

**UNIVERSITY OF ZULULAND**



**Teacher conceptualisations of pedagogical content knowledge in  
relation to meaningful learning in the context of teaching of  
mathematics in primary schools**

**BY**

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## DECLARATION

I, **SITSULA TSHISIKHAWE MARY**, hereby declare that this thesis, submitted to the University of Zululand in fulfilment of the academic requirements for the award of the Doctor of Education in the field of Mathematics Education, is an original work done by me. I also declare that the work has neither submitted nor copied elsewhere and that all reference material contained therein has been duly acknowledged.

.....  
**SITSULA T.M**

.....  
**DATE**

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## **DEDICATION**

I dedicate this project to my late parents, my mom, Vho Muofhe Mundalamo and my father Vho Philemon Mundalamo for their simple spirited attitude towards my further education, their prayers and words of encouragement. This research project would not be a success without their support

## ABSTRACT

Teachers play a critical role in mathematics classroom. For the most part, learning mathematics is more than just learning concepts and skills. The indispensable goal of teaching at all levels is to involve learners in meaningful learning, which occurs when learners are making meaning. Teaching is a mutual procedure that is centered on learner's learning. The current research problem is identified in finding out teacher's knowledge of teaching mathematics that may try to stimulate learners' meaningful learning. The aim of this study was to identify levels of IP teacher's Mathematics Pedagogical Content Knowledge and their associated operational manifestations in the context of mathematics teaching and learning at Sibasa circuit in the Vhembe District.

This study adopted a Mixed-method research design to understand teacher's conceptualisations and facilitations of mathematics learning. The population of this study constitutes of teachers from intermediate phase at Sibasa circuit in the Vhembe district in Limpopo province, South Africa. Two instruments, questionnaires and interviews were to collect data for this study. Teacher's questionnaire distributed to 120 Intermediate Phase mathematics teachers, and in qualitative method, 10 participants were interviewed to ensure that the researcher collected data from people who have the relevant information.

The results showed that teachers' Mathematics Pedagogical Content Knowledge is generally quite weak which fundamentally limit meaningful learning. Drawing from these findings, the author proposed Tshisi model (Figure 3.3), as a model to facilitate meaningful mathematics practice at the intermediate phase. It is anticipated that the findings of this study will give teachers and curriculum developers' new insights into emerging issues on meaningful teaching in mathematics classroom. The study gave general recommendations for improving teacher's mathematical knowledge towards teaching Mathematics.

**Key words:** Mathematics, Meaningful teaching and learning, Content knowledge, Mathematics Pedagogical Content Knowledge, Intermediate Phase

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## LIST OF ABBREVIATIONS

ANA	Annual National Assessment
CAPS	Curriculum Assessment Policy Statement
CK	Content knowledge
IP	Intermediate Phase
MK	Mathematics knowledge
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
PBL	Problem-based learning
PCK	Pedagogical content knowledge
PK	Pedagogical content knowledge
MKT	Mathematical knowledge for teaching
MPCK	Mathematics pedagogical content knowledge
PIRLS	Progress in International Reading Literacy Study
RNCS	Revised National Curriculum Statement
SCK	Specialized content knowledge
STD	Standard deviation
SPSS	Statistical Product and Service Solution
TIMSS	Trends in International Mathematics

KCL Knowledge of content and learners

KCT Knowledge of content and teaching

# CHAPTER 1

## INTRODUCTION

### 1.1 FOCUS AND BACKGROUND

It is worthy to mention that the intentions of teaching and learning mathematics is to make sure that learners attain and relates mathematical knowledge and skills in ways that are meaningful to their own lives. However, the subject does not seem to be progressing in terms of learner performances. This is evident as a result of learners' poor performance in mathematics at secondary school level (South African Government, media statements, 2015: 1). Report on the Annual National Assessment of 2016 shows that the results of school-based tests, external common tests or examinations and the annual national assessment (ANA) from 2011 up to 2016. It is evident from these assessments that learners are performing far under the expected grade levels. Venkat and Spaul (2015) argued that one of the problems in this regard is that teachers' own mathematics knowledge may be one of the reasons why the children do not develop competence. As a result, teachers may have challenges to recognize what are the most significant mathematical skills for learners to be learnt and how to adjust their teaching to address these.

Mathematical knowledge and skills are getting more significant in people's everyday life experiences. Moreover, mathematics performance in school are vital to higher education. So as to guarantee that learners gain the fundamental knowledge, skills and thinking required on a daily basis. Intermediate phase (IP) school mathematics teachers may, at the most basic level, has mastery of the mathematics pedagogical content knowledge that they may be required in order to teach. Consequently, no mathematics classroom can move beyond the quality of teachers' mathematics pedagogical content knowledge. Teachers' mathematics pedagogical content knowledge is a specific element needed for effective and meaningful mathematics teaching in order to achieve meaningful learning. Morris (2014) indicates that adequate teacher mathematics pedagogical content knowledge (MPCK) improves learner's meaningful learning. Furthermore, the CAPS IP mathematics curriculum documents also

emphasize the need for learners' meaningful mathematics learning and the acquisition of basic mathematical knowledge and skills. Teachers' mathematics pedagogical content knowledge may be an essential ingredient of meaningful and effective mathematics teaching particularly at the Intermediate Phase (IP) level. Intermediate Phase mathematics teachers are tasked to build a solid mathematical conceptual understanding for learners. Yet, many IP teachers themselves are reported not to hold a solid conceptual understanding of mathematical concepts and topics (Macdonald, 2014)

Mathematics education researchers (e.g. Macdonald, 2014:12) have pointed out other factors that may influence meaningful and effective teaching of mathematics but teachers' mathematical pedagogical factor is more important than any other factors stated by Macdonald, (2014). Consequently, Kola and Sunday (2015) indicate that the success of meaningful and effective mathematics teaching and learning process that influences learners' good mathematics performance depends more on teachers than learners themselves. Venkat & Spaul (2007) are of the view that primary school mathematics teachers are the most powerful agents for effective learning. Indeed, learning is critical to learners' development at all school levels (Zhang, 2015). In this regard, Morris (2012) argues that teachers' knowledge of teaching mathematics is paramount and suggests that strong teachers; MPCK supports effective teaching and learning.

This study describes MPCK as the blending of mathematics knowledge and pedagogical knowledge simultaneously because in order to teach mathematics well, teachers need to be able to translate their understanding of mathematics content into pedagogy in ways that make it accessible to learners (Brandt, Lunt & Rimmasch, 2012). In concurrence, Tanner (2003) avers that in order to teach mathematics effectively, teachers need to create experiences that help learners make sense or meaning of mathematics they are learning. Therefore, teachers need MPCK appropriate to specific grade or level to facilitate effective learning.

Schuck (2016) reports that there is a challenge of finding confident and competent teachers of mathematics in the IP. Therefore, there is a need for IP teachers to be given an opportunity to develop their MPCK in order to promote effective teaching and learning. According to Tsafe (2013) one of the characteristics of good and effective mathematics teachers is that they possess strong MPCK. This then means that teachers' MPCK plays an important role for

effective teaching to happen. Even though various elements may impact the successful Mathematics learning, teachers' MPCK assumes a significant role for effective teaching to happen (Turnuklu & Yesidere, 2007).

Some studies (Hill & Bass, 2014; Blackley & Howell, 2015) have demonstrated that teachers' MPCK in the context of teaching and learning in the IP is pivotal to their capacity to provide effective teaching and learning. This study assumes that teachers with adequate mathematics content knowledge can teach mathematics effectively, whereas teachers who have insecurities about their MPCK may not teach mathematics successfully. It was against this background the study is undertaken.

## **1.2 PURPOSE OF THE STUDY**

The importance and value of teaching and learning mathematics as both a tool for school achievement and as a tool for performing everyday activities is a serious concern in the IP. Mathematics will be continually empowering people with mathematical knowledge and skills in order to understand better the world in which they live. However, mathematics teaching and learning in the IP does not adequately teach for meaningful teaching and learning, but for memorizing rules and procedures to perform or solve mathematical problems. The aims of teaching and learning mathematics are to encourage and enable learners to: recognize that mathematics forms part and parcel of learner's everyday lives both inside and outside the school. As a result, teachers have to teach mathematics with good understanding in order for learners to enjoy mathematics and acquire basic mathematical concepts and computational skills when solving mathematical problems. The biggest challenge in teaching mathematics in the intermediate phase is to ensure that learners are committed to learning with a meaningful learning mindset rather than rote learning (Haylock, 2010).

Cognitive psychology says that meaningful teaching establishes clear goals of for the mathematics that learners are learning and making connections between what learners already know and new information that we'd like learners to know. In other words, IP teachers need to help learners to activate what they already know about what teachers are going to teach as new information. Mathematics plays an important role in human life, therefore mathematics must be taught well and correctly. The success of teaching mathematics is

influenced by several factors such as teacher' thinking ability, analysing errors, generalizing, explaining and more. Cognitive psychology is one of the critical success factors in teaching mathematics with good understanding.

The researcher has also first-hand experience which suggests that to many IP teachers teaching mathematics means to memorize examples rules from the textbooks or learner's workbook and applying the correct rules without understanding. Effective and meaningful mathematics teaching entails more than teaching subject matter from the textbook and departmental document. The researcher then argues that at the heart of teaching and learning mathematics meaningfully and effectively, mathematical topics and concepts need to be explained thoroughly in order for the learners to understand.

Teaching mathematics to intermediate learners should be meaningful to optimize meaningful learning. Meaningful learning teaches learners important cognitive skills they will use throughout their life. In teaching mathematics, it is important to know how learners learn. If we teach learners in a connected and related way, learners will then make sense of what they are learning. Amongst other subjects, mathematics is a scarce and priority subject. Therefore, mathematics teachers are required to teach it effectively and meaningfully to promote meaningful learning.

Once teachers teach mathematics in a meaningful method with full clarity and clear explanation of the facts and procedures related to the topic. They will be able to relate that knowledge with every relevant piece of information about that mathematical topic and possess a confidence of applying that information properly in their everyday life experiences. Generally meaningful teaching is very good compared to rote method of teaching. There is no doubt that learners who were taught mathematics in a meaningful way are really clear with all the facts and are conveniently able to transfer their knowledge to activities requiring problem solving with greater success compared to those who were taught by rote method. According to Ma (2013:885) the notion of profound understanding of fundamental mathematics involves both expertise and an understanding of how to communicate with learners. In short teaching mathematics requires teachers the ability to connect mathematics with learners' real life experiences.

It is very simple for a learner to understand the meaning of “fraction” if the teacher connects or associates meaning of fraction with the meaning in which the child is familiar, for example how many slice of bread you eat in the morning for breakfast? Indeed, teaching children mathematics using their everyday experiences promote meaningful learning. The purpose of the study, therefore, was to investigate teachers’ MPCK in teaching and learning context within the IP of Vhembe District. Therefore, this study aims to identify levels of MPCK and their associated operational manifestations

### **1.3 RATIONALE OF THE STUDY**

Research on teachers’ mathematics Pedagogical Content Knowledge (MPCK) and its influence on mathematics teaching and learning at IP level have recently become prominent. Taylor (2013) conducted a study on how children learn mathematics and the implications for teaching. One of the most important aims of mathematics at this level is to develop a positive attitude towards mathematics. In short learners should understand appreciate the functionality of mathematics.

The quality of mathematics is essential at the IP, hence is a source of worry to teachers themselves, and even the Department of Basic Education. However, suggestions, recommendations on how to teach mathematics have been raised by many experts in the field of mathematics department and also researchers in mathematics, yet the problem tends to persist. It is against this background that the study intends to investigate teachers’ MPCK in relation meaningful learning in the context of teaching and learning of mathematics. Internally, teachers experienced lack of sufficient mathematics pedagogical content knowledge supported teaching, as result that had a negative effect on their teaching and learning of mathematics.

The literature shows that MPCK is a special combination of teacher’s mathematical content knowledge and pedagogy (Abell, 2007). Therefore, teachers MPCK in mathematics classroom experiences is the key point to quality teaching which results in meaningful learning. However, the researcher experience as a mathematics teacher and researcher on mathematics education reveal that primary mathematics teachers lack sufficient knowledge of mathematics, content and its pedagogy which the study refers to as mathematics Pedagogical Content Knowledge. In this regard, teachers’ low level of confidence in their PCK seems to result in meaningless teaching and learning.

Taylor (2013) mentioned that mathematics teachers should in some sense know what they are being expected to teach and also how to teach it. It is therefore imperative for primary mathematics teachers to have adequate mathematical pedagogical content knowledge. McNamara (2011) study showed that teachers with adequate mathematics pedagogical content knowledge may teach in more interesting and dynamic way whilst those without content knowledge may struggle to teach mathematics. The Author conceptualizes MPCK as a key element in improving mathematics teaching, particularly at the IP. MPCK is unique in blending different components namely mathematics knowledge, content knowledge and pedagogical knowledge into one professional activity. It is worth noting that teachers' MPCK in the mathematics classroom influences how learners learn mathematics.

In the same light, more research on primary teacher's knowledge of teaching mathematics is needed in order to understand and improve the learning of mathematics learning. Walker (2016) reports that Pedagogical content knowledge (PCK) in mathematics teaching is understood as an essential component of the knowledge base required to deliver meaningful and effective teaching. Teachers' perspective on effective teaching of mathematics at primary level should be explored at teacher preparation level. Hill, Rowan & Ball (2005) investigated how teachers' knowledge for teaching adds to learners' Mathematics accomplishment.

Their findings suggested that teachers' mathematics pedagogical content knowledge enhance meaningful learning. However, teacher's MPCK is often found to be weak for primary mathematics teachers compared to secondary school mathematics teachers. The objective of this study was to develop a model of strategies and support systems for primary school teachers to develop their knowledge of mathematics teaching. Additionally, is meant to increase awareness of teacher's mathematics pedagogical content knowledge in the department of education and schools in general. Every subject teacher must possess an adequate knowledge for his or her teaching area, particularly in mathematics, as it is one of the core subjects in the IP.

Mathematics is part of our everyday life experiences and activities and is used by everyone every day. It is also recognized as one of the important tools of science and technology. The Trends in International Mathematics and Science Study (TIMSS) 2019 marks the seventh cycle of the study which provides 24 years of trends. TIMSS 2019 reports overall achievement as well as extensive background information in which teaching and learning mathematics and science take place. However, the Second International Mathematics Study (TIMMS) in 1989

pointed out that knowledge and skills of mathematics concepts and techniques are crucial in everyday life experiences and in our working places as already mentioned above. Therefore, mathematics should be taught in a meaningful and effective way to learners especially at the IP level. This then suggests that mathematics should be portrayed to learners as being practical, meaningful and relevant in their everyday lives. In this regard, the National Council of Teachers in mathematics has challenged the teachers to be more effective and also to understand deeply the mathematics they are teaching so that learners may acquire good mathematical knowledge and skills. Taylor (2013) mentioned that Mathematics teachers should in some sense know what they are being expected to teach and also how to teach it. It is therefore imperative for primary Mathematics teachers to have adequate Mathematics pedagogical content knowledge. Yuanita, Zulnaldi, Zakaria (2018) argued that learners must be equipped with knowledge and high-level skills and teachers must possess sufficient knowledge of what and how to teach.

In the same way Higgins (2014) argued that teaching mathematics for understanding of at the IP level is supposed to be the bedrock and foundation of mathematical competency at secondary and tertiary levels of education. Abramovich (2019) stated that though the necessity of mathematical learning at the primary, secondary, and tertiary schools is common knowledge, the question on how to teach mathematics is controversial. Briefly making progress on mathematical knowledge for teaching and how to teach is dynamic and continuous process.

Abramovich, Burns, Campbell, and Grinshpan (2016) highlighted that a 'teacher has a special responsibility of teaching content knowledge to learners and therefore should have adequate mathematical content knowledge to help learners understand the subject matter'. Teachers' knowledge of what to teach and how to teach has a tremendous impact on learner achievement.

The researcher believes that effective mathematics teaching requires skilful and knowledgeable teachers. As stated above, mathematics is important for all of us. Yet mathematics teachers are said to lack MPCK for teaching mathematics in ways that enhance learner's meaningful learning (Smith, 2004). The study sought to explore the manner in which teachers conceptualize the construct meaningful learning in the context of their MPCK.

The reason why this researcher focused on mathematics teacher, rather than any teacher of other teaching subject, was the fact that amongst other subjects, mathematics is a scarce and priority subject. It is important to support teachers of scarce skills subjects. Adler and Setati (2002:39) postulated that mathematics provide some of the knowledge that we all require in order to deal with personal and social problems. In this regard the role of intermediate phase teachers is to inspire learners to look beyond memorizing examples from the textbook or workbook to become critical problem solvers.

Once teachers failed to teach mathematics in a meaningful, it would be very difficult for him or her to correct learner's errors and misconceptions. McNamara, (2011) study showed that teachers with adequate Mathematics pedagogical content knowledge may teach in more interesting and dynamic way whilst those without content knowledge may struggle to teach Mathematics. Therefore, IP mathematics teachers are required to teach mathematics in effective and meaningful way to promote meaningful learning.

The researcher offers mathematics extra-classes after school and during weekends. Therefore, the researcher obtained experience as an IP mathematics teacher for more than 25 years, observed that primary learners are learning mathematics in a meaningless way that cannot make sense to them due to the manner in which mathematics is taught.

The Author conceptualizes MPCK as a key element of improving Mathematics teaching particularly at Intermediate Phase level. MPCK is unique blending of different components viz. Mathematics knowledge, content knowledge, and pedagogical knowledge. It is worth noting that teachers' MPCK in Mathematics classroom influence how learners learn Mathematics.

#### **1.4 STATEMENT OF THE PROBLEM**

This research has been prompted by some problems that have been observed regarding the teaching of mathematics in the IP. Generally, IP mathematics teachers lack adequate pedagogical mathematical content knowledge. This is the knowledge of what to teach and how to teach mathematics in a way that is meaningful and sensible to learners. The literature shows that MPCK is a special combination of teacher's mathematical content knowledge and pedagogy (Abell, 2007). Therefore, teacher's MPCK in Mathematics is the key point to quality teaching which results in meaningful learning. However, the researcher experience as a

Mathematics teacher and researcher on Mathematics education reveal that primary Mathematics teachers lack sufficient knowledge of Mathematics, content and its pedagogy which the study refers to as Mathematics Pedagogical Content Knowledge. In this regard, teachers' low level of confidence in their MPCK seems to result in poor teaching.

### **1.5 AIM OF THE STUDY**

The aim of this study was to explore teacher conceptualizations of pedagogical content knowledge in relation to the meaningful learning in the context of teaching and learning of mathematics in primary schools.

### **1.6 RESEARCH OBJECTIVES**

The following research objectives were formulated to address the above aim:

To determine:

- a. the current state of mathematics teaching in the Intermediate Phase in the Sibasa Circuit, Limpopo Province;
- b. intermediate phase teachers' perceptions about the importance of mathematical pedagogical content knowledge (MPCK) in the teaching of mathematics; and
- c. intermediate phase teachers' conceptualisation of mathematics pedagogical content knowledge (MPCK) that is associated with meaningful teaching and learning of mathematics.

### **1.7 RESEARCH QUESTIONS**

The following research questions were formulated for this research:

- a. What is the current state of mathematics teaching in the Intermediate Phase (IP) in the Sibasa circuit, Limpopo Province?
- b. What are IP teachers' perceptions about the importance of mathematical pedagogical content knowledge (MPCK) in the teaching of mathematics; and
- c. How do IP teachers conceptualise the relationship between pedagogical content knowledge (PCK) in relation to the meaningful teaching and learning of mathematics

## **1.8 METHODOLOGICAL SCOPE**

Yin (2003:21) expressed that a research design is an outline or a definite arrangement on which one plans directing an exploration. Similarly, Cresswel (2011) characterized research design as it is utilized to structure the exploration and to show how the entirety of the significant parts of the research venture (the examples or gatherings, measures, medications or projects, and strategies for task), consolidate trying to address the focal research questions.

McMillan and Schumacher (2010:117) likewise affirm that the “objective of a sound research outline is to give discoveries that are regarded to be believable”. As the character of the exploration theme of this study is engaging and logical, the analyst decided to utilize a qualitative research structure. The research outline uncovers the motivation behind the investigation. Yin (2003) showed that the most useful or appropriate research design for those researches that are addressing a subject about which there are high levels of uncertainty and ignorance about the subject, and when the problem is not very well understood.

Since mathematics is powerful and incredibly important subject as it was already stated in 1.3 (Rationale of the study) above, it is imperative for teachers to support learners in building mathematics knowledge and skills by teaching them mathematics in a way that make sense to them. This study is exploratory in nature as it provides insights into teachers' conceptualization of the mathematical pedagogical content knowledge (MPCK) that is associated with meaningful learning of mathematics. It explores the experiences of educators with meaningful mathematics teaching. More details of methodological scope will be presented in Chapter 4 of Research design and methodology.

## **1.9 DELIMITATION OF THE STUDY**

This study focused on teachers of mathematics at the IP in the Sibasa circuit, Limpopo Province of the Republic of South Africa. The investigation covered the period from 2016 to 2017. This study did not involve teachers outside the IP although they may have other

challenges pertaining to the teaching of mathematics. Thus, the study focused only on the teaching of mathematics at Grades 4-6.

### **1.10 LIMITATIONS OF THE STUDY**

This study was constrained to an intentional determination of 18 Intermediate Phase schools at Sibasa circuit. The sample didn't speak to all Intermediate Phase schools in Limpopo Province as Nieuwenhuis (2007:55-56) puts in this way, "in light of the particular social, political financial and social encounters supporting each investigation, the discoveries can't be summed up". However, they bring us more prominent clearness on how individuals make importance of marvels in a particular setting, along these lines adding to more noteworthy comprehension of the human condition. This research was confined by the accompanying certainties: Firstly, Intermediate Phase mathematics teachers were deliberately chosen on account of their teaching experiences in mathematics so as to gather rich information. Secondly, teachers who teach other subjects may not be honest in their responses due to their different personal reasons and this may also impact the investigation. Nonetheless, the researcher assured the respondents that it was a purely academic exercise, and no one would be penalised for the responses given.

### **1.11 INTENDED CONTRIBUTION TO THE BODY OF KNOWLEDGE**

This study set out to gather information that would assist stakeholders in education to find solutions related to teachers' pedagogical mathematical content knowledge pertaining to mathematics teaching and learning in the IP. It is believed that the findings of the study will assist Intermediate Mathematics teachers of Vhembe district in finding out ways on how they can improve their teaching and learning conditions. The study could prompt the arrangement of reasonable abilities and skills for helping teachers on how teaching can be effective. This study seeks to assist the following stakeholders:

- Circuit managers to understand the problem of teachers teaching mathematics at intermediate phase.
- Department of Basic Education with the provision a better understanding of the kind of mathematics teaching and learning that results in meaningful learning. The findings

study will assist teachers to improve their mathematics pedagogical content knowledge to enhance their teaching and learning situation.

- Curriculum developers to develop and deliberately curriculum for improving teacher's mathematics pedagogical content knowledge.

It was further envisaged that the findings of this study would add to the existing knowledge with respect to meaningful and effective teaching and learning in mathematics, and prompt further research by other scholars. Learning procedural skills by rote, without achieving conceptual insight, is a problem in mathematics learning. Thus, this study was deemed important insofar as it sought to provide information that would help most teachers understand how to approach and address mathematics problems in their classrooms.

## 1.12 OPERATIONAL DEFINITIONS OF KEY TERMS

The key terms that are used in this research are explained below:

**Teacher's knowledge for teaching** refers to all profession –related matters which are possibly relevant to a teacher's activities.

**Mathematical knowledge for teaching** refers to a specialized knowledge mathematics teachers need to support their learners learning that goes beyond the mathematics that anybody may require

**Pedagogical knowledge** refers to the specialised knowledge of teachers that is aimed in creating and facilitating effective teaching and learning environments for all learners.

**Pedagogical content knowledge (PCK)** refers to the incorporation of the matter to be taught, of how the learners learn the matter, of how best to teach the matter, of the materials that are suitable for teaching the matter, and of how the matter fits into the curricula.

**Common content knowledge:** Common content knowledge is the mathematical knowledge and skills that are used in settings other than teaching, with an example of such knowledge being that of the algorithm required for multiplying two numbers together (Ball, Thames & Phelps, 2011).

**Content knowledge:** Refers to the body of knowledge and information that teachers and that learners are expected to learn in a specific period.

**Horizon content knowledge:** Refers to an orientation and familiarity with the discipline that contribute to the teaching of the school subject at hand, providing teachers with sense for how the content being taught connects with the broader subject matter.

**Problem-solving:** Refers application of mathematical operations and individual's critical thinking skills through details of a problem to reach a solution.

**Problem-based learning (PBL):** Refers to learner-centred instructional approach in which learners learn through the experience of solving problem which allows development of other desirable skills and attributes.

**Relational understanding:** Refers to an understanding how and why the rules and procedures work.

**Teacher's knowledge and skills** refers to the teacher's awareness, or understanding, of a circumstance or fact, and their ability to use and/or to apply that awareness, or understanding, of a circumstance, or fact, in context.

**Teacher's pedagogical skills** refers to the teacher's ability to plan, and to provide, a set of learning opportunities that offer access to crucial concepts and skills for all learners, as well as to the teacher's ability to assess learner learning.

**Mathematical knowledge for teaching (MKT):** MKT refers to the mathematical knowledge that is used in the classroom. Exceeding the knowledge of formal mathematics, it is the mathematical knowledge that one needs for carrying out one's work as a teacher of mathematics (Krauss et al., 2008).

### **1.13 ETHICAL CONSIDERATIONS**

According to McMillian and Schumacher (1993) in the context of education, ethics are particularly important in such a way that they deal with benefits about what is right or wrong, proper or improper, good or bad. Ethical matters emphasis the rights of participants in a research activity. Apart from the participants ethical matters also relate to the researcher. These ethical matters mostly interchange around intellectual ownership, plagiarism and how it may influence research results. In this study the researcher will therefore consider the following ethical measures:

### **1.13.1 Informed consent**

Participants were informed that the questionnaire was not meant for work or department matters but for research purposes and therefore their names and school names could be kept unknown.

### **1.13.2 Confidentiality**

The second ethical consideration which was recognized in this study was confidentiality. McMillan & Schumacher (2011) define confidentiality as ensuring that identifying information that could be used to link respondents to their responses are kept private from the public. No information regarding any participant should be divulged to the public or any other unauthorized personnel. In this study the researcher made sure that no participant's name was written on the responses, instead pseudonyms were used. Participants' information was also protected and safely kept so that no one would access it without the participants' permission.

### **1.13.3 Avoidance of harm to participants**

The third ethical point of consideration was the avoidance of harm to participants. It entails being sensitive when asking questions and other issues discussed during an interview session through avoiding asking sensitive question (Neuman, 2012). In this study the researcher ensured avoidance of harm through making sure that all the participants are informed of the contents of the interview before starting. The interview sessions were also conducted in places where the participants where free and comfortable to avoid any emotional or physical discomfort.

### **1.13.4 Voluntary participation**

The researcher informed the participants participation is absolutely voluntary and that participants were free to discontinue terminate their participation at any time. Therefore refusal to participate or decision to withdraw will not result in any penalties or loss of benefits to which the participant is otherwise entitled.

## **1.14 Structure of the thesis**

### **Chapter 1: Introduction**

It includes the background of the study, purpose, rationale, problem statement, objectives and research questions.

### **Chapter 2: Literature Review**

The chapter discussed on what previous researchers have reported regarding teachers' mathematical content knowledge, curriculum knowledge, pedagogical knowledge and assessment in mathematics.

### **Chapter 3: Theoretical Framework**

This chapter presents the theoretical framework of the study.

### **Chapter 4: Research Methodology**

It describes the research paradigm adopted in the study, research design, research methods, the research approach, participants, instrumentation, procedures, and data analysis techniques and procedures.

### **Chapter 5: Data presentation and Interpretation**

This chapter presents the results, the interpretation thereof and the findings of the study.

### **Chapter 6: Summary, Conclusion and Recommendation**

This final chapter of the thesis gives a summary of the whole study, conclusion, implications of the study, as well as recommendations for classroom practice and further research.

The chapter that follows presents a literature review for this study.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 INTRODUCTION**

The previous chapter provided a background and introduction to the study. This chapter reviews literature pertinent to this study. The purpose of this review of literature is to document the importance of the topic. A qualitative review simply introduces the purpose of the study and the initial broad questions that are to be reformulated during data collection (McMillan & Schumacher, 2011). Both primary and secondary literature was studied in order to establish what other scholars have already gathered with regard to the research topic. Mouton (2012:34) contends that “you should start with a review of the existing scholarship or available body of knowledge to see how other scholars have investigated the research problem that you are interested in”.

The main goal of this chapter was to review literature pertinent to aspects that are supposedly related with the meaningful mathematics teaching and learning. This review of literature has three main aspects: (a) teachers’ understanding of the current state of mathematics teaching in South Africa, (b) IP teachers’ perceptions about the importance of mathematical content knowledge (MCK) in the teaching of mathematics and (c) conceptualization of the relationship between pedagogical content knowledge (PCK) and the meaningful teaching and learning of mathematics.

#### **2.2 CURRENT STATE FOR MATHEMATICS TEACHING IN INTERMEDIATE**

##### **PHASE (IP)**

This section will provide the background information on the intermediate teachers’ current state of mathematics teaching as follows:

##### **2.2.1 Mathematical Understanding**

Teaching for understanding is frequently perceived as to be a significant teaching and learning target. This implies that learners should know and comprehend mathematical

thoughts and facts including mathematical knowledge and skills. The aim of teaching mathematics should shift from memorization specific skills and knowledge to meaningful learning. Be that as it may, understanding may mean various things to various learners. The Common Core Standards in Mathematics emphasized the significance of relational understanding as a key aspect of mathematical expertise. My experience as mathematics teacher showed that many IP teachers do not understand conceptual understanding. Awfully many imagine that if learners know all the definitions, formulas and rules, at that point they possess such understanding. The question is: What does mathematical understanding look like?

One sign of mathematical understanding is the capacity to justify, in a way suitable for to the learner's mathematical development, why a specific mathematical statement is true or where mathematical rule comes from. There is a huge difference between a learner who can gather a memory helper to say the formula of finding the area of rectangle as it is  $A = L \times B$  and a learner who can clearly clarify or explain where  $L \times B$  comes from.

The study discussed teacher's conceptions about pedagogical content and its influence on their mathematics teaching and learning.

Cotton (2017) in his findings he further highlighted that what teachers know about math affects what they do. In his study one of the respondents described mathematics as "cut and dried". According to the respondents "cut and dried" is a process of following procedures and producing right answers. In other words cut and dried did not provide opportunities for learners to explore or engage in mathematical understanding, instead it emphasized memorizing and using specified procedures.

Cotton also emphasized that in order for teachers to cover each area of mathematics, every activity offers helpful step by step guidance including activating teaching and learning with learner's real life objects and experiences. Teachers communicate ideas about mathematics in the tasks they give learners, from the kinds of uncertainties that emerge in their classes and the ways in which they respond to those uncertainties, as well as from messages about why pupils should learn particular mathematical topics or concept. Finally, in addition to all of

this, understanding mathematics is colored by one's emotional responses to the subject and one's inclinations and sense of self in relation to it.

### **2.2.2 Teacher's knowledge for Meaningful Teaching**

In contextual mathematics teaching and learning, there is an extensive research that investigates the knowledge needed for teaching mathematics. Most authors agree with Shulman that the teaching of mathematics is a complex task as supported by Petersen and Hyde (2017) when they reinforce this by highlighting aspects of knowledge needed for teaching mathematics as predictors of high school math performance. Ball (2003) articulated these issues in terms of dilemmas for teaching mathematics in the Intermediate Phase.

The dilemmas that Ball's was referring to includes representing the content, respecting children as mathematical thinkers, and creating and using the community. Ball (2003) stated, In mathematics teaching, figuring out powerful and effective ways to represent particular ideas implies giving serious attention to both the mathematics and children. This is more easily said than done (Ball, 2003). The importance of her work highlights the complexity of the knowledge needed for teaching mathematics. Ball (2003) concluded that in order to teach mathematics effectively, teachers need to know the content of mathematics they are teaching and how to teach it. Ball identified the "pedagogical content knowledge" for mathematic teaching, which this study refers it as mathematics pedagogical content knowledge (MPCK).

Similarly, Fennema & Franke (1992) cited on Maboya (2014) claims that mathematics teachers need mathematics knowledge, pedagogical knowledge and knowledge of learner's cognitions in mathematics. These aspects can be presented discretely, they emphasise that it is the interaction between each component, within the context of classroom teaching that results in meaningful teaching of mathematics.

Hill (2007) agrees that the teaching of mathematics is complex, by describing it as being "multifaceted." Hill argued that meaningful teaching of mathematics includes the teacher's ability to solve mathematics problems their learners are expected to solve, to understand content for teaching, to understand the content from the learner's point of view.

The National Council of Teachers of Mathematics (2011) statement confirms the complexity by stating that indeed, teachers for mathematics requires a broad range of knowledge in

order to be effective i.e. - knowledge of content, knowledge of learners and knowledge of effective teaching strategies. In other words, the importance of teacher's knowledge about mathematical content and how learners learn mathematics is important. As supported by Tambara (2015) states that effective teachers of mathematics need to know the content they are teaching as well as having knowledge of the contexts and the characteristics of the learners with whom they are working with.

According to Mills (2015) describes important aspects of knowledge for teaching mathematics as, knowledge of mathematics, knowledge of representations, knowledge of learners as well as knowledge of the curriculum. Mills (2015) stated that knowledge about mathematics and learners affects the core teaching of tasks. It means that knowledge about mathematics and learners shapes how teachers select activities and resources, how they present material in class, how they interact with learners and how they assess learner's progress. While there are several varied yet similar assertions and descriptions about the knowledge needed for teaching mathematics, there is widespread agreement that the task of teaching mathematics is indeed complex and multifaceted.

#### **a) Meaningful mathematics teaching and learning**

Teaching and learning are among the five elements of knowledge advocated by Novak (2010). These two are inseparable and interdependent on each other especially in a classroom context. Where there is teaching there is learning. Teaching means imparting knowledge to someone who is less or not knowledgeable in a particular domain while learning according to Wikipedia, refers to acquiring new, or modifying existing knowledge, behaviour, skills, values or preference and may involve synthesizing; involves an appropriate interaction of new knowledge (new concepts) with the learner's existing or prior knowledge. Dreyer (2008) used the phrase "effective teaching" as he refers to meaningful teaching. Dreyer also supports that effective teaching facilitates meaningful learning. Tambara (2015) claimed that meaningful teaching is more than just reading or taking information from the textbook and give it as it is to learners'. In other words, meaningful teaching involves finding out what works for your learners" and trying different techniques that will enable them to succeed in their lives.

Fosnot, & Dolk (2002:175) affirmed that 'mathematics teaching takes place so that the learners *can learn* mathematics involved'. This study strongly stated that learners need to

learn mathematics in a way that make sense to learners. However traditionally, ‘the teaching of mathematics has relied heavily on exposition by the teacher together with consolidation and practice of fundamental skills and routines by the learner’ (Orton and Forbisher 2004:11). As a result, the fault of this approach is that, for some children, exposition never leads to mastery; many of the procedures, or routines, are not remembered correctly, or they are not remembered at all, and sometimes the procedures, or routines, are confused (Mji & Makgato, 2006).

The general agreement among educationists, learning theorists and psychologists is that if we are to produce learners who develop into thinking and problem- solving adults, we need to use teaching methods that foster these competencies (Anthony and Walshaw, 2009). The purposes of the current research study provides six effective mathematics teaching practices according to (NCTM, 2014) that facilitate meaningful learning. These six effective mathematics teaching practices can be implemented in the classroom as part of one’s daily teaching practice. Following an overview of the six practices, teachers will engage in a set of activities (e.g., exploring rich mathematical tasks, analyzing narrative and classroom instruction, analyzing learners work samples) that will highlight a subset of the practices in more depth. The six practices are:

- **Establish Mathematics goals to focus learning.** Meaningful teaching of mathematics establishes clear goals for the mathematics that learners are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions in order to promote meaningful learning.
- **Implement tasks that promote reasoning and problem solving.** Meaningful teaching of mathematics engages learners in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies towards meaningful learning
- **Use and connect mathematical representations.** Meaningful teaching of mathematics engages learners in making connections with learners’ everyday experiences among mathematical representations their everyday experiences to deepen understanding of mathematics concepts and procedures.

- **Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among learners to build shared understanding of mathematical ideas by analyzing and comparing learner approaches and arguments.
- **Pose purposeful questions.** Meaningful teaching of mathematics uses purposeful questions to assess and advance learners' reasoning and sense or meaning making about important mathematical ideas and relationships.
- **Build procedural fluency from conceptual understanding.** Meaningful teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that learners, over time, become skillful in using mathematical knowledge and skills flexibly as they solve contextual and mathematical problems.

The 6 practices stated above relate quite well with Klipatrick, Swafford, & Findell, (2001)

five "intertwining strands" of mathematical proficiency, namely:

- **Conceptual understanding.** Comprehension of mathematical concepts, operations, and relationship play an important role in the teaching and learning of mathematics
- **Procedural fluency.** Developing procedural fluency is a critical part of mathematics teaching in order to ensure that learners are adequately prepared for meaningful learning." Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem
- **Strategic competence.** It connects declarative, procedural, and conditional knowledge.
- **Adaptive reasoning.** The capacity to think logically about the relationships among concepts and situations and to justify and ultimately prove the correctness of a mathematical procedure

**Productive disposition.** Mathematics teachers needs to provide opportunities that take into account to see sense in mathematics, to perceive it as both useful and worthwhile within school context and in real life situation

Ausubel, (1977) defined meaningful learning as it involves the acquisition of new meanings. New meanings, conversely, are the end-products of meaningful learning. That is, the emergence of new meanings in the learner reflects the prior operation and completion of a meaningful learning process. Similarly, De Bernardi et.al, (2018) defined meaningful learning

as connecting new learning to an existing framework of knowledge. One way to understand mathematics as meaningful and interesting implies connecting mathematics to learners' interests and everyday life experiences. According to Boo and Hoh (2011) meaningful learning involves the appropriate interaction of new knowledge and is long-lasting. Boo and Hoh further alluded significant learning as profound learning and repetition learning as shallow learning.

## **b) Conceptualization of Teacher Knowledge in Mathematics**

Monk (2014) showed that studies dating from the 1960s revealed an assumption implicitly regarding teacher's knowledge. Monk study states that effectiveness in teaching resides simply in the subject matter knowledge that a teacher has accrued. What teachers do behind doors of their classroom is very much reliant on what knowledge they have about mathematics and on what they understand about the teaching and learning of mathematics. Shulman's presidential address delivered to the American Educational Research Association membership (1986) launched increased attention to subject matter knowledge unique to teaching. Subject matter knowledge plays an important role because meaningful teaching rests on teachers understanding of the subject they are teaching. Generally it is obvious that if one is to teach mathematics he or she should really know about mathematics. Subject matter makes mathematics accessible and understandable to the learner.

Maboya (2014) reframed the study of teacher knowledge in ways that attend to the role of content in teaching. Maboya (2014) defined three categories related to teacher content knowledge and later on, in a related Harvard Education Review article (2012), he specified seven categories of a knowledge base for teaching: knowledge of content; knowledge of curriculum; pedagogical content knowledge; knowledge of pedagogy; knowledge of learners and learning; knowledge of contexts of schooling; and knowledge of educational philosophies, goals, and objectives.

According to Wilson and Shulman (2009) content knowledge component includes both the amount of the subject knowledge as well as the organizing structure of the subject. In other words content knowledge is "beyond knowledge of the facts or concepts of a domain". The author suggests that teachers must know and be able to explain under what conditions a

particular proposition can hold true. Shulman and his colleagues postulated that teachers should have knowledge of the substantive structures of a discipline, “the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts of the syntactic structure and the set of ways in which truth or falsehood, validity or invalidity, are established”.

Shulman and his colleagues discuss two types of teachers’ beliefs as another dimension that enhances meaningful mathematics learning. Teacher’s beliefs are critical part of the teacher’s competence Teachers’ beliefs contribute to how teachers teach in in their classrooms. However, teachers aren’t always realize some of their beliefs in teaching context teacher. As a results, beliefs form part of the process of understanding how teachers shape their work is substantial to the understanding of their teaching approaches and their selections in the classroom.

This study acknowledged teacher’s beliefs as resilient influences on classroom practices. Understanding the natural surroundings of teacher beliefs together with their influence on primary mathematics teaching results in teacher creativity development. In mathematics, teachers’ beliefs about teaching and learning is acknowledged as an important aspect towards development of mathematics pedagogical content knowledge.

Graeber & Tirosh (2008) indicated that researchers differed in definitions of the term Mathematics Pedagogical Content and referred to it as a different aspects of subject matter knowledge for teaching. They further stated that theoretical framework relied on case studies of six first-year English teachers in secondary school. Three beginning English teachers who lacked professional preparation for teaching and three teachers who graduated from a fifth-year teacher education program were chosen in order to explore the source and nature of pedagogical content knowledge in English. By contrasting these two groups of teachers, Grossman tried to generalize about teacher knowledge for teaching (Grossman, 2013).

Based on her study, Grossman reorganized the seven categories defined by Shulman into four main categories: subject-matter knowledge, general pedagogical knowledge, pedagogical content knowledge, and knowledge of context. The subject matter content knowledge was composed of the same components Tambara (2015): knowledge of content, syntactic structure of a discipline, and substantive structures (Shulman, 2011). Conversely,

if teachers' beliefs are not aligned with teachers' teaching practice, it will therefore inhibit learners meaningful mathematics learning

### **2.3 IP TEACHERS' PERCEPTIONS ABOUT THE IMPORTANCE OF MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE (MPCK).**

A critical challenge faced by intermediate phase compared to senior phase is Mathematics teacher's lack of mathematical pedagogical content (MPCK). Mathematics Pedagogical Content Knowledge guide teacher to determine what and how to teach learners. The effective learning of mathematics is a learning that enables the learners to acquire specific skills and knowledge, and which promotes the obtaining of better learner outcomes than might otherwise have been possible (Russ, Sherin & Sherin, 2011).

Teacher's lack of mathematics pedagogical content knowledge raises questions about mathematics teacher's perceptions, current practices, and support needs with regard of 'what' and 'how' to teach learners for relational versus instrumental understanding. Learners with a relational understanding can learn new concepts better, retain previous knowledge and they are able to connect what they were taught to their real life expediencies. Teachers; need to deploy a wide range of knowledge for teaching such as: knowing what, knowing how to teach it, and also starting to get a feel for what kids might think about as they work on mathematical tasks (Ball, 2014). When you start introducing adding fractions with different denominators to learners they add as whole numbers like this  $\frac{2}{3} + \frac{1}{4} = \frac{3}{7}$ . Ask yourself why. What knowledge of fractions would learners be missing? How can you as teacher help them understand fraction notation, and more importantly, the relationship between part and whole in that relationship? The evidence of meaningful learning happens when teachers are able to connect mathematical concepts to learners' real life experiences as well as establishing connection and relationship amongst different concepts.

Therefore, one could say that indeed meaningful mathematics learning revolves knowing more than collections of facts, rules, formulas and procedures to solve mathematical problem. A significant concern in mathematics is about learners' understanding of facts, procedures, rules, formulas and the ability to apply and connect their knowledge into new topic. In the light of the above statements learners must have a sense and meaning of what mathematics they are learning.

The author has an impression that meaningful learning is conceptualized in terms of its impact on the learner. It means that quality teaching is basically on what learners' knows, understands, and capable to apply or transfer it to new situations. The issue of meaningful mathematics is aligned with teachers' perceptions about mathematical content knowledge (MCK).

### **2.3.1 Transforming Mathematical knowledge for teaching (MKT) into classroom practice**

The concept of mathematical knowledge for teaching (MKT) was introduced by Ball (2008). Building on Shulman's (1986) notion of pedagogical content knowledge (PCK). When teaching and learning of mathematics takes place, MKT plays a vital role for meaningful learning. However, IP teachers lack specialization in their teaching subjects compared to SP teachers. Or they may have not learned mathematics in their academic years of learning. As such they may even not know how to teach it in their classroom practice in order to help learners learn it in a meaningful way that makes sense. Others may have even learned mathematics that is not related to what they are currently teaching in the class. The authors' view maintains teaching mathematics without mathematical knowledge then that particular teaching is said to be insignificant.

Mathematics educators are expected to know his or her subject matter deeply. It is on basis that the study suggests that once a teacher is knowledgeable about the subject matter, it then results to learners understanding of mathematics. For this reason teacher's mathematical knowledge is a significant factor in learners' successful learning. Unfortunately in primary school level it is not in that way due to the fact that there is no specialization for the teaching subjects compared to secondary school level.

Ball (2008) study showed that MKT gives teachers a better understating of what to teach and how to teach it, hence she argues that MKT is concerned with the tasks involved in teaching and the mathematical demands of these tasks. Hill (2012) and Rockoff, Jacob, Kane & Staiger (2011) propose that the MKT of teachers can be linked to attainment of meaningful learning. Given the state of learner meaningless learning in mathematics in the intermediate phase, and the explanations provided as to why learners are not achieving in Chapter 1, an investigation of the MKT may be able to provide useful knowledge on what intermediate teachers need to do in order promote meaningful learning. Hurrel (2013) stated that MKT

contributes to effective teaching and it therefore would not seem unreasonable to suggest that if we want to improve teacher effectiveness the development of MKT is an important factor.

This study acknowledged that indeed mathematics knowledge for teaching is essential to learners' meaningful learning. What logically flows from this relationship is that high level teacher knowledge influences learners' meaningful learning through classroom practice. Hill (2008a: 41) define classroom practice as a "composite of several dimensions that characterize the rigor and richness of the mathematics content of the lesson, including the presence or absence of mathematical errors, mathematical explanation and justification, mathematical representation, and related observables". Nevertheless, teaching is all activities that teachers must do in provision of achieving meaningful learning of their learners. These activities involve mathematical ideas, skills of mathematical reasoning and connecting mathematics with learner's real life. The presumption is that teacher knowledge is essential for quality and meaningful learning. Clearly, teacher knowledge alone is not enough to ensure successful implementation of reforms in mathematics.

### **2.3.2 Classroom practice in teaching Mathematics at intermediate phase**

The classroom is a dynamic environment, bringing together students from different backgrounds with various abilities and personalities. Therefore an effective teacher requires the implementation of creative and innovative classroom practices in order to meet learners' individual needs.

Teacher's classroom practices are obviously proposed to improve adapting, however could possibly do as such, contingent upon the degree of adequacy. Classroom practice is a key part of "direct guidance" have by and large been appeared to positively affect how teachers teach mathematics (OECD, 2009). TIMSS (2011) study showed that classroom practices enhance meaningful teaching.

Hattie (2010) findings suggested several highly effective classroom practices as: teacher clarity, classroom discussion, feedback formative assessments and metacognitive strategies. Generally, teacher classroom practices contribute significantly to learners' meaningful

learning. The study suggests that it is worthwhile to acknowledge effective classroom practices influence meaningful learning in mathematics. In turn these classroom practices make teaching a more maintainable profession and also give teachers an opportunity to develop deeper understanding of classroom challenges and difficulties. Teacher's practices are therefore closely associated with teachers' strategies for handling and managing challenges and difficulties in their classroom. Moreover, they shape learners learning atmosphere and encourage learners to learn mathematics as active participants rather than passive participants.

It is noted that teacher classroom practices can be described as meaningful acquisition of the skills, knowledge, attitudes and values that the learner needs to learn. The study suggests effective classroom practices must be seriously taken into account for the improvement of meaningful teaching. Mathematics can be therefore regarded as meaningful rather than meaningless, as interesting than uninteresting, enjoyable than disillusioning when teacher's classroom practices are well planned and organized.

Despite meaningful teaching, classroom practices have a major and positive outcome on reducing problem behavior in the classroom. Generally, classroom with frequent disruptive behaviors have less teaching and learning time and thus results in poor performance.

### **2.3.3 Mathematical knowledge for teaching (MKT) in classroom practice at intermediate phase**

The concept of mathematical knowledge for teaching (MKT) was introduced by Ball (2008). Building on Shulman's (1986) notion of pedagogical content knowledge (PCK). Hill (2012) conducted an interactive work session to investigate the mathematical knowledge and skills that are needed in the teaching of mathematics. Hill (2012) and Kim (2013) define MKT as the mathematical knowledge, skills, and habits of mind entailed in the work of teaching. It is the crucial mathematical knowledge that teachers apply during teaching to improve the teaching and learning of mathematics (National Mathematics Advisory Panel, 2008).

Ball (2008:395) argue that "it seemed obvious that teachers need to know the topics and the procedures that they teach but they "decided to focus on how teachers need to know that content. In addition, they wanted to determine what else teachers need to know about mathematics and how and where teachers might use such mathematical knowledge in practice.

Ball (2008) suggest that MKT gives teachers a better understating of what to teach, and when and how to teach it, hence they argue that MKT is concerned with the tasks involved in teaching and the mathematical demands of these tasks. Hill (2012) and Rockoff, Jacob, Kane & Staiger (2011) propose that the MKT of teachers can be linked to attainment of meaningful learning. Given the state of learner meaningless learning in mathematics in the intermediate phase, and the explanations provided as to why learners are not achieving in Chapter 1, an examination of the MKT of intermediate phase teachers may be able to provide useful knowledge on what intermediate teachers need to do in order promote meaningful learning. Hurrel (2013) states that MKT contributes to effective teaching and it therefore would not seem unreasonable to suggest that if we want to improve teacher effectiveness the development of MKT is an important factor.

This study acknowledged that mathematics knowledge for teaching is essential for quality and meaningful teaching. Logically teachers' knowledge influences learners' meaningful learning. Hill (2008) argues that there is a positive relationship between teacher's knowledge for teaching and meaningful learning.

According to Ball (1991) the work of teaching is everything that teachers must do to support meaningful learning of their learners involve mathematical ideas, skills of mathematical reasoning, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency. Teachers Mathematical Knowledge for Teaching (MKT) is therefore regarded as tool to address the challenge of learning mathematics in ways that make mathematics accessible and teachable.

#### **2.3.4 The teacher's strategies and practices: the use of curriculum materials**

When curricula reform takes place, a wide variety of materials and textbooks are usually on offer to meet the needs of a range of potential purchasers or users. Such resources are closely aligned with the goals of the curricula reform. Often, two practices pertain to the use of such materials and textbooks (Blackley and Howell 2015)). On the one hand, there are teachers who will take the time to go through the wide range available, and to select the materials that they find appealing and useful. Cobb et al (2011) notes that this kind of teacher is likely to adopt a more trusting and adherent view, using the tasks and recommendations in the curriculum materials to more comprehensively guide their instructional practices. On the other hand, some teachers will select materials containing elements with which they are

familiar (Remillard, 2009). Such teachers will use the curriculum materials as a source of tasks, without altering their teaching and learning practices, thus weakening the materials' ability to support the curriculum reform-related practices (Hoover et al 2016). The textbooks and curricular materials were intended to be aligned with the new curriculum.

The curricular materials and textbooks included activities appearing in the learner's textbook with suggestions about how to teach the activities, and about how to cope with the areas where learners might struggle to learn the content.

The teacher's interaction with the above-mentioned materials begins at the preparatory stage. How and which materials the teacher selects is influenced by the teacher's beliefs, content knowledge and knowledge of the curriculum (Davis, 2009). The rationale and meaningfulness for the teacher of a syllabus, or of a curriculum, the materials, media or textbooks (i.e. knowledge of the curriculum) also affects what is learned and taught (Hameyer, 2014).

## **2.4 CONCEPTUALISATION OF THE RELATIONSHIP BETWEEN PEDAGOGICAL CONTENT KNOWLEDGE (PCK) AND THE MEANINGFUL TEACHING AND LEARNING OF MATHEMATICS**

Ball and Forzani (2009) argued that the majority of teachers currently teaching at

Intermediate Phases "express, lack of confidence in their Pedagogical Content Knowledge, and such teachers are not managing the demands on how to adapt their teaching to address learners" meaningful teaching and learning. In the teaching of mathematics, Aunio, Aubrey, Godfrey, & Liu, (2012) stressed how the depth of teachers' understanding of PCK is a major determinant of teachers' choice of examples, explanations, exercises, items, and reactions to children's work. This results in meaningful and effective mathematics teaching and learning (Aunio *et al.*, 2012).

### **2.4.1 Subject matter knowledge**

As observed above, it would be unwise to say that mathematical subject matter knowledge is not important to the teaching of mathematics, although conventional content knowledge seems to be insufficient for the skilful handling of the mathematical tasks of teaching. In the current study, subject matter knowledge is not discussed in as much detail as is PCK, because the focus of the study is on IP teachers, who, in this case, had a matric qualification,

and who, in general, had mastery of adequate mathematical content to be able to teach in this phase. The teachers concerned had a greater need for PCK than they did for subject matter knowledge. The domain of subject matter knowledge has three components: common content knowledge; specialised content knowledge; and knowledge at the mathematical horizon (Hill and Ball, 2009).

- **Common content knowledge:** The construct of MKT refers to the mathematical knowledge and skills that is used in settings other than teaching, such as the knowledge of the algorithm that enables the multiplying together of two numbers (Maboya, 2014).
- **Specialised content knowledge:** This form of knowledge refers to the mathematical knowledge and skills that are unique to teaching, such as knowing how the algorithm to multiply two numbers together relates to place value and the distributive property (Maboya, 2014).
- **Knowledge at the mathematical horizon:** This form of knowledge refers to an awareness of how mathematical topics are related over a span of mathematics included in the curriculum, for example knowing how the algorithm to multiply two numbers together is related to multiplying two polynomials together (Ball et al., 2003). The implication of the above is that teachers need to know more than merely the common content of mathematics.

#### 2.4.2 Knowledge of content and teaching (KCT)

The other component of PCK, which is called KCT, refers to the knowledge of content and teaching of mathematics. The knowledge of content and teaching consists of knowing how to prepare for instruction, and of mastering the different modes of delivering instruction. Such knowledge also includes the knowledge of how mathematical topics are connected (Brent and Simmt, 2008). As has already been pointed out, there is, currently, an increased emphasis on developing teachers' capacities to convey excellent learner results, and, subsequently, thought is continually given to the issue of what constructs powerful guidance in the classroom. Reynolds & Muijs (2009) give significant experiences into the highlights of classroom practices in which mathematics is instructed effectively. Their consideration distinguished the following as being significant qualities of teacher's content knowledge:

- A high level of opportunity to learn;

- An academic orientation emanating from the teacher;
- Effective classroom management;
- Heavily interactive teaching;
- The rehearsal of existing knowledge and skills; and
- The use of a variety of activities on a set topic.

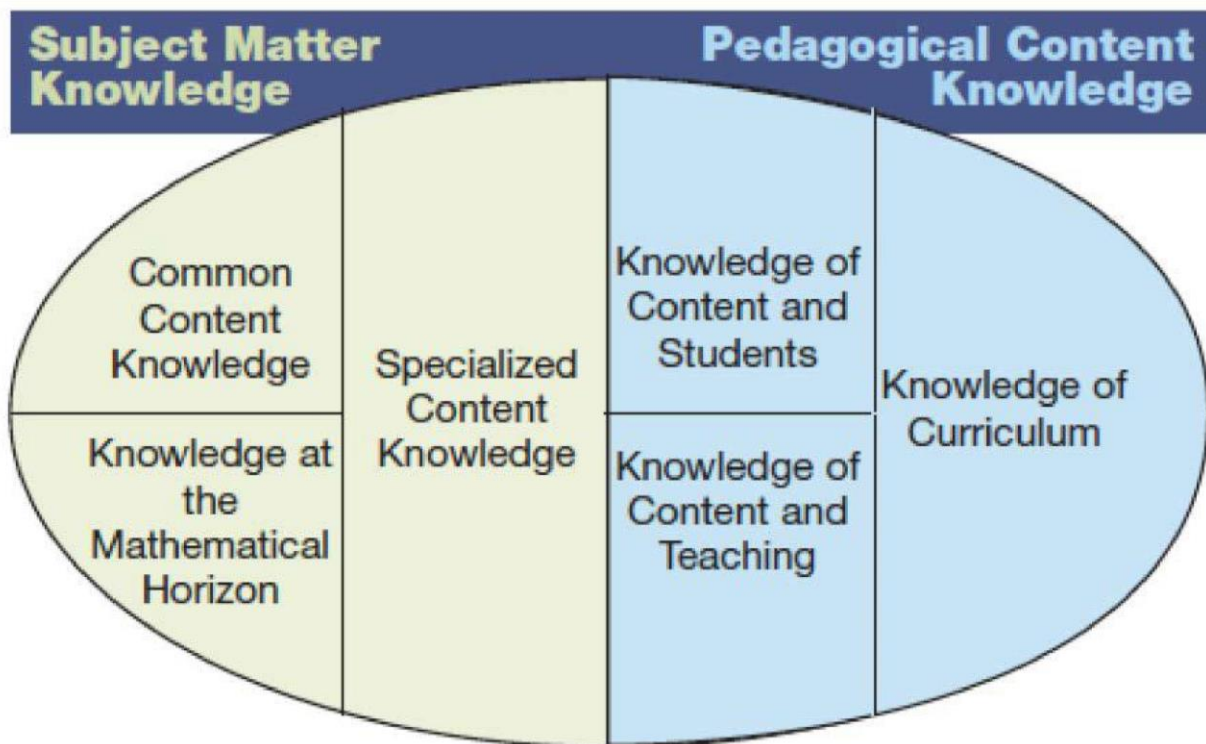
Teacher's content knowledge in mathematics classroom is significantly imperative to the improvement of teaching and learning. Content knowledge by and large alludes to the realities, ideas, speculations and rules that are taught and learnt in a particular grade or class level. Teachers who are exceptionally learned in content information will in general be increasingly successful teachers. As demonstrated via Carroll (2007) study the instructor should be proficient so as to understand the route toward educating and learning. This is a profound established system that incorporates the role part played by the teacher.

### **2.4.3 The importance of teachers' PCK in mathematics teaching**

Hill (2008) study showed that teachers' knowledge of teaching plays an important role in generous connection among PCK. Teachers with high PCK were noted as giving preferred guidance to their learners over did educators with lower PCK. Teachers play important roles in mathematics classrooms. Beyond that, teachers serve various roles in the classroom such as:

- To avoid numerical blunders and errors;
- To apply knowledge to help progressively thorough clarifications and thinking, and utilize learner mathematical thoughts than would some way or another have been conceivable
- To make rich mathematical situations for students
- To be incredulous of commanded educational program, and to put significant time in distinguishing and blending exercises from supplemental resource material
- To provide high-skill numerical exercises;
- To select models shrewdly to guarantee fair open doors for learning

The previously mentioned components were considerably more factor among lower knowledgeable teachers. Once in a while, an educator with low-level PCK would show an attributes, however they were not continually shown during teaching and learning process. There is developing acknowledgment of the way that PCK information alone doesn't ensure improved learning, and activities are being made to characterize the different types of information that are required for teaching (Tirosh, 2009). As Ball, Thames and Phelps (2008) examined PCK, they additionally started to see its various spaces (see Figure 2.1 underneath).



**Figure 2.1 Model of Teachers' Mathematical Knowledge for Teaching (Ball, 1991)**

- As indicated by Ball, Lubienski & Mewborn (2011), the contention for the move in center from mathematical knowledge to different kinds of knowledge is that, in spite of the fact that these significant professions depict what the teacher knows, they don't address the specific type of knowledge 'call up' as they teach mathematics. Thus, researchers' have started to attempt the challenges measures that teachers must draw upon when teaching for comprehension (Yuanita, Zulnaidi, and Zakaria 2018).

The argument, up to this point, has recognized that teachers need to have a significant comprehension of essential mathematics (as summarized in Figure 2.1). Be that as it may, the literature has likewise indicated that significant content alone isn't adequate for the compelling instructing of mathematics. A powerful teacher should likewise have a profound and wide information on the showing procedure, learners and the educational program. Equipped with this information, teachers can interface their insight into substance as far as anyone is concerned of the students, the educational plan, and the kind of encouraging that is required, so as at last to address the objective of improving learners' learning (Shuhua, 2014).

#### **2.4.4 PCK in the teaching of Mathematics in intermediate phase**

In the teaching of mathematics teachers must possess pedagogical content knowledge to teach effectively. Abdullah (2011) identified pedagogical content knowledge (PCK) as one of the most important knowledge bases that teachers should possess in order to teach effectively. He maintained that having knowledge of the subject matter is not enough. Teachers therefore need to possess pedagogical content knowledge as well. In Bansilal, & Wallace (2011) PCK include "knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purposes and values and their philosophical and historical bases. This has led researchers to consider PCK as important as the subject matter knowledge in the teaching and learning of mathematics. According to Bansilal, & Wallace (2011) PCK components are teacher's subject matter knowledge, knowledge of pedagogy and how the teacher transforms this knowledge into different structures that empower learners' in various learning conditions to comprehend the topic. Carroll (2011) study recognized that PCK structures is hard to disconnect and to gauge it. Barko (2013) has proposed that PCK structures as one of a kind and unique knowledge of teacher insight.

Pedagogical Content Knowledge emphasised the manner in which teachers relate their subject matter knowledge to their pedagogical knowledge and how subject matter knowledge is part of the process of pedagogical reasoning. Brandt, Lunt, & Rimmasch (2012) defined PCK as the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations in a word, the ways of representing and formulating the subject that make it comprehensible to other. It includes an understanding of what makes the learning of specific concepts easy or difficult, the

conceptions and preconceptions that learners of different ages and backgrounds bring with them to the learning environment (Brandt, Lunt, and Rimmasch, 2012).

Pedagogical content knowledge elements according to Corcoran (2011) are: subject matter, knowledge of instructional strategies, knowledge of learners' conceptions, and an understanding of what makes the learning of a specific topic difficult or easy for learners. Corcoran (2011) revealed teachers' knowledge base, curriculum knowledge which involves awareness of how topics are arranged both within a school year and over a given longer period and ways of using curriculum resources, such as textbooks, to organize a programme of study. Pedagogical content knowledge, is an important element of this study. It is an amalgam of a teacher's knowledge base that Da Ponte and Chapman (2013) say that it includes the following knowledge:

- Knowledge of context, curriculum and assessment
- Knowledge of learners learning
- Knowledge of instructional strategies and representations of mathematics  
Knowledge of learners understanding about concepts in mathematics.

Cooper (2010) puts it in this way, these three knowledge bases connect together to inform teacher practice: namely, subject matter content knowledge, pedagogical knowledge and knowledge of context. Subject matter content knowledge is described as knowledge that is unique to mathematics teachers. Cooper (2010) definition of pedagogical content knowledge is therefore adopted as theoretical framework in which he sees it as an amalgam of a teacher's knowledge base that includes:

- Knowledge of the representation of subject matter for teaching
- Knowledge of relevant instructional strategies
- Knowledge of learners' conceptions (preconceptions and misconceptions).

These three elements mentioned above were consciously integrated when observing how the two teachers displayed them when teaching functions in their respective Grade 3 mathematics classes (Clarke, 2013). The choice of the three elements of PCK were influenced by the fact that they form the core of what (Crespo, and Nicol, 2014) indicated as

teachers" PCK that would enable teachers to transform the subject matter in such a way that their learners would be readily able to access the content. Firstly the teacher needs to have a good grasp of the subject matter before being able to transform it.

Second, the teacher needs a teaching strategy to use to make the subject accessible to the learners. Thirdly, the teacher needs to have an idea of possible learners' conceptions that the learners may have about the topic in order to prepare explanations that will help to eliminate or reinforce the conceptions as is necessary (Cobb, Yackel, and McClain, 2011). Generally, PCK elements cannot be done in isolation. It is the intersection of knowledge of pedagogy, knowledge of learners' conceptions and subject matter knowledge (Clarke, 2013).

Shulman (1987:12, cited in Cobb et al., 2011) stated that PCK included those special attributes that a teacher should possess in order to help him/her to guide learners to understand content matter in a manner that will be personally or individually meaningful. Shulman further argued that PCK can only be described as truly effective and meaningful when it positively impact learners learning

Shulman (1987:12, cited in Cobb et al., 2011) also mentions the fact that pedagogical content knowledge is the best knowledge base of teaching.

#### **2.4.5 Knowledge of Mathematics teaching at intermediate phase**

According to Leatham, Lawrence, & Mewborn (2014), the new curriculum that is currently used calls for learners for being active participants in mathematics lessons. It is desirable that active involved enables learners to express their mathematical ideas freely. Therefore, it is imperative for teachers to make their lessons more learner-centred rather teacher centred. The choice of the instructional strategy or method to be used by the teacher is very important. Different lessons require different teaching methods. According to Krainer (2011), the correct choice of such an instructional strategy does not depend on the teachers' knowledge of the subject matter only but also on learners' level of understanding. Since this research investigated the use of instructional strategies. It is important to know what "good" teaching strategies are used for meaningful learning to take place.

Leatham et al., (2014) study showed that teachers' success in teaching mathematics means to integrate a range of various aspects of teaching: sound knowledge PCK, always select teaching strategies that are appropriate for different learners. Krebs (2010) asserts that, pedagogical content knowledge is central to effective teaching. Evidence to this effects can

be seen in the classroom practices when learners are actively involved in the learning process. However the method of teaching plays an equally important role if meaningful learning is to take place (Crawford and Witte, 2013)

Krebs further posits that good instructional strategies should: (i) actively engage the learners; (ii) assist them in using their prior knowledge and skills to solve problems in mathematics; (iii) motivate the learners to participate during the lesson; and also; (iv) create an appropriate learning environment. According to Lester (2013) excellent teachers of mathematics are aware of a wide range of effective teaching strategies and techniques for teaching and learning mathematics that promote the learners' meaningful learning. Such teachers usually choose teaching strategies that tend to create the best learning experience for every learner. Maurer & Neuhold (2012) involves knowing how teachers take advantage of different teaching approaches that make a learning experience most appropriate for the learners.

This includes being flexible and adjusting instruction that takes into account various learning styles, abilities and interests. Knowing how to best teach a concept so that the learners will receive the best learning experience speaks to the essence of PCK. The different teaching approaches employed vary from teacher to teacher and in differing contexts, but consistently revolve around similar principles for each approach. Lorenzo (2015) indicated that although expert teachers differ in their actual style of teaching and management, they all use instructional strategies that (i) maximize learners' time and engagement in learning tasks; and (ii) encourage learners' active participation during lessons. In addition, (iii) they ensure that learners understand the work they are required to do; and, (iv) they set tasks and activities at the right level to ensure high rates of success. Expert teachers also (v) create a positive and supportive classroom environment; (vi) they are good managers of behaviour; and (vii) are skilled in motivating learners to learn". The current study suggests that it is important for teachers to use appropriate strategies that encourage discussion and justification of ideas in the content of the topic so as to demonstrate mathematical understanding

On the other hand teachers need to support their learners through guided practice until they are independent, and also challenge misconceptions that learners may have about a given topic in mathematics (Kennedy, 2013). According to Barrows (2010), the choice of a teaching strategy must also encourage a conducive learning environment that allows learners to listen to other learners.

#### **2.4.6 Levels of Teachers' MPCK on their Conceptualisations of the Notion of Meaningful Mathematics Learning**

Kilpatrick, Swafford & Findell (2001) showed that teachers' level of MPCK profoundly influences how she/he understands how to do mathematics. They further argued that if a teacher's understanding of doing mathematics involves memorizing facts, rules, formulae and procedures to determine the answers to questions then she will teach in a particular way. By contrast, if a teacher regards doing mathematics as a sense-making problem-solving activity then her teaching approach will be quite different.

In a study conducted by Davis, & Simimt (2006) that explored and described the level of mathematics pedagogical content knowledge of three teachers, focusing on the topic of functions at secondary level mathematics, it was found that the teachers who participated in that study lacked conceptual knowledge of topics on functions. Hence their lessons were inaccurate and lacked substantial clarity for the learners. In turn this inhibited understanding.

These three elements of MPCK in this study were also used by Clarke (2013) in a study involving an investigation of pre-service mathematics teachers' knowledge of pedagogical content knowledge where he viewed the three elements of MPCK as the legs of "a three-legged chair. The seat of the chair represents the MPCK and each one of the legs represents subject matter knowledge, knowledge of learners and knowledge of instructional strategies (Barko, 2013:21). 'It is reasoned that the seat needs equal support from each leg while the legs need help from the seat to stand firmly' (Davidoff and Van den Berg, 1990:77). They are mutually dependent to each other and they cannot be treated discretely although each has its own features.

The MPCK elements that were well-thought-out in the study involved mathematics knowledge, pedagogical knowledge and content knowledge. In particular the study sought to explore and describe the elements of teacher's MPCK that facilitates meaningful mathematics learning. For teachers to be successful in their classroom practices teacher's level of MPCK should be taken into account. There are different levels of teacher's MPCK

According to Steinbring & Scherer (2014:61), Level 1: indicates wrong explanations of concept. In other words the teacher is not quite clear" about the concept which results in learners misunderstanding of the concept. Despite wrong explanation the teacher asks low

level questions which failed him/her to detect learners' difficulties with the topic. For Level 2: the teacher explains the correct mathematical concepts but gives the same type of examples to support his explanations, no variety of examples are given (Steinbring, and Scherer, 2014). The teacher is well aware of learners' difficulties but does not probe asking questions that allow learners to voice out their ideas about the topic. Level 3: The teacher's explanations of concepts are more accurate and clear along with the incorporation of suitable examples. As a results learners' participation is seen to be positive through the provision of relevant activities.

The following are the notion of meaningful mathematics learning:

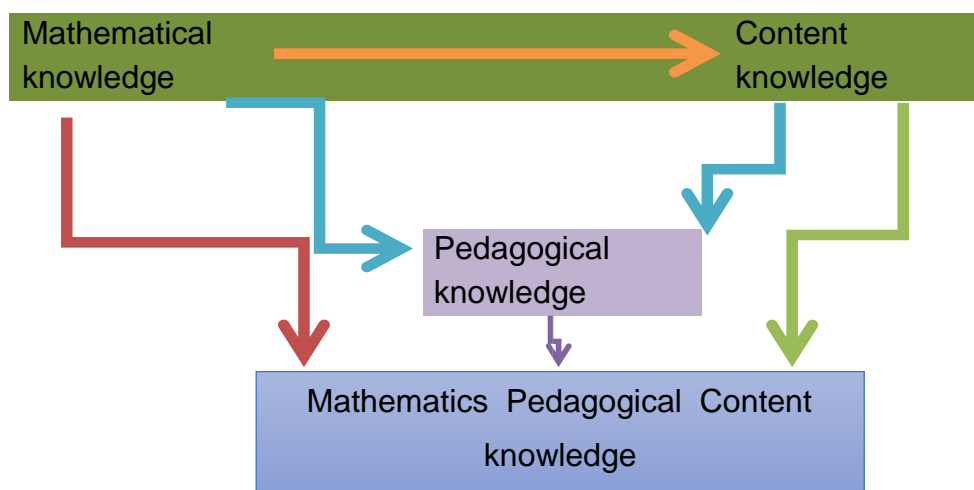
- Adequate knowledge of the subject matter: The teacher demonstrates deep and thorough conceptual understanding.
- Effective instructional strategies; It is imperative for the teacher to use appropriate activities which are connected with learner's real life experiences.
- Knowledge about learners' conceptions: Learners' prior knowledge as a good foundation for the current knowledge to be learnt. Learners' difficulties and misconceptions are seriously taken into considerations.

The framework for the analysis of MPCK used in that study uses knowledge of teaching strategies, knowledge of learners and the curriculum. Two of these components thus knowledge of teaching strategies and knowledge of learners were incorporated. The remaining sections of this chapter will discuss each of the three identified elements of MPCK and the conceptual framework used in this study. It will draw attention to how other researchers, in their respective studies, observed MPCK elements as suggested. The Researcher presented her view of dimensions that constitutes mathematics pedagogical content knowledge as follows:

- I. Mathematics knowledge (MK): mathematical knowledge is widely acknowledged as one of the critical attributes of mathematics teachers, thus, it is not surprising that this mathematics knowledge is a significant contributing element of effective and meaningful mathematics teaching (Chapman, 2004).
- II. Content knowledge (CK): Content knowledge is the "what" of teaching, or the

knowledge about the subject matter to be taught (Lorenzo 2015). Teachers' content knowledge is important in teaching mathematics.

- III. Pedagogical knowledge (PK): Pedagogical knowledge is the process of imparting the knowledge to the learners by employing techniques and strategies that will make the least in the class in terms of understanding to get a clear picture of the lesson (Tsafe, 2013). Similarly, pedagogical knowledge is a strategy and style which allow the teacher to present his lesson in a stimulating way (De Corte, 2011). Figure 2.1: Dimension of effective mathematics teaching and learning. The following diagram shows how the three components of MPCK are related to each other in order to enhance effective and meaning mathematics teaching.



**Figure 2.2: Model of Knowledge types of effective Mathematics teaching**

Fennema, Carpenter, Franke, Levi, Jacobs & Empson (2013) argued that MPCK in mathematics teaching context is a unique blending of three types of knowledge viz. mathematics knowledge, content knowledge and pedagogical knowledge as shown in figure 2.1 above. In mathematics teaching and learning context these three types of knowledge cannot be treated separately, they complement each other. The study recognizes the harmonization of mathematics, content and pedagogical knowledge of might yield effective teaching practices. However, De Graaf, & Kolmos (2011) studies showed that effective teaching entails various knowledge, skills, and abilities that enable teachers to create teaching environment that supports learner's meaningful learning.

The study regarded MPCK as the heart of effective and meaningful mathematics teaching and learning to happen in the intermediate phase. This study is founded on the premise that teacher's mathematics pedagogical knowledge is a strong indicator of his/her mathematics classroom practices in intermediate phase. In other words what teachers do behind mathematics classroom is said to be governed by what level of

MPCK teachers have (De Corte, 2011). From the viewpoint of the study teacher's high level of MPCK is described as the valuable knowledge of effective and meaningful mathematics teaching and learning.

No lesson, no matter how well planned, can be successful if the elements of effective and meaningful teaching are not in place. Based on this argument MPCK plays a very critical role for effective mathematics teaching and learning to happen. Davidoff and Van den Berg (1990) argued that without adequate mathematics pedagogical content knowledge, teachers would not be in a position to deal with their mission of teaching effectively and such would not cater for meaningful learning. Likewise, Ball (2013:101) affirms that a teacher who possesses good Mathematics pedagogical content knowledge can break down mathematical knowledge into less polished and abstract forms that make it accessible to all different learners.

#### **2.4.7 The response of intermediate phase learners to activities facilitated to enhance meaningful learning in mathematics.**

This section will provide the literature review on the response of intermediate phase learners to activities facilitated to enhance meaningful learning in mathematics

#### **Teaching for understanding**

The teaching of mathematics for understanding helps teachers to attend and solve challenges associated with the teaching and learning of mathematics, so that learning outcomes achieved. There is a contrast between figuring out "how" to do mathematics and seeing "how and why" arithmetic functions. Learners who figure out how to do mathematics can't respond to the inquiry why. It is important for teachers to require some more talk in homerooms doesn't really improve student understanding. Better understanding is subject to

specific educational methodologies deliberately centered on building up a talk culture that evokes explanation and produces accord inside the homeroom network in helping learners to comprehend mathematics they learn. Learners who comprehend arithmetic they get familiar with their advantage, support and accomplishment in mathematics improves. D'Ambrosio (2016) indicated that more talk in classroom doesn't really improve learner understanding. Better understanding is subject to specific educational methodologies deliberately centered on building up a talk culture that evokes explanation and produces solution inside the classroom

It is crucial to teach mathematics through critical problem solving, thinking and reasoning skills. Hiebert & Carpenter (2012) hold the view that in order to think mathematically there is a lot more to do than just doing arithmetic or solving number problems. It is a whole way of viewing at a problem, strip off down to make it simple and then analyze the problem. As a result the teacher then identified the mathematics and problems that learners should learn and solve. According to Kahan, & Wynberg (2016), making mathematics to be problematic means posing problems that are just within the level understanding of learners and allowing them to struggle or fight to find the correct solutions. He or she make sure that learners have sufficient understanding of the problem, but without telling them how to solve it. In other words a teacher refrains from stepping in and doing too much of the work which is expected to be done by learners. Sufficient understanding in mathematics implies making connections between ideas, facts or procedures, with a view to linking new knowledge to existing knowledge

Learners will then make sense of the problem and eventually they will produce the expected solution to the problem. Through the guidance of the teacher, learners think about their work and those significant thoughts that were developed. This implies that teachers can encourage learners to reflect on what they or their contemporaries have asked or proposed in order to build on and extend their own understanding and solicit contributions from everyone.

#### **2.4.8 Learning activities to support meaningful mathematics learning**

The purpose of presenting this sub-topic is to introduce the full range of learner learning activities for teachers to consider when teaching mathematics. In doing so, I attempt to

scaffold teachers' effective activities to promote meaningful learning. Essentially, these mathematics activity types are designed to facilitate thoughtful and creative instruction by teachers. The author conceptualized seven ranges of action types for mathematics that are gotten from the National Council of Teachers of Mathematics' (NCTM's) process standards (Reiss and Törner, 2011). The activity types provided below are by no means an exhaustive list, but it will help teachers in thinking through how best to design and deliver high impact learning experiences for learners.

### ***The "Consider" Activities Type.***

As mathematics teachers the most effective way of teaching learners to learn in a way that make sense for them, is allow them to build their own knowledge by connecting new knowledge and skills into their pre-existing intellectual framework of understanding. Learners who construct their own knowledge for themselves learner better than those who learn from their teachers. Constructivist theory states that knowledge does not simply arise from experience, rather it arises from the interaction between experiences and current knowledge structure.

***The "Practice" Activity Types:*** The primary role of a teacher is to teach learners to learn. As an assessment tool it can give teachers a sense where learners are not clear about the topic being taught. On the other hand it is often very important for a learner to be able to practice computational techniques or other algorithm-based strategies, in order to automate these skills for later and higher-level mathematical learning (Hoover et al, 2010:582)".

***The "Interpret" Activity Types:*** In the discipline of mathematics, individual concepts and relationships can be quite abstract, and at times can even represent a bit of unknown to learners. Teachers therefore need to spend some time explaining and clarifying these concepts for the development of learners' mathematical argument related to why they think that something is true or wrong. Placing mathematics in an argument is one way of the most basic, and potentially effective ways of teaching mathematics.

**The "Produce" Activity Types:** When teachers are dynamically involved in teaching mathematics, they become interested makers of their own teaching, rather than gathering information from different source such as text books and departmental documents. The truth and the beauty of mathematics is based on teachers when they become effective and deeply understand the mathematics they are teaching.

**The "Apply" Activity Types:** The usefulness of mathematics in the world can be found in its realistic or accurate application. As teachers, linking mathematics with learners' every day experiences is important for meaningful teaching and learning to happen. The study maintains that learning mathematics is profound or beautiful, interesting and make sense for both teachers and learners if is connected or applied to learners" real life experiences.

**The "Create" Activity Types:** The highest levels of mathematics teaching activities often engaged learners in very resourceful and creative thinking process. As mathematics teachers we need to create a classroom that recognizes creativity. Teachers should also take into consideration what is important to learners. Learners' interest is a good starting point on what drives their own thinking and reasoning capacity. As a result learners take ownership of their own learning process.

#### **2.4.9 Learning activities challenges**

The fundamental task of the teacher is to involve learners in teaching activities that are likely to result in a meaningful learning. The question may rise: What effects learning activities have on learners learning? This is an extremely important and exhausting inquiry to answer. The question doesn't give a particular answer because of following challenges.

*Challenge 1: Various learning activities might be effective differently on learners learning.*

Every learner in the class is unique with his or her special social, emotional, intellectual, and physical qualities. For this reason they must be understood by their teachers in their uniqueness hence the way how they learn must be also respected. It is essential to take note of that diverse learning activities may be viable contrastingly on learner's learning. This implies learners react contrastingly to various learning activities.

### *Challenge 2: Teaching activities as a system of interacting structures*

Learning activities are designed by the teacher for the purpose of creating conducive teaching environment. Whereas some teaching activities encourage and motivate learners for conceptual thinking while others prompt learners to engage in logical discussion (Lorenzo, 2015). This suggests that the response to learners to a specific teaching activity cannot be measured by means of comparing the results of learners. The results of a particular activity depend on how teachers interact with different aspects for teaching.

### *Challenge 3: Generating Appropriate Measures*

Effecting learning of mathematics generate clear measures for mathematics. However there is no appropriate measure or a tool to identify which teaching activity is suitable for specific topic or concepts. Fundamental assumption behind research on the effectiveness of Mathematics teaching is based on the fact that teacher's mathematics pedagogical content knowledge is a strong tool of generating measures in mathematics classroom.

## **2.5 CONCLUSION**

The primary goal of teaching and learning mathematics is to appreciate the usefulness, power, and beauty of mathematics within and outside the school environment. The usefulness of mathematics on daily basis and the importance of it is unquestionable and undeniable. As a result of conceptualization of the notion meaningful teaching and learning teacher's genuine conceptual understanding and adequate teacher knowledge of teaching mathematics needed is fundamentally important. Teacher's knowledge of teaching is necessary both in terms of research and in the professional development of teachers to guide them towards meaningful teaching. Abadzi (2013) agrees that to teach mathematics effectively requires a major support and guidance for teachers particularly in primary schools level. In fact, a view that intermediate teacher should be generalists rather than specialists has always been held within the education sector DBE (2013). On the contrary, DoE, (2014) indicated that subject specialization has been a preserve and privilege to senior phase teachers as opposed to intermediate phase teachers. While teachers' educational experience and credentials are often used by policy-makers and researchers as measures of teacher quality, these characteristics explain little of the variation in teacher effectiveness in terms of improving learning (Grouws, and Cebulla, 2016)

## CHAPTER 3

### THEORETICAL FRAMEWORK FOR THE STUDY

#### 3.1 INTRODUCTION

The theoretical question explored in this section of theoretical framework is how meaningful teaching in the intermediate phase enhances effective learning of mathematics. The aim of this study was to explore teachers' conceptualizations of meaningful learning in mathematics and how learners respond to activities facilitated to enhance meaningful learning in the IP. The aim of this section is to give an overview of learning recommended by different studies in connection of mathematics learning

Theories help teachers to conceptualize learning correspondence, advance relational connections among teachers and learners', help teachers to execute proficient morals and apply an effect on how teachers respect themselves. As indicated by Ingarrison et al. (2004) learning in general clarified as a procedure that includes the enthusiastic, intellectual and natural impacts and encounters for picking up, upgrading, or making changes in a person's values, abilities, skills, and world perspectives.

A learning theory is an endeavor to explain how animals and human beings learn, thereby helping to understand the inherently complex process of learning. It is of importance for teachers to have thorough understanding of the various learning. It is of significance for teachers to have intensive understanding of the different learning theories to understand how to support, motivate and inspire learners to succeed in learning

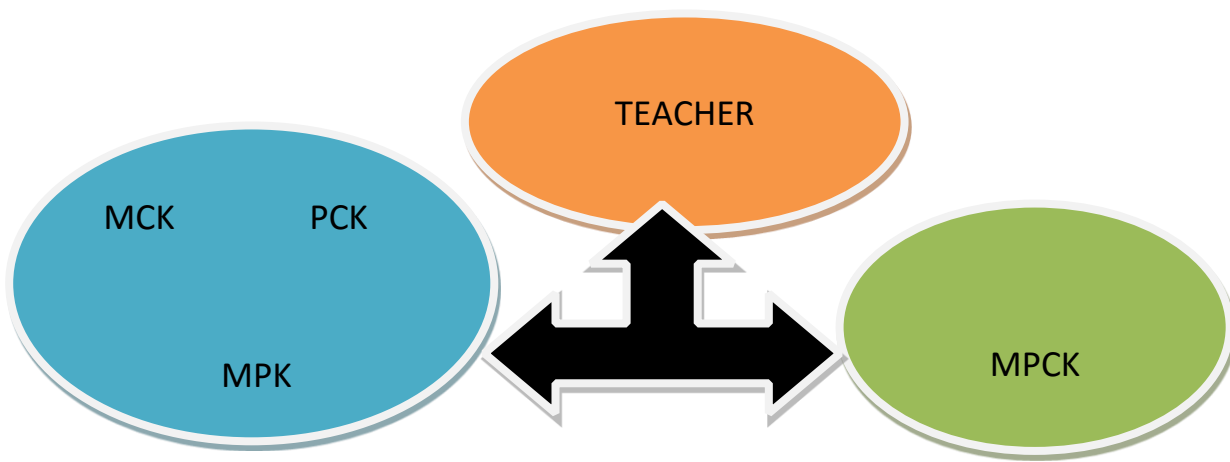
#### 3.2 Mathematics Pedagogical Content Knowledge (MPCK)

Cooper (2010) defines pedagogical knowledge (PK) as the knowledge of how to teach. In the teaching and learning process, teachers must demonstrate that they realize the subject they teach with a significant or specialization in the subjects that they teach. Effectiveness mathematics teaching resides not simply in mathematical content knowledge a teacher has accrued as stated above, but in "how" this knowledge is used in classrooms Hill, Rowan and Ball 2005).

Teaching mathematics is a complex enterprise involving both cognitive and socio- systemic factors (Haylock and Manning, 2014). In this regard effective teaching of this subject requires a proper choice of resources, suitable pacing and sequencing of content, creating classroom environments that promote learner participation and stimulate learner's interests, motivating learners, challenging and extending learner thinking and understanding. However, Hattie (2012) reported that the majority of IP teachers, particularly in mathematics, generally employed poor pedagogical practices which resulted in poor mathematics learning. Indeed, meaningless learning would result from meaningless teaching. As Howie (2013) also reported, mathematics teaching and learning at IP level were often focused on meaningless learning rather than meaningful learning.

Effective and meaningful mathematics teaching is “more than a collection of activities” but “coherent, focused on important mathematics and well-articulated” (Howie, 2013:112). Overall, it is critically important as mathematics teachers to teach learners through the provision of proper pedagogical practices in order to make mathematics meaningful to learners. All pedagogy rests on a conception of knowledge acquisition. This means that teachers' pedagogical choices must be informed by the conceptions that they hold about mathematical knowledge. In order for learners to interpret learning contexts created by teachers and adapt teaching methods that maximize learner understanding, pedagogical knowledge is of importance.

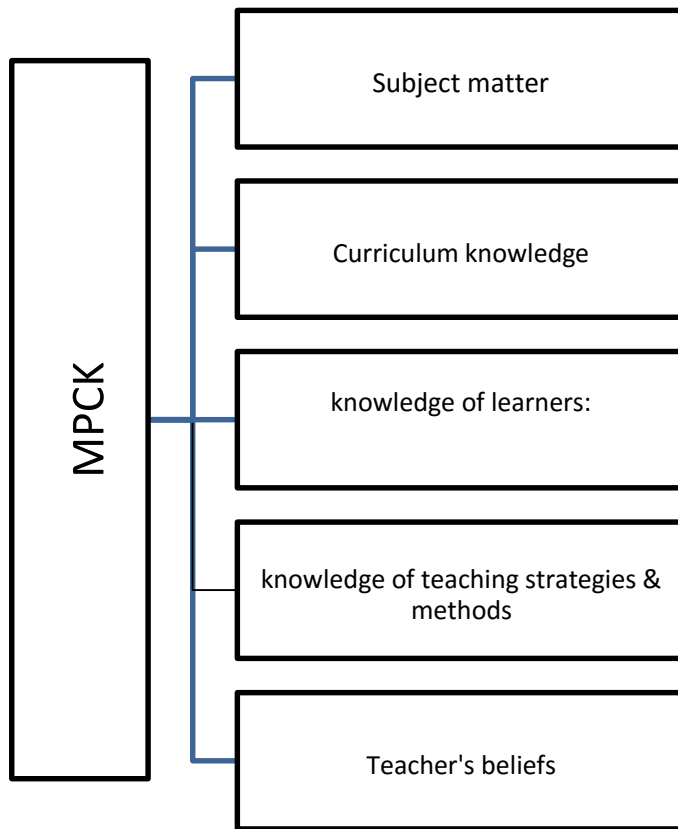
In this study, MPCK is seen as the blending of the knowledge of mathematical content and the teacher's pedagogical knowledge. The integration and relationship of this knowledge is used to define and describe knowledge needed for teaching mathematics successfully. The relationships amongst the types of knowledge used in this study are elaborated below



**Figure 3.1 The relationships amongst the three types of knowledge**

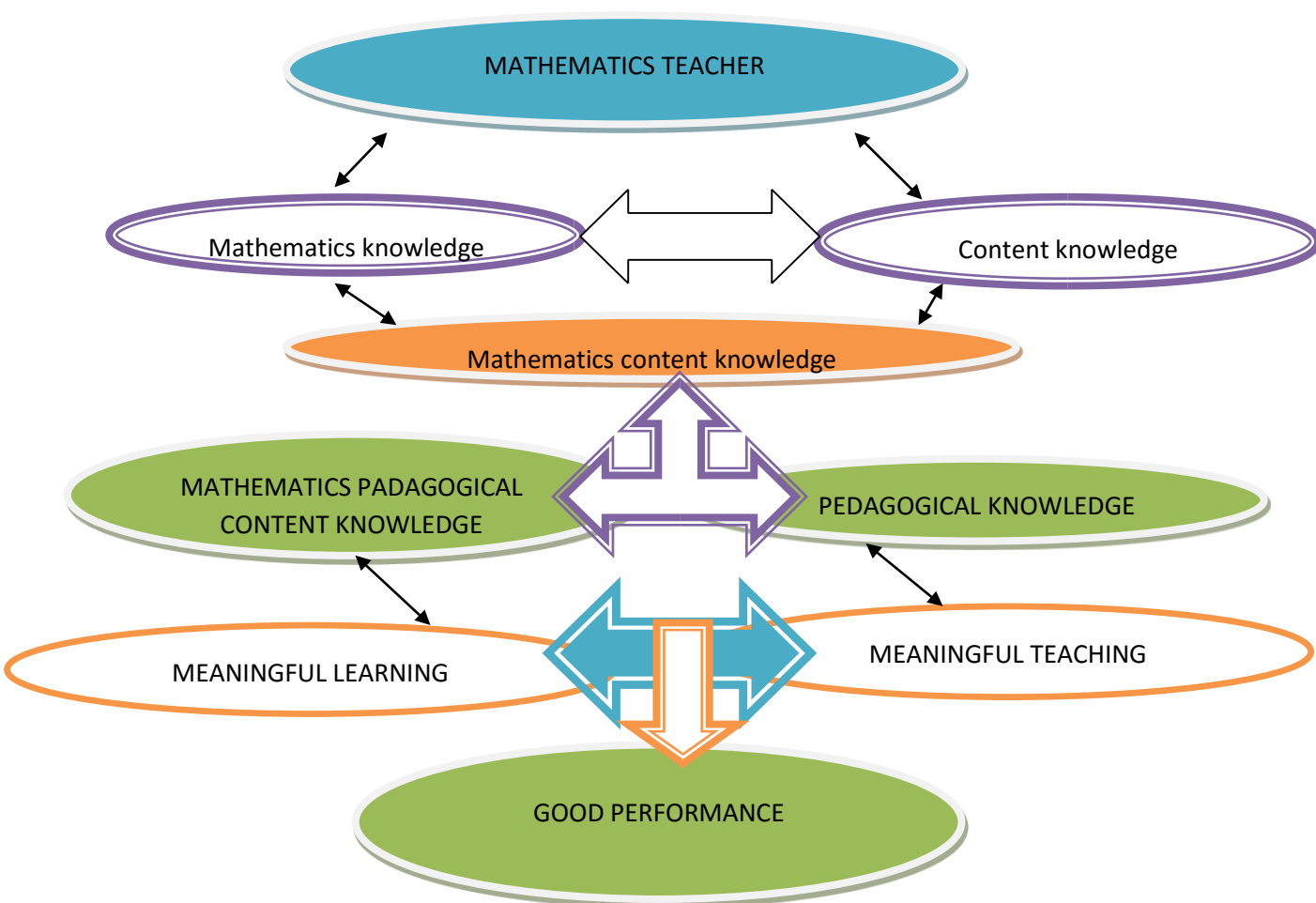
In Figure 3.1, shows the three types of knowledge domains which constitute the components for MPCK: namely, mathematics content knowledge (MCK); mathematics pedagogical knowledge (MPK); and pedagogical content knowledge (PCK).

Over the past few decades, MPCK has gained prominence in directing effective and meaningful teaching – in particular, by suggesting that what and how the teacher teaches mathematics is determined by his/her MPCK. In this regard, the researcher is of the view that IP mathematics teachers should, at the most basic level, have an adequate mathematics pedagogical content knowledge required, for effective and meaningful mathematics teaching and learning to happen. However, the study was aware that MPCK components as they are cannot make effective mathematics teaching and learning to be of high standard. Even though a mathematics teacher needs to possess adequate MPCK of the subject, there are other attributes that contribute to the effective teaching and learning of the subject such as indicated in Figure 3.2.



**Figure 3. 2: MPCK components**

Figure 3.2 show that MPCK is a tangled mix that is based on comprehensible and generative understandings of other aspects or factors that make up mathematics teaching and learning to be meaningful to learners. The view of MPCK being part of effective mathematics teaching and learning was the main focus of this study. It is against this background that the study aimed at formulating a model or strategy to improve IP teachers' mathematics pedagogical content knowledge (MPCK) for effective and meaningful mathematics teaching and learning. Therefore, the study proposed the following model to assist IP mathematics teachers to make teaching and learning of mathematics interesting and enjoyable (see Figure 3.3).



**Figure 3.3: TSHISI MODEL: A model to facilitate meaningful mathematics teaching practice in the IP**

The above diagram shows the theoretical position taken in this research.

**Mathematics teacher.** Teachers are considered the most important input element to effective teaching and learning (Maboya, 2014). It is widely accepted that teachers of mathematics need deep understanding of mathematics (Ball, 2008). Teacher knowledge has long been a focus of educational research (Maboya, 2014). Among the types of knowledge that have been shown to be important for teachers to develop are: content knowledge, pedagogical content knowledge (Hino, 2010).

According to Ball & Bass (2003) knowing mathematics for teaching requires that teachers be able to “unpack” ideas that are typically compressed into highly abstract and usable forms and to recognize connections both within and across mathematical domains (Howie, 2013). For example, accurate mathematical explanations, presenting appropriate definitions, making connections between representations, responding productively to learner’s mathematical questions and curiosities, making mathematical judgments about instructional materials, and interpreting and making mathematical judgments about learners’ questions, solutions, and insights (Ball and Bass, 2005). Hiebert & Grouws (2009) identified four categories of MKT, all of which are drawn on by teachers when they “unpack”, recognize connections, and make decisions about what to do next.

The four categories of MKT are: 1) common mathematical knowledge, 2) specialized mathematical knowledge, 3) knowledge of mathematics and learners, and 4) knowledge of mathematics and teaching. These skills are especially needed when teachers are required to interpret and make judgments about learner’s problem-solving methods.

### ***Knowledge of Mathematics***

It is generally accepted that secondary teachers come to teaching with a higher level of subject knowledge than IP teachers. In other words, “secondary teachers proved themselves that they know the subject they teach with a major in the subject they teach or credits equivalent to a major in the subject” (Zazkis and Zazkis, 2011:87). Yet, for IP teachers there is a general notion that they teach all subjects, without acknowledgement of knowing the subjects. Hiebert & Grouws (2009) showed that a lack in subject knowledge could lead to teaching difficulties. Mathematics knowledge is widely acknowledged as one of the critical attributes of mathematics teachers, thus, it is not surprising that mathematics knowledge was reported to be one of the significant factors that promote effective and meaningful mathematics teaching and learning (Hiebert, and Grouws, 2009). However, it is noticeable that teachers’ knowledge of mathematics alone is insufficient to support their attempts to teach effectively

***Content knowledge.*** First and foremost, teacher content knowledge is very important. One of the aspects of a highly qualified teacher from the No Child Left Behind Act, ((2002) is to be knowledgeable as content of the subject taught thus the knowledge about the subject matter to be taught.

First and foremost, the model above (figure 3.3) shows mathematics knowledge and content knowledge placed on the same level with a ray between them. A ray indicates that mathematics knowledge lays a foundation of content knowledge. A good mathematics teacher needs sufficient knowledge of mathematics (Fennema and Franke, 1992). He or she needs to have a profound understanding of basic mathematics and to be able to perceive connections between different concepts and fields (Fennema and Franke, 1992). Fey (2009) perceives content knowledge as the subset of mathematics knowledge in the sense that a teacher's content knowledge is related to what mathematical topics and concepts learners learn on that particular level. In this regard,

Hill (2002: 22) contends that the subject content knowledge is the “what” of teaching, or the “knowledge about the subject matter to be taught.” As a result, content knowledge influences how teachers engage learners with the subject matter and also evaluate instructional materials because having a flexible, thoughtful and conceptual understanding of mathematics knowledge is critical to effective teaching (Howie, 2010).

Most importantly, Figure 3.3 shows two rays which are running on opposite directions towards the same point (pedagogical knowledge). According to Gamoran, & Drake, (2009) the practice of teaching is a complex task to perform. PK includes the “how” of teaching in general, or the knowledge about teaching methods that pertain to topics and situations (Yu, 2005). According to Zazkis, & Zazkis (2011) PK determines the most effective means to teach particular topics or problems consistent with the learners' interest and ability. However, one of the challenges of teaching mathematics is teacher's lack of or insufficient pedagogical knowledge. For mathematics content knowledge to be effective and meaningful to learners, pedagogical knowledge is of paramount importance. In the mathematics classroom the three dimensions depend on each other. One cannot exist without the other two.

### **3.3 MATHEMATICS TEACHING APPROACHES AT INTERMEDIATE PHASE**

The theoretical position taken in this research shows that mathematics teacher is an important factor for meaningful and effective teaching and learning to happen. Teachers need knowledge of mathematics, pedagogical knowledge and knowledge of learners' cognition in mathematics (Haylock, 2010). This study is confined inside constructivism hypothesis in light of the fact that the point is to see how teachers teach learners for important learning. This implies that learners derive meaning from what they have been taught and also connect it in

their everyday lives. Apart from teacher's MPCK, Baroody (2013) also identified the following four qualitatively different approaches that inform quality and meaningful teaching in primary schools. Apart from teacher's MPCK, Baroody (2013) also identified the following four qualitatively different approaches that inform quality and meaningful teaching in primary schools

### **3.3.1 The Mathematics Skills approach**

The skills approach focuses on memorisation of basic skills (Yu, 2005). This approach is based on the assumption that numerical knowledge is simply a collection of useful information (facts, rules, formulas and procedures). In the skills approach, a teacher simply tells children that, for instance, to add addends you start adding from the units, tens hundreds and so on. Children then complete numerous computations with the procedure until it is memorised by rote.

As practice is performed without context (a reason) at a largely symbolic (abstract) level, the skills approach is not purposeful (in the sense that instruction builds on children's interests and creates a genuine need to learn and practice mathematics), nor is it typically meaningful (Tambara, 2015). As children are seldom engaged in any real numerical thinking, the skills approach is almost never inquiry based, as it involves a repetitive practice. Even though the foregoing discussion asserts that the skills approach focuses on memorisation, it is important that Foundation Phase teachers encourage the children to learn the multiplication tables in order for them to apply the knowledge when solving mathematics tasks. Wilson, Cooney, & Stinson (2005) argue that in mathematics, an exclusive focus on basics leaves children without the understanding that enables them to use mathematics effectively. A focus on "process" without attention to skills deprives children of the tools they need for fluid, competent performance.

### **3.3.2 The Mathematics Conceptual Approach**

Barrows (2010) argues that the focus of the conceptual approach is on the meaningful memorisation of skills rather than procedural memorisation. This approach is based on the assumption that mathematics constitutes a network of skills and concepts. In other words children are viewed as capable of understanding mathematics if told or showed why procedures work. The aim of this approach is for teachers to help children to acquire needed

facts, rules, formulas, and procedures in a meaningful way (i.e., with comprehension). The teacher guides children towards understanding and mastery of skills.

In the conceptual approach, symbolic procedures, such as addition of addends are illustrated by actual teacher demonstration. Children may even be encouraged to imitate an illustrated programme themselves with calculating. Thus, although instruction and practice is often without context, an effort is made to promote meaningful learning. From the discussion above, it is important that Foundation Phase teachers should guide children towards understanding and mastery of skills. Teachers should provide them with tasks/activities that will help them to acquire those skills. Children would then be able to solve problems themselves.

### **3.3.3 The Mathematics Problem solving approach**

The problem-solving approach focuses on the development of numerical thinking (reasoning and problem solving). This approach is based on the assumption that mathematics is, at heart, a way of thinking, a process of inquiry, or a search for patterns in order to solve problems (Barrows, 2010). Children are viewed on the one hand, as using intuitive thinking and possessing incomplete knowledge and, on the other hand, as naturally curious creatures that can and must actively construct their own understanding of mathematics. The aim of mathematics instruction is to immerse numerical novices in mathematics inquiry (solving what are to them real and challenging problems) so that children can develop more mature ways of thinking and incidentally discover and construct more complete mathematics knowledge.

The teacher as a wise partner in this enquiry pushes the process along but does not entirely, or even largely, set the agenda or control the enquiry. The learning content such as the formal procedure for addition in word sums is secondary to developing children's thinking processes (Beswick, 2012). In the light of the discussion above, it is clear that learning using this approach will encourage children to construct their own understanding of mathematics and children will be able to investigate and to solve real and challenging problems that they may come across.

### **3.3.4 The Mathematics Investigative approach**

According to Beswick (2012) the investigative approach focuses on meaningful memorisation of skills and development of numerical thinking. Like the conceptual approach, mathematics is viewed as a network of skills and concepts. Also, like the problem-solving approach, it is viewed as a process of inquiry. Children's active construction of understanding is mediated, guided, and prompted by the teacher most often through planned activities. This study offers a theoretical argument of teacher's mathematics for teaching as a framework for conceptual understanding. Mathematical investigation approach can be used to encourage meaningful construction of procedures and concepts and the development of numerical thinking.

The teacher uses indirect means to help children to construct knowledge. For example, a teacher might guide children to reinvent a procedure such as the algorithm for addition of word sums. The teacher might then encourage children to invent their own procedures for solving the problem (Abramovich et al 2016). That may well involve using calculating or drawings. Children can then be encouraged to represent the problem and their informally determined solution symbolically and look for shortcuts to their concrete procedures. The investigative approach involves purposeful, meaningful and inquiry based instruction. The teacher encourages self-invention of problem solutions by building on children's informal knowledge. The children would begin to enjoy what they do and develop a love thereof.

### **3.3.5 Mathematics: the enjoyable way**

Boaler (1993, cited by Barrett, 2005) claims that "mathematics is no longer mainly conceived as a collection of abstract concepts and procedural skills to be mastered, but primarily as a set of human sense making and problem-solving activities based on numerical modelling of reality". Indeed, children should learn by understanding and not by rote. In order to understand mathematics, the teaching of concepts through everyday language and the use of the immediate environment is critical and essential. The old method of making children learn by rote, passively and with repetition is no longer encouraged in the reformed curriculum. Teachers who still follow this approach are themselves a barrier to teaching mathematics, which further compounds children's problems. Such educators should be retrained in order to help them to help and support the children they teach.

Borko (2004) alleges that in mathematics, different from other sciences, objects don't have a tangible existence. In other words, mathematics can only be presented symbolically.

However, in mathematics classes, teachers use counters to develop children's" skills of adding and subtracting. It is precisely this symbolic representation that needs clear and simple language together with examples from the children's immediate milieu to understand mathematics. The role of the teacher in this regard is of vital importance. Attention should not, therefore, be focused on the symbols and their meaning, but rather on the activity of the symbolising and meaning making (Cobb et al., 2010). Therefore, the teacher who lacks language or skills to impart knowledge will be a barrier to children. The situation becomes even direr where the medium of instruction is a second language. This is a major impediment for South African children and therefore strategies should be devised to improve mathematics teaching.

### **3.4 CONCLUSION**

In this chapter, an attempt was made to describe the concept of theoretical framework on teaching of mathematics at the intermediate phase. In this chapter, two perspectives played a critical role in explaining and justifying the need to ground the teaching and learning of mathematics in constructivism. The researcher also gave a literature account on how theory on teacher knowledge has been problematised, and thus evolved over time, in an attempt to close literature and research gaps that exist on the relationship between teacher knowledge and classroom practice. Research developments on efforts to bring together teacher knowledge of mathematics and mathematics classroom practice, the main constructs in the research question, were brought to the fore.

Contributions to a theory on mathematics teacher knowledge and practice by various scholars such as Lee Shulman (PCK), Liping Ma (PUFM), Tim Rowland, Peter Huckstep and Anne Thwaites (knowledge quartet framework) were discussed to indicate how they culminated into and helped to shape theory on MKT, the main construct of my research question. The detailed discussion of constructivism, which is the most current psychological approach to learning, was critical, because it is the theory of learning and development that is the basis of the current reform movement in teaching and learning. MKT was discussed, focusing on its role, and on its importance in regard to the effective teaching of mathematics, and, hence, the need for the development of the teacher's MKT. This discussion was extended by exploring beliefs about what mathematics teaching and learning is. The next chapter presents the methodology employed in this research to answer my research question.

## **CHAPTER 4**

### **RESEARCH METHODOLOGY**

#### **4.1 INTRODUCTION**

This chapter describes the methodology that was followed in carrying out this study. This chapter has covered the following: the study area, research design, target population, sample size and sampling procedures, research instruments, validity and reliability of data, data gathering procedures, data analysis, ethical considerations, limitations and delimitations of the study.

#### **4.2 RESEARCH PARADIGMS**

Leedy & Ormrod (2014) explain that research methodology is the creation and development of techniques and strategies to collect data, the development of methods to investigate and improve the psychometric properties such as reliability and validity of the data obtained by means of these techniques and the analysis of such data. In this study, research methodology involves a selection of appropriate research approaches, research methods, sampling procedures, respondents and instruments for collecting and analysing data. To Eyisi (2016: 91) research involves “the combination of reasoning and experiences.” Eyisi further explains that in educational research, “different research approaches are used by educational researchers based on the data collection and analysis used at a given time” (Eyisi, 2016: 91).

Typically, a researcher conducts his or her research by either using quantitative (a positivist orientation), qualitative (a social constructivist approach) or mixed methods (a combination of both quantitative and qualitative) research approaches (Johnson and Christensen, 2019). In this regard, a research paradigm is envisioned as a set of philosophical assumptions or beliefs a researcher has about the phenomenon he or she wishes to study or understand. Accordingly, a research paradigm is characterised by its “methods of data collection and analysis as well as methodological approaches” (Eyisi, 2016: 92). Thus, qualitative and quantitative research differ “in their paradigmatic approaches with respect to their epistemological (ways of knowing and enquiry in nature of reality) and ontological foundations (what is to be known and assumptions about the nature of reality)” (Eyisi, 2016: 92). Eyisi (2016: 92) further explains this as follows:

In ontological orientations, qualitative and quantitative researchers are constructivism and objectivism respectively in terms of their strategies. However, in epistemological orientation, quantitative researchers are objectivists and positivists in their research approach while qualitative researchers are subjectivists and anti-positivists in their research approach.

To Grouws, & Cebulla, (2016), a research paradigm encompasses four distinct terms which are: ethics (axiology), epistemology (types of knowledge), ontology (nature of reality), and methodology (ways of knowing). Nonetheless, there is a view that both paradigms seek to achieve the same goal, namely to “explain” the phenomena they study, *albeit* through paradigmatically different techniques, procedures, ways of gathering and analysing data (Eyisi, 2016). However, the point is noted that “since qualitative and quantitative research approaches are based on divergent theories and assumptions, one should be more advantageous than the other and vice versa, depending on the nature of research and data collection methods” (Eyisi, 2016: 92). Each of these research paradigms will now be explained:

#### **4.2.1 Qualitative Research Paradigm**

According to Neumann (2014) the qualitative research paradigm is an approach that examines the empirical world by interpreting the real world from the perspective of the subject of his or her investigation. Qualitative researchers seek to understand phenomena in their entirety in order to develop a complete understanding of the person, programme or situation. Leedy & Ormrod (2014) also emphasise that qualitative research centres on phenomenon that occur in natural settings with, and despite, the difficulties of studying phenomena in their real-world settings without manipulation to suit the researcher. According to Leedy & Ormrod (2014) the purposes of qualitative research studies are (a) to reveal the nature of certain situations, settings, processes, relationships, systems, or people (*description*), (b) to (i) gain new insights about a particular phenomenon, (ii) develop a new concept or theoretical perspective about the phenomenon, and/or (iii) discover the problems that exist within the phenomenon (*interpretation*), (c) to test the validity of certain assumptions, claims, theories, or generalizations within real-world contexts (*verification*), and (d) provide a means through which a researcher can judge the effectiveness of a particular policy, practice, or innovation (*evaluation*).

Accordingly, qualitative research approaches have the advantage of allowing for more diversity in responses as well as the capacity to adapt to new developments or issues during the research process itself. While qualitative research can be expensive and time-consuming to conduct, many fields of research employ qualitative research techniques that have been specifically developed to provide more succinct, cost-effective and timely results. The strength of qualitative research is its ability to provide complex textual descriptions of how people experience a given research issue.

It provides information about the human side of an issue in terms of variables such as behaviours, beliefs, opinions, emotions, and relationships of individuals which may be different from individual to individual even when they have experienced the same phenomena. In the view of (Haertel 2013) the reliance of qualitative research on the collection of non-numerical primary data, such as words and pictures, by the researcher who serves as an instrument himself makes qualitative research well suited for providing rich factual and descriptive information. Furthermore, because in qualitative research, theory emerges from data, the emergent of theory allows the researcher to construct and reconstruct theories where necessary, based on the data he or she generates, instead of testing data generated elsewhere by other researchers.

The strength of qualitative research methods lies in their use of open ended and probing questions which gives the participants the opportunity to respond in their own words, rather than forcing them to choose from fixed responses, as quantitative research methods do. In particular, the two qualitative research techniques, namely focus-group and individually based in-depth interviews can generate a lot of useful information. According to Eyisi (2016) qualitative research approach seeks to tell the story of a particular group's experiences in their own words, and is therefore focused on narrative (while quantitative focuses on numbers).

#### **4.2.2 Quantitative Research Paradigm**

The quantitative research paradigm entails the use of statistical methods to measure and analyse data. According to Leedy & Ormrod (2014), statistics are a very simple matter but are a powerful tool in the hands of a skilful researcher who is able to view the nature and interrelationships among different variables more understandably. Through statistics, the

researcher is able to conceptualize what otherwise might be incomprehensible. As Eyisi (2016: 94) points out, “the use of statistical data for the research descriptions and analysis reduces the time and effort which the researcher would have invested in describing his result.”

The second advantage of quantitative research approaches is that “the use of scientific methods for data collection and analysis make generalization possible” (Eyisi, 2016: 94). This advantage of quantitative research leads to another benefit, namely one of reliability. Accordingly, “since the research approach basically relies on hypotheses testing, the researcher need not to do intelligent guesswork, rather he would follow clear guidelines and objectives” (Eyisi, 2016: 94). For this reason, it is argued that quantitative research “is conducted in a general or public fashion because of its clear objective and guidelines, and can therefore be repeated at any other time or place and still get the same results” (Eyisi, 2016: 94). Typically, the use of experimental designs with in-built checks and balances – mainly through the use of control groups, give quantitative research approaches another measure of transparency and objectivity, which one does not normally see in qualitative research. Finally, it is usually argued that the requirement for quantitative researchers to “distance” or “detach” themselves from the researched situation is taken be a strength insofar as bias and objectivity are concerned – as Eyisi (2016: 94) explains: The issue of a researcher being biased with either his data collection or data analysis will be highly eliminated when the researcher is not in direct contact with the participants, that is, he collects his data through either telephone, internet or even pencil-paper questionnaire. There is full control for alternatives such as interpretations, explanations, and conclusions. In other words, the objectivity of the researcher will not be compromised. Secondly, this may perhaps guarantee respondent anonymity. Thus, it is against these perceived benefits that quantitative research approaches find favour with researchers in the positivist and post positivist research orientations, whereas social constructivism and critical theory tend to undergird qualitative research approaches.

#### **4.2.3 The Mixed Methods (Blended) Research Paradigm**

The mixed methods (blended) research paradigm has emerged as a practical way to address many research problems in education as Fai (2010: 6) puts it, “mixed methods research has been hailed as a response to the long-lasting, circular, and remarkably unproductive debates discussing the advantages and disadvantages of quantitative versus qualitative research as

a result of the paradigm “wars”. Thus, proponents of mixed methods research strive for an integration of quantitative and qualitative research strategies (Cresswell, 2011: 7). Both quantitative and qualitative research paradigms outlined have strengths and shortcomings. According to Neuman (2014) researchers often combine elements from qualitative and quantitative research approaches in order to address their research problems more fully – that is, better than the use of only one approach would be able to deliver. There are various reasons for combining these two approaches in research - for example, qualitative approaches usually yield rich information about the social processes in specific settings (Neuman, 2014).

However, cognisance must always be taken that these two research paradigms are based on fundamentally different assumptions about the world, the purpose of research, research methods, the researcher’s role, and the importance of the context (Maree, 2007). For instance, Neuman (2014) points out that a quantitative researcher assumes that concepts can be conceptualised as variables, and that objectives and precise measures can be developed that attach numbers which capture important features of the social world.

#### **4.2.4 The Research Paradigm Chosen for the Study**

This study followed the mixed methods research paradigm for addressing the research questions set out in chapter one. Accordingly, both the qualitative and quantitative approaches to data collection, analysis and interpretation were followed. It was envisaged that, when used jointly, these approaches would contribute to improving the trustworthiness of the results of this study and the subsequent generalisations and recommendations. As Fai (2010: ) states, mixed methods research has been hailed as a response to the long-lasting, circular, and remarkably unproductive debates discussing the advantages and disadvantages of quantitative versus qualitative research as a result of the paradigm ‘wars’. Accordingly, proponents of mixed methods research strive for an integration of quantitative and qualitative research strategies (Fai, 2010).

The reason for using the qualitative approach in this study was to gain insight from heads of the intermediate phase departments regarding PCK in the context of teaching and learning of mathematics by intermediate phase teachers. So, both focus-group discussions and individually based in-depth interviews were employed to obtain data from intermediate phase mathematics teachers and their learners. For the teachers, they were asked to describe how

their MPCK assisted them in their teaching of mathematics in the intermediate phase, while the learners were asked about how well their teachers facilitated their meaningful learning of mathematics. Thus, the qualitative research approach was found to be suitable for collecting this form of data.

However, it was also important to collect some quantitative data to complement and support the data collected through qualitative methods. The frequencies of occurrence of occurrences in each theme or category identified were recorded. The categories with high frequency occurrences were regarded as significant to the problem investigated and reflected the mathematics teachers' PCK in the context of teaching and learning of mathematics in intermediate phases. Thus, the quantitative aspect of the research was used to make it possible to establish whether or not there was a relationship between the intermediate phase teachers' MPCK and their conceptualisations of the notion of meaningful learning in mathematics. Accordingly, the data gathered from the questionnaires were translated into tabular form to allow for statistical expression, analysis and interpretation.

#### **4.3 RESEARCH DESIGNS**

According to Creswell (2012), a research design is a plan or structured framework of how you intend to conduct the research process in order to solve a problem. In concurrence Niewehuis (2013) also avers that a research design is a plan or strategy specifying the how and from where the participants was be selected, the data gathering techniques to be used, procedures for data collection and analysis. As stated earlier, this study was framed around a mixed methods research paradigm, which involved the collection and analysis of both qualitative and quantitative data. In this regard, the qualitative aspect of the research design allowed for understanding the context-specific and subjective meanings related to Vhembe District intermediate phase teachers' conceptualisations and facilitation of meaningful mathematics learning. Although subjective interpretation could result in potential misconceptions and self-deceptions, Stringer (2013) argues that it may also lead to the discovery of new, unanticipated insights.

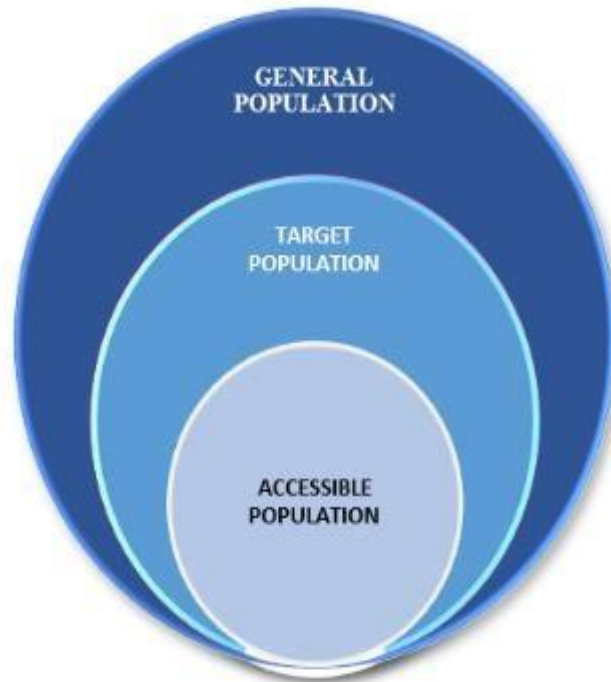
As the study was premeditated to explore teachers' conceptualizations of meaningful learning in mathematics and how learners respond to activities facilitated to enhance meaningful learning in the Intermediate Phase, it was felt necessary to apply a descriptive approach.

This would help present the responses as put by mathematics teachers in a more descriptive manner. It was also in view of the fact that perceptions and conceptualisation could be easily be understood only by attaching descriptive connotations.

For the quantitative aspect, statistical analysis was the main approach employed. The quantitative research design was used to assess teachers' perceptions about the importance of mathematical pedagogical content knowledge (MPCK) and teachers' conceptualization of the mathematical pedagogical content knowledge (MPCK) that is associated with meaningful teaching and learning of mathematics. In general, quantitative research design is an excellent way of finalizing results and proving or disproving a hypothesis. After statistical analysis of the results, a comprehensive agreement is reached, and the results can be legitimately discussed and published.

#### **4.4 GENERAL, TARGET AND ACCESSIBLE POPULATIONS**

Neumann (2006: 221) defines population as the group upon which the researcher is interested in making inferences. To Creswell (2013: 162) population is defined as a set of entities in which all the measurements of interests to the practitioner or researcher are presented. Asiamah, Mensah & Oteng-Abayie (2017) refer to the *general*, *target* and *accessible* populations. To Asiamah, et al (2017: 1612) see the general population as "characteristically crude in the sense that it often contains participants whose inclusion in the study would violate the research goal, assumptions, and/or context." Accordingly, they define the general population as the entire group that shares the basic attributes of interest about which some information is required, while the target "is more refined as compared to the general population on the basis of containing no attribute that controverts a research assumption, context or goal" (Asiamah, et al., 2017: 1612). The accessible population has all the attributes of the target population, except that it is closest to the researcher and therefore, for practical reasons, becomes the population from which the study sample is drawn. Diagrammatically, Asiamah, et al. (2017: 1611) illustrate the relationships among these three concepts as in Figure 4.1.



**Figure 4.1: A Conceptualization of the Relationship between General, Target and Accessible Populations** [Source: Asiamah, et al (2017: 1611)].

For this study the target population comprised all intermediate phase mathematics teachers in the Sibasa circuit in Vhembe district consisting of 120 intermediate phase mathematics teachers from grade 4 to grade 7. Intermediate Phase schools of Sibasa circuit were purposively identified for the study because of its accessibility to the researcher.

#### **4.5 SAMPLING PROCEDURE**

The study used two types of sampling procedures which are purposive and simple random sampling methods. Purposive sampling means that respondents are chosen on the basis of their knowledge of the information desired (Asiamah, et al., 2017: 1608). Moreover, random sampling was used in choosing sample unit from the entire population of intermediate mathematics teachers. For the quantitative segment of the study, purposive sampling technique was used (Leedy and Ormrod, 2014). This resulted in the selection of a research sample of 120 intermediate phase mathematics teachers.

Through random sampling process 10 teachers from 120 mathematics teachers were selected. To avoid bias when choosing 10 teachers to be involved in the interview process, pieces of paper were written interviewee 1 up to interviewee 120 were put in a box. During the process of collecting questionnaires a teacher was requested to pick a piece of paper from the box. After picking it up the researcher stapled it with his or her questionnaire. Those teachers who picked papers written interviewee 1 up to 10 were involved in interview discussion. With the support from the Circuit Manager, Principals and Heads of Department 120 questionnaires were easily distributed and collected to all 120 mathematics teachers.

#### **4.6 DATA COLLECTION INSTRUMENTS**

Data collection is not just a process of gathering data; it is also a process of creation of gathering information in unique ways related to the purpose of study (Leedy andOrmrod, 2014: 332). This section describes the instruments developed for data collection and the procedures followed in this study.

##### **4.6.1 Questionnaire Survey**

As already stated, a questionnaire was developed and used for the study. As the researcher desired to collect quantitative information concerning the intermediate mathematics teachers' PCK in the context of teaching and learning of mathematics, a Likert-type questionnaire was developed to collect data to address the research questions of this study. The questionnaire was made up of 11 close-ended items for the teachers and learners. According to Cohen, Manion & Morrison (2011) close-ended questions are quick to compile and easy to code, and do not discriminate unduly on the basis of how articulate the respondents are. For this study, the Likert-type scale was administered from this study. The questionnaire was developed based on the following components of mathematics teachers' knowledge:

- a. Knowledge of mathematics
- b. Knowledge of mathematical representations
- c. Knowledge of teaching and decision making

This study used a questionnaire to collect data for the purpose of investigating IP teachers' MPCK and its relationship to the teaching and learning of mathematics. A structured questionnaire consisted of six sections. Section "A" contained items addressing the

participants' background and biographical information, such as teachers' qualifications, teaching experience, interest and motivation towards mathematics.

#### **4.6.2 Interview Schedule**

Interviewing is one of the most common methods of data collection used by researchers to inform them about social life. Interviewing could thus be regarded as a universal mode of systemic enquiry (Neumann, 2006). In particular, the face-to-face interview technique is seen as a pipeline for extracting and transmitting information from the interviewee to the interviewer (Creswell, 2011). However, learning about these „closed worlds“ depends on the ability of the interviewer to maximize the flow of validity and reliable information, while reducing the distortions in the interviewee's recollection of events. Open-ended interviews enable the interviewer to obtain an inside view of the social phenomenon that also explore other avenues that emerge from the interaction. Furthermore, the use of open-ended interviews encourages a two-way communication, allowing for confirmation of what is already known, as well as provide reasons for answers. In addition, it is also often possible for interviewees to more easily discuss sensitive issues when they can trust the interviewer. Another advantage of using interviews is that, in some cases, interviewers need not have to be particularly skilled in the art of interviewing. In this study, the researcher conducted interviews with mathematics teachers, focusing on their individual circumstances in their own settings.

The interviews aimed to understand the mathematics teachers' PCK in the context of teaching and learning of mathematics in the intermediate phase.

#### **4.7 DATA ANALYSIS**

Steps on how to process and analyse qualitative data are documented by several authors (Leedy and Ormrod, 2014). Firstly, all the data must be transcribed – and then read and re-read for the researcher to become familiar with the data, while notes are made to capture recurring concepts, common themes, events and other patterns in the data. These are then labelled and coded, before being sorted and categorised. Then the analysed categories are ready to be written up in a report. Further, Leedy and Ormrod (2014: 178) point out that the term „analysis“ basically means the resolution of a complex whole into parts. For qualitative research, data analysis involves reducing a wealth of available data, or data that one has collected, to manageable proportions.

This is done systematically, searching and arranging the interview transcriptions, field notes and other materials that are accumulated to increase the researcher's own understanding of them and to enable one to present what one has discovered. Regarding the qualitative data in this study, a simple qualitative data analysis method was used involving content analysis which, according to Creswell (2011) entails comparing the words used in the answers of the respondents. Initially, the researcher studied her field notes and reduced the taped verbal interviews into written transcripts

For their part, quantitative studies emphasise the use of numerical measures to arrive at specific findings. The data gathered in this study were grouped into categories and then analysed using the Statistical Product and Service Solutions (SPSS) version 25.0. Thereafter, the results were interpreted and presented using tables, pie charts and figures. The percentage of the research sample responding to each question was given. The data were presented according to the responses and/or the views of the respondents. Numerical scores were assigned to them to indicate possible relationships in the responses of the respondents.

#### **4.8 MEASURE OF QUALITY CONTROL IN QUANTITATIVE APPROACH**

This section provides information on the quality control measures related to the quantitative aspects of the research.

##### **4.8.1 Validity**

Validity means the correct procedures have been applied to find answers to a question. According to Cohen, Manion and Morrison (2013) validity indicates how worthwhile a measure is likely to be in a given situation. Validity shows whether the instrument is reflecting the sureness with which conclusions can be drawn. Validity also refers to extent to which an instrument measures something. In this study, the researcher was being wary of internal validity threats such as history, attrition, subject and effect.

##### **4.8.2 Reliability**

Reliability refers to the quality of measurement procedure that provides repeatability and accuracy. Grouws (in Cohen at al., 2013) state that quantitative research assumes the possibility of replication. According to Cressswel (2008), a reliable measuring instrument is that if repeated under similar conditions would present the same results. Morse and Richards

(2002) indicate that test validity is the extent to which inferences and uses made on the basis of scores from an instrument are reasonable and appropriate.

Leedy & Ormrod (2012) indicate that reliability is concerned with the consistency, stability and repeatability of the informants accounts as well as the investigators' ability to collect and record information accurately. In summary, reliability focuses on the consistency of the measuring instrument. The questionnaire is the right tool because the aspect of mathematics teachers' pedagogical content knowledge in the context of teaching and learning of mathematics in intermediate phases requires unstructured questions and this is exactly what questionnaires do.

#### **4.9 MEASURES TO ENSURE TRUSTWORTHINESS OF QUALITATIVE DATA**

Presented below are the quality control measures taken related to the qualitative aspects of the research.

##### **4.9.1 Credibility**

According to Creswell, (2011:80), the qualitative investigator's equivalent concept, i.e. credibility, deals with the question, "How congruent are the findings with reality? Nieuwenhuis (2007) affirms that ensuring credibility is one of most important factors in establishing trustworthiness. For qualitative researchers, validity and reliability were about the credibility and trustworthiness of the research. The researcher ensured that data collected was credible through peer researchers' involvement which includes amongst others the supervisors and the assistance from several other researchers involved in the study. The researcher engaged in preliminary visits to the sampled schools to establish a relationship of trust and mathematics educator checking before she started with data collection to ensure that the responses from the participants are validated.

##### **4.9.2 Dependability**

In addressing the issue of reliability, the positivist employs techniques to show that, if the work were repeated, in the same context, with the same methods and with the same participants, similar results would be obtained. Thus, the research design may be viewed as a "prototype model". Such in-depth coverage also allows the reader to assess the extent to which proper research practices have been followed. So as to enable readers of the research report to develop a thorough understanding of the methods and their effectiveness.

In general dependability refers to the provision of evidence such that if it were to be repeated with the same or similar participants in the same or similar context, its findings would be similar (Creswell, 2011:55). In this study dependability was ensured by independent coder who was used to verify data.

#### **4.9.3 Confirmability**

The concept of confirmability is the qualitative investigator's comparable concern to objectivity. Critical steps must be taken to ensure as far as possible that the work's findings are the result of experiences and ideas of the informants, rather than the characteristics and preferences of the researcher. Confirmability refers to the potential for congruency of data in terms of accuracy, relevance or meaning (Creswell, 2011:98).

The role of triangulation in promoting such confirmability must again be emphasised, in this context to reduce the effect of investigator bias.

#### **4.9.4 Transferability**

Transferability refers to the ability to apply the findings in other contexts or to other participants (Creswell, 2011:12). Creswell further indicated that transferability is achieved by providing a detailed, rich description of the setting studied, so that readers are given sufficient information to be able to judge the applicability of findings to other settings which they know. Again, to verify the honesty of the data, transcripts of data analysis were submitted to a critical reader in order to guard against the researcher's bias.

#### **4.10 CONCLUSION**

This chapter has presented the various aspects of the research methods used in this study, including the research paradigm, design, target population and sampling techniques. In addition, the chapter has also presented the instrumentation, data collection procedures and data analysis techniques. The chapter that follows presents the results and major findings of the study.

## **CHAPTER 5**

### **DATA PRESENTATION, INTERPRETATION AND RESULTS**

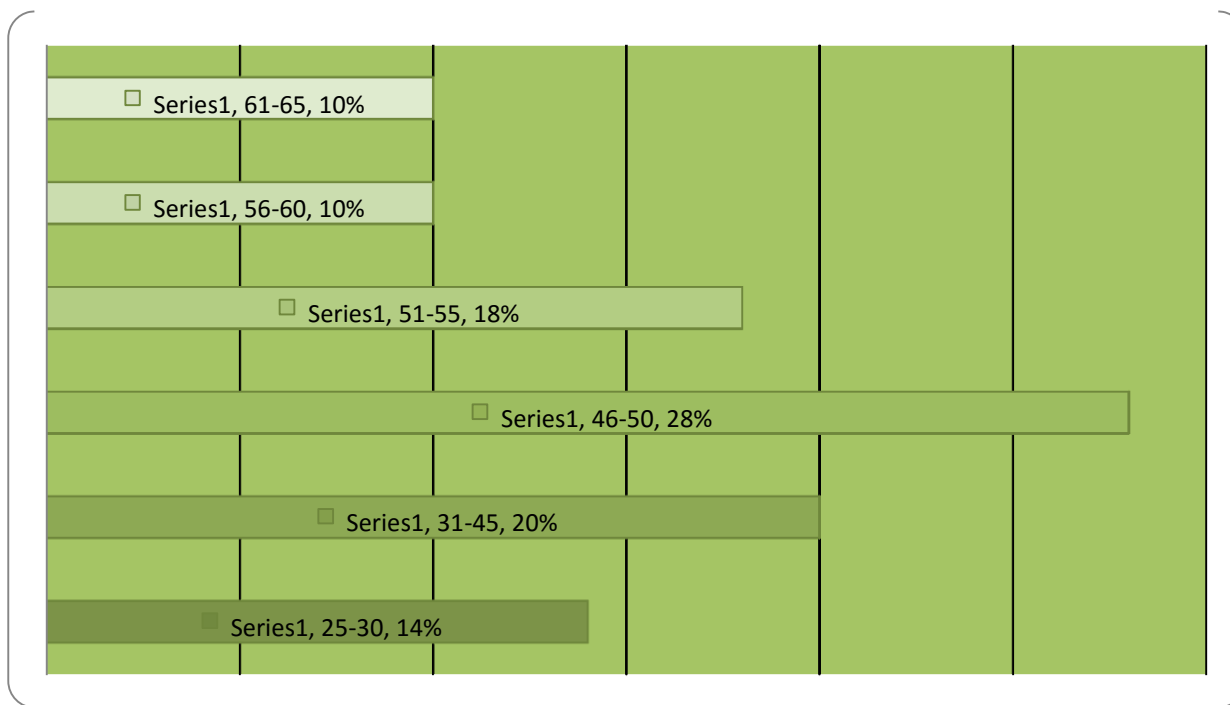
#### **5.1 INTRODUCTION**

This chapter presents the processed data collected by means of the questionnaire and interview protocol described in the previous chapter. The data are interpreted and the results presented, starting with demographic information about the participants, followed by a presentation and analysis of data collected through questionnaire and interview. The purpose of this chapter is to present an analysis and interpretation of the collected data, in order to answer research questions which seeks to establish effective approaches and strategies that IP teachers may use to enhance meaningful mathematics teaching and learning. The responses were collected from Grade 4- 6 through questionnaire and individual interview. The rationale behind the data collection was to give credibility to meaningful mathematics teaching and learning. Additionally, such analysis is crucial to determine mathematical topics that may be problematic in the teaching and learning of mathematics. Each item in the questionnaire was interpreted in terms of the findings on the observed essential aspects and approaches used to enhance meaningful mathematics learning, with the incorporation of the principles of effective and meaningful teaching.

#### **5.2 BIOGRAPHICAL INFORMATION**

Biographical information is important for research and contains information about the people who are participants in a particular study. Regardless of its length and target audience, biographical information contains basic facts like time, places, gender, educational qualification, ethnicity, age, and other pertinent characteristics of the participants in a research project. Biographical information identifies the type of people who have participated in the research. This information can be used to determine and influence the usability of the results and findings reported.

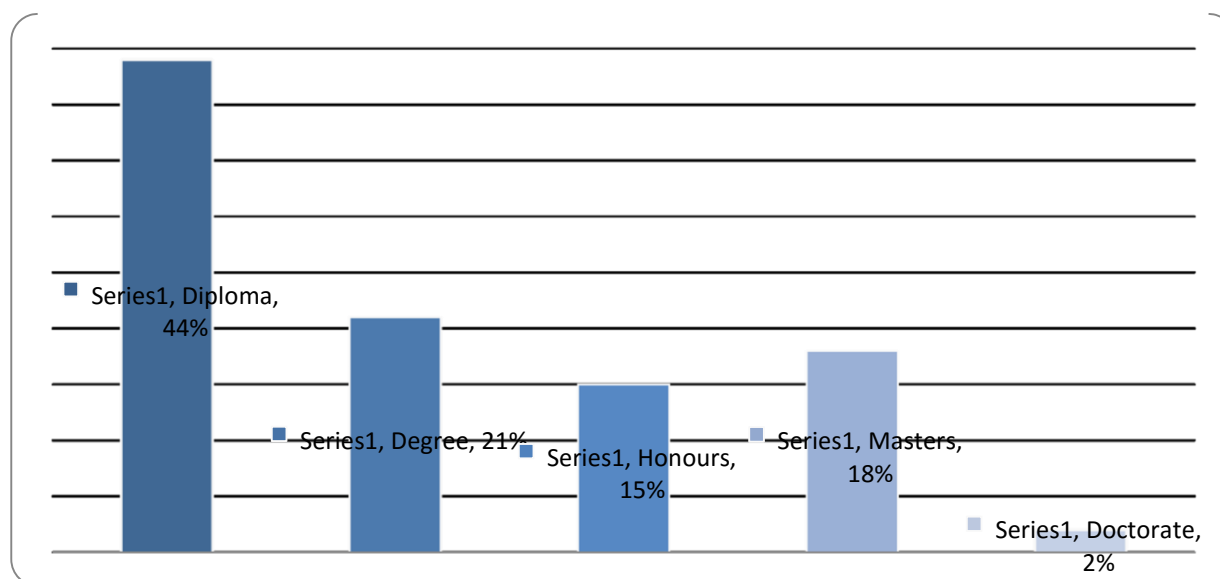
### 5.2.1. Age



**Figure 5.1: Age distribution of the respondents (n=120)**

Looking at Figure 5.1 respondents of various ages were well distributed in figure 5.1. Mathematics teacher characteristics can either influence positively or negatively by his or her age. It is expected that as teachers advance in age, particularly in the intermediate phase. Generally the majority of respondents were the matured age group promising source of potential teachers. An empirical study shows that teachers who ranged from 35 to 55 are more knowledgeable in the aspect of teaching. What obviously emerges from this finding is that teachers in this sample are greater likely to influence effective learning. Although there was no significant difference between teacher's age and learner's mathematics learning. In line with this finding teachers age is positively related to how and what teachers teach in the class. The way how older teachers teach might differ significantly compared to young teacher's especially young teachers for this generation.

## 5.2.2 Education Qualifications



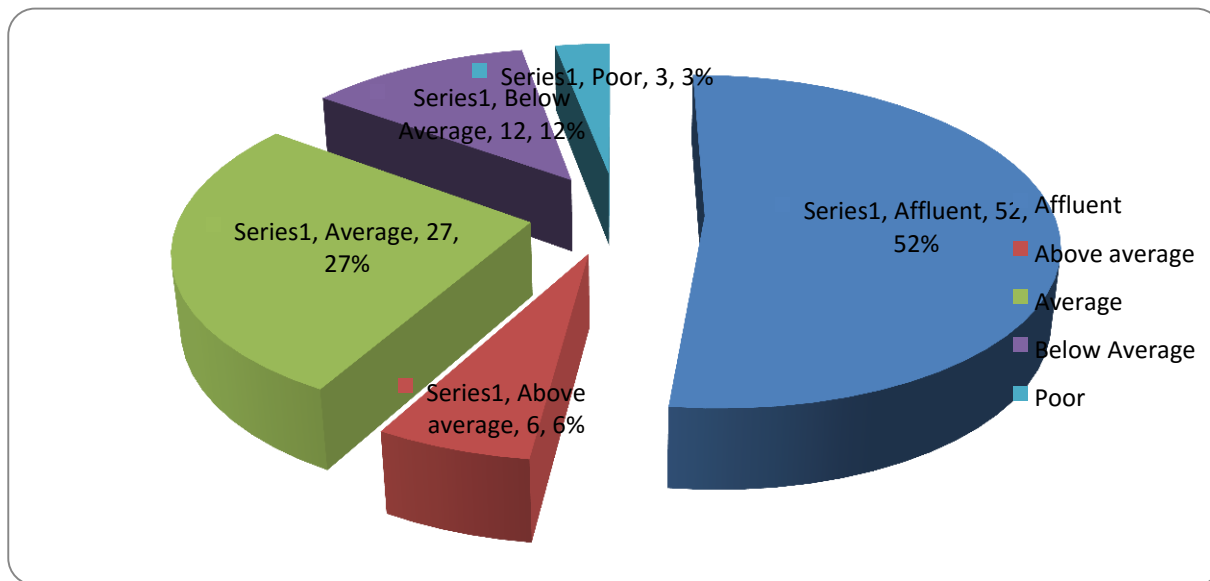
**Figure 5. 2: Educational information of the respondents (n=120)**

The distribution of the respondents' educational qualifications is illustrated in Figure 5.2.

The Figure shows that 44% of the diploma held recognitions in teaching; 21% held degree capabilities, while 15% held Honors degrees; 18% held Masters Qualifications and 2% held doctoral degrees. In South Africa, a Diploma in teaching is the base capability for enlistment with the South African Council for Educators (SACE) as qualified teachers. Thus, it might be said that the profile in Figure 5.2 shows that all the respondents were properly qualified as teachers without specialization. Effective and meaningful mathematics teaching entail more than viewing teaching profession from the perspective of content or pedagogy. Ball (2003) affirms that teaching of mathematics is a complex task which requires several aspects of knowledge needed for teaching. This finding revealed that intermediate teachers are viewed as generalist compared to senior teachers. This study argues that we can no longer reasonably expect intermediate teachers to be subject matter generalists, they must specialize especially in major subjects such as mathematics. Subsequently, too many primary teachers are given the post of a subject leader responsible for coordinating a curriculum area without being a specialist.

In the light of this finding the study argues that subject specialist are desperately needed in our Intermediate phase schools. However, being a subject specialist does not mean you have all the answers but they will be far more qualified to make sense of the subject and to teach it.

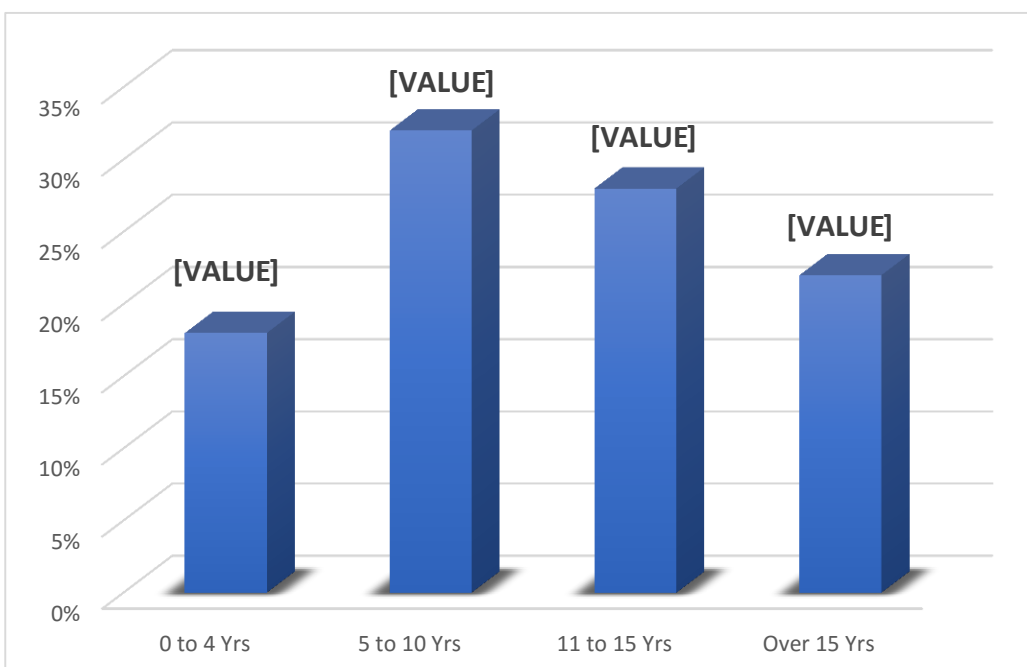
### 5.2.3 Economic status of the respondent



**Figure 5. 3: Economic status of the respondents (n=120)**

Figure 5.3 shows that the respondents belonged (or came) from different economic conditions, although the majority of respondents were upper middle class. Teacher's economic status is measured by the combination of education as showed in (Figure 5 2 above), income and occupation. It is argued that apart from teacher's knowledge of teaching there are various aspects that can affect learning in one way or another. For instance, teacher's economic status affects learning. Literature showed that teachers who are frequently stressed about money have more negative attitudes about their classroom issues and had rates of chronically absenteeism which then results, in poor learning. It implies that teacher's economic status influence classroom climate on effective mathematics teaching and learning

## 5.2.4 Teaching Experience



**Figure 5.4: Respondents' years of teaching experience (n=120)**

Figure 5.4 displays the years of teaching experience of the respondents.

The majority of teachers (32% + 28% = 60%) had teaching experience falling between 5 and 15 years, while 15% had more than 15 years of teaching experience and 18% had less than 5 years of teaching experience.

The results showed that the respondents were all experienced in the teaching of mathematics in the intermediate phase. However, the number of years for grade currently teaching were not indicated. The author's experience as an IP mathematics teacher for three decades revealed that due to the view that IP teachers are generalists as it is already elaborated in the findings of Figure 5.2 (Educational qualification), the allocation or distributions of teaching subjects are not stable depending on different reasons. The finding of this item is significant for IP leaders to be educated about the increased benefits of specific subject teaching experience and consider it in their decisions about allocation of subject's assignment. Indeed, mathematics teachers are expected to be more and deeply knowledgeable in their teaching subject. Once the teacher has knowledge about the subject

matter, teaching and learning .becomes more beautiful and profound for both teachers and learners

### 5.2.5 Employment Status

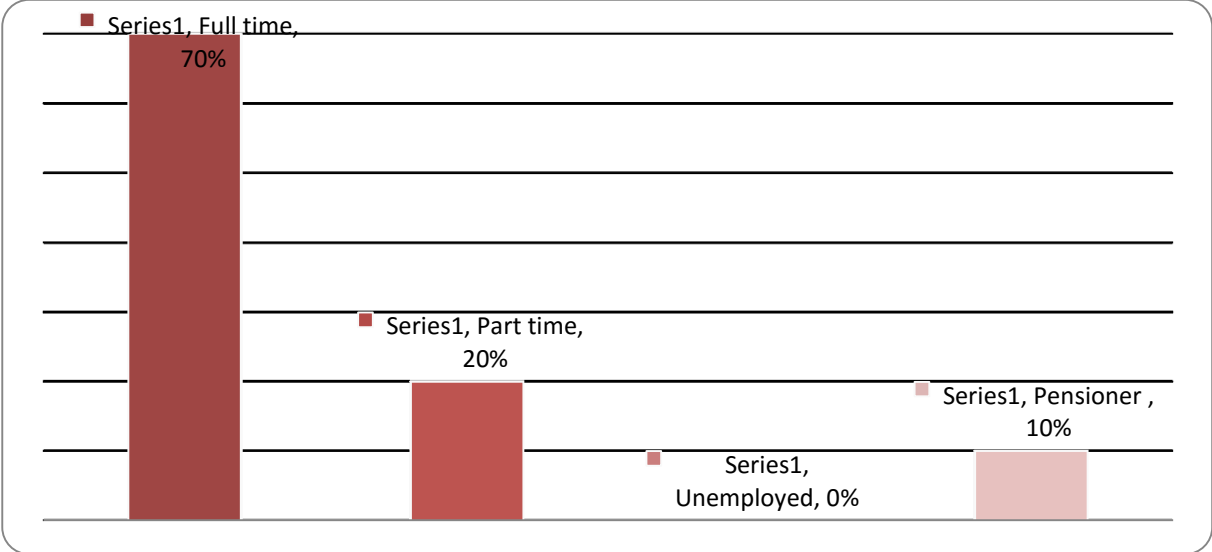
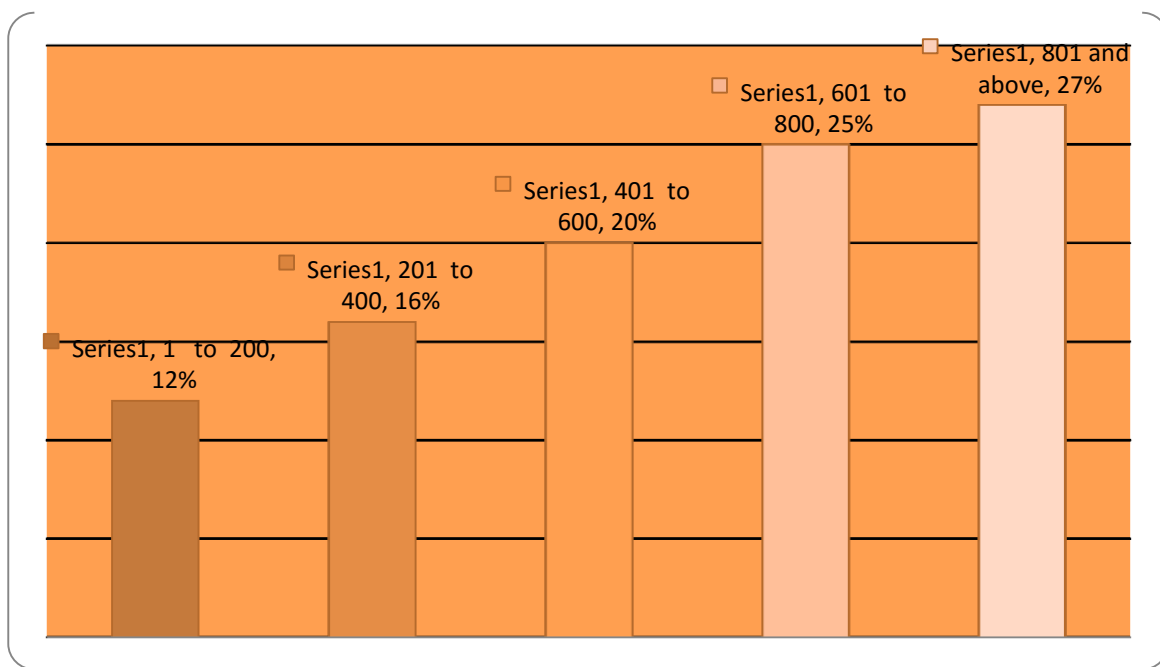


Figure 5. 5: Employment status of the respondents (n=120)

Teaching is a form of public service which requires expert knowledge and specialized skills acquired and maintained through rigorous and continuing study. With regard to employment status aspect in this study, Figure 5.5 shows that 70% of the respondents were full time employees, while 20% were part time or temporary employed and 10% were on post-retirement contracts. This is a good profile because it allows flexibility with regard to the 30% who were not permanent employees. The results showed teachers' occupational status is seen as average in all schools of Sibasa circuit. They all have professional knowledge. However the lack of implementation of professional knowledge i and practices by intermediate phase teachers is a cause of concern particularly in mathematics classroom. Some teachers are unable to teach in ways that involves variety of other aspects of teaching. However, their ability to do so requires more than their professional knowledge. In the context of mathematics classroom various knowledge of teaching is needed. Hill (2007) puts it in this way: teaching mathematics is complex, describing it as being 'multifaceted'. The author argues that professional knowledge alone is not enough for effective learning.

### 5.2.6 School Enrolment



**Figure 5. 6: The enrolment at respondents' schools (n=120)**

The learner enrolments of the schools from which the respondents were drawn are given in Figure 5.6. This ranged from schools that had only a maximum of 200 learners to schools

with over 800 learners. The percentages of respondents from the different sizes of schools increased correspondingly with the sizes of the schools, starting with 12% for schools with a maximum of 200 learners and ending with 27% for schools with learner enrolments greater than 800. The maximum recommended learner teacher ratio for South African intermediate phase learners is 40:1 and for senior phase is 35:1. The results showed that in most of the IP classes in Sibasa circuit the number of learners exceeds the optimum level. Such that it reduces learner's ability to pay attention and increases class violence. The results showed that overcrowding of classrooms in Sibasa circuit significantly affect mathematics teaching and learning. The results showed keeping that overcrowding of classrooms significantly affect effective teaching and learning. Because keeping them all engaged becomes a serious problem The problem of overcrowding classes is a more demanding and taxing, as there is no one size fits all solution. It's about finding the best solution for that particular school

### **5.3 TEACHING OF MATHEMATICS IN PRIMARY SCHOOLS**

One of the main purposes of a theoretical or conceptual framework is to guide a researcher in the interpretation of his or her results (Imenda, 2014). Accordingly, in interpreting the results of this study, the researcher was guided by the conceptual model presented in Figure 3.3. This model emphasizes teaching modalities based on constructivism. Imenda (2018, p. 92) characterizes constructivist teaching as comprising the following seven features:

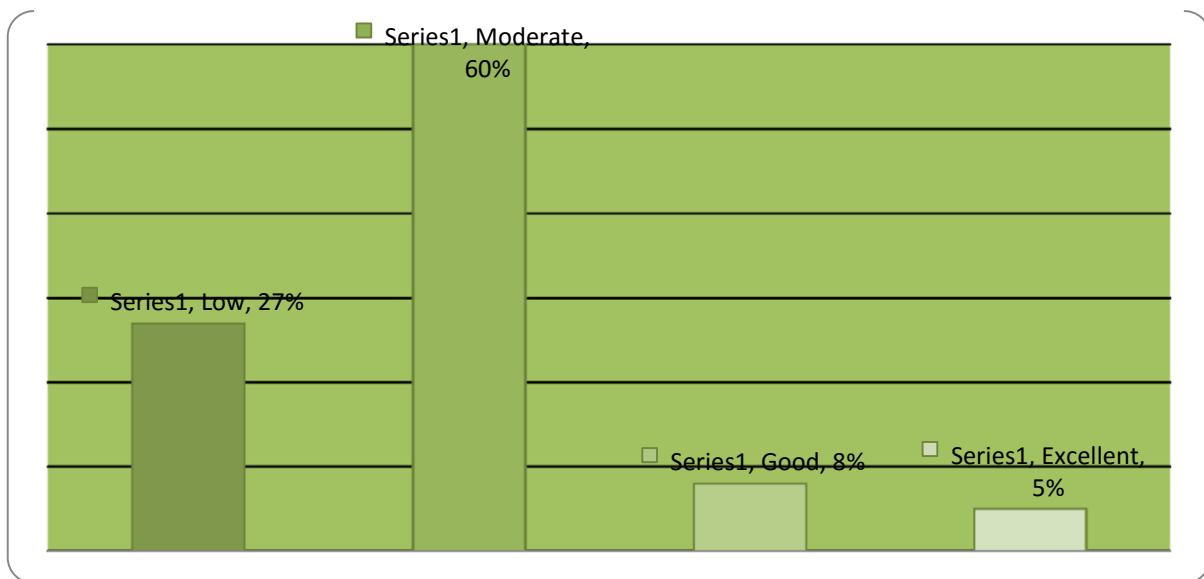
- a) Learning should be an active process. Keeping learners active doing meaningful activities results in high-level processing, which facilitates the creation of personalized meaning.
- b) Learners should construct their own knowledge rather than accepting that given by the instructor
- c) Collaborative and cooperative learning should be encouraged to facilitate constructivist learning: Working with other learners gives learners real-life experience of working in a group, and allows them to use their metacognitive skills.
- d) Learners should be given control of the learning process. There should be a form of guided discovery where learners are allowed to make decision on learning goals, but with some guidance from the instructor.

- e) Learners should be given time and opportunity to reflect. When learning online, students need the time to reflect and internalize the information.
- f) Learning should be made meaningful for learners. The learning materials should include examples that relate to students, so that they can make sense of the information. Assignments and projects should allow learners to choose meaningful activities to help them apply and personalize the information.
- g) Learning should be interactive to promote higher-level learning and social presence, and to help develop personal meaning.

Accordingly, teachers' responses were measured in respect of the extent to which they reflected these features.

The first research objective of this study sought to determine the current state of mathematical teaching in the Intermediate Phase. There were ten items formulated to address this research objective. These results from these ten items are presented and interpreted below.

### 5.3.1 The Quality of Mathematics Teaching

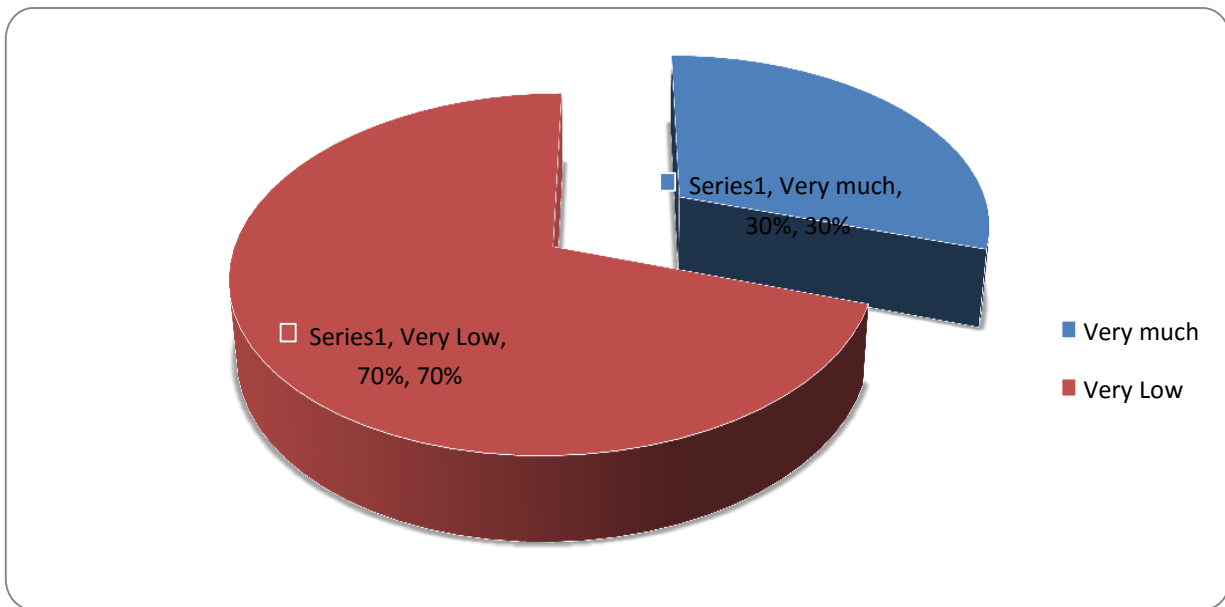


**Figure 5.7: The quality of mathematics teaching at the school (n=120)**

The respondents were asked to rate the quality of mathematics teaching in their schools and the results are reflected in Figure 5.7. The results showed that the majority of the respondents (60%) felt that the quality of mathematics teaching at their schools was just moderate and

27% rated it as low. This raises a red flag, although this admission could also be used as a springboard for interventions to address the limitations together with the teachers. It is likely that the teachers, themselves, will indicate areas of concern – out of which practical steps could be taken to address the areas requiring improvement. What is important from this result is that the respondents themselves are aware that the quality of mathematics teaching is of importance value.

### 5.3.2 Collaboration with other Teachers



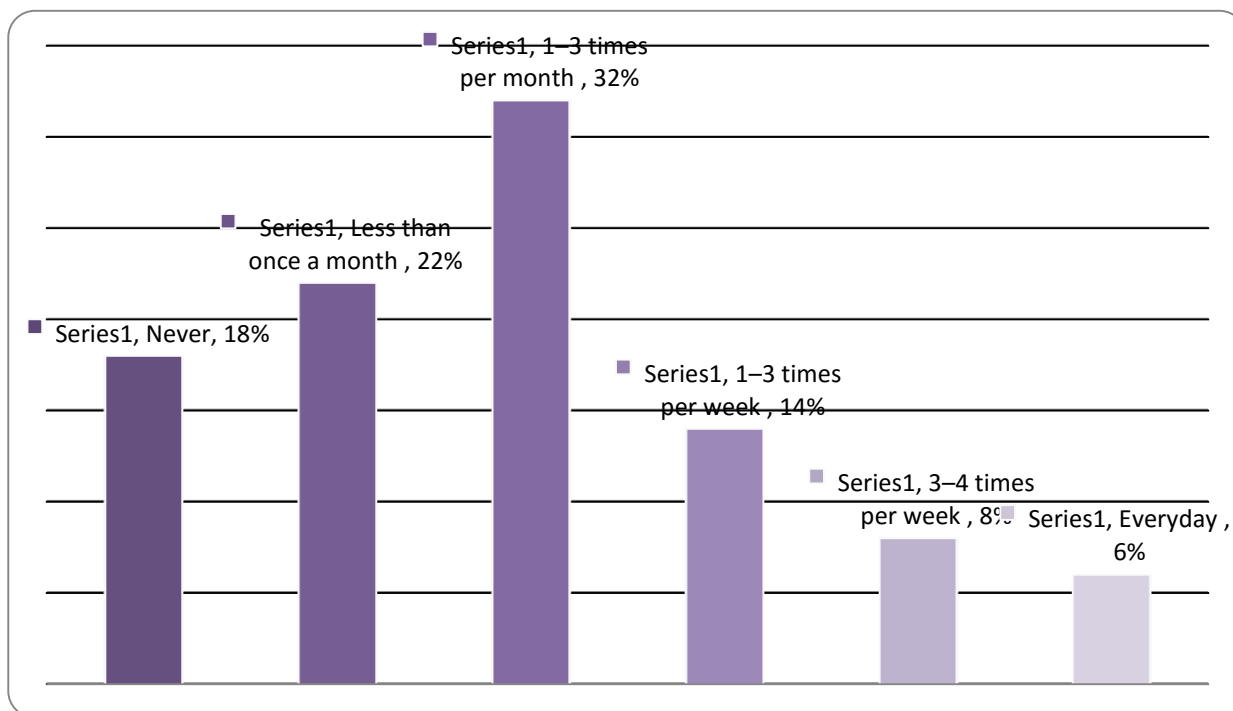
**Figure 5. 8: How often teachers met with other mathematics teachers in the school (n=120)**

Figure 5.8 presents the results as to how often the teachers in the research sample met with other teachers with respect to the teaching of their subjects, including mathematics.

Up to 70% of the respondents reported meeting with other mathematics teachers “very low” and 30% “very much” (Figure 5.8). Thus, this shows that by far the majority of the respondents hardly met with their colleagues for the purpose of discussing their teaching. Cooperation and collaboration between and among teachers is one of the indicators of the present-day indicators of continuing professional teacher development which, in turn, tends to improve the quality of teaching and learning. The theoretical framework around teacher’s views on

meaningful mathematics teaching we believe that collaboration among teaching subject is crucial for effective learning. Research shows that teachers can learn and improve their own practices through sharing ideas and challenges. The study assumes that teachers can also acquire knowledge needed for teaching mathematics through working collaboratively with their colleagues. It's not surprising why the quality of teaching mathematics as rated by the respondents in section 5.3.1 is rated being poor. If our ultimate goal as mathematics teachers is learner's accomplishment, consider collaboration on the capacity to share the responsibilities regarding our learner's learning. The more regularly teachers meet, the better the opportunity that learner must be effective in their learning process. So why teachers are not meeting in a normal manner regardless of the obvious advantages? It may be the case that some of teachers don't know about the advantages, or basically haven't put the time or effort into collaboration process.

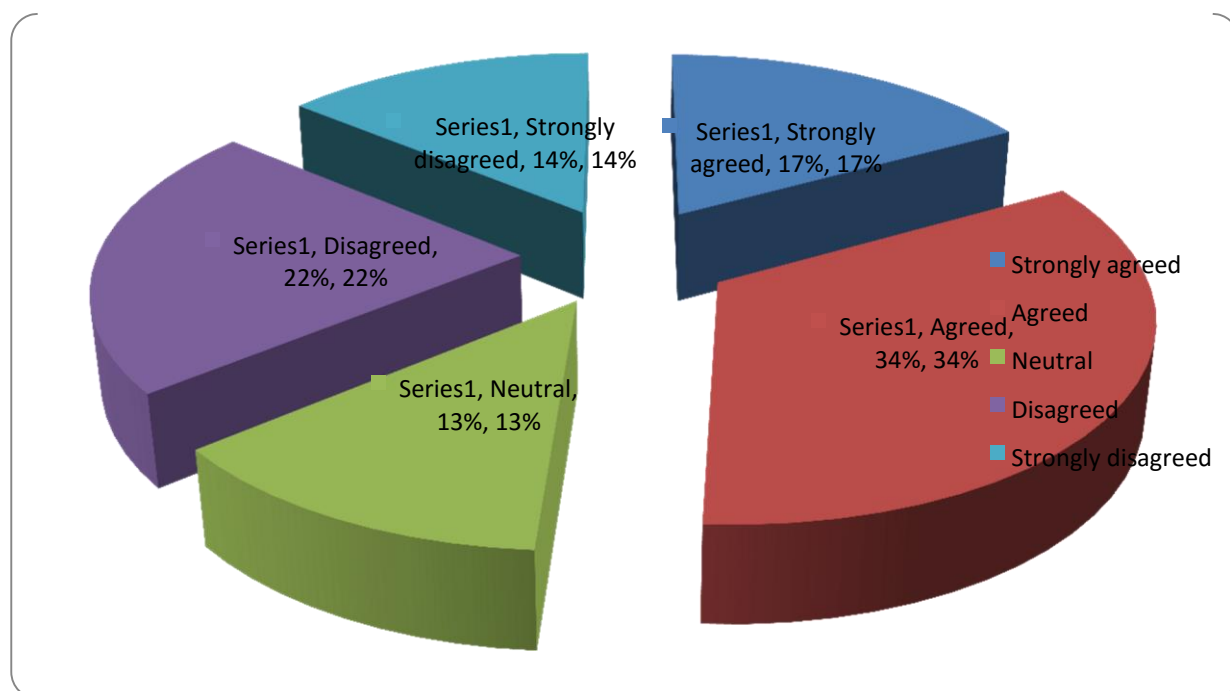
### 5.3.3 Presenting the correct Rules and Procedures



**Figure 5.9: Teaching mathematics by presenting the correct rules and procedures (n=120)**

a) Another common practice in mathematics teaching is whereby teachers focus on making learners to follow specific rules and procedures. In this study, 18% of the respondents said they “never did so”, while 22% did so less than once per month and 32% reported to be doing so 1-3 times monthly. On the other hand, 6% reported to be doing so daily, 8% 3-4 times per week and 12% 1-3 times per week. The overall results confirms (Hill, 2007) statement which shows that indeed teaching mathematics is complex and describe teaching mathematics as ‘multifaceted’. Likewise The National Council of Teachers of mathematics (2011) confirms this complexity by stating that teachers of mathematics require a broad range knowledge in order to be effective. Learners develop their mathematical awareness as they begun to discover rules and procedures to solve mathematical problems. Learner’s attention skills play a critical role in the learning and recognition of mathematical rules and procedures. The study showed that the use of mathematical rules and procedures help learners to compute math more effectively and efficiently. As mathematics teachers lets strive to teach learners mathematical rules and procedures for understanding. According to constructivist approaches to teaching and learning knowledge is a mental representation that a learner constructs in his or her mind (Imenda, 2018). As a result Learning should be an active process, keeping learner’s active doing meaningful activities results in high-level processing, which facilitates the creation of personalized meaning.

### 5.3.4 Use of Poor Teaching Methods

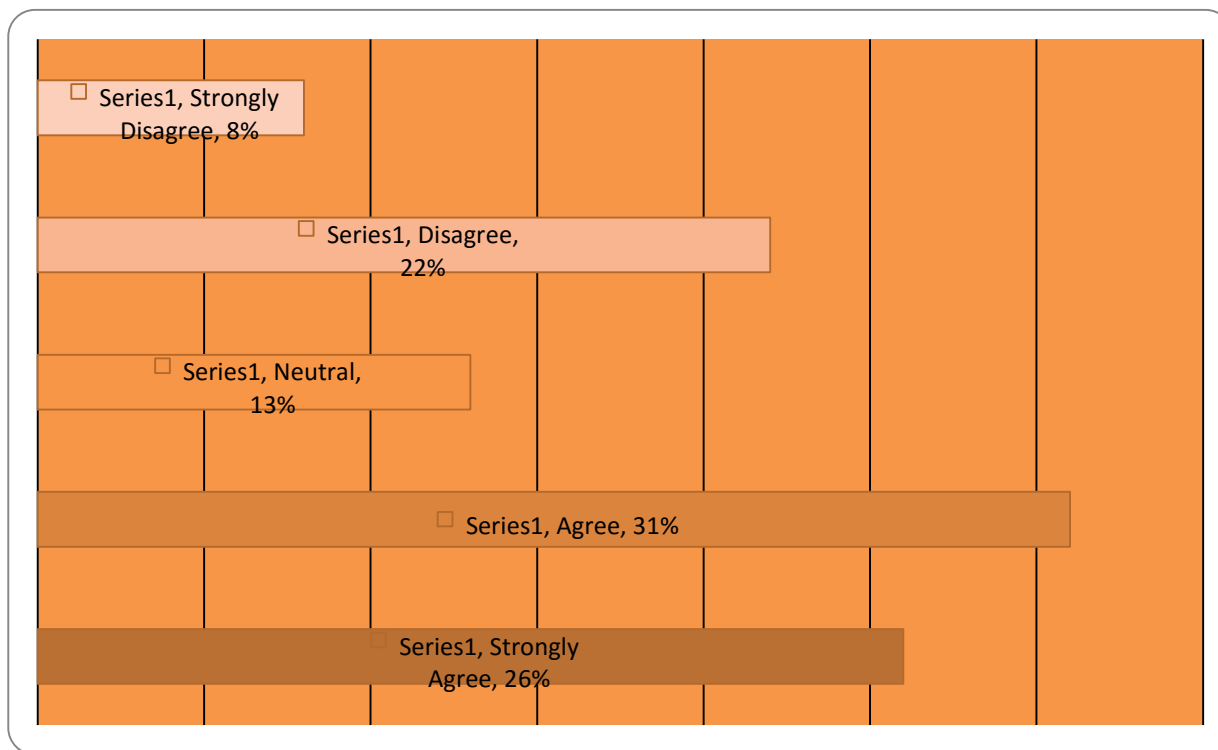


**Figure 5. 10: The effect of poor teaching methods in mathematics classroom (n=120)**

There is a view that, currently, much of the poor performance in mathematics is caused by teachers using poor teaching methods. This question was put to the respondents and Figure 5.15 shows that 17% strongly agreed and 34% agreed that poor teaching methods were responsible for poor learner performance in mathematics – giving a total of 51%. On the other hand, 14% strongly disagreed and 22% disagreed with the view that the currently reported poor learner performance in mathematics was due to poor teaching (total of 36%). However, poor teaching methods is affected by many factors such as teacher’s lack of MPCK.in which this study regards it as driving force for effective learning. Teaching according to Hattie (2012) is a process of knowledge dissemination from teacher to the learner. Therefore the teachers’ role is to stimulate learner to learn at least as expected.

The results demonstrated that effect of utilizing poor teaching methods in mathematics classroom brings about poor learning. The reason of this investigation shows that different strategies, systems and methods of teaching mathematics is essential so as to comprehend and improve learning of mathematics.

### 5.3.5 Problem-Solving is Just about an answer



**Figure 5. 11: On the perspective that problem-solving is just about an answer (n=120)**

A total of 30% (8% strongly disagree and 22% disagree) did not support the view that problem solving is just about an answer, suggesting that problem-solving went beyond just about an answer. In contrast, a total of 57% (31% agree and 26% strongly agree) held the view that problem-solving was just about an answer. This result shows that the majority of the respondents had a limited understanding of the various situations that require the application of problem-solving exercises and skills. This has implications for the quality of mathematics teaching in the schools. If teachers are not aware of the full range of application of problem-solving skills, this limits the experiences that learners can have under the guidance of such teachers. As Cobb et. al (2002) suggested, the purpose of engaging in problem solving is not just to solve specific problem to get the answer, but as means of developing mathematical thinking as a tool for both classroom and day living. Problem solving is more than getting the answer only, however more than a tool for teaching and reinforcing mathematical knowledge and. helping learners to meet mathematical challenges.

In spite of the fact that, is a critical skills which can enhance logical reasoning. Consequently, meaningful learning begins with a problem to be solved, and the problem is posed in such a way that learners need to apply rules and procedures to solve a specific problem. Rather than seeking a single correct answer, learners interpret and analysis the problem and gather needed information, rules and procedures for that specific problem. And finally evaluate options and present conclusions.

### 5.3.6 The relationships between selected variables

<b>(MCK) in the teaching of mathematics</b>				
<b>Age of teacher</b>	<b>Yes</b>		<b>No</b>	
	<i>F</i>	<i>%</i>	<i>F</i>	<i>%</i>
<b>25-30 years</b>	48	98.3	1	<b>1.7</b>
<b>31-35 years</b>	51	98.2	1	<b>1.8</b>
<b>Older than 36</b>	21	96.2	1	<b>3.8</b>
<b><math>\chi^2</math> value</b>	0.412			
<b>Degrees of freedom</b>	2			
<b><i>p-value</i></b>	<b>0.814</b>			

**Table 5. 1: The relationship between the age of the teachers and mathematical content knowledge (MCK) in the teaching of mathematics (n=120)**

In addition to the above descriptive statistics, the researcher also conducted inferential statistical testing to establish whether or not some relationships were statistically significant. In this regard, the chi-square ( $\chi^2$ ) statistic was used. The chi-square test of independence is designed to test the relationship between two variables when the data are in the form of frequencies (De Vos, 2010). It is noteworthy that some of the categories for the different variables have few frequencies, and these were grouped with other categories. All the independent variables could not be cross-tabulated against all the dependent variables. In this case, the researcher wished to ascertain whether or not the age of the teachers was significantly related to their mathematical content knowledge (MCK) in the choice of teaching strategies. The result of this analysis is presented in Table 5.1 above

According to Table 5.1, a  $\chi^2$  value of 0.412 (for 2 degrees of freedom) was calculated. This value is not statistically significant at the 1% level of significance; therefore, it may be inferred that there was no statistically significant relationship between the teachers' age and their mathematical content knowledge (MCK) in the teaching of mathematics.

### 5.3.7 The relationship between teaching experience and MCK

Teaching experience	Disciplinary method		Stay after school		Depriving of privileges		Homework	
	Explanation							
	F	%	F	%	F	%	F	%
<b>0-4 years or less</b>	9	37.5	2	8.3	1	4.2	12	<b>50.0</b>
<b>5-10 years</b>	7	26.9	6	23.1	1	3.8	12	<b>46.2</b>
<b>11-15 years</b>	14	30.4	0	0.0	4	8.7	28	<b>60.9</b>
<b>16 years or more</b>	10	29.4	1	2.9	4	11.8	19	<b>55.9</b>
<b><math>\chi^2</math>value</b>	16.788							
<b>Degrees of freedom</b>	9							
<b>p-value</b>	<b>0.050</b>							

**Table 5. 2: Relationship between the teaching experience of the teachers and MCK in teaching (n=120)**

Concerning whether or not a statistically significant relationship existed between teaching experience and a teacher's MCK, the results are reflected in Table 5.2

Table 5.2 shows that the calculated  $\chi^2$  value was 16.788. At 9 degrees of freedom, this was not statistically significant at 99% confidence interval. Therefore, it may be said that there was no significant relationship between teaching experience and MCK.

### **5.3.8 Answer to the First Research Question: What is the current state of mathematics teaching in the Intermediate Phase (IP) in the Sibasa circuit, Limpopo Province?**

Given that this study focused on teacher quality as one of the critical factors determining the learning process in mathematics classroom. The results showed that the quality of mathematics teaching of IP schools at Sibasa circuit is poor due to pedagogical challenges. Surprisingly, all teachers were well or had sufficient experience in terms of teaching mathematics. Alternatively, majority of IP mathematics teachers generally employed poor pedagogical practices which resulted in poor mathematics learning. Lenhart (2010) defines pedagogical knowledge (PK) as the knowledge of “how” to teach.

This is evident as the majority of teachers express, lack of confidence in their Pedagogical Knowledge, and those teachers were unable to manage the demands on how to adjust their teaching to address learners’ meaningful teaching and learning. At the same time overcrowding classes and lack of teacher’s collaboration is a serious concern within the circuit which also contribute to poor teaching. The study acknowledged that poor teaching of mathematics in IP deprive learner’s access to meaningful learning.

The quality of mathematics teaching is a central goal in mathematics classroom. It provides an opportunity for learners to construct their own knowledge rather than accepting that given by their teachers (Imenda 2018). Certainly, it is critically essential as IP mathematics teachers to teach learners through the facility of appropriate pedagogical practices in order to make mathematics meaningful to learners. The study claims that teachers should in some sense know “what” they are expected to teach and also “how” to teach it. For this reason it is crucial for IP teachers to have pedagogical knowledge. Although pedagogical knowledge alone is not enough. Different studies have shown that teaching mathematics is more complexity because teachers of mathematics require a broad range of knowledge and other different aspects in order to be effective in their teaching.

Briefly, to improve quality of teaching especially at intermediate phase where teachers are a greater number of generalists than specialists is an issue of considerable debate within the Department of Education. This is evident as Education Angie Motshekga said ‘quality of math teaching is poor’. She additionally further showed there is a general access to education at schools, tragically enough there is no access to quality teaching. The study aimed on creating an opportunity to expose IP mathematics teachers, with a more prominent attention being

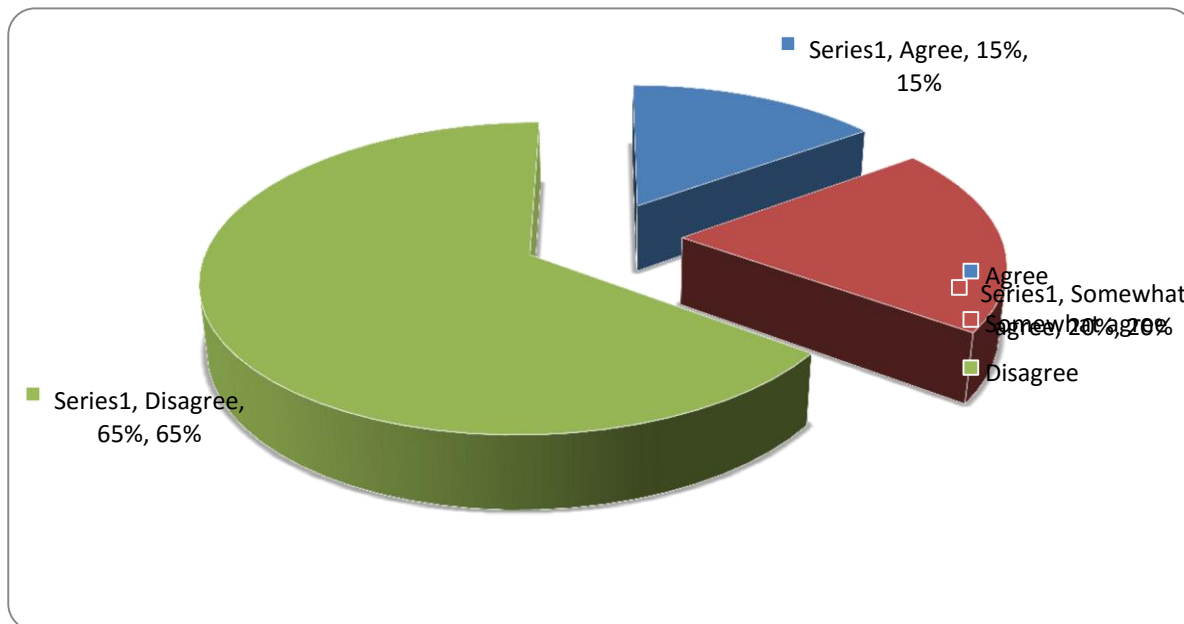
put on teacher's knowledge of teaching mathematics which the study alludes it to Mathematics Pedagogical Content Knowledge.

A sound knowledge of mathematics viewed as elements for effective teaching which at that point brings about successful learning. For this study MPCK is the major form of professional development for all Intermediate Phase. Its objective is the consistent and continuous improvement of teaching within Intermediate Phase schools with the goal that their learners think and learn. Generally, effective or quality teaching should be interesting, thought-provoking, enjoyable and meaningful

#### 5.4 INTERMEDIATE PHASE TEACHERS' PERCEPTIONS ABOUT THE IMPORTANCE OF MATHEMATICAL CONTENT KNOWLEDGE IN THEIR TEACHING

The following items addressed the second research objective, seeking to establish intermediate phase teachers' perceptions about the importance of mathematical content knowledge in their teaching. The results are presented under the various sub-headings below.

##### 5.4.1 Possession of Adequate Mathematical Knowledge

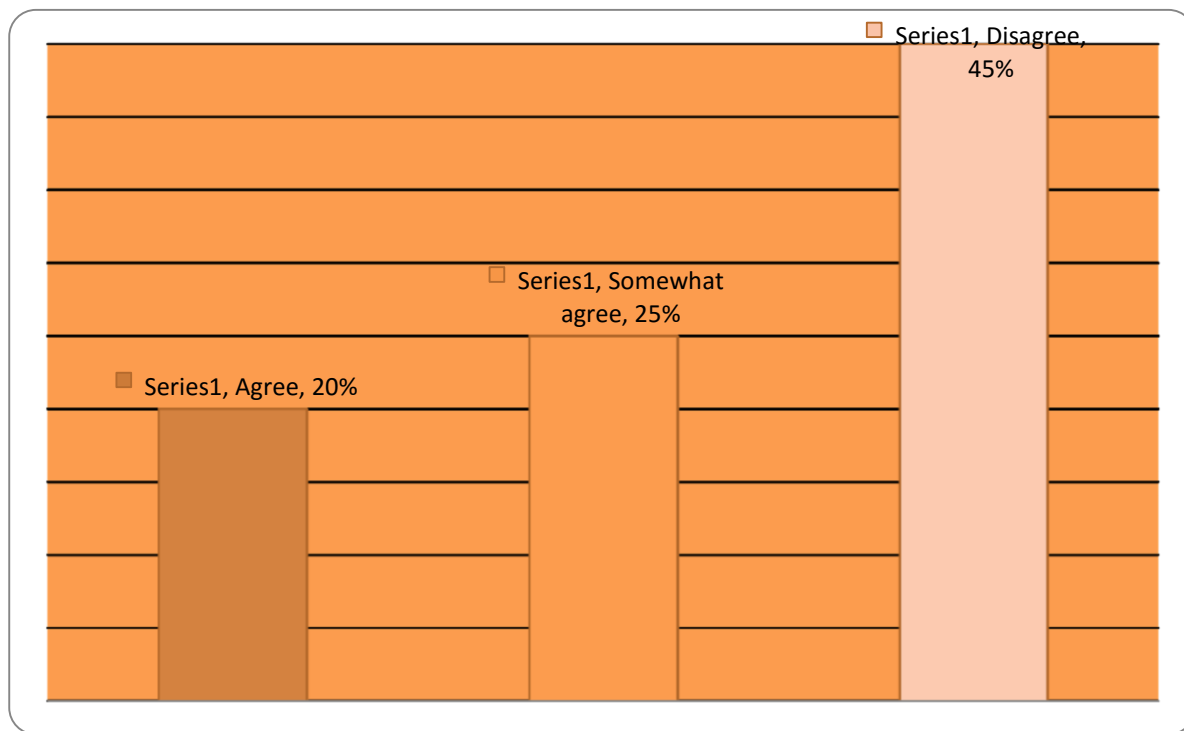


**Figure 5. 12: Intermediate Phase mathematics teachers do not need to possess enough mathematics knowledge to be an effective mathematics teacher (N120)**

The first of these items sought to find out the respondents' views as to whether or not IP teachers need to possess enough mathematics knowledge to be effective mathematics teachers. Figure 5.12 presents the results. It is clear that teachers of mathematics must know the mathematics they are to teach and also "how" to teach it.

Ball et al (2008) describe a categorization of the mathematical knowledge needed for teaching into five broad types which were described in more in detail in chapter 2 (Figure 2.1 *Model of Teachers' Mathematical Knowledge for Teaching* (Ball, 1991). At the point when teachers lack mathematical knowledge, they often end up relying on tricks instead of mathematical understandings. Figure 5.12 shows that 65% of the research sample teachers disagreed that intermediate Phase mathematics teachers did not need to possess enough mathematics knowledge to be an effective mathematics teacher. This was the majority response and implies that they recognized. However, the results showed lack of teacher's mathematics knowledge is major problem particularly in primary as result of subject generalists.

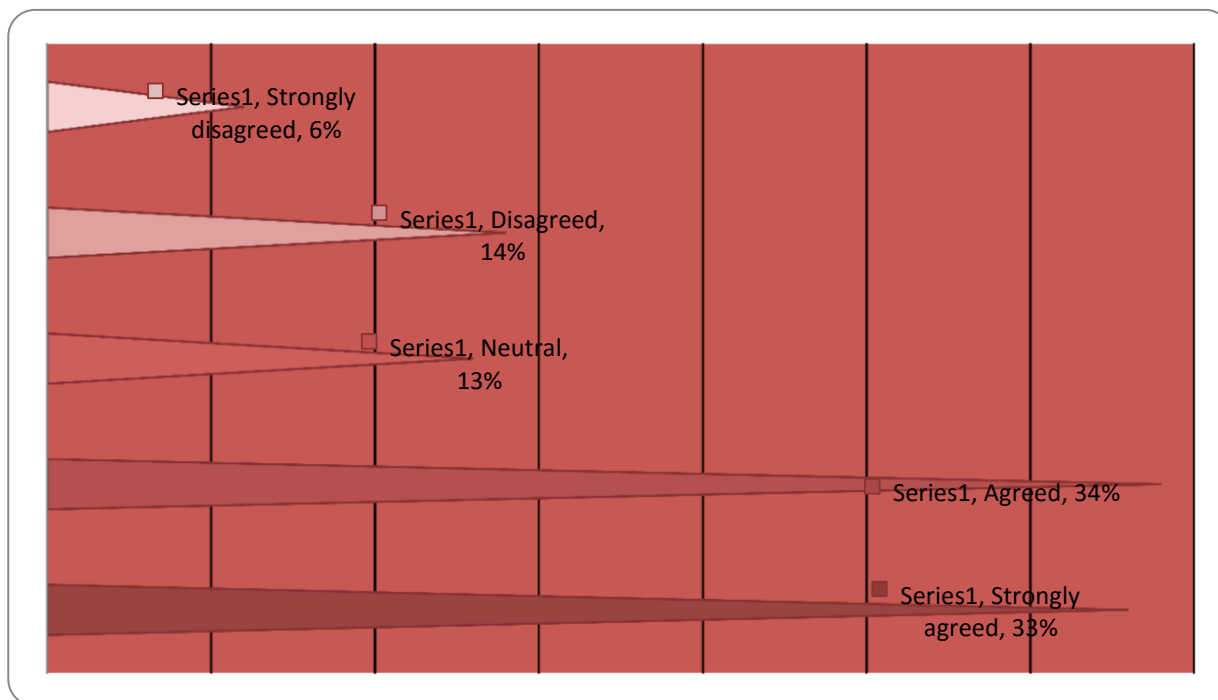
### 5.4.2 Self-Adequacy in the Knowledge of Subject Content of Mathematics



**Figure 5. 13: I feel quite confident about my knowledge of the mathematics content that I teach (n=120)**

The respondents were asked to rate themselves about their knowledge of mathematics content. In response, 45% disagreed with this statement, suggesting that they did not feel confident about their own knowledge of the mathematics content (see Figure 5.21). This was the single highest response, and it suggests that 45% of the teachers were struggling with the mathematics content that they were teaching. In the circumstances, it is unlikely that one can confidently and adequately teach the subject content that one is not clear about. Indeed, teaching mathematics with little content knowledge affects one's ability to exhibit the necessary pedagogical content knowledge to teach the subject well, so remains a very serious and pressing challenge.

### 5.4.3 Relationship between Knowledge of Subject Content and Pedagog



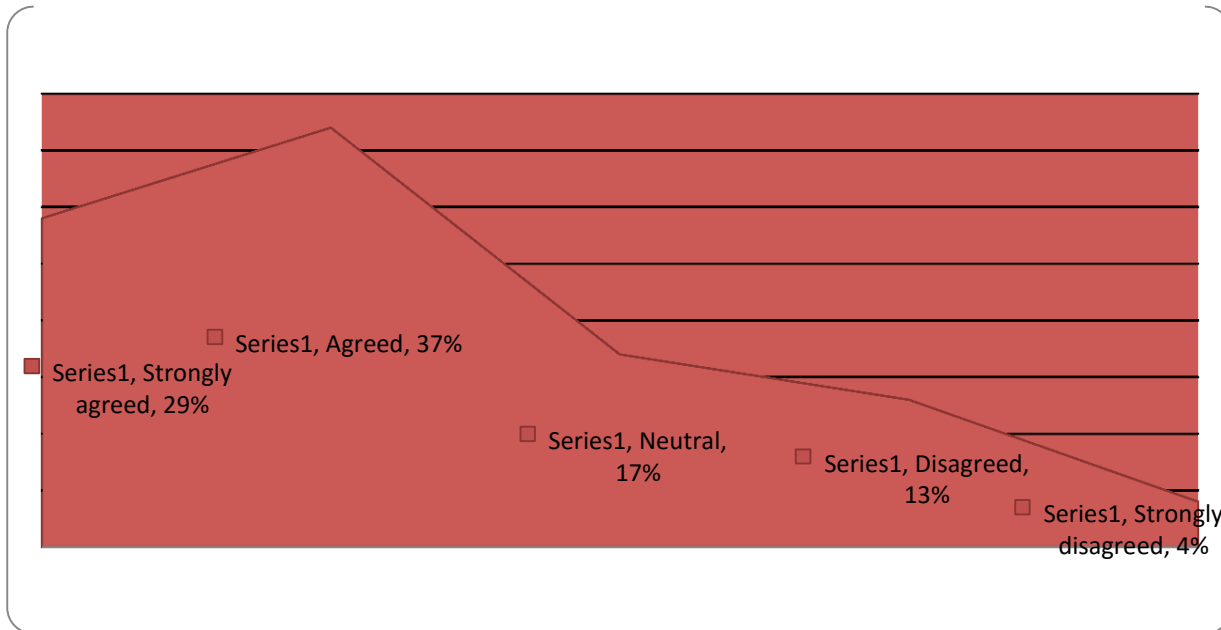
**Figure 5. 14: Teachers need to know the subject matter very well to teach effectively (n=120)**

The previous question (section 5.4.2) is related to this one, where the respondents were specifically asked to express themselves on the notion that a teacher needs to know his or her subject matter very well to be able to teach effectively. As already discussed in literature review in chapter 2 subject knowledge is the actual knowledge teachers are expected to teach, for example you may need have to have a thorough understanding of decimals in order to be able to them. And pedagogy is understanding how decimals can be taught. In teaching and learning context, subject and pedagogy knowledge complement one another. The two cannot be treated separately. To this question, the majority of the respondents agreed with this notion. Although there were some dissenting voices.

A teacher's possession of good understanding of subject and pedagogy knowledge could help learners gain relevant knowledge of numerical procedures, terms, concepts and operations, many others did not think so. Generally, the results support the conceptual framework of this study (Figure 3.3). Furthermore, this result supports literature that contends

that teachers need to possess mathematical content knowledge to teach the subject effectively (Barko, 2013).

#### 5.4.4 Weak Content and the Risk of Wrong Explanations



**Figure 5.15: Teachers who are weak in the subject run the risk of giving wrong answers and explanations to learners – thereby undermining effective and meaningful learning of the subject (n=120)**

This item focused on the contention that a teacher with weak content knowledge of the subject risked giving wrong answers and explanations of mathematical concepts to learners, thereby undermining effective and meaningful learning of the subject. In this regard, a total of 66% of the respondents acceded to this view (Figure 5.23); 29% strongly agreed with the statement while 37% agreed with it. In contrast, 4% strongly disagreed and 13% disagreed with this notion. Nonetheless, the majority view was that one needed to have a good command of the subject matter to be able to provide meaningful learning experiences to learners. The results revealed that teachers see the need for a good command of the subject matter for one to portray the subject to learners correctly. This result supports Barko (2013) who contends that teachers who are weak in the subject run the risk of giving wrong answers and explanations to learners – thereby undermining effective and meaningful learning of the subject

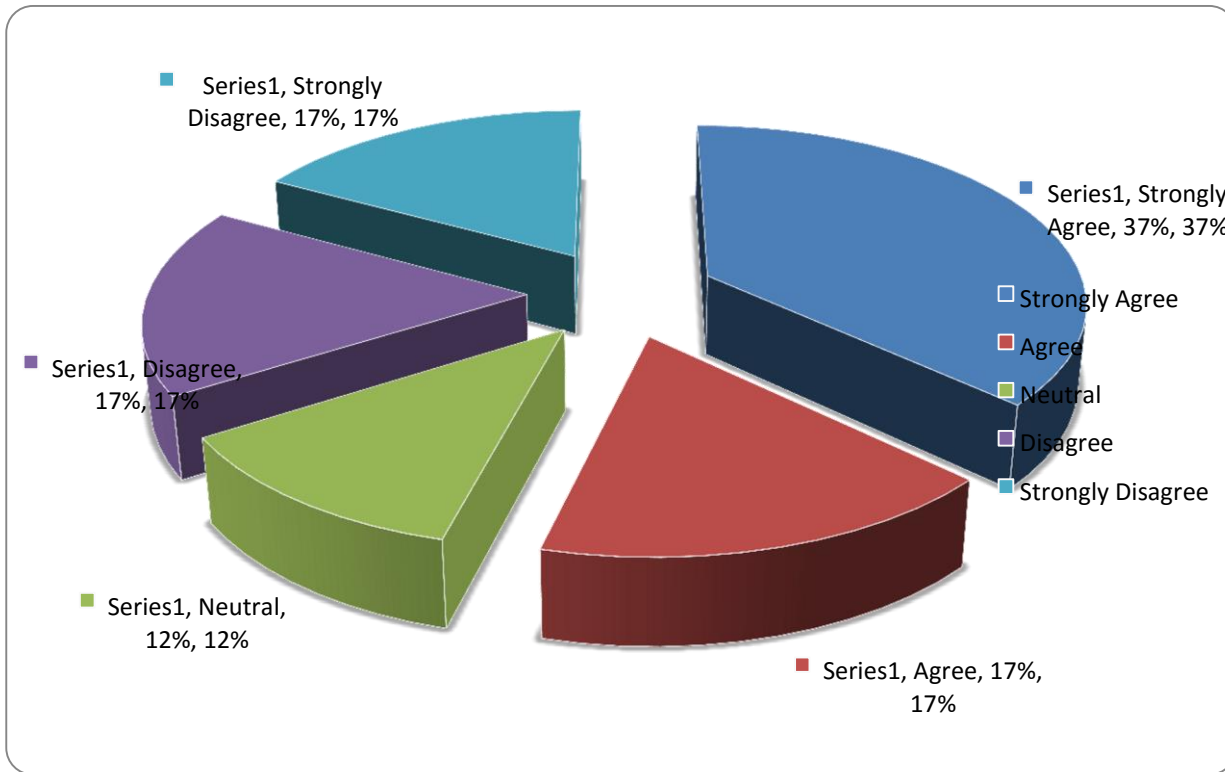
An adequate teacher's content knowledge developed through mathematics pedagogical content knowledge (MPCK). On the other hand, lacking MPCK brings about weak content. Eventually, the majority of primary teachers do not know the content well enough to teach mathematics. As result wrong explanations of mathematical concepts and rules may happen.

#### **5.4.5. The Importance of Teacher's Knowledge and Understanding of the Subject**

Teacher's knowledge of the subject is distinguished as a significant indicator of the quality of teaching. If the teacher possesses a high level of specialised subject knowledge he/she will have a huge scope of factual facts about the subject and good understanding of the theoretical aspects of the subject. It is imperative as teachers to know the subject we teach. Deep knowledge of the subject knowledge has a strong impact on the quality of teaching.

Continuing with the importance of the teacher's knowledge and understanding of the subject the results embraced the notion that: meaningful mathematics teaching requires adequate teacher's knowledge and understanding of the subject. This is an overwhelming and coheres well with theoretical framework of the study which, in turn, was based on both theory and empirical findings from earlier researchers.

#### **5.4.6 Weak Knowledge of Specific Aspects of the Mathematics Curriculum**



**Figure 5.16: IP teachers' mathematical knowledge on key topics, such as fractions, decimal fractions and integers is generally weak**

Figure 5.25 indicates that a total of 54% supported this view by strongly agreeing (37%) and agreeing (17%) with the statement, as compared to a total of 34% who strongly disagreed (17%) and disagreed (17%) with this notion. This means that 34% of the respondents do not believe that IP teachers' mathematical knowledge on key topics, such as fractions, decimal fractions and integers is generally weak, thus they cannot not assist their learners overcome the misconceptions that the learners may have. In general, most teachers experience difficulty with specific aspects of the mathematics curriculum, while they can manage with other topics. For this study, aspects of the mathematics curriculum which were identified as possible areas of difficulty for teachers were fractions, decimal fractions and integers. Thus, the respondents were asked to express themselves on whether or not, to their knowledge, IP teachers' mathematical knowledge on key topics, such as fractions, decimal fractions and integers were really generally weak.

Given the educational demographics of the sample, this is quite worrying. Nonetheless, the overwhelming message here is that special attention should be given to up skilling teachers in the content areas where they are weak, so that they become more confident, proficient and capable of teaching these sections, rather than skipping them or portraying them wrongly to learners. If not taught properly, this could be a source of misconceptions which may be quite difficult to redress and overcome.

#### 5.4.7 Relationship between the type of school and the application of Mathematical Content Knowledge

Further to the descriptive statistics presented above, the researcher conducted a two  $\chi^2$  tests as a way of corroborating some of the results reported above. In the first test, the researcher sought to establish whether or not there was a statistically significant relationship between the type of school and how teachers' mathematical content knowledge was applied to the teaching of the subject. This was done with a view to ascertaining whether or not a school or departmental teaching philosophy could influence the teachers' orientation to teaching the subject.

Table 5.3 presents the results of this analysis.

Type of school	Application of Mathematical Content Knowledge			
	Yes		No	
	<i>F</i>	%	<i>F</i>	%
<b>Farm</b>	20	100.0	0	<b>0.0</b>
<b>Peri-urban</b>	18	100.0	0	<b>0.0</b>
<b>Urban</b>	82	96.4	3	<b>3.6</b>
<b><math>\chi^2</math>value</b>	1.040			
<b>Degrees of freedom</b>	2			
<b><i>p</i>-value</b>	<b>0.595</b>			

**Table 5.3: Relationship between the type of school and the application of Mathematical Content Knowledge (n=120)**

In Table 5.3 a  $\chi^2$  value of 1.040 (for 2 degrees of freedom) was calculated. This value is not statistically significant at the 99% level of significance. Thus, it was inferred that there was no statistically significant relationship between the type of school at which teachers taught and their application of the mathematical content knowledge. Instead, therefore, it may be the case that the extent to which teachers are willing to keep themselves up to date with the latest developments in the field of education and training could play an important role in the expression of their MCK in their teaching.

#### 5.4.8 Relationship between a teacher’s willingness to keep up to date with the latest developments and the expression of one’s MCK in his/her teaching

Most effective teachers are those who try to keep up to date with developments in their subject and pedagogy. This view was put to the respondents and the results are presented in Table 5.4.

Willingness				
Interpretation	Yes		No	
	<i>F</i>	%	<i>F</i>	%
<b>Yes</b>	119	98.5	2	<b>1.5</b>
<b>No</b>	1	100.0	0	<b>0.0</b>
<b><math>\chi^2</math> value</b>	0.015			
<b>Degrees of freedom</b>	1			
<b><i>p</i>-value</b>	<b>0.902</b>			

**Table 5. 4: Relationship between the willingness to keep up to date with the latest developments and the expression of one’s MCK in his/her teaching (n=120)**

A  $\chi^2$  value of 0.015 (for 1 degree of freedom) was calculated – and this was too low in relation to the critical value and, therefore, not statistically significant at the 99% level of significance. Thus, the inference was that there was statistically significant relationship between teachers’ willingness to keep themselves up to date with the latest developments in the field and the expression of their MCK in the teaching. It is somewhat strange that up to 98.5% of the respondents expressed willingness to keep up to date with their MCK and the pedagogy related thereto, but this did not translate itself in their teaching strategies.

#### **5.4.9 Answer to Research Question 2: What are IP teachers' perceptions about the importance of mathematical pedagogical content knowledge (MPCK) in the teaching of mathematics?**

The results indicated that IP teachers' level of MPCK was profoundly low. This study showed that MPCK in mathematics teaching context is a unique combination of three types of knowledge viz. mathematics knowledge, content knowledge and pedagogical knowledge. However, due to teacher's lack of combining these three types of knowledge stated teachers failed to give clear explanations on mathematical key topics such as fractions, decimals and integers. In turn this could withdraw learners' good understanding of the topic.

Eventually, teachers were unsuccessful to exhibit deep and through conceptual understanding on key topics of teaching commendably and such would not accommodate for meaningful learning. The study showed that the success of a teacher in teaching a specific mathematics topic depends on the high level of teacher's mathematics pedagogical content knowledge (MPCK). It should be noted that what teachers do behind mathematics classroom door is more governed by the level of MPCK that teachers have. Briefly, MPCK gained critical status towards meaningful learning. In this regard, the study is of the view that IP mathematics teachers should, at the most basic level, have an adequate mathematics pedagogical content knowledge required, for meaningful learning to happen.

Research showed that MPCK strongly influences how teachers understand 'what' and 'how' to do Mathematics. In other words IP mathematics teachers should therefore deeply understand the mathematical ideas such as concepts, procedures, rules, manipulation of numbers and symbols, reasoning skills that are central to the grade levels they will be teaching. Apart from that, they should be able to communicate these ideas in appropriate manner and how to represent and connect mathematical ideas so that learners may comprehend them in a way that make sense for them.

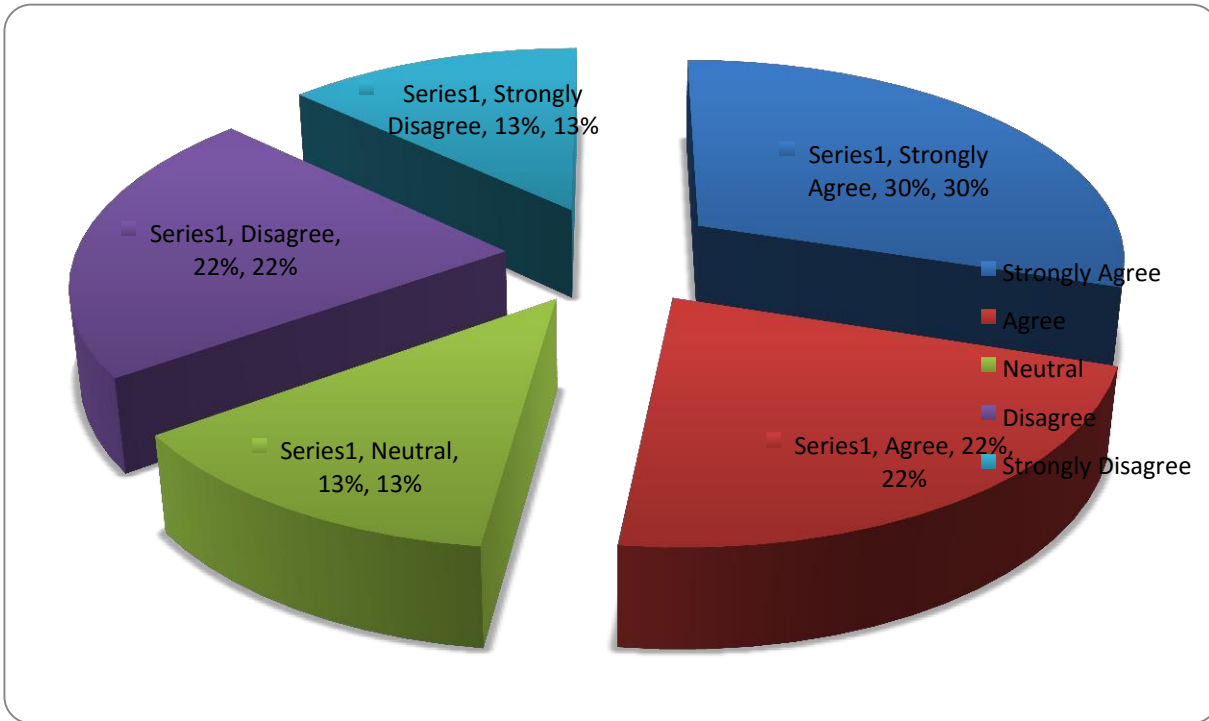
### **5.5 INTERMEDIATE PHASE TEACHERS' CONCEPTUALISATION OF TEACHING**

#### **STRATEGIES ASSOCIATED WITH MEANINGFUL LEARNING IN MATHEMATICS**

The results which addressed the third research objective, seeking to establish intermediate phase teachers' conceptualisation of teaching strategies associated with meaningful learning

in mathematics. All items were interpreted based on *Teaching strategies associated with meaningful learning in mathematics*.

### 5.5.1 Using a Variety of Teaching Strategies

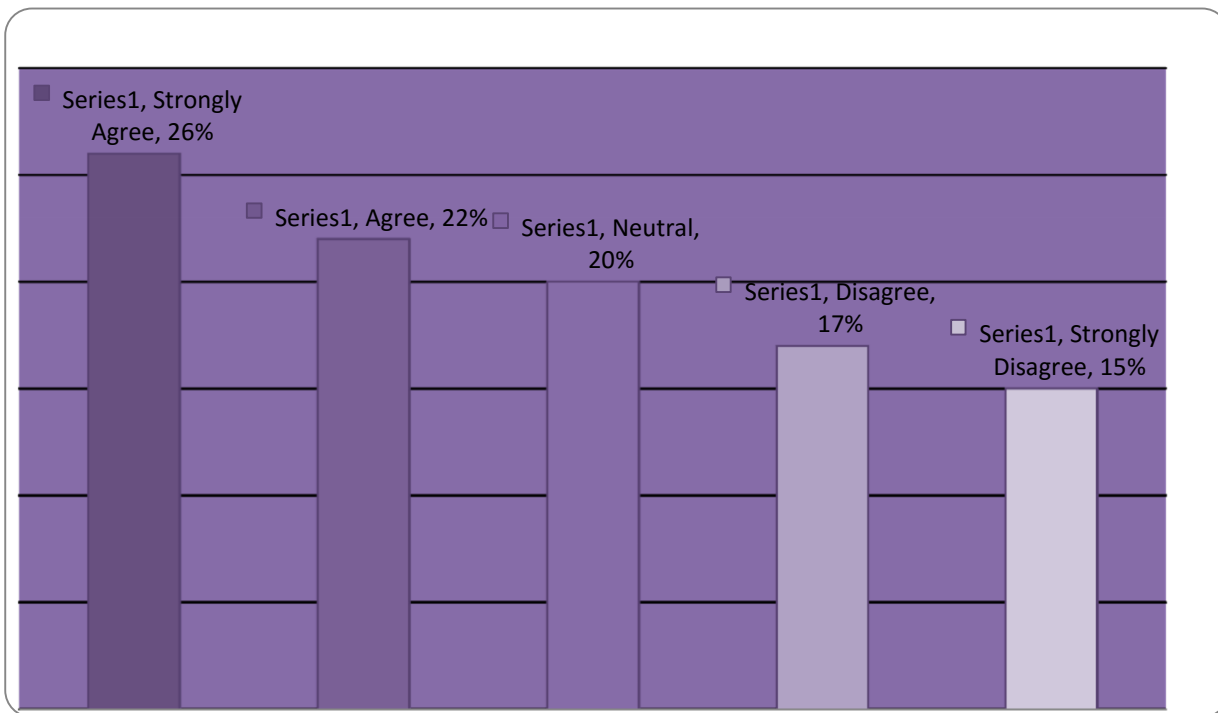


**Figure 5.17: Mathematics teachers need to demonstrate competence of a variety of teaching strategies and the knowledge of their learner's cognitions in mathematics (n=120)**

On the question of whether or not to teach effectively, mathematics teachers need to demonstrate competence of a variety of teaching strategies as well as the knowledge of their learners' cognitions in mathematics, a total of 52% consented to this view, as opposed to 35% respondents who held a dissenting view (Figure 5.19). Although 52% respondents saw the importance of using a variety of teaching strategies it is worrying that so many others (35% who disagreed and 13% who could not decide) did not expressly see this need to be competent in, and actually employ, a variety of teaching strategies as a way of meeting the individual and collective needs of their learners

Indeed, mathematics teachers need to demonstrate competence of a variety of teaching strategies associated with meaningful teaching to enhance meaningful learning. On the question of whether or not to teach effectively, mathematics teachers need to demonstrate competence of a variety of teaching strategies in mathematics, majority of respondents consented to this view, as opposed to minority respondents who held a dissenting view of this item. The respondents realized that using variety of teaching strategies is important for relational understanding.

### 5.5.2. Knowledge of Teaching Mathematics at the Foundation Phase

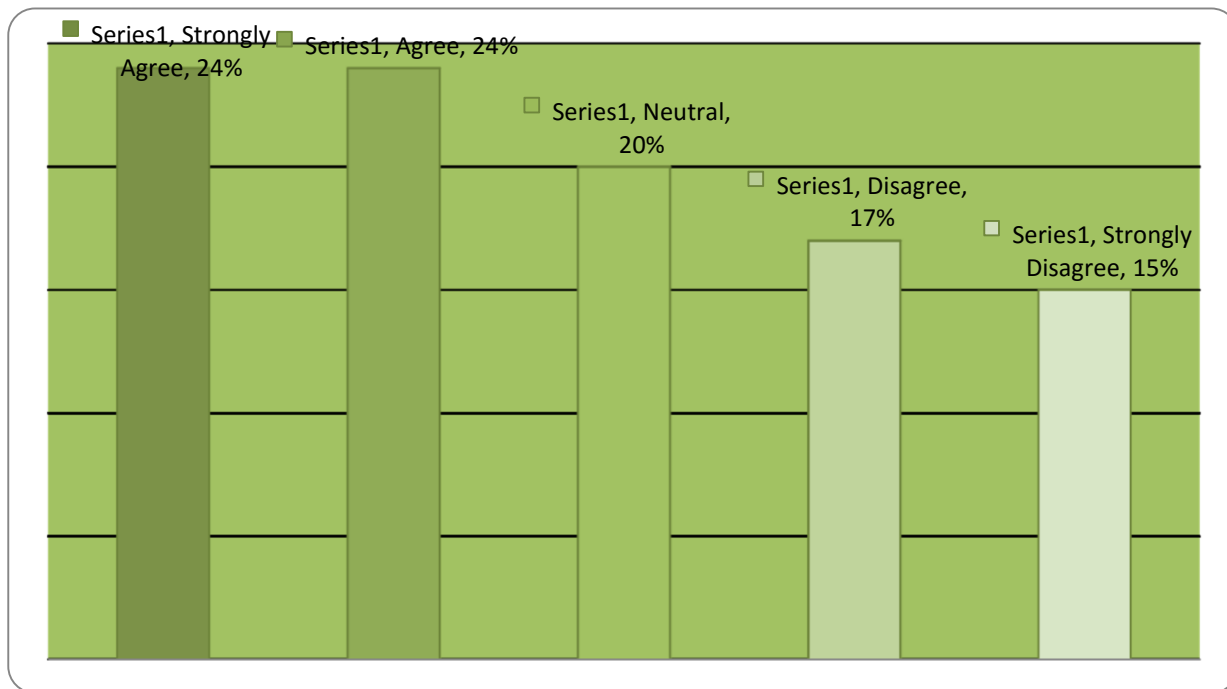


**Figure 5. 18: Knowledge of teaching mathematics at Foundation Phase level can assist IP teachers adopt appropriate strategies for meaningful mathematics (n=20)**

Teacher’s good understanding of learner’s prior knowledge is important. On the other hand, it is step forward for teachers’ suitable strategies for teaching. It is well documented that learners build on what they have already know. The results showed that the majority of the respondents agreed that the knowledge of teaching mathematics at FP could assist IP teachers adopt appropriate strategies for meaningful teaching.

This result supports Brandt, et al. (2012) who contend that the knowledge of teaching mathematics at FP level can assist IP teachers adopt appropriate strategies for meaningful mathematics teaching. It is therefore important for teachers to assess prior knowledge and skills in order teachers adopt appropriate strategies associated with meaningful learning in mathematics. Certainly, contemporary educational thought supports the view that a teacher must have a good understanding of his or her learners’ thinking capabilities for him or her to be able to offer them appropriate instruction (Da Ponte and Chapman, 2013; Imenda, 2018) So, it is pleasing to see that the affirmation of this view attracted more takers than the opposing view since for any teaching and learning to become meaningful is the existence of intellectual interaction between the current and prior knowledge already existent in the learner intellectual structure.

### 5.5.3 Using problem-solving



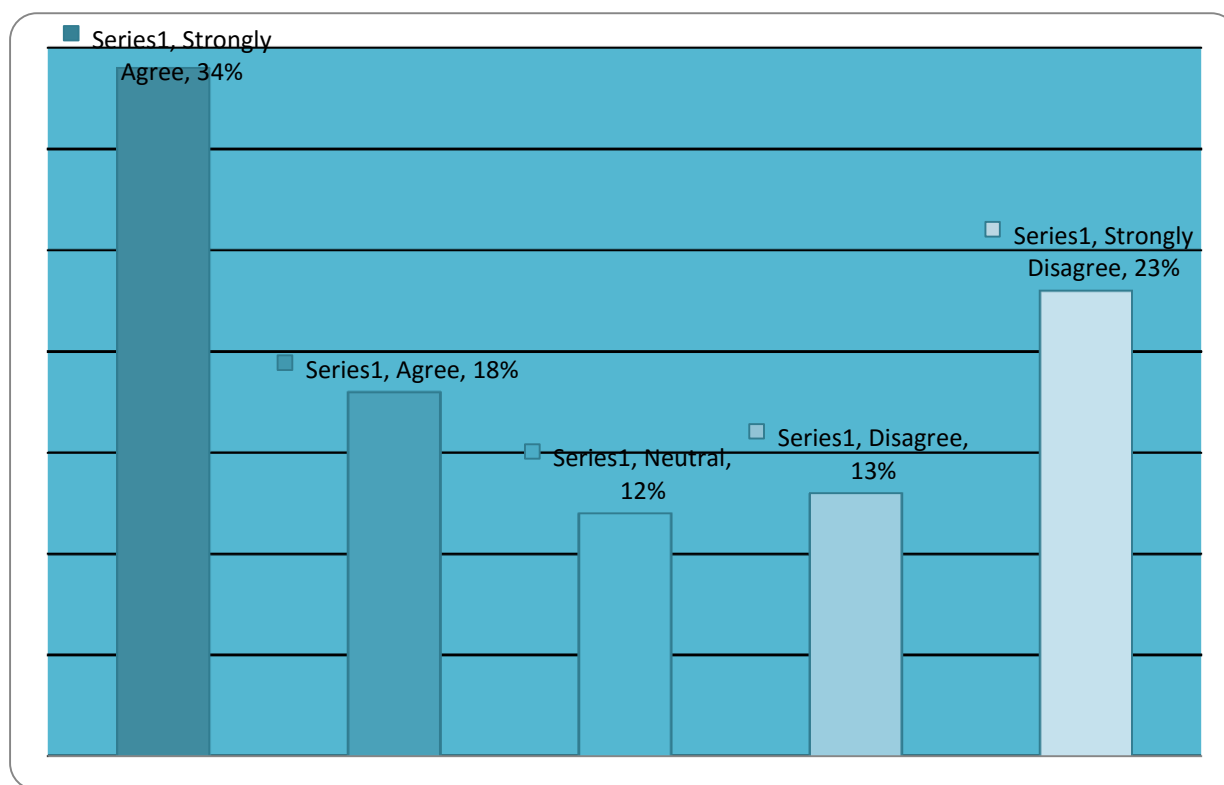
**Figure 5.19: Using problem-solving as vehicle for teaching mathematics can lead to meaningful teaching of mathematics (n=120)**

Figure 5.19 shows that a total of 48% of the respondents (24% each strongly agreeing and agreeing) were of the view that using problem-solving as a vehicle for learning mathematics could lead to meaningful learning of mathematics. In contrast, a total of 32% did not think so – and it is quite surprising that 20% could not decide one way or another on this item.

Teaching using problem-solving strategy serves as a vehicle for teaching mathematics effectively and efficiently. The first of these items sought to find out the respondents' views about using problem solving in promoting meaningful teaching in mathematics classroom. The results showed that teaching using problem-solving strategy as a vehicle for teaching mathematics could lead to meaningful learning of mathematics. Yet, this is one of the critical strategies that should be commonly used in the mathematics classroom that might enhance meaningful teaching.

The results are in agreement with the National Council of Teachers of Mathematics (NCTM, 1980) emphasized that problem solving is the focus of mathematics teaching because, they say, it encompasses skills. Similarly, Grouws & Cebulla (2016) indicated that meaningful learning focus on teaching mathematical topics through problem solving contexts which is characterized by the teacher helping learners construct a meaningful understanding of mathematical ideas. With the use of problem solving strategy mathematical concepts that were used to describe the situation were decimals, integers and fractions. The study recognized problem solving as one of the most important skill base that give learners experience of meaningful mathematics learning.

### 5.5.4 Using Objects from the Child's Own Environment concepts in mathematics enhances meaningful learning(n=120)



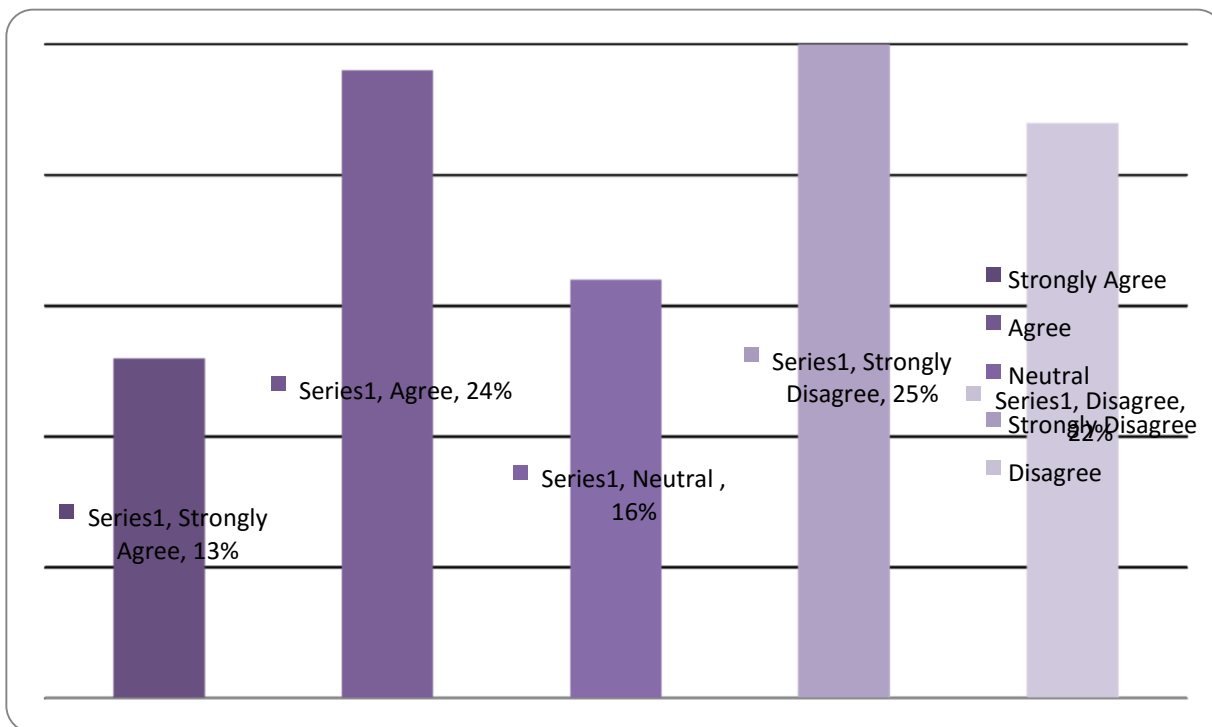
**Figure 5.20: Using objects from the child's own environment when teaching mathematical concepts in mathematics enhances meaningful teaching (n=120)**

One hallmark of good teaching is that a teacher would use objects and other materials that are familiar to learners. In this regard, majority of the respondents agreed with the statement that using objects from the child's own environment when teaching concepts in mathematics enhances meaningful teaching, while the minority disagreed. Although the majority response is in favour of this statement, it is not clear why other respondents would disagree with it. Certainly, one would have expected a much higher level of consensus on this statement. The results showed that teaching mathematics concepts using objects from the child's own environment enhances meaningful teaching and learning. The teaching of Mathematics is made interesting to learners when teacher is able to connect mathematical concepts to real life problems and experiences as well as establishing connection between the various forms

of Mathematical concepts and other subjects. One way to achieve meaningful mathematics teaching is to show relationships among mathematical concepts and other subjects.

One key point or strategy of good teaching in mathematics classroom that can help learners to understand mathematics better is basically based on using objects and materials that are familiar to learners' real life situation. Connecting mathematics in real life make mathematics fun particularly in primary schools. The Mathematics teachers' ability to connect Mathematics to real life problems is related to meaningful teaching. The current study aims to identify levels of MPCK in relation to meaningful learning in the context of teaching mathematics in primary schools.

### 5.5.5 Fostering connections between child's informal knowledge and the abstract.



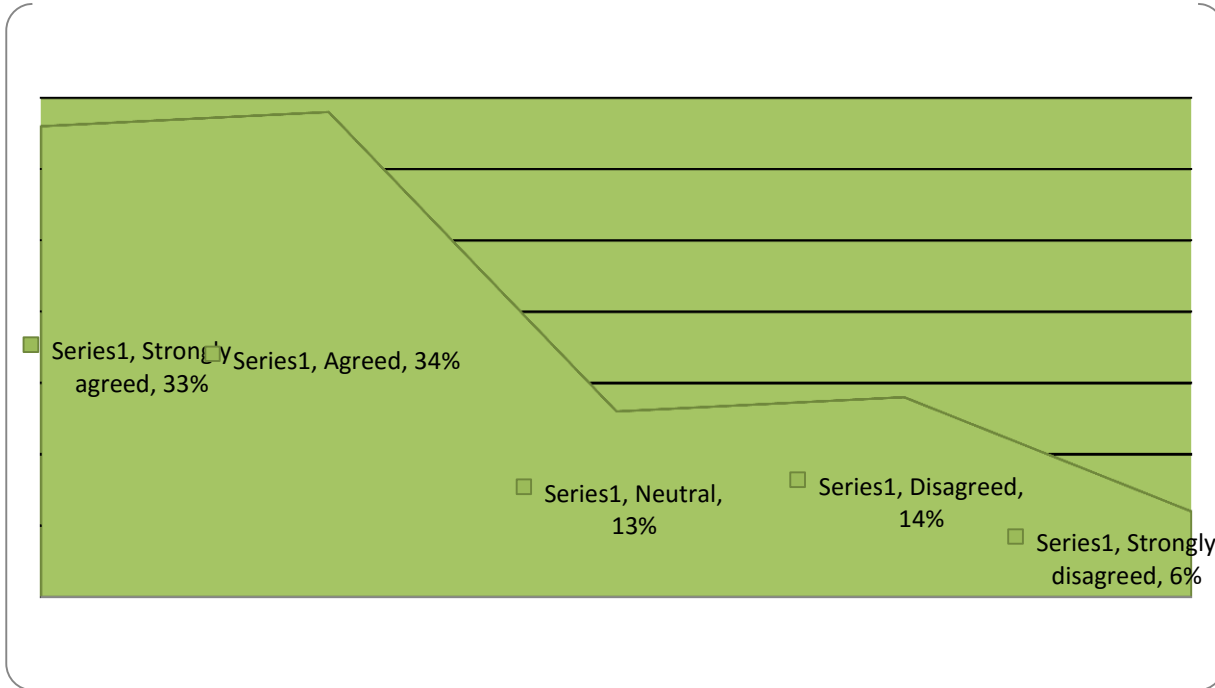
**Figure 5.21: Good teaching attempts to foster connections between child's informal knowledge and the abstract and arbitrary of symbolisms (n=120)**

Figure 5.21 shows that a total of 37% of the respondents supported this view, while 47% did not think that good teaching attempted to foster connections between the child's informal knowledge and the abstract and arbitrary system of symbolisms.

The idea that an effective teacher is one who is able to foster connections between concrete and abstract concepts has been around for some time (Swartz, and Perkins. 2016). With regard to this item, the respondents were asked to react to the statement that good teaching attempts to foster connections between the child's informal knowledge and the abstract and arbitrary system of symbolisms. This result is not good because it shows that close to half of the teachers who participated in this study did not believe in something as important to good teaching as enabling learners to make connections between concrete and abstract concepts. This is one of the most important aspects of mathematics teaching, given that a lot of mathematics is about symbolisms, that is, using abstract symbols to represent concepts.

So, it is unfortunate to see the majority of respondents not supporting the notion that good teaching attempts to foster connections between the child's informal knowledge (concrete concepts) and the abstract and arbitrary system of symbolisms. The purpose of teaching through concrete is to ensure learners develop a tangible understanding of mathematical concepts and skills they learn. Apart from this, concrete objects used to help learners understand abstract concepts in the domain of mathematics (Maurer and Neuhold, 2012). Consequently, teaching through concrete focuses on learning mathematics in an enjoyable way, making connections between the concrete and the abstract. Thus, learners are more actively engaged mathematics and group problem solving. Malan & Ndlovu (2014) summarize several benefits of concrete objects as follows: (1) they provide an additional resource in learning mathematics. (2) They help children connect with real-world knowledge, and (3) they help increase memory and understanding. The good side of IP teachers for using concrete objects is to facilitate meaningful learning.

### 5.5.6 Participation in Hands-On Activities

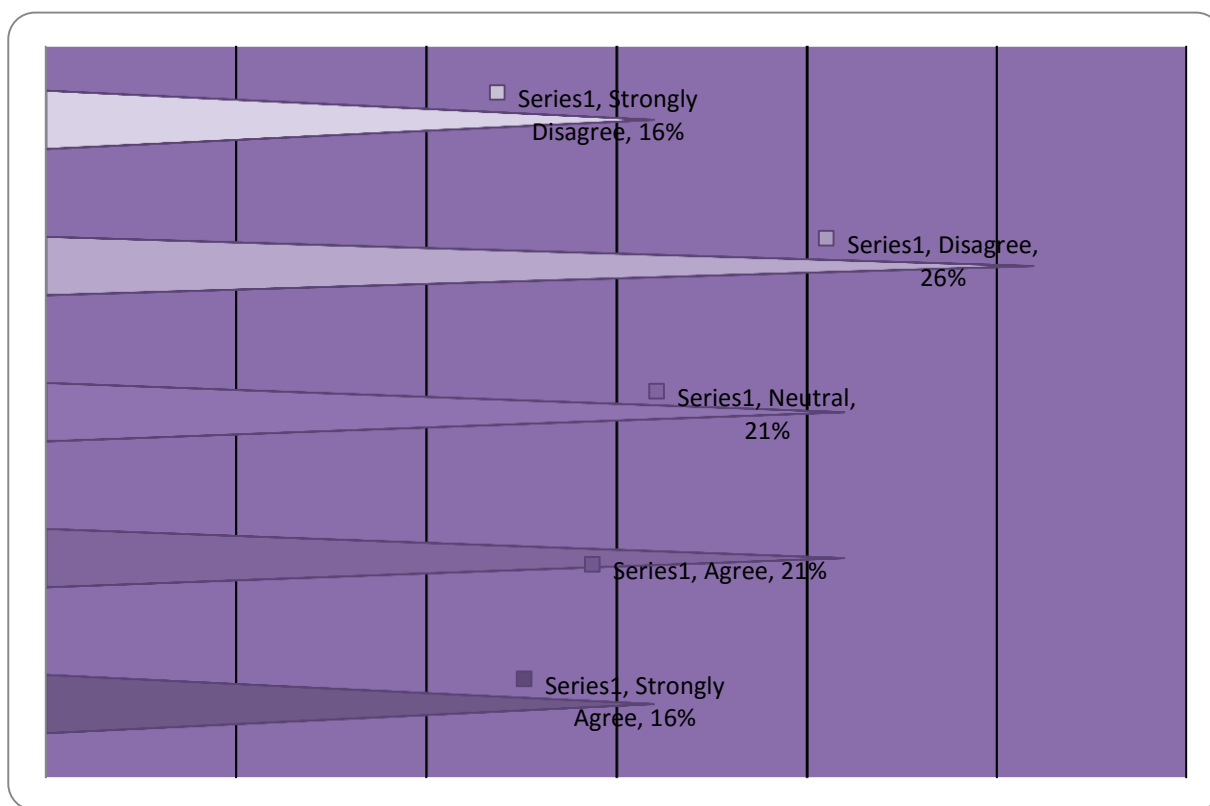


**Figure 5. 22: Meaningful mathematics learning takes place in classrooms where children participate in hands-on activities, instead of merely listening to the teacher (n=120)**

Constructivism, as a central aspect of the theoretical framework of this study, advocates active participation of learners in learning activities. In this regard, the respondents were asked to agree or disagree with the statement that meaningful mathematics learning takes place in classrooms where children participate in hands-on activities, instead of merely listening to the teacher'. From this response profile, it could be said that the majority view was well-aligned with the conceptual framework of this study. The above results support the contention by Ball & Forzani (2015) that meaningful mathematics learning takes place in classrooms where children participate in hands-on activities, instead of merely listening to the teacher. For the purpose of this study, participation in Hands on Activities is related to meaningful learning in the context of the context of teaching mathematics.

The study showed that learners learn and retain information better when they are given hands on activities. Reiss, & Torner (2011) showed that learning mathematics is profound or beautiful and make sense to learners if learners participate in hands on activities. This implies that hands on activities allow learners to uses their senses while learning. It should be noted that meaningful learning does not have an automatic meaning but the meaning emerges even through hands on activities (Wenglinsky, 2000).

### 5.5.7 Teaching Mathematics for Relational Understanding



**Figure 5. 23: Teaching mathematics for relational understanding is one good way of to promote meaningful mathematics learning (n=120)**

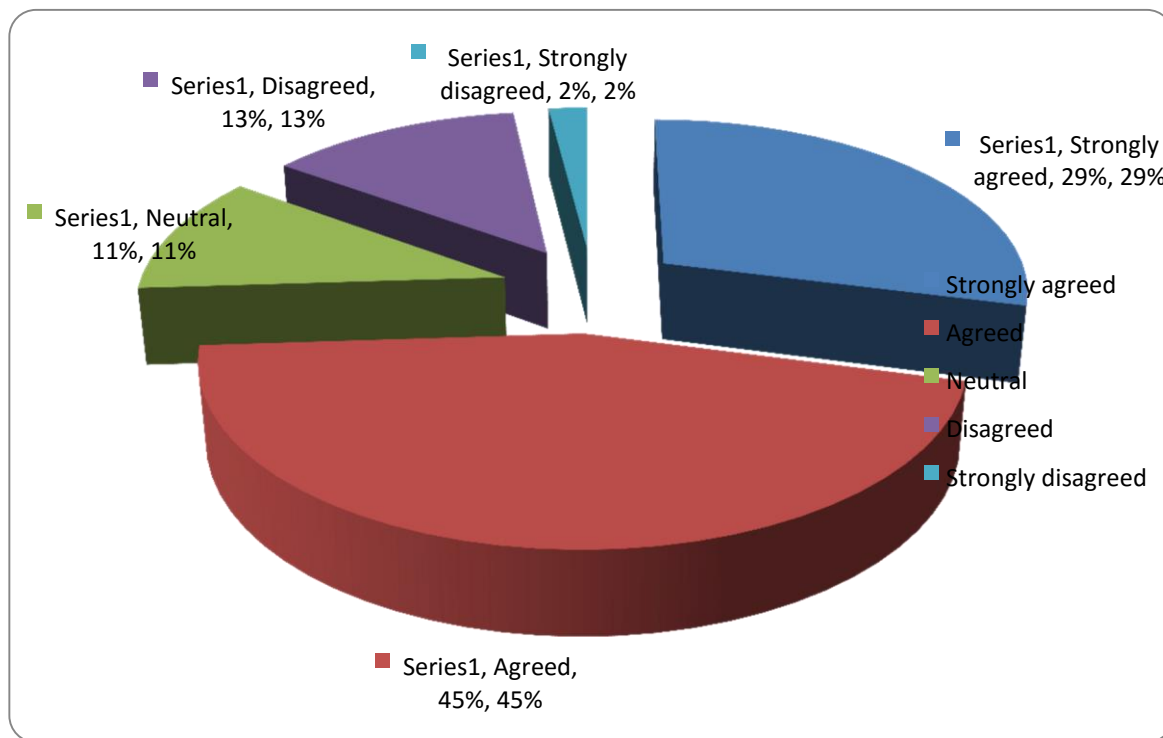
Relational understanding refers to having a mathematical rule, procedure and knowing ‘how’ to use it, and ‘why’ it works. Teaching for relational understanding make learners to learn new concepts easier. However, teachers who taught relationally make connections between and within concepts and skills. Therefore, learners with a relational understanding learn new

concepts easier and linked them with the previous concepts. The results of the study showed a dissenting view. This result is in contrast to the perspective held by Brandt, et al. (2012) that teaching mathematics for relational understanding is one good way to promote meaningful mathematics learning.

This implies that teaching mathematics to intermediate learners should be relational in order to optimize meaningful mathematics learning. The National Research Council (2012: 118) argued that relational understanding refers to an integrated and functional grasp of mathematical ideas". When a learner "understands the meaning and underlying principles of mathematical concepts," he or she has relational knowledge in mathematics (Fleisch and Schoer, 2011: 94). For this study relational understanding is often thought of to be a better alternative to instrumental understanding, there are advantages and disadvantages of both. One of the general advantages to relational understanding is easily adjusted and interconnects when new task is introduced.

Likewise, Howie (2013) statement confirms this advantage by stating that indeed relational understanding interconnects. It promotes the construction of knowledge out of learners' every day experience, feelings and exchanges with other learners, making connections and along with making adjustments to accommodate the new learning with previous mental structures (Mercer, 2010). Above all, learners who are taught relationally are more likely to remember the procedures because they have truly why they work, they are more likely to retain their understanding longer, more likely to connect new learning with previous learning, and they are less likely to make careless mistakes. Finally, teaching mathematics is effective and meaningful when it positively impacts a learner's understanding of the subject. In the light of this, learning becomes an active process. In other words teaching strives to involve learners in the learning process more directly than in the other methods. In particular, learners are engaged in such higher order thinking tasks as analysis, synthesis, and evaluation

### 5.5.8 Building Knowledge on Clear, Correct Answers



**Figure 5.24: Teaching should be built around problems with clear, correct answers around, and around ideas that most learners can quickly grasp (n=120)**

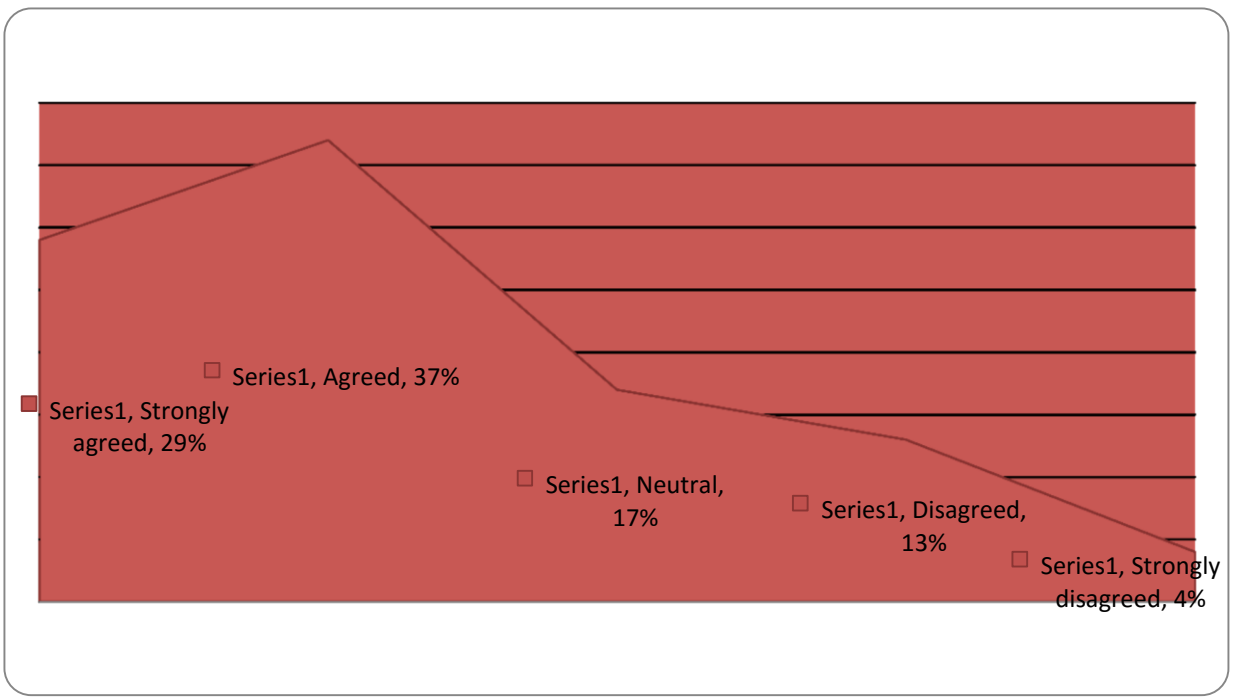
The view that teaching should be built around problems with clear, correct answers, and around ideas that most learners can quickly grasp were supported by the respondents, but emphasis was not placed on how to build knowledge on clear and correct answers. Conceptual understanding is required to build knowledge on clear and correct answers. Evidence for how to build knowledge on clear and correct answers in each item is the teachers' level of MPCK and their associated operational manifestations. In this study, the overwhelming majority supported this notion, which suggests that they have not yet made the pedagogic shift from direct (teacher-centred) teaching to more facilitative (learner-centred) forms of teaching.

D'Ambrosio (2016) revealed that teacher-centred does not necessarily enhance learner understanding. As a result many of the procedures, or routines, are not remembered correctly, or they are not remembered at all, and sometimes the procedures, or routines, are

confused (Mji and Makgato, 2006). Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating new knowledge and understanding about the real world every day Texas Education Agency (2006). The biggest challenge in teaching mathematics in the intermediate phase is to ensure that learners are committed to learning with a meaningful learning mindset rather than rote learning (Haylock, 2010).

The majority of IP teachers are understandably threatened by the transition of the pedagogic shift. However, in mathematics classroom there is not a one size fits all method that works for all learners in all situations. The majority of IP teachers are understandably threatened by the transition of the pedagogic shift. However, in mathematics classroom there is not a one size fits all method that works for all learners in all situations. Therefore, I found it advisable for teachers to use the combination of the two forms of teaching to use both meaningful and rote learning.

**5.5.9 Being Grounded in Various Aspects of Teaching**



**Figure 5. 25: Apart from mathematical knowledge, effective teaching demands that teachers know other aspects of teaching (n=120)**

This item sought to establish the views of respondents concerning the idea that, apart from teachers' conceptualisation of teaching strategies associated with meaningful learning in mathematics there are other aspects of teaching in relation to meaningful learning.

De Graaf, & Koloms (2011) postulated that effective teaching entails various knowledge, skills, and abilities that enable teachers to create a teaching environment that supports learner's meaningful learning. Kola & Sunday (2015) indicated that the success of meaningful and effective mathematics teaching and learning process is influenced by number of factors, but teachers' MPCK plays an important role for effective teaching and learning. Fennema & Franke (1992) cited on Maboya (2014) claim that mathematics teachers need knowledge of mathematics, pedagogical knowledge and knowledge of the learner's cognitions in mathematics. While these aspects can be presented discretely, they emphasise that it is the interaction between each component, within the context of classroom teaching that results in meaningful mathematics teaching.

#### **5.5.10. Statistical Analysis**

Descriptive statistics tends to describe a big chunk of data with summary charts and tables but do not attempt to draw conclusions about the population from which the sample was taken (DeCaro, 2003). During the course of this study, measures of location variability and shape were calculated across all the Likert scaled items. The numerical presentation of the questionnaires that were completed is shown as Valid N in the table below. Given the five-point Likert scale used ranging from 1= Strongly Disagree to 5 = Strongly Agree, the two major types of statistics used to describe the data were the *Measures of Central tendency* (mainly, the arithmetic Mean [ $\bar{X}$ ]) and the *Measures of Dispersion* (mainly, the Standard Deviation [SD]), along with the mean differences ( $\bar{X} - \Delta$ ). The „sample t-test“ statistic was applied to the data, at the 95% Confidence Interval, comparing means on the teaching strategies associated with the notion of „meaningful learning in mathematics“. In interpreting the results, higher mean values (in Column 4, Table 5.5) below, were taken to suggest greater agreement with the statements, reflecting the particular teaching strategy and indicating lesser perceived influence.

Strategy	t	Pvalue		SD	$\Delta$	95% Confidence Interval of the Difference	
						Lower	Upper
1. Using problem-solving	-1,22	0,89	2,88	1,08	-0,12	-0,31	0,08
2. Knowledge of teaching mathematics at Foundation Phase	-0,14	0,55	2,99	1,01	-0,01	-0,19	0,17
3. Ability to understand learners and their learning needs	3,97	0,00	3,33	0,90	0,33	0,17	0,49
4. Motivation to teach mathematics in the IP	-0,26	0,60	2,98	0,98	-0,02	-0,20	0,15
5. Using objects from the child's own environment	4,34	0,00	3,35	0,89	0,35	0,19	0,51
6. Teaching mathematics in ways that positively impact a learner's understanding	16,25	0,00	4,11	0,75	1,11	0,97	1,24
7. Teaching mathematics for relational understanding	3,77	0,00	3,37	1,06	0,37	0,18	0,56
8. Ensuring that learner is able to demonstrate the intended knowledge and skills	17,24	0,00	4,33	0,84	1,33	1,17	1,48
9. Demonstrating competence of a variety of teaching strategies	8,78	0,00	3,51	0,64	0,51	0,40	0,63
10. Participating in CPTD	-12,48	0,00	2,02	0,85	-0,98	-1,14	-0,83
11. Having learners participate in hands-on activities	14,43	0,00	4,05	0,790	1,05	0,91	1,20
12. Maintaining a quiet classroom	10,85	0,00	4,00	1,001	1,00	0,82	1,19

**Table 5.5: Results of one sample t-test in comparing means of teaching strategies associated with „meaningful learning in mathematics (n=120)**

The mean values of these 4 labels are observed to be very high; L8 (4.33) was the highest, followed by L6 (4.11), thereafter L11 (4.05) and L12 (4.00). This suggests that the participants perceive “Effective teaching and learning are achieved,” (L8) (Mean= 4.33) to be most influential on their success rates in meaningful learning mathematics, followed by “Knowledge of teaching Mathematics” (L6) (Mean=4.11), and then “Appropriate teaching and learning materials improve interest in mathematics”, and finally, “Flexibility and variability in teaching approaches”.

Furthermore, the participants perceive “Mathematics teachers need to demonstrate” (L9) (Mean= 3.51), to have a higher influence on their success rates in bridging mathematics, followed by “Teaching mathematics for relational understanding” (L7) (Mean= 3.37), thereafter, “foster meaningful mathematics learning”, (L3) (Mean= 3.33). Other factors closely followed those mentioned, “appropriate strategies for meaningful mathematics learning” (L2) (Mean= 2.99), “Motivation to teach mathematics in the IP” (L4) (Mean= 2.98), and “Using problem-solving as a vehicle for learning mathematics can lead to meaningful learning in mathematics”, (L1) (Mean= 2.88) also affects the participants’ success rates in mathematics. Lastly the participants perceive, “Ineffective teaching strategies affect poor performance in mathematics” as least influential on the success rates in knowledge in mathematics, hence, the low Cronbach alpha earlier computed.

Standard deviation is a statistic that tells you how tightly all the various examples are clustered around the mean in a set of data. A very high standard deviation indicates more dispersion of agreement among the participants; this was true of the label, “Using problem-solving as a vehicle for learning mathematics can lead to meaningful learning in mathematics” (Label 1) (Std. Dev. =1.077). This means most participants have a varied agreement regarding this factor. This was followed closely by, “Knowledge of teaching mathematics at Foundation Phase level can assist Intermediate Phase (IP) teachers adopt appropriate strategies for meaningful mathematics learning”, (Label 2) (Std. dev. = 1.007) was next in that order as can be seen from the table. The Lowest standard deviation, indicating less dispersion of agreement amongst the participants was recorded for the influence of

“Mathematics motivation (tasks and skills related)”. The Mean difference measures the difference between the mean computed value and the test value of 3.

### 5.6.1 Knowledge about mathematics teaching versus PCK teaching strategies

Table 5.6 shows the relationship between one’s mathematical content knowledge and his/her choice of teaching strategies.

ITEMS	n	At least disagree	Neutral	At least agree	Mean	S.D.
1. Knowing mathematics involves the ability to remember formulas and procedures	120	24.1%	22.1%	53.9%	3.33	1.186
2. The textbook is the best resource to use when teaching mathematics	120	31.7%	30.8%	37.5%	3.03	1.092
3. The role of the mathematics teacher is to transmit knowledge and ensure that the learners have received this knowledge	120	16.4%	7.7%	76%	3.85	1.147
4. Correct answers are more important than the method used to obtain them	120	83.8%	8.6%	7.6%	1.81	0.955
<b>Total average</b>					<b>3.005</b>	<b>1.095</b>

**Table 5.6: The relationship between mathematical knowledge and teaching strategies (n=120).**

On a 0-5 Likert type scale, Item 4 has the lowest mean score of 1.81 and standard deviation 0.955, showing that teachers refuted the idea that answers are more important than the method. Indeed, only 7.6% of the respondents acceded to the statement. Item 2 produced a disappointing result with more teachers agreeing (than disagreeing) with the statement that

textbooks were the best resource to use when teaching mathematics. However, one has to be very careful in interpreting this result, bearing in mind that there are limited resources in many of the participating schools – resulting in a heavy reliance on the use of prescribed textbooks. Perhaps, this explains the very high response rate under Neutral, which could mean that although some of the teachers could have been dissatisfied with relying of textbooks as the dominant resource to use in teaching mathematics, they did not have much else to use beyond the prescribed textbooks.

Affirmative responses to items 1 and 3 are an antithesis to constructivist teaching. In this case, 53.9% and 76% of the respondents agreed with these statements 1 and 3, respectively – suggesting that they were firmly anchored in the transmission mode of teaching mathematics. By and large, this result conclusively answers the first research question. There is very limited constructivist teaching taking place in the IP in Sibasa. Instead, the majority of the teachers are teaching from the front in the traditional way, rather than employing strategies that encourage learner-based construction of meaning.

### **5.6.2 The relationship between the age of the teachers and disposition to teach mathematics for meaning**

The researcher wished to test whether or not a statistically significant relationship existed between the respondents' ages and how they taught mathematics, with specific reference to allowing their learners to learn the subject meaningfully. The results are reflected in Table 5.7.

meaningful teaching								
Age	Observation		Selfassessment		Group assessment		Oral questions	
	<i>F</i>	%	<i>F</i>	%	<i>F</i>	%	<i>F</i>	%
<b>25-30 years</b>	5	8.8	4	7.0	6	10.5	42	<b>73.7</b>
<b>31-35 years</b>	7	13.5	2	3.8	6	11.5	37	<b>71.2</b>
<b>Older than 36</b>	1	4.0	1	4.0	4	16.0	19	<b>76.0</b>
<b>X<sup>2</sup>value</b>	2.764							
<b>Degrees of freedom</b>	6							
<b><i>p</i>-value</b>	<b>0.838</b>							

**Table 5. 7: The relationship between the age of the teachers and meaningful mathematics teaching (n=120)**

Table 5.7 shows that a  $\chi^2$  value of 2.764 (for 6 degrees of freedom) was calculated. This value was not statistically significant at 99% confidence interval, indicating that there was no significant relationship between age and the disposition to teach mathematics through approaches that encourage meaning-making. The table also indicates that the older teachers (76.0%) prefer to use more oral questions in their assessment process than the younger teachers (73.7%). However, the percentages are not far from each other to suggest a significant difference.

### **5.6.3 Relationship between the type of school and meaningful learning in**

#### **Mathematics**

With respect to whether or not the type of school a teacher was working in was significantly related to the disposition to teach for meaning, the results are presented in Table 5.8.

Awareness of corporal punishment				
Type of school	Yes		No	
	<i>F</i>	%	<i>F</i>	%
<b>Farm</b>	20	100.0	0	<b>0.0</b>
<b>Peri-urban</b>	18	100.0	0	<b>0.0</b>
<b>Urban</b>	82	97.6	2	<b>2.4</b>
<b><math>\chi^2</math> value</b>	0.679			
<b>Degrees of freedom</b>	2			
<b><i>p</i>-value</b>	<b>0.712</b>			

**Table 5. 8: Relationship between the type of school and meaningful learning in Mathematics (n=120)**

Table 5.8 shows a calculated  $\chi^2$  value of 0.679. At 2 degrees of freedom, this value was not statistically significant at the 99% confidence interval. Therefore, it may be concluded that there was no significant relationship between the type of school at which teachers teach and the disposition to teach mathematics for meaning.

### **5.7 Answer to Research question 3: How do IP teachers conceptualise the relationship between pedagogical content knowledge (PCK) and the meaningful teaching and learning of mathematics?**

The relationship of IP teachers between pedagogical knowledge and the meaningful teaching and learning of mathematics displayed by respondents was moderate. The potential explanation was due to a challenge faced by IP teachers" including lack of teachers' background of knowledge use of the content, lack of teachers" ability to understand learners and their learning needs, lack variety of teaching strategies and as a result, drill and rote learning were used. Depaepe, Verschaffel, & Kelchtermans (2013) study also showed that many IP teachers are reported not to hold a solid conceptual understanding of mathematical concepts and topics. Briefly, the study revealed teachers' low level of confidence in their MPCK. From the viewpoint of the study teacher's high level of MPCK is described as the valueless knowledge of effective and meaningful mathematics teaching and learning.

The study confirms Figueiredo, Gomes & Rodrigues (2018) statement: without adequate mathematics pedagogical content knowledge, teachers would not be in a position to deal with

their mission of teaching effectively and such would not cater for meaningful learning. On the contrary, this study affirms that a teacher who possesses good Mathematics pedagogical content knowledge can teach mathematics in a meaningful way that make it accessible and meaningful to all learners with different abilities and capabilities.

In a nutshell, the Author conceptualizes MPCK as a key element in improving mathematics teaching, particularly at the IP. Apart from this teachers' high level of MPCK profoundly influences how she/he understands "what" and "how" to do mathematics. From the viewpoint of the study teacher's high level of MPCK is described as the valuable knowledge of effective and meaningful mathematics teaching and learning

## **5.8 REFLECTION ON THE SUCCESS OF THE RESEARCH**

In this chapter, I presented, interpreted and analysed the collected data. Numeric information was summarised using tables, graphs and figures. Furthermore, open-ended questions have been grouped into related categories and were explained. Qualitative data from the interviews have also been comprehensively presented, interpreted and explained. However, the quantitative and qualitative results were presented separately and then integrated at the end. The analysis of the interviews either confirmed or refuted the results of the data collected by the questionnaires. The findings of the quantitative and qualitative data in this study sometimes contradicted each other. The findings of the study, recommendations and conclusions, will be presented in the following chapter.

## CHAPTER 6

### FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

#### 6.1 INTRODUCTION

This chapter presents the discussion and conclusion following the findings in the previous chapter (chapter 4). In this chapter the researcher begins with a general overview of the teachers' conceptualizations of meaningful learning in Mathematics and how learners respond to activities facilitated to enhance meaningful learning in the Intermediate Phase in the Vhembe District, which shows that the aims were achieved.

The study reveals that several elements contribute to Intermediate Phase teachers' conceptualization and facilitation of meaningful mathematics learning. This chapter (Chapter six), will present the findings, recommendations and concluding remarks.

However, findings and recommendations will be done concurrently.

#### 6.2 MAJOR FINDINGS OF THE STUDY AND THEIR RECOMMENDATIONS

The main focus of this study was to explore teachers' conceptualizations of meaningful learning in mathematics and how learners respond to activities facilitated to enhance meaningful learning in the IP in the Vhembe District.

This section presents the findings of the study, with regard to the formulated research questions in section 1.7. However, the findings and recommendations of this study are consistent with other previous research results.

##### 6.2.1 What is the current state of mathematics teaching in the Intermediate Phase (IP) in the Sibasa circuit, Limpopo Province?

The findings of the study revealed that the quality of mathematics teaching was just moderate and rated it as low. Figure 5.8 showed that the average percent learner performance of the learners taught by the respondents was generally low – ranging from 10% to 33%. This generally demonstrates the pedagogical challenges faced by teachers in the teaching of mathematics which results in learners' meaningless mathematics learning.

### 6.2.1.1 Recommendations

In regards for the findings in 6.2.1 the study recommends the following:

a) **Pedagogical shift:** The study recommends that new methods/ approaches/ways of mathematics teaching can go a long way in improving meaningful mathematics learning in all phases. Therefore in order for teachers to overcome pedagogical challenges, they should move away from traditional teacher-centred classroom practices, such as telling method and drill and practice activities, into a more learner-centred context that allows learners to work collaboratively and cooperatively to develop meaningful learning skills.

The older teachers prefer to use more oral questions in their assessment process than the younger teachers. The results of the respondents show that if pedagogical understanding is profound, then the degree of meaningful learning will be unlimited. The study accepted that a high level of pedagogical knowledge is critical for competent teaching hence for meaningful learning

b) **Create effective learning environment:** The study recommends that IP teachers should create environments where knowledge is constructed by the learner. This environment enables learners to build their Mathematical knowledge and understanding of the subject. However there was no significant relationship between the type of school at which teachers teach and the disposition to teach mathematics for meaning. But there is very limited constructivist teaching taking place in the IP in Sibasa. Instead, the majority of the teachers are teaching from the front in the traditional way, rather than employing strategies that encourage learner-based construction of meaning.

Based on this background the study found it worthwhile for the classroom to be defined as a constructivist classroom (Imenda, 2018:75). Constructivism, as a central aspect of the conceptual framework of this study, advocates active participation of learners in learning activities.

c) **Various teaching strategies:** Analysis shows that ineffective and lack of various teaching strategies affect poor performance in mathematics” as least influential on the success rates in knowledge in mathematics, hence, the low Cronbach alpha earlier

computed. Likewise Figure 5.30 shows that the knowledge of teaching mathematics at the FP could assist IP teachers to adopt appropriate strategies for meaningful mathematics learning. Therefore the study recommends that if teachers are able to incorporate various methods and strategies into their teaching of mathematics, learners may find learning mathematics more stimulating and enjoyable.

d) **The use of concrete materials:** One hallmark of good teaching is that a teacher uses concrete objects or materials that are familiar to learners. The use of concrete materials is positively related to increases in learners' meaningful Mathematics learning. This study recommends that IP teachers should use manipulative materials regularly in order to give learners hands-on experience that helps them construct useful meanings for the Mathematical topics or concept they are learning. It is also important that learners use materials in meaningful ways rather than in a rigid and prescribed way that focuses on memorization rather than on relational understanding.

## **6.2.2 What are IP teachers' perceptions about the importance of mathematical pedagogical content knowledge (MPCK) in the teaching of mathematics?**

The findings based on this formulated research question revealed that IP teachers are reported to hold low level of MPCK. Even & Tirosh (2015) argued that MPCK in mathematics teaching context is a unique blending of three types of knowledge viz. mathematics knowledge, content knowledge and pedagogical knowledge

### **6.2.2.1 Recommendations**

In regards to teachers' low level of MPCK, the study recommends the following:

a) **Mathematics coaching:** Mathematics coaching is a powerful vehicle for helping teachers to build their Mathematical Knowledge for Teaching (MKT). According to Ball (2008) MKT is a specialized type of knowledge that teachers need for teaching mathematics successfully and effectively. MKT encompasses abilities such as analysing the learner thinking that led to an incorrect answer, identifying the mathematical understanding a learner does not yet have, and deciding how the best represent a mathematical idea so that it can be understood by learners (Ball 2008). As coaches partner with teachers in the context of planning for,

implementing, and evaluating instruction, teachers will therefore have multiple opportunities to:

- Grow teachers, and is especially effective in growing Mathematical Knowledge for Teaching
- Strengthen their Content knowledge (CK) for the purpose of making mathematics accessible for learners.
- Increase teachers understanding of how learners learn mathematics.
- Implementing effective strategies that promote thinking, reasoning, and making sense of mathematics.
- Strengthen teacher's decision making processes for future teaching situations or other topics.

b) **Teachers' collaboration:** The study showed that 70% of the respondents reported meeting with other mathematics teachers "very low" and 30% "very much" (Figure 5.11). Thus, this shows that by far the majority of the respondents hardly met with their colleagues for the purpose of discussing their teaching. Collaboration between and among teachers is one of the indicators of the present-day indicators of continuing professional teacher development which, in turn, tends to improve the quality of teaching and learning. Consequently, the study recommends that IP teachers should frequently meet with their colleagues especially those who teach the same subject for the purpose of discussing their teaching experiences. As a result teacher collaboration benefits are:

- *Increased Educational Effort.* Since teachers who collaborate on instruction are all of the same colour, they can increase the level of educational accuracy to match the basic know-hows they want learners to meet.
- *Increased Understanding of Learner information or evidence.* Consequently, they will have a sense of shared responsibility for analysing failure (such as meaningless mathematics teaching and learning) and

celebrating success (such as meaningful mathematics teaching and learning).

- *More Creative Lessons Plans.* When teachers communicate and share ideas, they also share a large repertoire of teaching strategies that encourage creative teaching which will then results into relational understanding.
- *Less Teacher Isolation.* The opportunity to share ideas and information truly combats professional loneliness and frustrations which improves staff morale and professional satisfaction.

### **6.2.3 How do IP teachers conceptualise the relationship between pedagogical content knowledge (PCK) and the meaningful teaching and learning of mathematics?**

The findings showed that lack of conceptual understanding about mathematical knowledge for teaching which comprises content knowledge and pedagogical content knowledge affected the level of PCK negatively. Fortunately, lack of conceptual understanding of mathematical knowledge of teaching can be corrected. This statement was confirmed by Stronge, Ward, & Grant (2011) by saying that PCK can be built through courses, fieldwork, experience, and professional development. Therefore there is a need for teachers especially those who teach intermediate phase who do not necessarily have a specialization in core area to be given an opportunity to develop their PCK.

#### **6. 2.3.1 Recommendations**

***Teachers as Learning Specialists or Experts:*** The results of the study showed that lack of flexibility in the implementation of CAPS was a major factor that contributed to poor mathematics performance. Teachers as professionals in the field of teaching, they are expected to process and evaluate new knowledge relevant for their core professional practice and to regularly update their knowledge of teaching mathematics to improve their practice and to meet new teaching demands.

***Continuing Professional Teacher Development:*** This recommendation concurs with the perspective of Crawford & Witte (2013) who aver that continuing professional teacher development is crucial for effective mathematics teaching in the IP. Teachers

can therefore learn through studying, by doing and reflecting, by collaborating with other teachers, by looking closely at learners and their work, and by sharing what they see. It gave “substantial opportunities for teachers to learn mathematics (Hill and Ball 2009: 70)

### **6.3 RECOMMENDATION FOR FURTHER RESEARCH**

This study focused on IP mathematics teachers’ conceptualisations and facilitation of meaningful Mathematics learning in Sibasa Circuit of Vhembe District. Since the study was mainly focused on the IP mathematics teachers in Sibasa circuit of Vhembe District only, it could be ideal if similar studies could be extended to other circuits of Vhembe District and even the other districts of the Limpopo Province.

It is also recommended that further studies should involve all mathematics stakeholders within the Department of Education from National level down to school level so that the various problems can be probed from all perspectives. Nevertheless, the findings of this study could be vital in improving meaningful mathematics teaching and learning in the Intermediate phase Mathematics in Sibasa Circuit.

### **6.4 CONCLUSION**

This study explored IP teachers’ conceptualizations and facilitation of meaningful learning in mathematics in the Vhembe District. IP learners experienced difficulties in learning mathematics because of how teachers teach mathematics. The previous section discussed a number of recommendations that will assist IP mathematics teachers for making teaching and learning more meaningful.

Firstly, it suggests that teaching should focus on understanding, not memorization of facts and procedures. If we are to improve the quality of meaningful learning in critical core content areas, we need to resist some old traditions in professional learning. Instead, teachers should acknowledge and expand the understandings of specialists who develop proficiency in subject matter teaching.

Secondly, teachers should also commit themselves to high quality professional development targeted to develop this expertise. If teachers can do this, they will support the growth of

themselves as a person and a professional who are knowledgeable and effective. Concurrently, they will contribute to the realization of the goals and priorities of meaningful mathematics learning.

Lastly, at the heart of effective and meaningful content teaching is the teachers' Pedagogical Content Knowledge (MPCK). Yet, majority of IP teachers at Sibasa circuit were reported not to hold a solid conceptual understanding of mathematical concepts and topics. It is also well documented by other studies that many teachers exhibit weaknesses and lack a deep conceptual understanding of mathematics pedagogical content knowledge (Ball, Hill & Bass, 2005; Hill et al., 2008). There is a general notion that IP teachers teach all subjects, without acknowledgement of knowing the subjects compared to SP teachers who teach the subject according to what he or she specialized on. It is therefore important for IP teachers to acquire MPCK to make a positive change in the teaching and learning of Mathematics.

This study concurs with other studies with the view that MPCK in mathematics teaching is a unique blending of three types of knowledge viz. mathematics knowledge, content knowledge and pedagogical knowledge. MPCK is different from the math knowledge needed in other professions. For this study meaningful mathematics learning depends more on how teachers integrate mathematics knowledge, content knowledge and pedagogical knowledge. Much demand and emphasis is therefore placed on the teacher high level of MPCK can be useful. In conclusion, it is believed that the above recommendations stated in section 6.2 and together with Figure 3.3: TSHISI MODEL: A model to facilitate meaningful mathematics teaching practice in the IP level will go a long way for improving teachers' conceptualizations of meaningful learning in Mathematics.

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## APPENDICES

**APPENDIX A**  
**TEACHER QUESTIONNAIRE**

**SECTION A: BIOGRAPHICAL INFORMATION** (*Please complete all questions*)

**Gender of the respondents**

Male	
Female	

**Age distribution of the respondents**

18- 25 years	
26-30 years	
31-45 years	
46-50 years	
Over 51 years	

**Educational information of the respondents**

Below matric	
Matric	
Diploma	
Degree	

**Grade currently teaching**

Grade 4	
Grade 5	
Grade 6	
Grade 7	

**How many years of teaching do you have altogether, including this year**

0-4	
5-10	
11-15	
More than 15	

**Employment status of the respondents**

Full time	
-----------	--

Part time	
Unemployed	
Pensioner	

**Indicate your overall performance average level of interest in Mathematics teaching**

Affluent	
Above average	
Average	
Below average	
poor	

**Indicate the category of your school. Please tick the box that matches your answer**

Public	
Private	

**What is the enrolment at your school?**

1- 200	
201-400	
401-600	
601-800	
801+	

## SECTION B: INTERMEDIATE PHASE TEACHERS' CURRENT STATE OF MATHEMATICS TEACHING IN SOUTH AFRICA

For the following statement, please tick the box that matches your view most closely

### 1. Rate the quality of mathematics teaching at your school

Low	
Moderate	
Good	
Excellent	

### 2. How often do you meet with other mathematics teachers in your school?

Very much	
Very Low	

Please, tick in the appropriate box to indicate your level of agreement with the following statements.

Statement	Never	Less than 1 per month	1 times per month	1 times per week	3 times per week	Everyday
3. How often do you ask learners to do pen- and paper calculations, and to practise?						
4. How often do you demonstrate to the class a procedure on the chalkboard, and then let the learners practise?						
5. How often do you teach your learners mathematics by focusing on rules and procedures?						

Please, tick in the appropriate box to indicate your level of agreement with the following statements.

Statement	Strongly agree	agree	Neutral	disagree	Strongly disagree
7 Presently, one factor contributing to poor mathematics performance is use of poor teaching methods					
6. No matter how hard we try, many learners are just not able to learn mathematics					
7. My understanding of problem-solving is that it is just about word sums					
8. There isn't much we can do because the problem of poor mathematics performance is not only experienced in South Africa, it is universal					
9. Lack of flexibility in the implementation of CAPS (the Curriculum and Assessment Policy Statement) is a major factor that contributes to poor mathematics performance					

## SECTION C: INTERMEDIATE PHASE TEACHERS' PERCEPTIONS ABOUT THE IMPORTANCE OF MATHEMATICAL CONTENT KNOWLEDGE IN THEIR TEACHING

1. Intermediate Phase mathematics teachers do not need to possess enough mathematics knowledge to be an effective mathematics teacher

Agree	
Somewhat agree	
Disagree	

2. I feel quite confident about my knowledge of the mathematics content that I teach.

Agree	
Somewhat agree	
Disagree	

	Strongly agree	agree	Neutral	disagree	Strongly disagree
3. Teachers need to know the subject matter very well to teach effectively					
4. Teachers who are weak in the subject run the risk of giving wrong answers and explanations to learners – thereby undermining effective and meaningful learning of the subject					
5. The teaching process of mathematics topics starts from the teacher's knowledge and understanding of the subject					
6. In South Africa, IP teachers' mathematical knowledge on key topics, such as fractions, decimal fractions and integers is generally weak, thus they cannot assist their learners overcome the misconceptions that the learners have					
7. At the moment, systematic mistakes develop when teachers are unable to assist learners who encounter new problems and end up incorrectly generalising mathematical procedures because they do not know what to do.					

8. Most teachers' failure to use knowledge associated with mathematics is a major factor that contributes to poor mathematics performance					
9. Only teachers with adequate mathematical knowledge can help learners to gain relevant knowledge of numerical procedures, terms, concepts and operations					

**SECTION D: INTERMEDIATE PHASE TEACHERS' CONCEPTUALISATION OF TEACHING STRATEGIES ASSOCIATED WITH „MEANINGFUL LEARNING IN MATHEMATICS“**

Statement	Strongly agree	agree	Neutral	disagree	Strongly disagree
1. Using problem-solving as a vehicle for learning mathematics can lead to meaningful learning in mathematics					
2. Knowledge of teaching mathematics at Foundation Phase level can assist Intermediate Phase (IP) teachers adopt appropriate strategies for meaningful mathematics learning					
3. The ability to understand learners and their learning needs is important for teachers to foster meaningful mathematics learning					
4. Motivation to teach mathematics in the IP could lead to meaningful learning in mathematics					
5. Using objects from the child's own environment when teaching concepts in mathematics enhances meaningful learning					
6. Teaching mathematics is effective and meaningful when it positively impacts a learner's understanding of the subject					
7. Teaching mathematics for relational understanding is one good way to promote meaningful mathematics learning					
8. Effective teaching and learning are achieved when the learner knows, understands and can demonstrate the intended knowledge and skills after the teaching process					
9. Mathematics teachers need to demonstrate competence of a variety of teaching strategies and the knowledge of their learners' cognitions in mathematics					
10. Continuing professional teacher development is crucial for effective mathematics teaching in the IP					
11. Meaningful mathematics learning takes place in classrooms where children participate in hands-on activities, instead of merely listening to the teacher					
12. Learners engage in meaningful mathematics learning when they sit, listen carefully and watch you, the teacher,					

demonstrate and explain mathematics, before you ask them to practise what they have seen and heard.					
13. A quiet classroom is required for effective, meaningful mathematics learning to take place					
14. Effective/good mathematics teachers demonstrate the correct way of solving a problem					
15. In order to promote meaningful mathematics learning, learners should learn and master basic number facts before they do problem-solving					
16. How much students learn depends on how much background knowledge they have, that is why the teaching of basic facts is necessary for meaningful mathematics learning to take place					
17. For teachers to adopt appropriate learning strategies which promote meaningful mathematics learning, it is important to identify children who experience barriers to learning mathematics					
18. Teaching should be built up around problems with clear, correct answers, and around ideas that most students can quickly grasp.					
19. Good teaching involves knowing how learners think and, therefore, how to prepare appropriate instructional opportunities					
20. Apart from mathematical knowledge, effective teaching demands that teachers know many other aspects of teaching					
21. It is important that teachers are able to identify their learners' problems and present mathematics topics in ways that are easily understood by their learners					
22. Group work enhances meaningful mathematics learning through active learning and child interaction					
23. Good teaching attempts to foster connections between the child's informal knowledge and the abstract and arbitrary system of symbolisms					
24. The effective use of analogies, representations using symbols, giving appropriate examples, explanations or demonstrations are all very important elements in teaching for meaningful understanding of mathematics					
25. Teacher's command of the subject matter and teaching					

strategies plays a very critical role for effective mathematics teaching and learning to happen.					
26. Knowledge of teaching strategies can be generically applied to all teaching subjects					

**APPENDIX B**  
**INTERVIEW SCHEDULE**

**Have you ever attended workshops that focus on teacher development? If so did the workshops cover the teaching and learning of Mathematics?**

.....  
.....  
.....  
.....

**Which approaches do teachers use when teaching computations in mathematics?**

.....  
.....  
.....  
.....

**Did the teacher clearly explain the purpose of the class session and instructional activities?**

.....  
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**Which aspects of mathematics were most problematic?**

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**How do they resolve the problems?**

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**How do teachers identify children who experience mathematics problems?**

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**What else do they do to help children understand mathematics computations?**

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**Did the teacher have effective classroom management skills?**

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.....  
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.....

**Did the instructional strategies allow adequate learner participation?**

.....  
.....  
.....  
.....

**THANK YOU FOR PARTAKING, BAIE DANKIE, NDI A LIVHUWA**

**APPENDIX C**  
**INFORMED CONSENT**

I.....on this day of.....2017 hereby  
consent to:

Being interviewed on the topic: **AN INVESTIGATION INTO THE MATHEMATICS TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE IN THE CONTEXT OF TEACHING AND LEARNING OF MATHEMATICS IN PRIMARY SCHOOLS.** Follow-up interview if necessary

1. The use of data derived from these interviews by the interviewer in a research report as she deems appropriate
2. I also understand that;
  - I am free to end or to recall my consent to participate in this research at any time.
  - Information given up to this point of participation could however still be used by the researcher.
  - Anonymity is granted by the researcher, and the data will under no circumstances be reported in a manner that will reveal my identity.
  - I may refrain from answering questions should I see an invasion of my privacy.
  - I will be given an original copy of the agreement.

Interviewee

Interviewer

Date

.....

.....

.....

**APPENDIX D**

**PERMISSION TO CONDUCT RESEARCH**

P.O. Box 1233

Thohoyandou

0950

11 February 2016

The Principal  
Department of Education  
Vhembe District  
Sir/Madam

**PERMISSION TO CONDUCT RESEARCH**

I am a Doctor of Education at the University of Zululand and engaged in a research project in Primary schools in the Vhembe District. My research study is entitled: **“AN INVESTIGATION INTO THE MATHEMATICS TEACHERS’ PEDAGOGICAL CONTENT KNOWLEDGE IN THE CONTEXT OF TEACHING AND LEARNING OF MATHEMATICS IN PRIMARY SCHOOLS”**. The aim of my research is to explore teachers’ conceptualizations of meaningful learning in Mathematics and how learners respond to activities facilitated to enhance meaningful learning in the Intermediate Phase in the Vhembe District.

The Department of Education has approved the administration of this research. Your school has been chosen to form part of the study. I therefore request permission to conduct this research. You are further assured that data collected during the investigation will be highly confidential and will only be used for the purpose of my research. For further information about this study, please contact my supervisor, Dr A Krishnannair, at this number: 035 902 6950. Thanking you in anticipation.

Yours Faithfully

.....

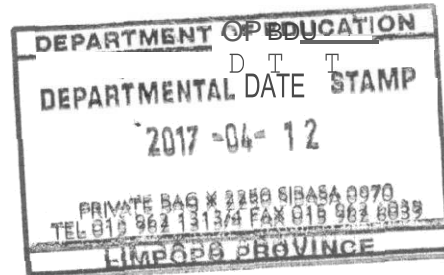
Sitsula T.M

**APPENDIX E**  
**GRANTED PERMISSION TO CONDUCT RESEARCH**  
**VHEMBE DISTRICT**



**LIMPOPO**  
PROVINCIAL GOVERNMENT  
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF  
**EDUCATION**



**REF: 14/7/R**  
**ENG: MATIBE M.S**  
TEL: 015 962 1029

MST.MSITSULA  
P.O.BOX 2794  
SIBASA  
0970

**PERMISSION TO CONDUCT RESEARCH IN VHEMBE OISTRICT.**

1. The above matter refers.
2. You are hereby informed that your request for permission to conduct research on “*An Investigation into the Mathematics Teacher’s pedagogical content knowledge in the context of Teaching and learning of Mathematics in Primary Schools*” has been granted.
3. You are expected to adhere to research ethical considerations, particularly those relating to confidentiality, anonymity and informed consent of your research subjects.
4. Kindly inform circuit managers and principals of selected schools prior to commencing your data collection.
5. Wishing you the best in your study.

  
DISTRICT DIRECTOR

12/04/2017  
DATE

## APPENDIX F

### PARTICIPANT INFORMATION SHEET

My name is Sitsula T.M. I am a researcher at University of Zululand. I would like to invite you to participate in this project; research topic is **“AN INVESTIGATION INTO THE MATHEMATICS TEACHERS’ PEDAGOGICAL CONTENT KNOWLEDGE IN THE CONTEXT OF TEACHING AND LEARNING OF MATHEMATICS IN PRIMARY SCHOOLS**”

*What will I have to do if I take part?*

If you agree to take part, we will ask you to answer some questions. There aren’t any right or wrong answers; we just want to hear about your opinions. The discussion should take about an hour at the longest. Please note that some of the questions will relate to your personal history and experiences in the Department of Education.

*Do I have to take part?*

No, **taking part is voluntary**. If you do not want to take part, you do not have to give a reason, and no pressure will be put on you to try and change your mind. You can pull out of the discussion at any time. Please note: If you choose not to participate, or pull out during the discussion, this will **not** affect your current prison sentence or your chances of parole.

*If I agree to take part, what happens to what I say?*

All the information you give us **will be confidential** and used for purposes of this study only. The data will be collected and stored in accordance with the Data Protection Act 1998 and will be disposed of in a secure manner. The information will be used in a way that will not allow you to be identified individually. DoE authorities will not be able to link any information provided to you. **However, we must inform management if:**

1. You disclose details of any potential offence within this institution, which could lead to adjudication. So, you should not mention anybody’s name during this discussion;
2. You disclose details of any offence for which you have not yet been arrested, charged or convicted;

3. Something you have said leads us to believe that either your health and safety, or the health and safety of others around you, is at immediate risk;
4. Something you have said leads us to believe that there is a threat to security.

**In these situations, we will inform a member of DoE staff, who may take the matter further.**

*What do I do now?*

Think about the information on this sheet and ask me if you are not sure about anything. If you agree to take part, sign the consent form. It will not be used to identify you. It will be filed separately from all other information. If, after the discussion, you want any more information about the study, tell your personal officer, who will contact me.

**If you feel upset after the discussion and need help dealing with your feelings, it is very important that you talk to someone right away.**

**The contact details for the person to talk to are:**

Name: Sitsula T.M

Supervisor :Dr A Krishnannair

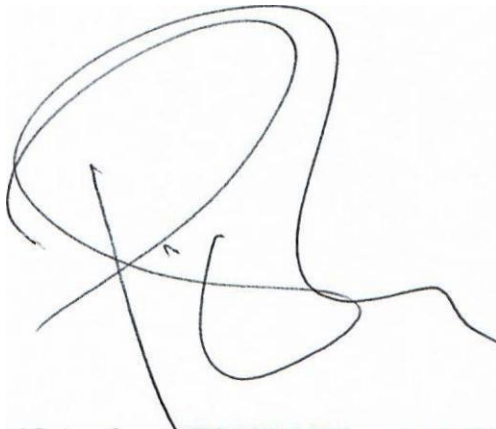
Co-supervisor: Prof N. Imenda

**THANK YOU VERY MUCH FOR YOUR HELP!**

**5 MAY, 2020**

This is to certify that I, Dr P Kaburise, of the English Department, University of Venda, have proofread the research report titled – **AN INVESTIGATION INTO THE MATHEMATICS TEACHERS’ PEDAGOGICAL CONTENT KNOWLEDGE IN THE CONTEXT OF TEACHING AND LEARNING OF MATHEMATICS IN PRIMARY**

**SCHOOLS** – by Sitsula Tshisikhawe Mary (student number: 201640100). I have indicated some amendments which the student has undertaken to effect, before the final report is submitted.

A handwritten signature in black ink, appearing to be 'P. Kaburise', with a large loop at the top and a long tail extending to the right.

**Dr P Kaburise (0794927451)**

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**Dr P Kaburise: BA (Hons) University of Ghana (Legon, Ghana); MEd University of East Anglia (Cambridge/East Anglia, United Kingdom); Cert. English Second Language Teaching, (Wellington, New Zealand); PhD University of Pretoria (South Africa)**



