level 3 showed that the majority of learners were unable to verbally explain their procedures as to how they had arrived at the final answer. In this respect, learners were able to express themselves in writing but they were unable to verbally explain what they had written when interviewed. One learner said *"… It is better to write it down than explaining*". This finding may be associated with the results of observations (see Section 5.2.2.1) as they show that learners spent most of their time listening to the teacher. Thus, spending more time listening to the teacher may have negative effects on learners' verbal expression, as they will have less time to talk about what they learned. The findings of the study, in this respect, corroborate the findings of Evbuomwan (2013), as he also noted that learners had difficulties with explaining what they had written using correct geometric language.

Another finding of this study was that problem-solving skills regarding level 4 questions were poorly developed in both post-test performance and learners' responses on semi-structured interviews. Some learners responded as *"I did not answer this question"* while others responded as *"This question was difficult to me".* Judging from learners' responses when interviewed about how they have answered questions 6.2.2 and 6.2.3, it became obvious that learners had difficulties in reaching level 4 of Van Hiele's. This may imply that manipulative objects were unsuccessful in helping learners reach level 4 as the majority of learners were still unable to perform geometry proof. Similar findings were found by Jamhari and Wongkia (2018), who in their study observed that manipulative objects were unable to help learners write geometry proofs.

5.3 Implications of findings

Learners' poor performance in geometry is a concern to learners, parents, mathematics teachers and the Department of Education in South Africa. The implications for teachers and curriculum planners from the findings of this study are: firstly, the incorporation of manipulative objects enhances learners' problem-solving skills and help them understand geometry to such an extent that they are able to apply the knowledge of geometry theorems to find the size missing angles (level 3). Teachers should therefore incorporate manipulative objects in their instructions to enhance learners' problem-solving skills to the extent that they are able to reach level 3 of Van Hiele's model.

Further, there has been a concern regarding learners' inability to meet the curriculum needs which is to reach levels 3 and 4 as they are expected in FET phase. Thus, curriculum planners need to pay careful attention to the incorporation of manipulative objects in helping learners reach level 3. Also, curriculum planners should encourage teachers to incorporate manipulative objects when teaching geometry theorems. This finding also concurs with Van Hiele (1959) notion that incorporation of manipulative objects enables learners to reach high levels.

The majority of learners have difficulties with writing geometry proof which means that they have difficulties with reaching level 4 of Van Hiele's model. Thus, in contrary to the theory's recommendation with reference to manipulative objects helping learners reaching level 4, this finding indicates that these manipulative objects which were used and the manner in how they were used may not help learners to reach level 4. However, this implication is only limited to geoboards, and not all types of manipulative objects.

Furthermore, learners had difficulties with verbally explaining what they had written. Teachers should encourage learners to speak using correct geometry language in their classroom practice.

In an hour-long lesson, learners can spend up to 81% of their time being engaged when learning geometry with the assistance of manipulative objects. Thus, teachers should consider manipulative objects to engage learners in learning geometry.

5.4 Contribution of the study

Manipulative objects like geoboards are commonly used when learning geometry which is done in primary school as noted (Bayram, 2004; Scott, 1983). However, the results of this study indicated that geoboards may also be used even with secondary school learners. The results in this respect support the recommendation made by Boling (1991) that manipulative objects should be across the grades. Thus, the findings of this study may be helpful to mathematics teachers in primary and secondary schools as well as curriculum developers as they are going to consider the potential that manipulative objects like geoboards have in improving learners' performance in geometry. Further, improvement in learners' performance in geometry may also improve learners' performance in mathematics (Cassim, 2006). Thus, this investigation hopes to improve learners' performance in mathematics by improving their performance in geometry through manipulative objects.

5.5. Recommendations for future study

Based on the findings of this study, the following future investigations are recommended:

- Further research can be conducted to examine how manipulative objects enhance learning of other geometry related concepts such as parallelograms in lower grades.
- This study suggests future work on the assessment of learners' attitudes and perceptions towards the use and integration of manipulative objects into the learning of geometry.
- The study suggests future work on effects of the simultaneous use of geoboards and GeoGebra in basic geometry in relation to students with learning difficulties.

5.6 Conclusions

The aim of this study was to investigate the effects of manipulative objects as scaffolders towards learning of geometry in grade 11. The investigation was guided by three research questions namely: 1. Does the incorporation of manipulative objects enable learners to engage in solving geometry problems? 2. What effect do manipulative objects have on learners' engagement time in geometry learning at grade 11? 3. To what extent are manipulative objects able to scaffold learners in reaching level 3 and level 4 of Van Hiele's model of geometry thinking? The mixed-methods sequential explanatory design was used to answer the research questions. Employing mixed-methods sequential explanatory design implied that the process of data collection, analyses and discussion of the findings must first be done quantitatively then the qualitative analysis may follow later as stated by Ivankova et al. (2006). Written tests and observations were used to collect qualitative data while semi-structured interviews were used to collect qualitative data.

"Does the incorporation of manipulative objects enable learners to engage in solving geometry problems?"

The analyses of the results emanated from written tests and semi-structured interviews with respect to research question 1, suggest that manipulative objects may engage learners in geometry problem-solving. The enhancement of geometry problem-solving abilities with respect to mobilization, organization, isolation and combination was observed as learners' performance improved after the introduction of manipulative objects. However, the improvement observed was specifically based on "to find" type of questions. This finding was also supported by the results of interviews, as the overall results of semi-structured interviews showed that manipulative objects helped them to understand geometry. Thus, it can be concluded from the findings of this investigation that manipulative objects may engage learners in geometry problem-solving.

What effects do manipulative objects have on learners' engagement time in geometry learning at grade 11?

Observations were conducted to investigate the effects of manipulative objects on engagement time. No evidence of learners' engagement time in geometry, neither engagement time on using manipulative objects were detected in the reviewed literature. The findings of this study with respect to the effects that manipulative objects have on geometry learning reveal that the introduction of manipulative objects in the learning of geometry may yield the engagement time of 48.6 minutes in an hour-long period. The findings further reveal that learners spend most of their time listening and using geoboards (manipulative objects). Thus, it can be concluded that manipulative objects may positively affect learners' engagement.

To what extent are manipulative objects able to scaffold learners in reaching levels 3 and level 4 of Van Hiele's model of geometry thinking?

The results of the analyses that emanated from written tests and semi-structured interviews also revealed that scaffolding from manipulative objects may help learners reach level 3. This means that manipulative objects may help learners apply the knowledge of geometry theorems to find the size of the missing angles. Further, although learners had correctly answered level 3 questions in the post-test, however, the result emanated from thematic analyses from interviews indicated that learners were unable to verbally explain what they had written.

The findings also reveal that manipulative objects were unsuccessful to help learners reach level 4 as learners had difficulties with writing geometry proofs in pre-and post-test.

Thus, it may be concluded from the findings of this study that manipulative objects may only help learners to reach level 3; different scaffolding and approach may be needed to help learners reach level 4 of Van Hiele's model of geometry thinking.

5.7 Limitations of the study

Due to time limit and financial constraints, this study focused only on the effects of manipulative objects as scaffolders in the learning of geometry in grade 11 learners selected from one public school of King Cetshwayo District in KwaZulu-Natal province in South Africa. Furthermore, the study employed a convenient form of sampling. Thus, the findings of this study may not be generalised to all public schools in KwaZulu-Natal province. However, the findings of this study provided an in-depth understanding of how manipulative objects affect the learning of geometry in the eleventh grade.

In the post-test learners only wrote questions 5 and 6. In the pre-test however, learners wrote the whole second paper of mathematics with six questions in total. Thus, the presence of four other questions in addition to questions 5 and 6 may have affected learners' performance in the pre-test. Further, although question papers were collected soon after the completion of the pre-test, however, since this was a common paper which was written by all grade 11 learners in KZN, it is possible that some learners could get these papers from their friends in other schools nearby and seek for assistance from relevant people. The presence of a camera in the classroom may also have affected learners' behaviour.

REFFERENCES

Alex, J. K., & Mammen, K. J. (2014). An assessment of the readiness of grade 10 learners for geometry in the context of curriculum and assessment policy statement (CAPS) expectation. *International Journal of Educational Sciences, 7*(1), 29-39.

Alex, J. K., & Mammen, K. J. (2016a). Geometrical Sense Making: Findings of Analysis Based on the Characteristics of the van Hiele Theory among a Sample of South African Grade 10 Learners. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(2).

Alex, J. K., & Mammen, K. J. (2016b). Lessons Learnt from Employing van Hiele Theory Based Instruction in Senior Secondary School Geometry Classrooms. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(8).

Anstrom, T. (2006). Supporting students in mathematics through the use of manipulatives. *American Institutes for Research*.

Arıcı, S., & Aslan-Tutak, F. (2015). The effect of origami-based instruction on spatial visualization, geometry achievement, and geometric reasoning. *International Journal of Science and Mathematics Education, 13*(1), 179-200.

Armah, R. B., Cofie, P. O., & Okpoti, C. A. (2017). The Geometric Thinking Levels of Pre-service Teachers in Ghana. *Higher Education Research, 2*(3), 98-106.

Armah, R. B., Cofie, P. O., & Okpoti, C. A. (2018). Investigating the Effect of van Hiele Phase-Based Instruction on Pre-Service Teachers' Geometric Thinking. *International Journal of Research in Education and Science, 4*(1), 314-330.

Baker, L. (2006). Observation: A complex research method. *Library trends, 55*(1), 171-189.

Baki, A., Kosa, T., & Guven, B. (2011). A comparative study of the effects of using dynamic geometry software and physical manipulatives on the spatial visualisation skills of pre-service mathematics teachers. *British Journal of Educational Technology, 42*(2), 291-310.

Bakker, A., Smit, J., & Wegerif, R. (2015). Scaffolding and dialogic teaching in mathematics education: Introduction and review.

Balinha, F., & Mamede, E. (2018). Young children working with geometric figures. *Journal of the European Teacher Education Network, 13*, 11-22.

Barkley, E. F., & Major, C. H. (2020). *Student engagement techniques: A handbook for college faculty*: John Wiley & Sons.

Bayram, S. (2004). *The effect of instruction with concrete models on eighth grade students'geometry achievement and attitudes toward geometry.* Middle East Technical University.

Beltran, R. S., Regala, R. M. P., & Bog-ot, J. P. T. (2018). Visual Modelling Approach (VMA) in Learning Fractions. *International Journal of Humanities and Social Sciences, 10*(4), 56-62.

Biyane, G. (2007). *Students' perception of space at the Central University of Technology (Bloemfontein) and Free State School of Nursing Campuses: an exploratory study.*

Bobis, J., Way, J., Anderson, J., & Martin, A. J. (2016). Challenging teacher beliefs about student engagement in mathematics. *Journal of Mathematics Teacher Education, 19*(1), 33-55.

Boling, A. N. (1991). They Don't Like Math? Well, Let's Do Something! *Arithmetic Teacher, 38*(7), 17-19.

Bossé, M. J., & Adu-Gyamfi, K. (2011). Stop teaching and let students learn geometry. *Mathematics Teacher, 105*(4), 293-297.

Bouck, E. C., Satsangi, R., Doughty, T. T., & Courtney, W. T. (2014). Virtual and concrete manipulatives: a comparison of approaches for solving mathematics problems for students with autism spectrum disorder. *Journal of Autism and developmental disorders, 44*(1), 180-193. doi: 10.1007/s10803-013-1863-2

Bouck, E. C., Satsangi, R., & Park, J. (2017). The Concrete–Representational–Abstract Approach for Students With Learning Disabilities: An Evidence-Based Practice Synthesis. *Remedial and Special Education*, 0741932517721712.

Braynard, D., Hanekom, S., & Brynard, P. (2015). Introduction to Reaserch (3rd ed.). Pretoria: Van Schaik Publishers.

Briesch, A. M., Hemphill, E. M., Volpe, R. J., & Daniels, B. (2015). An evaluation of observational methods for measuring response to classwide intervention. *School Psychology Quarterly, 30*(1), 37.

Brown, R., & Redmond, T. (2016). Constructing Classroom Contexts That Engage Students in the Learning of Mathematics: A Teacher's Perspective. *Mathematics Education Research Group of Australasia*.

Bruner, J. S. (1966). *Toward a theory of instruction* (Vol. 59): Harvard University Press.

Bruner, J. S. (1975). From communication to language—a psychological perspective. *Cognition, 3*(3), 255-287.

Bruner, J. S. (1976). Peekaboo and the Learning of Rule Structures. I: Bruner, JS, Jolly, A. & Sylvia, K.(Eds.) Play (s. 277-285): New York: Penguin Books.

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for research in mathematics education*, 31-48.

Burns, A., & de Silva Joyce, H. (2005). Explicitly supporting learning: An overview. *Teachers' voices, 8*, 2-7.

Calderwood, C., Ackerman, P. L., & Conklin, E. M. (2014). What else do college students "do" while studying? An investigation of multitasking. *Computers & Education, 75*, 19-29.

Caldwell, J. H., Huitt, W. G., & Graeber, A. O. (1982). Time spent in learning: Implications from research. *The Elementary School Journal, 82*(5), 471-480.

Carden, J., & Cline, T. (2015). Problem solving in mathematics: the significance of visualisation and related working memory. *Educational psychology in practice, 31*(3), 235-246.

Carifio, J. (2015). Updating, Modernizing, and Testing Polya's Theory of [Mathematical] Problem Solving in Terms of Current Cognitive, Affective, and Information Processing Theories of Learning, Emotions, and Complex Performances. *Journal of Education and Human Development, 4*(3), 105-117.

Carroll, J. (1992). *Using the geoboard for teaching primary mathematics.* Paper presented at the Mathematics: meeting the challenge.

Carroll, J. B. (1989). The Carroll model: A 25-year retrospective and prospective view. *Educational researcher, 18*(1), 26-31.

Cassim, I. (2006). *An exploratory study into grade 12 learners' understanding of Euclidean Geometry with special emphasis on cyclic quadrilateral and tangent theorems.*

Castro, S. (2017). Algebra tiles effect on mathematical achievement of students with learning disabilities.

Catala, A., Sylla, C., Theune, M., Brooks, E., & Read, J. C. (2018). *Rethinking children's co-creation processes beyond the design of TUIs.* Paper presented at the Proceedings of the 17th ACM Conference on Interaction Design and Children.

Chafouleas, S. M., Sanetti, L. M. H., Jaffery, R., & Fallon, L. M. (2012). An evaluation of a classwide intervention package involving self-management and a group contingency on classroom behavior of middle school students. *Journal of Behavioral Education, 21*(1), 34-57.

Chi, M. T., Siler, S. A., Jeong, H., Yamauchi, T., & Hausmann, R. G. (2001). Learning from human tutoring. *Cognitive Science, 25*(4), 471-533.

Chilton, P. (2014). *Language, space and mind: The conceptual geometry of linguistic meaning*: Cambridge University Press.

Chimuka, A., & Ugorji, I. O. (2016). The impact of teaching and learning circle geometry using geogebra on grade 11 student's problem solving skills. *AMESA, 1*(2), 434-446.

Ciesielska, M., Boström, K. W., & Öhlander, M. (2018). Observation methods *Qualitative Methodologies in Organization Studies* (pp. 33-52): Springer.

Clements. (2003). Teaching and learning geometry. *A research companion to principles and standards for school mathematics*, 151-178.

Clements, & McMillen. (1996). Rethinking" concrete" manipulatives. *Teaching children mathematics, 2*(5), 270-279.

Cockett, A., & Kilgour, P. W. (2015). Mathematical Manipulatives: Creating an Environment for Understanding, Efficiency, Engagement, and Enjoyment. *TEACH Collection of Christian Education, 1*(1), 5.

Connelly, L. M. (2014). Ethical considerations in research studies. *Medsurg Nursing, 23*(1), 54-56.

Cope, L. (2015). Math manipulatives: Making the abstract tangible. *Delta Journal of Education, 5*(1), 10-19.

Creswell, J. W. (2014). *A concise introduction to mixed methods research*: Sage Publications.

Crowley, M. L. (1987). The van Hiele model of the development of geometric thought. *Learning and teaching geometry, K-12*, 1-16.

De Wet, J. J., Monteith, J. L., Venter, P. A., & Steyn, H. S. (1981). Navorsingsmetodes in die opvoedkunde:'n inleiding tot empiriese navorsing: Durban: Butterworths.

Dean, B. A. (2018). The interpretivist and the learner. *International Journal of Doctoral Studies, 13*, 1-8.

Delport, C., De Vos, A., Fouche, C., & Strydom, H. (2005). Research at grassroots. *For the social science and human service*.

Department of Education. (2011). *Curriculum And Assessment Policy Statement (CAPS)*. Ndabase Printing Solution Retrieved from http://www.education.gov.za.

Department of Education. (2014). *Reports - National Senior Certificate*. Pretorea: Government Printers Retrieved from http://www.education.gov.za/Resources/Reports.aspx.

Department of Education. (2015). *Reports - National Senior Certicicate* Pretoria: Government Printers Retrieved from http://www.education.gov.za/Resources/Reports.aspx.

Department of Education. (2016). *Reports - National Senior Certificate* Government Printers Retrieved from ttp://www.education.gov.za/Resources/Reports.aspx.

Department of Education. (2017). *Reports- National Senior Certifiacte* Pretoria: Retrieved from www.education.gov.za.

Department of Education. (2018). *Reports- National Senior Certifiacte* Pretoria: Retrieved from www.education.gov.za.

Dignath, C., & Büttner, G. (2018). Teachers' direct and indirect promotion of self-regulated learning in primary and secondary school mathematics classes–insights from video-based classroom observations and teacher interviews. *Metacognition and Learning, 13*(2), 127-157.

Doias, E. D. (2013). *The Effect of Manipulatives on Achievement Scores in the Middle School Mathematics Class.* Lindenwood University.

Dorier, J.-L., Gutiérrez, Á., & Strässer, R. (2003). *Geometrical thinking.* Paper presented at the Proceedings of CERME.

Dörnyei, Z. (2007). *Research methods in applied linguistics: Quantitative, qualitative, and mixed methodologies*: Oxford University Press Oxford.

Droit-Volet, S., & Wearden, J. H. (2015). Experience sampling methodology reveals similarities in the experience of passage of time in young and elderly adults. *Acta Psychologica, 156*, 77-82.

Epelboim, J., & Suppes, P. (2001). A model of eye movements and visual working memory during problem solving in geometry. *Vision research, 41*(12), 1561-1574.

Erdogan, T., Akkaya, R., & Celebi Akkaya, S. (2009). The Effect of the Van Hiele Model Based Instruction on the Creative Thinking Levels of 6th Grade Primary School Students. *Educational sciences: theory and practice, 9*(1), 181-194.

Erickson, D. K. (1999). A problem-based approach to mathematics instruction. *Mathematics Teacher, 92*(6), 516-521.

Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American journal of theoretical and applied statistics, 5*(1), 1-4.

Evbuomwan, D. (2013). *An investigation into the difficulties faced by Form C students in the learning of transformation geometry in Lesotho secondary schools.*

Fedewa, A., Davis, M. A., & Ahn, S. (2015). Effects of stability balls on children's on-task behavior, academic achievement, and discipline referrals: A randomized controlled trial. *American Journal of Occupational Therapy, 69*(2), 6902220020p6902220021-6902220020p6902220029.

Ferguson, S., & McDonough, A. (2010). The Impact of Two Teachers' Use of Specific Scaffolding Practices on Low-Attaining Upper Primary Students. *Mathematics Education Research Group of Australasia*.

Fredricks, J. A., & McColskey, W. (2012). The measurement of student engagement: A comparative analysis of various methods and student self-report instruments *Handbook of research on student engagement* (pp. 763-782): Springer.

Furner, J. M., & Marinas, C. A. (2011). Geoboards to Geogebra: moving from the concrete to the abstract in geometry.

Fuys, D., Geddes, D., & Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education. Monograph, 3*, i-196.

Fyfe, E. R., McNeil, N. M., & Borjas, S. (2015). Benefits of “concreteness fading” for children's mathematics understanding. *Learning and Instruction, 35*, 104-120.

Goodman, S. G., Seymour, T. L., & Anderson, B. R. (2016). Achieving the performance benefits of hands-on experience when using digital devices: A representational approach. *Computers in Human Behavior, 59*, 58-66.

Graham, P., & Chick, H. (2015). The 20 Matchstick Triangle Challenge: An Activity to Foster Reasoning and Problem Solving. *Australian Mathematics Teacher, 71*(1), 9-17.

Greeno, J. G. (2017). Forms of understanding in mathematical problem solving *Learning and motivation in the classroom* (pp. 83-112): Routledge.

Greenwood, C. R. (1991). Longitudinal analysis of time, engagement, and achievement in at-risk versus non-risk students. *Exceptional children, 57*(6), 521-535.

Gürsoy, B., & Özkar, M. (2015). Visualizing making: shapes, materials, and actions. *Design Studies, 41*, 29-50.

Habegger, W. V. (1994). Geometric discovery through interactive software: an honors thesis (HONRS 499).

Habegger, W. V., & Emen, J. W. (1994). Geometric Discovery Through Interactive Software.

Hall, N. (1998). Concrete representations and the procedural analogy theory. *The Journal of Mathematical Behavior, 17*(1), 33-51.

Harbour, K. E., Evanovich, L. L., Sweigart, C. A., & Hughes, L. E. (2015). A brief review of effective teaching practices that maximize student engagement. *Preventing School Failure: Alternative Education for Children and Youth, 59*(1), 5-13.

Hintze, J. M. (2005). Psychometrics of direct observation. *School Psychology Review, 34*(4), 507.

Hock, T. T., Tarmizi, R. A., Yunus, A. S. M., & Ayub, A. F. (2015). Understanding the primary school students’ van Hiele levels of geometry thinking in learning shapes and spaces: A Q-methodology. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(4), 793-802.

Hopstaken, J. F., Van Der Linden, D., Bakker, A. B., & Kompier, M. A. (2015). A multifaceted investigation of the link between mental fatigue and task disengagement. *Psychophysiology, 52*(3), 305-315.

Imenda, S., N, & Muyangwa, M., M. (2006). *Introduction To Research In Education And Behavioural Sciences*. Pretoria in RSA: Publication and Design Services, Tswana University of Technology.

In'am, A. (2016). Euclidean Geometry's Problem Solving Based on Metacognitive in Aspect Awareness. *International Electronic Journal of Mathematics Education (IEJME), 11*(4), 961-974.

Ivankova, N. V., Creswell, J. W., & Stick, S. L. (2006). Using mixed-methods sequential explanatory design: From theory to practice. *Field methods, 18*(1), 3-20.

Jackson, P. W. (1992). Conceptions of curriculum and curriculum specialists. *Handbook of research on curriculum, 3*, 40.

Jamhari, & Wongkia, W. (2018). *CircleBoard-Pro: Concrete manipulative-based learning cycle unit for learning geometry.* Paper presented at the AIP Conference Proceedings.

Johns, B. H., Crowley, E. P., & Guetzloe, E. (2017). Engaged time in the classroom. *Focus on Exceptional Children, 41*(4).

Johnson, E. L. (2018). *A New Look at the Representations for Mathematical Concepts: Expanding on Lesh's Model of Representations of Mathematical Concepts.* Paper presented at the Forum on Public Policy Online.

Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational researcher, 33*(7), 14-26.

Jojo, Z. (2017). Disrupting a learning environment for promotion of geometry teaching. *Africa Education Review, 14*(3-4), 245-262.

Kahu, E. R., & Nelson, K. (2018). Student engagement in the educational interface: Understanding the mechanisms of student success. *Higher Education Research & Development, 37*(1), 58-71.

Kamina, P., & Iyer, N. N. (2009). From concrete to abstract: Teaching for transfer of learning when using manipulatives.

Karakus, F., & Aydin, B. (2017). The Effects of Computer Algebra System on Undergraduate Students' Spatial Visualization Skills in a Calculus Course. *Malaysian Online Journal of Educational Technology, 5*(3), 54-69.

Karakuş, F., & Peker, M. (2015). The effects of dynamic geometry software and physical manipulatives on pre-service primary teachers' van Hiele levels and spatial abilities. *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 6*(3), 338-365.

Karatas, I., & Baki, A. (2017). The effect of learning environments based on problem solving on students' achievements of problem solving. *International Electronic Journal of Elementary Education, 5*(3), 249-268.

Karimah, R., Kusmayadi, T., & Pramudya, I. (2018). *Analysis of difficulties in mathematics learning on students with guardian personality type in problem-solving HOTS geometry test.* Paper presented at the Journal of Physics: Conference Series.

Keraf, Y. L. (2017). Penggunaan media papan berpaku untuk meningkatkan hasil belajar matematika. *BASIC EDUCATION, 6*(8), 824-830.

Kong, Q.-P., Wong, N.-Y., & Lam, C.-C. (2003). Student engagement in mathematics: Development of instrument and validation of construct. *Mathematics Education Research Journal, 15*(1), 4-21.

Kovanovic, V., Gašević, D., Dawson, S., Joksimovic, S., & Baker, R. (2016). Does time-on-task estimation matter? Implications on validity of learning analytics findings. *Journal of Learning Analytics, 2*(3), 81-110.

Kroll, T., & Neri, M. (2009). Designs for mixed methods research. *Mixed methods research for nursing and the health sciences, 31*.

Lambdin, D. (2003). Benefits of teaching through problem solving. *Teaching Mathematics Through Problem Solving: Grades PreK, 6*, 3-14.

Lamborn, S., Newmann, F., & Wehlage, G. (1992). The significance and sources of student engagement. *Student engagement and achievement in American secondary schools*, 11-39.

Lane, E. S., & Harris, S. E. (2015). A new tool for measuring student behavioral engagement in large university classes. *Journal of College Science Teaching, 44*(6), 83-91.

Larbi, E., & Mavis, O. (2016). The Use of Manipulatives in Mathematics Education. *Journal of Education and Practice, 7*(36), 53-61.

Larkin, K. (2016). Mathematics Education and Manipulatives: Which, When, How? *Australian Primary Mathematics Classroom, 21*(1), 12-17.

Larvor, B. (2019). From Euclidean geometry to knots and nets. *Synthese, 196*(7), 2715-2736.

Laski, E. V., Jor'dan, J. R., Daoust, C., & Murray, A. K. (2015). What makes mathematics manipulatives effective? Lessons from cognitive science and Montessori education. *SAGE Open, 5*(2), 2158244015589588.

Lim, C. P., Zhao, Y., Tondeur, J., Chai, C. S., & Tsai, C.-C. (2013). Bridging the gap: Technology trends and use of technology in schools. *Journal of Educational Technology & Society, 16*(2), 59-68.

Lincoln, Y. S., Lynham, S. A., & Guba, E. G. (2011). Paradigmatic controversies, contradictions, and emerging confluences, revisited. *The Sage handbook of qualitative research, 4*, 97-128.

Luneta, K. (2015). Understanding students' misconceptions: an analysis of final Grade 12 examination questions in geometry. *Pythagoras, 36*(1), 1-11.

Mackenzie, N., & Knipe, S. (2006). Research dilemmas: Paradigms, methods and methodology. *Issues in educational research, 16*(2), 193-205.

Makhubele, Y. E. (2014). Misconceptions and resulting errors displayed by Grade 11 Learners in the Learning of Geometry.

Maričić, S., & Stamatović, J. (2018). The effect of preschool mathematics education in development of geometry concepts in children. *Eurasia Journal of Mathematics, Science and Technology Education*.

Mason, M. (2009). The van Hiele levels of geometric understanding. *Colección Digital Eudoxus, 1*(2).

Maulyda, M. A., Hidayati, V. R., Rosyidah, A. N. K., & Nurmawanti, I. (2019). Problem-solving ability of primary school teachers based on Polya's method in Mataram City. *PYTHAGORAS, 14*(2).

Maxwell, J. A. (2012). *Qualitative research design: An interactive approach* (Vol. 41): Sage publications.

Menghini, M. (2015). *From practical geometry to the laboratory method: the search for an alternative to Euclid in the history of teaching geometry.* Paper presented at the Selected Regular Lectures from the 12th International Congress on Mathematical Education.

Meyer, D. Z., & Avery, L. M. (2009). Excel as a qualitative data analysis tool. *Field methods, 21*(1), 91-112.

Mildenhall, P., Swan, P., Northcote, M., & Marshall, L. (2008). Virtual manipulatives on the interactive whiteboard: A preliminary investigation.

Morse, J., & Richards, L. (2002). The integrity of qualitative research. *Morse J, Richards L. Read me first for a user's guide to qualitative method. California: Sage*, 23-41.

Mushipe, M. (2016). *Effects of integrating GeoGebra into the teaching of linear functions on Grade 9 learners' achievement in Mopani district, Limpopo Province.*

Naidoo, J., & Goba, B. (2011). *Scaffolding the teaching and learning of mathematics.* Paper presented at the Proceedings of the 17th Annual Nafional Congress of the Associafion for Mathemafics Educafion of South Africa.

Naidoo, J., & Kapofu, W. (2020). Exploring female learners' perceptions of learning geometry in mathematics. *South African Journal of Education, 40*(1).

Newmann, F. M. (1992). *Student engagement and achievement in American secondary schools*: ERIC.

Norris, E., Shelton, N., Dunsmuir, S., Duke-Williams, O., & Stamatakis, E. (2015). Physically active lessons as physical activity and educational interventions: a systematic review of methods and results. *Preventive medicine, 72*, 116-125.

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia-Social and Behavioral Sciences, 55*, 720-729.

Ozmantar, M. F., & Roper, T. (2004). Mathematical Abstraction through Scaffolding. *International Group for the Psychology of Mathematics Education*.

Özsoy, G., & Ataman, A. (2017). The effect of metacognitive strategy training on mathematical problem solving achievement. *International Electronic Journal of Elementary Education, 1*(2), 67-82.

Padmore, E. A. (2017). *Examining the use of manipulative materials in teaching mathematics among Junior High School teachers in the Wa Municipality.* University of Education, Winneba.

Perry, D. R., & Steck, A. K. (2015). Increasing student engagement, self-efficacy, and meta-cognitive self-regulation in the high school geometry classroom: Do iPads help? *Computers in the Schools, 32*(2), 122-143.

Pfister, M., Opitz, E. M., & Pauli, C. (2015). Scaffolding for mathematics teaching in inclusive primary classrooms: a video study. *ZDM, 47*(7), 1079-1092.

Pham, L. T. M. (2018). Qualitative Approach to Research A Review Of Advantages and Disadvantages Of Three Paradigms: Positivism, Interpretivism And Critical Inquiry. *University of Adelaide*.

Piaget, J., & Cook, M. (1952). *The origins of intelligence in children* (Vol. 8): International Universities Press New York.

Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. *Developing research in mathematics education. Twenty years of communication, cooperation and collaboration in Europe*, 196-210.

Polya, G. (1962). *Mathematical Discovery, 1962*: John Wiley & Sons.

Pusey, E. L. (2003). The Van Hiele model of reasoning in geometry: a literature review.

Putri, H. E. (2015). The Influence of Concrete Pictorial Abstract (CPA) Approach to The Mathematical Representation Ability Achievement of The Preservice Teachers at Elementary School. *Internasional Journal of Education and Research, 3*(6), 24115881.

Radford, J., Bosanquet, P., Webster, R., & Blatchford, P. (2015). Scaffolding learning for independence: Clarifying teacher and teaching assistant roles for children with special educational needs. *Learning and Instruction, 36*, 1-10.

Rahman, A., & Ahmar, A. (2016). Exploration of mathematics problem solving process based on the thinking level of students in junior high school.

Ramlan, A. M. (2016). The Effect of Van Hiele Learning Model Toward Geometric Reasoning Ability Based on Self-efficacy of Senior High School Students. *Journal of Mathematics Education, 1*(2 July), 62-71.

Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly, 31*(3), 235-252.

Rofii, A., Sunardi, S., & Irvan, M. (2018). Characteristics of students' metacognition process at informal deduction thinking level in geometry problems. *International Journal on Emerging Mathematics Education, 2*(1), 89-104.

Rosen, L. D., Carrier, L. M., & Cheever, N. A. (2013). Facebook and texting made me do it: Media-induced task-switching while studying. *Computers in Human Behavior, 29*(3), 948-958.

Rosen, L. D., Cheever, N., & Carrier, L. M. (2015). *The Wiley handbook of psychology, technology, and society*: John Wiley & Sons.

Rouinfar, A., Madsen, A. M., Hoang, T. D. N., Puntambekar, S., & Rebello, N. S. (2012). *Comparing the development of students' conceptions of pulleys using physical and virtual manipulatives.* Paper presented at the AIP Conference Proceedings.

Ryan, A. M., Daum, D., Bauman, T., Grisez, M., Mattimore, K., Nalodka, T., & McCormick, S. (1995). Direct, indirect, and controlled observation and rating accuracy. *Journal of Applied Psychology, 80*(6), 664.

Ryan, G. (2018). Introduction to positivism, interpretivism and critical theory. *Nurse researcher, 25*(4), 41-49.

Saleh, M., Prahmana, R. C. I., & Isa, M. (2018). Improving the Reasoning Ability of Elementary School Student through the Indonesian Realistic Mathematics Education. *Journal on Mathematics Education, 9*(1), 41-54.

Scandrett, H. (2008). Using Geoboards in Primary Mathematics: Going... Going... Gone? *Australian Primary Mathematics Classroom, 13*(2), 29-32.

Schoenfeld, A. H. (1987). Pólya, problem solving, and education. *Mathematics magazine, 60*(5), 283-291.

Scott, P. B. (1983). A survey of perceived use of mathematics materials by elementary teachers in a large urban school district. *School Science and Mathematics, 83*(1), 61-68.

Sibiya, M. R. (2019). The effect of geoboard use on learners' motivation for learning of geometry theorems. *International Journal, 75*(6/1).

Sibiya, M. R., & Mudaly, V. (2018). The effects of the geoboard on learner understanding of geometry theorems. *International Journal, 74*(11/1).

Silverman, D. (2013). *Doing qualitative research: A practical handbook*: SAGE Publications Limited.

Sinatra, G. M., Heddy, B. C., & Lombardi, D. (2015). The challenges of defining and measuring student engagement in science: Taylor & Francis.

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM, 47*(3), 319-329.

Sinclair, N., Bussi, M. G. B., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2017). *Geometry education, including the use of new technologies: A survey of recent research.* Paper presented at the Proceedings of the 13th International Congress on Mathematical Education.

Smith, & Stephanie. (2015). Playing to engage: Fostering engagement for children and teachers in low socioeconomic regions through science and mathematics play-based learning.

Smith, J., & Firth, J. (2011). Qualitative data analysis: the framework approach. *Nurse researcher, 18*(2), 52-62.

Solvie, P., & Kloek, M. (2007). Using technology tools to engage students with multiple learning styles in a constructivist learning environment. *Contemporary Issues in Technology and Teacher Education, 7*(2), 7-27.

Spanjers, D. M., Burns, M. K., & Wagner, A. R. (2008). Systematic direct observation of time on task as a measure of student engagement. *Assessment for Effective Intervention, 33*(2), 120-126.

Storms, M. D. (1973). Videotape and the attribution process: Reversing actors' and observers' points of view. *Journal of personality and social psychology, 27*(2), 165.

Subedi, D. (2016). Explanatory sequential mixed method design as the third research community of knowledge claim. *Am J Educ Res, 4*(7), 570-577.

Sulistiowati, D., Herman, T., & Jupri, A. (2019). *Student difficulties in solving geometry problem based on Van Hiele thinking level.* Paper presented at the Journal of Physics: Conference Series.

Tabach, M., & Nachlieli, T. (2015). Classroom engagement towards using definitions for developing mathematical objects: the case of function. *Educational Studies in Mathematics, 90*(2), 163-187.

Toussaint, M. J., & Brown, V. (2018). Connecting the arcs motivational model to game design for mathematics learning. *Transformations, 4*(1), 19-28.

Unluer, S. (2012). Being an insider researcher while conducting case study research. *Qualitative Report, 17*, 58.

Usman, H., Yew, W., & Salleh, S. (2018). Effect of Van Hiele's model on pedagogical abilities of undergraduate mathematics education students in Niger State, Nigeria. *e-PROCEEDING*, 5.

van de Pol, J., Volman, M., Oort, F., & Beishuizen, J. (2015). The effects of scaffolding in the classroom: support contingency and student independent working time in relation to student achievement, task effort and appreciation of support. *Instructional Science, 43*(5), 615-641.

Van Hiele, P. M. (1959). The child's thought and geometry. *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele*, 243-252.

Van Hiele, P. M. (1984). The child's thought and geometry. *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele. Washington DC: NSF*.

Vygotsky, L. S. (1981). The genesis of higher mental functions. *The concept of activity in Soviet psychology*, 144-188.

Walle, V. d. (2004). The Van Hiele theory of geometry thought. Retrieved 29 January 2019, from https://www.google.com/imgres?imgurl=https://ylgarris.files.wordpress.com/2013/07/hypx.png&imgrefurl=https://ylgarris.wordpress.com/2013/07/15/the-van-hiele-levels-of-gemetric-understanding/&h=383&w=633&tbnid=Qy08vDrHgw7HWM:&q=The+van+hiele+theory+of+geometric+thought&tbnh=137&tbnw=226&usg=AI4_-kQxbpxvSYBavOz33_1Dvz8gu-VgaA&vet=12ahUKEwjqoeH3yZPgAhVS5uAKHX7sDyEQ9QEwAHoECAkQBg..i&docid=6OmimXIlTph3zM&sa=X&ved=2ahUKEwjqoeH3yZPgAhVS5uAKHX7sDy

EQ9QEwAHoECAkQBg#h=383&imgdii=8Z2JP4tvY8GRDM:&tbnh=137&tbnw=226&vet=12ahUKEwjqoeH3yZPgAhVS5uAKHX7sDyEQ9QEwAHoECAkQBg..i&w=633

Walsh, R. (2015). A purpose-built model for the effective teaching of trigonometry: a transformationc of the van Hiele model.

Wells, P. (1970). Creating mathematics with a geoboard. *The Arithmetic Teacher, 17*(4), 347-349.

West, L. (2011). *Using Physical and Virtual Manipulatives with Eighth Grade Geometry Students.* University of Nebraska-Lincoln.

Widodo, S. A., & Ikhwanudin, T. (2018). *Improving mathematical problem solving skills through visual media.* Paper presented at the Journal of Physics: Conference Series.

Wilkins, S. (2017). Ethical issues in transnational higher education: The case of international branch campuses. *Studies in Higher Education, 42*(8), 1385-1400.

Wilson, S. (2001). What is an Indigenous research methodology? *Canadian journal of native education, 25*(2), 175-179.

Wisniewski, Z. G., & Smith, D. (2002). How Effective Is Touch Math for Improving Students with Special Needs Academic Achievement on Math Addition Mad Minute Timed Tests?

Yazdani, M. A. (2008). The Gagne–van Hieles Connection: A Comparative Analysis of Two Theoretical Learning Frameworks. *Journal of Mathematical Sciences & Mathematics Education, 3*(1), 58-63.

Yeatts, K. (1991). Manipulatives: Motivating Mathematics.

Yvonne Feilzer, M. (2010). Doing mixed methods research pragmatically: Implications for the rediscovery of pragmatism as a research paradigm. *Journal of mixed methods research, 4*(1), 6-16.

Zakszeski, B. N., Hojnoski, R. L., & Wood, B. K. (2017). Considerations for Time Sampling Interval Durations in the Measurement of Young Children's Classroom Engagement. *Topics in Early Childhood Special Education, 37*(1), 42-53.

Zheng, P., Yu, S., Wang, Y., Zhong, R. Y., & Xu, X. (2017). User-experience based product development for mass personalization: A case study. *Procedia CIRP, 63*, 2-7.

Žilková, K., Guncaga, J., & Kopácová, J. (2015). (Mis) Conceptions about Geometric Shapes in Pre-Service Primary Teachers. *Acta Didactica Napocensia, 8*(1), 27-35.

Zuber-Skenitt, O. (1993). Improving learning and teaching through action learning and action research. *Higher education research and development, 12*(1), 45-58.

Appendix A

Partial Interval Recording for learners’ engagement time

Behavioural engagement: 1. Irrelevant behaviours 2. listening attentively 3. asking a question 4. answering a question 5. using geoboard 6. doing exercise 7. seeking for assistance 8. assisting other learners. Note that 15-s means 15 seconds.

Date: **Duration: 30 minutes (min)**

Time		Participants								
In min	**In 15-s intervals**	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
1	1									
	2									
	3									
	4									
2	5									
	6									
	7									
	8									
3	9									
	10									
	11									
	12									
4	13									
	14									
	15									
	16									
5	17									
	18									
	19									
	20									
6	21									
	22									
	23									
	24									
7	25									
	26									
	27									
	28									
8	29									
	30									
	31									
	32									

Time		Participants								
In min	**In 15-s intervals**	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
9	33									
	34									
	35									
	36									
10	37									
	38									
	39									
	40									
11	41									
	42									
	43									
	44									
12	45									
	46									
	47									
	48									
13	49									
	50									
	51									
	52									
14	53									
	54									
	55									
	56									
15	57									
	58									
	59									
	60									
16	61									
	62									
	63									
	64									

Time		Participants								
In min	In 15-s intervals	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
17	65									
	66									
	67									
	68									
18	69									
	70									
	71									
	72									
19	73									
	74									
	75									
	76									
20	77									
	78									
	79									
	80									
21	81									
	82									
	83									
	84									
22	85									
	86									
	87									
	88									
23	89									
	90									
	91									
	92									
24	93									
	94									
	95									
	96									

Time		Participants								
In min	In 15-s intervals	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
25	97									
	98									
	99									
	100									
26	101									
	102									
	103									
	104									
27	105									
	106									
	107									
	108									
28	109									
	110									
	111									
	112									
29	113									
	114									
	115									
	116									
30	117									
	118									
	119									
	120									

Appendix B

Semi-structured interview schedule for: Effect of manipulative objects on geometry learning in grade 11

1. Did manipulative objects support you to understand geometry, if yes, how?

2. Between test one and test two, which one was difficult, and why?
3. Explain with an aid of geoboard how you have answered 6.1?
4. Why is line AB//FC on the figure shown on 6.2?
5. Did you answer question 6.2.2 in the first test, if No, why?
6. Did you answer question 6.2.2 in the second test, if yes, how?

1. Did manipulative objects support you to understand geometry, if yes, how?

Learner B: Yes, because I was able to visualise and construct theorems on my own, in my own way. Manipulative objects helped me to easily identify and separate each theorem when their combined in a figure.

Learner E: Yes because I was able to assemble and separate theorems on the geoboard, because in the exam, the theorems are combined, so with geoboards I can do each one of them separately and they are amazing, it is easy to understand them.

Learner G: Yes, I now know how to demonstrate and draw different parts of the circle. It became easy for me to understand because I was demonstrating it practical. It also became easy for me to understand because I was able to separate each theorem in a figure with a combination of theorems like the ones we encounter in the examination.

Learner J: I did not understand geometry but because of geoboards I now have an understanding of what geometry is about. I was able to make use of elastics break a figure with many theorems on it down

Learner L: Yes, geoboards helped me because I was able to create my representation of theorems, unlike when given a figure with many theorems on the paper. But with geoboards I able to separate each theorem which helped to be not confused.

Learner O: Yes because they have given me a clue about geometry and also to support my geometry statement with corresponding reasons.

Learner Q: Yes because I was able to demonstrate and create angle on the geoboard so that I could understand theorems better.

Learner T: Yes because we were doing geometry practically so it became easy to understand.

2. Between test one and test two, which one was difficult, and why?

Learner B: *Test one was difficult because I did not have the knowledge that I have now. After using geoboards, I was able to understand geometry.*

Learner E: *Test one was difficult because there were no geoboards. But after using geoboards test numbers two became easy.*

Interviewer: *Don't you think that maybe the presence of other questions like question 1, 2, 3 and 4 in the test1 affected your performance in the first test?*

Learner E: *"No, in test 1, it was like I knew only 25% but in test 2 I knew about 75% of geometry".*

Learner G: *Test 1 because I did now know how to construct but with the geoboards, I could use the elastics to see how to construct a line. Geoboards helped, I realised that I did know much about theorems in test 1. I did not know much about how theorems are represented practically in test 1. Even the proofs were difficult to me but now I have a bit of an understanding.*

Learner J: *Test 1 was difficult because I did not have the knowledge that I have which I gained from geoboards.*

Learner L: *Test one was difficult because I did not understand much about geometry at that time. Now I am able to differential between the theorems because I have even done them practically on the geoboard.*

Learner O: *I think Test 1 was difficult. In test 1 I did not have basic understanding of geometry. But in test 2 I had a basic understanding of circle geometry even though the knowledge I have is now enough.*

Learner Q: *Test 1 was difficult because there was no geoboard. Geoboard makes understand easily angles and theorems.*

Learner T: *Test1 I think it is because I had a lack of knowledge before I write test 1. In test 2 I at least have knowledge but not that well.*

3. **Explain with an aid of geoboard how you have answered 6.1?**

Learner B: *Geoboards helped me to visualise different representations of theorem assessed in 6.1. I was able to see equal angle and sides.*

Interviewer: *can you explain how did you write your proof on the test?*

Learner B: *Join OP produced to K. Then I said let P1=V OP=OP because of radii. I do the same to the second triangle.*

Interviewer: But in the question you were asked to prove that angle VCW=2×angle P, do you think that the proof is complete?

Learner B: "Eish ,eish".

Learner E: I constructed line OB produced to K. On the geoboard I could see that OV =OP because they are radii. Two triangles were formed, OVP and OWP.

Angles P1, P2 and O1 and O2 were formed.

P1 is opposite to V they both equal to x; P2 opposite to W they are equal to y

Interviewer: Why is p1=V?

Learner E: Because, angle, angle P is, It is better to write it down than explaining.

Learner G: I constructed line OP produced to K Then OV=OW are radii.

Interviewer: How is this construction going to assist you in proving?

Learner G did not respond.

Learner J: Join OP produced to K. Then I said let P1=V OP=OP because of radii. I do the same to the second triangle.

Learner L: I first constructed a line so that I can have two triangles, then I proved starting between triangle OVP and OPW then I was able to find.

Learner O: I joined OP to form S, then P1 and P2; O1 and O2 were formed. OP=OA because radii, P1=V because of angles opposite equal side. Then O1 =2P1, I did the same on the second triangle.

Learner Q: I constructed line, angles P1, P2 and O1 and O2 were formed.P1 is opposite to V they both equal to x; P2 opposite to W they are equal to y.

Learner: I constructed OP to F; I saw that I now, have two triangles.

OP =OV because they are radii. Let P1=x, then P1=V=x because they angles opposite equal sides.

O1=P1+V reason is exterior angle of a triangle. The O1= 2x.

Then I said let P2=y, OW=OP they are radii.

P2=W=y angles opposite equal sides. The O2=W+P2 reason exterior angle of a triangle.

O1+O2=2x+2y, I then took out a common factor

O1+O2=2(P1+P2), then I concluded to say

Angle VOW=2P.

4. Why is line AB//FC?

Learner B: *For two lines to be parallel, alternating and corresponding angle must be equal.*

Learner E: *I think it is because we do have alternating angles which are angle A and C2.*

EC=EF which means that angle C2 and angle F are equal.

Learner G: *This question was difficult to me.*

Learner J: *A1=A3 because they are vertically opposite angles.*

Learner L: *Alternating angles are equal.*

Interviewer: *Which are those angles?*

Learner L: *Angle A and F.*

Interviewer: *How do you know that they are equal?*

Learner L did not respond.

Learner O: *When proving parallel line we prove alternating, corresponding and vertically opposite. C2 =A.*

Learner Q: *I did not answer this question.*

Learner T: *When talking about alternating, corresponding and vertical opposite angles angle, so angle A= angle C2 they alternating.*

Interviewer: *Ok, but how do all these angles assist you to prove that AB//FC?*

Learner T: *Because corresponding angles are equal.*

Interviewer: How do you know?

Learner T did not respond.

5. Did you answer question 6.2.2 in the first test, if No, why?

Learner B: *No, I found difficult to answer.*

Learner E: *No*

Learner G: *No I was difficult to me.*

Learner J: *I did not answer*

Learner L: *No because I did not know much about cyclic quadrilaterals.*

Learner O: *Yes*

Learner Q: *No because in the first test time was against me and I did not know how to prove.*

Learner T: *No, I did not have enough knowledge at that time and I did not focus in the classroom where the geometry was taught. But now as we were learning I was more focused. When the test 1 can it was something that I have never seen.*

6. **Did you answer question 6.2.2 in the second test, if yes, how?**

Learner B: Yes I answered it but I did not complete answering it.

Learner E: Yes but I was not sure. With the help of geoboard I could see with my own mind the different theorems which are there in the figure. I know that opposite angles of a cyclic quadrilateral are supplementary.

Learner G: Yes I answered it but I did not complete answering it

Interviewer: Can you please explain what you have written.

Learner G: No I don't remember.

Learner J: B2=C1 because angles on a same segment, then angle A=angle A. It is better to write it down than explaining.

Learner L: I tried to answer; I am not sure about what I wrote. Angle B2= C because they are on the same segment.

Learner O: When proving the cyclic I need to check angles on the same segment, O=2A angle at the centre is twice the angle at the circumference, B2=F angles on the same segment.

Learner Q: I tried to answer, E=C2 they are alternating angles.

Interviewer: Can you please explain how you have tried answering.

Learner Q: E=C2 they are alternating angles.

Interviewer: Ok, please explain further.

Learner Q: I only wrote this.

Learner T: Yes but I am not sure about I how I have answered. I said angle C1= B1 because they are subtended by the same segment, then O=2F=2C angle at the centre is twice the angle at the circumference.

Appendix C

Pre-Post Test to measure: learners' academic progress in problem solving and Van Hiele's levels (3 and 4) when geometry learning is scaffolded with manipulative objects.

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

[redacted]

GRADE 11

MATHEMATICS P2

COMMON TEST

JUNE 2019

MARKS: **100**

TIME: **2 hours**

This question paper consists of 8 pages and 2 DIAGRAM SHEETS.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. TWO DIAGRAM SHEETS for QUESTION 2.2, QUESTION 5.1, QUESTION 5.2, QUESTION 6.1 AND QUESTION 6.2 are attached at the end of this question paper. Detach the DIAGRAM SHEETS and hand in together with your ANSWER BOOK.
10. Write neatly and legibly.

 Please turn over

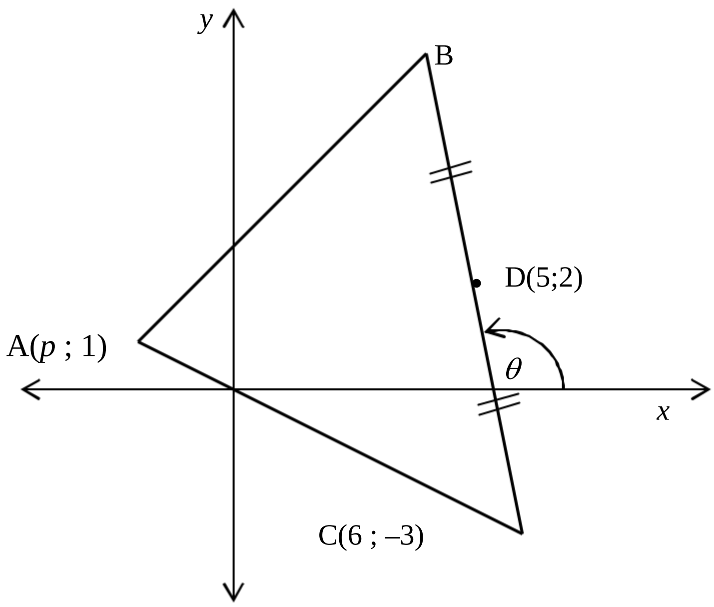
QUESTION 1

1.1 Consider the point K(–8 ; 3) in the Cartesian plane.

1.1.1 Write down the equation of the horizontal line passing through K. (1)

1.1.2 Write down the equation of the vertical line passing through K. (1)

1.2 In the diagram, A(p ; 1), B and C(6 ; –3) are the vertices of Δ ABC.
D(5 ; 2) is the midpoint of BC. A lies in the second quadrant.
DC forms an angle θ with the x-axis.



Determine the:

1.2.1 Gradient of BC. (2)

1.2.2 Size of θ, rounded off to ONE decimal place. (3)

1.2.3 Coordinates of B. (3)

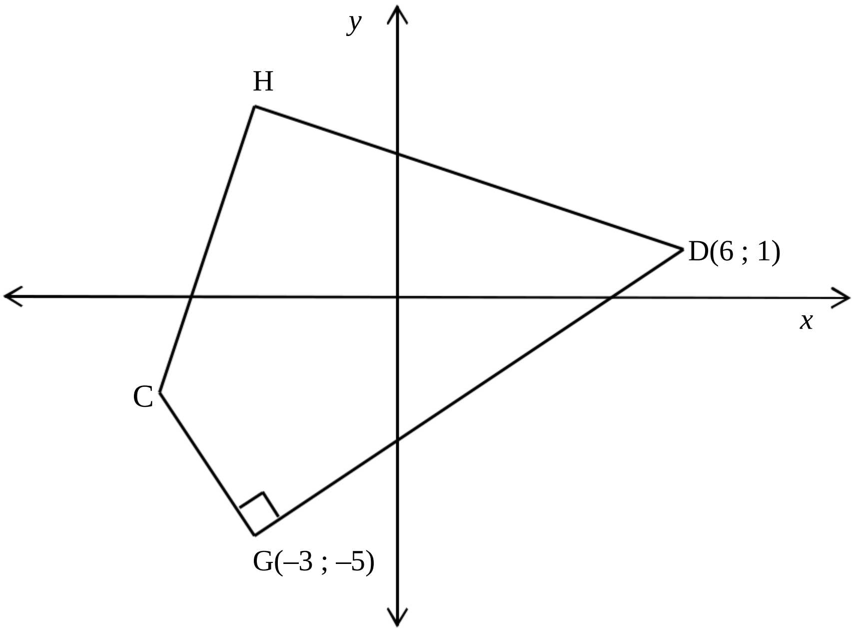
1.2.4 Value of p, if it is given that the length of AC = $4\sqrt{5}$ (5)

[15]

QUESTION 2

2.1 Calculate the value of q if K(–6 ; 9), L(–3 ; q) and M(–2 ; –1) are collinear. (4)

2.2 G(–3 ; –5), D(6 ; 1), H and C are the vertices of quadrilateral GDHC.
CG $\perp$ GD. The equation of CH is $y = 3x + 13$.



2.2.1 Determine the equation of CG. (4)

2.2.2 Calculate the coordinates of C. (3)

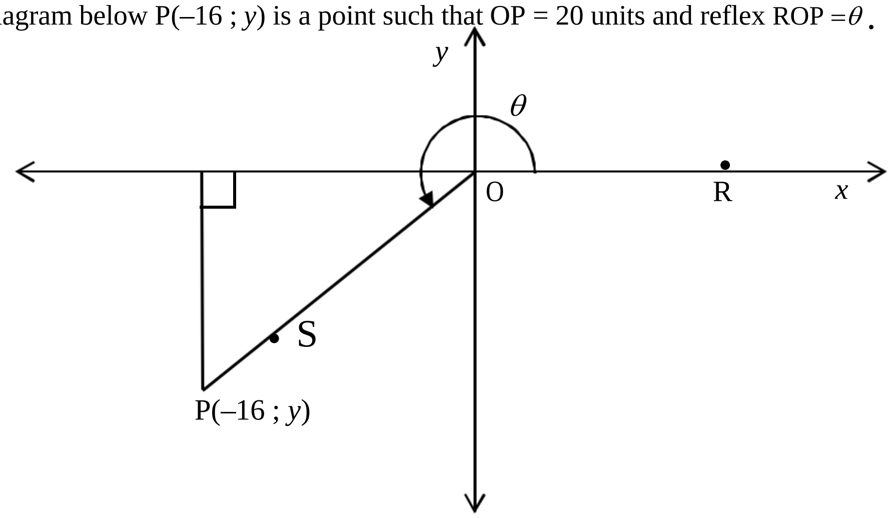
2.2.3 Calculate the size of $G\hat{C}H$. (5)

[16]

QUESTION 3

3.1

In the diagram below P(–16 ; y) is a point such that OP = 20 units and reflex $R\hat{O}P = \theta$.



3.1.1 Calculate the value of y. (2)

3.1.2 Determine the value of each of the following without using a calculator:

(a) $\sin(180^\circ - \theta)$ (2)

(b) $\cos(180^\circ + \theta)$ (2)

3.1.3 S is a point on OP such that OS = 15.
Determine the coordinates of S, WITHOUT using a calculator. (4)

3.2 Simplify, WITHOUT the use of a calculator: $\dfrac{\cos(-33^\circ).\tan 147^\circ}{2\cos 303^\circ.\sin 240^\circ}$ (7)

[17]

QUESTION 4

4.1 Use trigonometric identities to prove that $\frac{\sin^3 x + \sin x.\cos^2 x}{\cos x} = \tan x$ (3)

4.2 Solve for x if $\sin x = 0,412$ and $x \in [0^\circ ; 360^\circ]$. (2)

4.3 Consider the equation: $\tan 3x + 2,64 = 0$.

4.3.1 Determine the general solution of $\tan 3x + 2,64 = 0$ (4)

4.3.2 Hence, or otherwise, solve for x if $-90^\circ \leq x \leq 90^\circ$ (3)

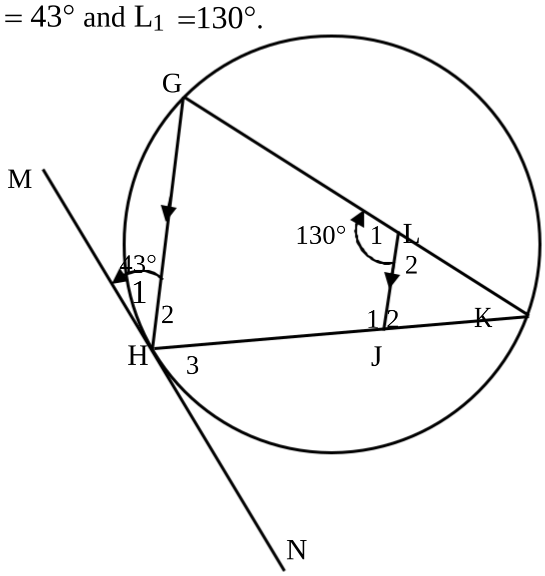
4.4 Solve for x if $4\sin^2 x + 7\cos x - 4 = 0$ and $x \in [0^\circ ; 360^\circ]$. (6)

[18]

GIVE REASONS FOR YOUR STATEMENTS AND CALCULATIONS IN QUESTIONS 5 and 6.

QUESTION 5

5.1 MHN is a tangent to circle GHK at H. L is a point on GK and J a point on HK such that LJ is parallel to GH. $\hat{H}_1 = 43°$ and $\hat{L}_1 = 130°$.



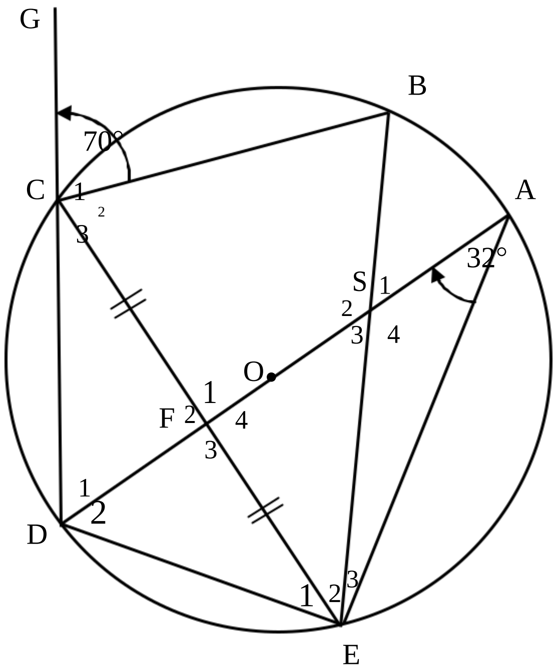
Calculate, with reasons, the size of:

5.1.1 $\hat{K}$ (2)

5.1.2 $\hat{H}_3$ (4)

5.2 A, B, C, D and E are points on the circle having centre O. DC is produced to G. Diameter AOD bisects chord CE in F, and intersects chord BE in S.

$\hat{A} = 32°$ and $G\hat{C}B = 70°$.



Calculate, with reasons, the sizes of the following angles:

5.2.1 $B\hat{E}D$ (2)

5.2.2 $\hat{C}_2$ (4)

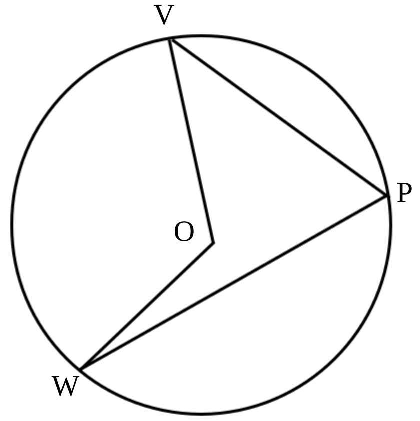
5.2.3 $\hat{D}_1$ (3)

5.2.4 $\hat{E}_3$ (3)

[18]

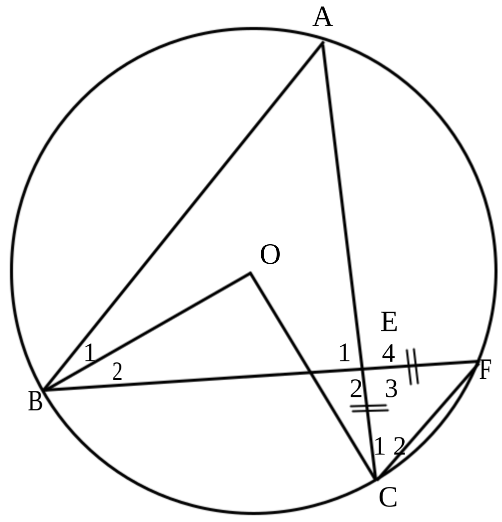
QUESTION 6

6.1 In the diagram, O is the centre of the circle. VP and WP are chords, and VO and WO have been drawn.



Use the diagram on the DIAGRAM SHEET to prove the theorem which states that an angle that an arc subtends at the centre of a circle is twice the size of the angle subtended by the same arc at the circle i.e. $V\hat{O}W = 2\hat{P}$. (6)

6.2 In the diagram, O is the centre of the circle. A, B, C and F are points on the circumference. AC and BF intersect in E and EF = FC.



Prove that:

6.2.1 AB || FC. (5)

6.2.2 OBCE is a cyclic quadrilateral. (5)

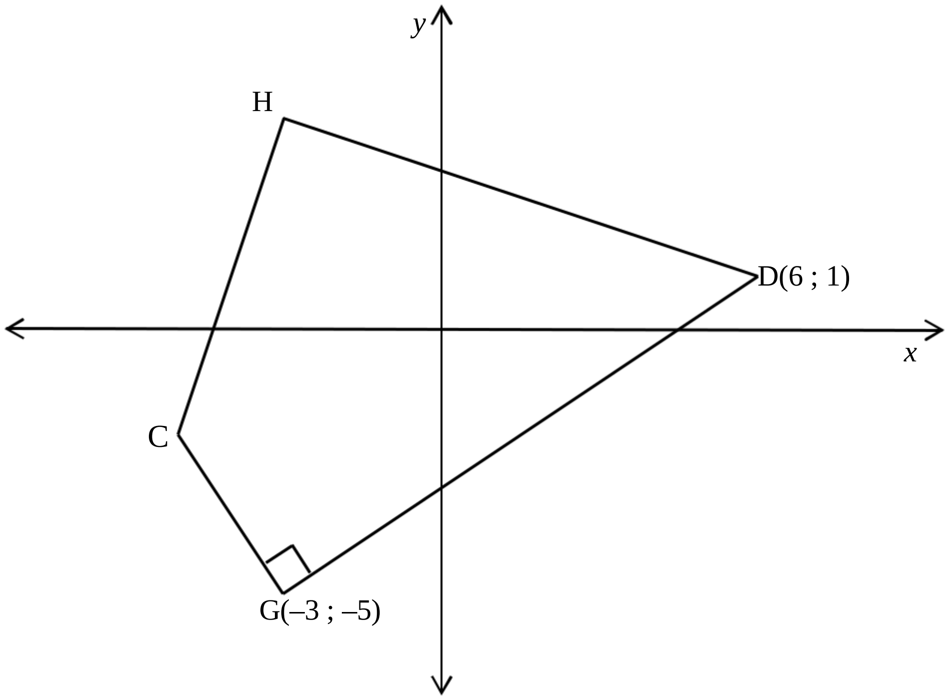
[16]

TOTAL: 100

NAME & SURNAME:

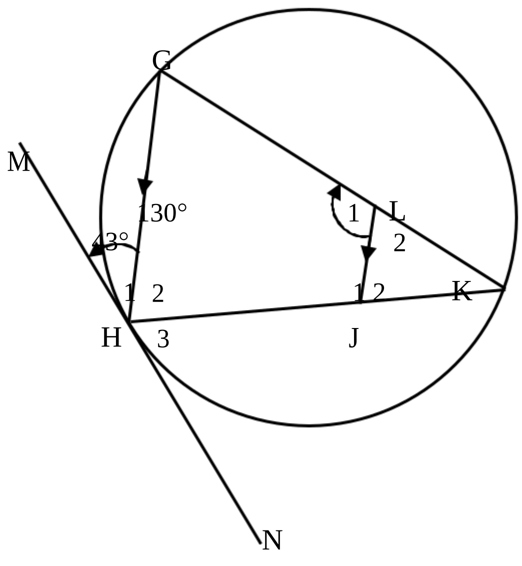
DIAGRAM SHEET 1

QUESTION 2.2

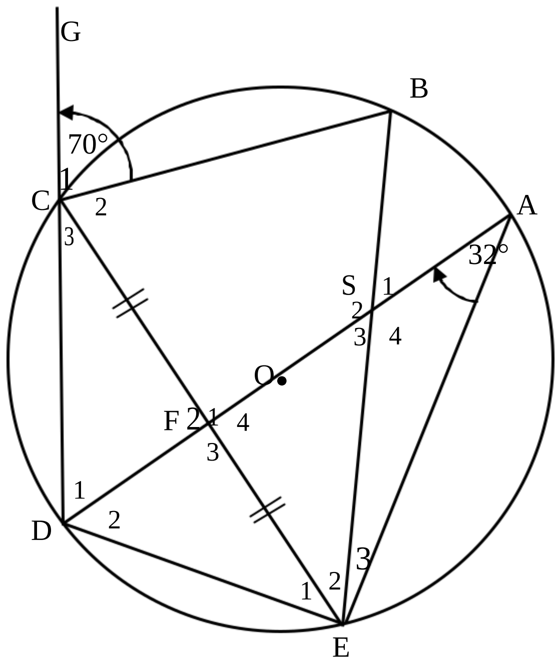


QUESTION 5.1

TEAR OFF



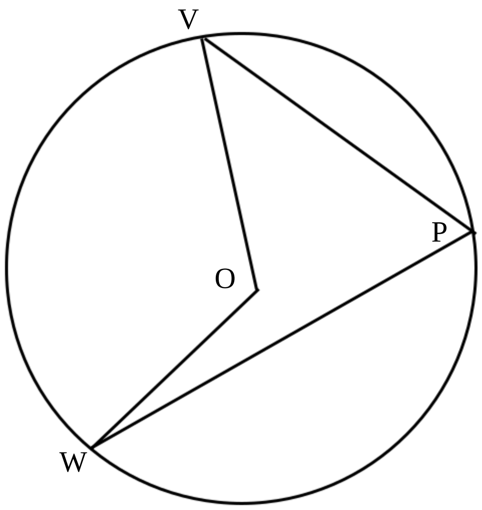
QUESTION 5.2



NAME & SURNAME:

DIAGRAM SHEET 2

QUESTION 6.1



QUESTION 6.2

TEAR OFF