

**THE UNIVERSITY OF ZULULAND**



**A STUDY OF FACTORS CONTRIBUTING TO UNDERACHIEVEMENT IN  
EXPONENTIAL AND LOGARITHMIC FUNCTIONS IN THE FURTHER EDUCATION  
AND TRAINING SCHOOL PHASE**

**BY**

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## ORIGINALITY DECLARATION

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I acknowledge that I have read and understood the University's policies and rules applicable to postgraduate research, and I certify that I have, to the best of my knowledge and belief, complied with their requirements. In particular, I confirm that I had obtained an ethical clearance certificate for my research (Certificate Number UZREC 171110-030 PGM 2017/366.....) and that I have complied with the conditions set out in that certificate.

I further certify that this thesis is original, and that the material has not been published elsewhere, or submitted, either in whole or in part, for a degree at this or any other university, except as follows:

.....NONE.....

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I declare that this research dissertation (*delete that which is not applicable*) is, save for the supervisory guidance received, the product of my own work and effort. I have, to the best of my knowledge and belief, complied with the University's Plagiarism Policy and acknowledged all sources of information in line with normal academic conventions.

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## ABSTRACT

This study sought to determine the NSC learners' level of understandings of exponential and logarithmic functions; grade twelve teachers' self- assessment of their readiness to teach exponential and logarithmic functions; the relationship between the educators' self-concept about their ability to teach exponential and logarithmic functions and the actual performance of their learners; and whether or not the educators' MCK and PCK impacted learner achievement in exponential and logarithmic functions. The study developed a conceptual framework from literature which consisted of two major components depicting **learner** and **educator** readiness. These models illustrated factors that could possibly affect the ability of the learner to succeed in understanding instruction related to exponential equations and logarithmic functions, as well as those that would prevent educators from delivering optimum instruction to learners.

This study used a mixed-methods research paradigm, as there was need to collect both quantitative and qualitative data in order to adequately answer the four research questions. The survey research design was used, and data were collected through a researcher-designed test (for learners) and a researcher-designed questionnaire for educators, focusing on their MCK and PCK. The research sample, consisting of nine school principals, nine mathematics educators, and 242 mathematics learners based in nine randomly selected schools, was drawn from a target population of high schools in the uMkhanyakude education district, KwaZulu-Natal Province, South Africa. Analysis was done using the SPSS version 23 software programme.

The results revealed that learners had basic understanding of exponential and logarithmic functions in most aspects of the topic, although their performance was border line. For the educators, although all they were suitably qualified in terms of their minimum requirements for registration with the South African Council for Educators (SACE), their performance on the same test taken by their learners was only marginally above the performance of their learners. The educators' responses to the question about their readiness to teach exponential equations and logarithmic functions were

mixed shedding some light on why many of them were unable to solve the same problems given to their learners. On the relationship between educators' self-concept about their ability to teach exponential and logarithmic functions and their learners' performance, the results showed that learners whose teachers considered themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions performed significantly lower than learners whose teachers considered themselves not to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions. The results of the questions which sought to establish the impact of educators' MCK and PCK on learner performance in exponential and logarithmic functions drew a blank, suggesting that there was no relationship between teachers' MCK and PCK, on one hand, and learner performance, on another.

## **KEY TERMS**

Learners' understanding, learners' misunderstanding, mathematical content knowledge, pedagogical content knowledge, conceptions and misconceptions, learners' achievement, educators' knowledge, knowledge of strategies, conceptual knowledge, procedural knowledge, exponential and logarithmic functions.

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## **DEDICATION**

This study is dedicated to my parents Ch. Khizer Hayat Virk (father) and Khalida Perveen Dudhra (mother) and my daughters for their encouragement and continued support from day one till the end. May this study be a source of inspiration throughout their lives?

## DECLARATION

I, Javed Khizer Mohammad, declare that this dissertation entitled, “A study of factors contributing to underachievement in exponential and logarithmic functions in the further education and training school phase”, is my own work and that all sources I have used or quoted have been indicated and acknowledged by means of complete references. It has not been submitted before for any degree or examination in any other university.

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Javed Khizer Mohammad

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8 May 2019



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Professor S.N Imenda (Supervisor)

8 May 2019

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# CHAPTER ONE

## STUDY ORIENTATION

### 1.1 INTRODUCTION

Learners' poor performance in mathematics is not a problem that is peculiar to South Africa only. Siyepu (2013:1) indicates that the poor performance of learners in mathematics is a global concern. This challenge is expressed by Khouyibaba that,

A significant number of students taking mathematics have not developed a deep conceptual understanding of the subject. They are unable to tell why a particular solution is the right one, as they only know how to follow algorithms or formulas to solve the problems. They never question why that particular method or algorithm works. This phenomenon does not only affect learners' ability to access further educational opportunities, but it also affects a country's skills sector. In rural poverty-stricken communities, education could be instrumental in enabling such communities to escape poverty. However, as learners struggle to perform well in mathematics and science subjects, rural communities are likely to remain in that state of life. (Khoyibaba, 2015: 928)

A lot has been said regarding the poor performance of learners in mathematics. Ali (2013:907) indicates, for instance, that the problem lies in the misconceptions that many learners develop about this subject, including the notion that it is a difficult subject. Consequently, learners pay little attention to mathematics, especially if they have no passion for the subject. Furthermore, research has found that the majority of teachers in the Further Education and Training (FET) phase of the school system (Grades 10-12), are under-qualified to teach the subject and therefore lack the mathematical content knowledge (MCK), pedagogical content knowledge (PCK), skills and understanding of the requirements of the Curriculum and Assessment Policy Statement (CAPS) (Shezi, 2008; Kriek & Grayson, 2009; Swanepoel, 2009). The 2013 and 2014 reports of UMALUSI (South Africa's quality assurance body for the General Education & Training school band) and the Department of Basic Education (DBE), respectively, elaborate on the problem of a lack of assessment skills and content knowledge on the part of

teachers. Additionally, the analysis of Grade 12 matriculation results of 2012, 2013 and 2014 shows that mathematics is the subject with the highest number of failures out of the approved subject units (DBE, 2013, 2014).

It is, therefore, evident that there are unending problems with learner performance in mathematics, and if the problems are not addressed the future of South Africa will be compromised. This study contended that in order for the problems to be resolved there should be studies undertaken to identify the contributing factors to the problem in specific situations. In this study the researcher investigated the factors that contribute to learners' underachievement in mathematical functions (exponential and logarithm) in the FET phase. The study focused mainly on selected rural schools in the uMkhanyakude district of the KwaZulu-Natal province.

## **1.2 BACKGROUND TO THE STUDY**

Over the years, there has been lack of agreement in the identification of factors associated with poor learner performance in mathematics in general, and in certain mathematical operations – such as exponential and logarithmic functions, in particular. In the 1970s and 1980s research focused on generic teaching skills and techniques, rather than on content. The widespread interest in and concern about educators' knowledge, what counts as mathematics educators' subject matter knowledge for teaching, and how this relates to learners' achievement have attracted a large number of research studies (McNamara, 1991; Tsang & Rowland, 2005; Cheang, Yeo, Chan, & Lim-Teo, 2007; Hurrell, 2013; Gonzalez & Gómez, 2014; Ogbonnaya, Mji & Mogari, 2014). Concurrently, attention has also been given to the issue of pedagogical content knowledge (PCK) as a possible important factor in the training and accreditation of educators.

Previous research on educators' knowledge has not reported the presence or a strong relationship between educators' knowledge and understanding of mathematics versus learner achievement (Begle, 1979; General Accounting Office, 1984; Monk, 1994; Goldhaber & Brewer, 2001). However, these studies were limited by the fact that, almost invariably, the researchers used proxy measures rather than attempting to

measure the educators' knowledge per se. Neither did they directly measure the relationship between the formal mathematics and the educators had learnt, and what they taught in their classrooms (Hill, Rowan & Ball, 2005). In undertaking their studies, most researchers were guided by Shulman's notion of PCK, and used techniques such as interviews, direct observations and educator tests. Most of these studies reported that educators lacked the command of basic mathematical knowledge. Out of these findings, the researchers argued that a strong relationship existed between educators' knowledge and learner achievement (Ball, 1990; Even, 1993; Fennema, Franke, Carpenter & Carey, 1993; Ma, 1999; Ball, et al., 2005; Hill, Rowan & Ball, 2005).

Other scholars have studied the impact of educators' knowledge on instruction (Fennema & Franke, 1992; Borko, Eisenhart, Brown, Underhill, Jones & Agard, 1992; Gencturk, 2012; Marshall & Sorto, 2012; Park, 2012; Altinok, 2013; Shepherd, 2013). In particular, in a longitudinal study carried out in rural Guatemalan primary schools by Marshall & Sorto (2012), the researchers sought to determine whether or not educator mathematical knowledge had an effect on student achievement, and reported that effective educators had different kind of mathematical knowledge. Jacob, Hill & Corey (2017) have reported "limited evidence of positive impacts on teachers' mathematical knowledge for teaching, but no effects on instructional practice or student outcomes." From these studies, there is a suggestion that strong educator knowledge could yield benefits for classroom teaching and learners' achievement.

This study investigated factors contributing to learner underachievement in exponential and logarithmic functions in selected South African schools. The study was carried out against the expectation that student enrolments in mathematics are likely to increase without a concomitant increase in qualified mathematics teachers (Centre for Development and Enterprise [CDE], 2015:3). Table 1.1 shows the estimated demand and supply of educators in coming years (CDE, 2015:15).

Table 1.1: Distribution of subjects among graduates and requirement: FET phase

FET SUBJECTS	TEACHER GRADUATED in YEAR 2009	CAPS REQUIREMENT	REMARKS
MATHEMATICAL LITERACY	3.0 %	8.7 %	SHORTAGE
MATHEMATICS	12.1 %	8.4 %	ENOUGH MORE
MECHANICAL TECHNOLOGY	0.6 %	0.2 %	SHORTAGE
PHYSICAL SCIENCES	6.5 %	5.6 %	ENOUGH

[Source, CDE, Technical Report, March 2015, p. 15]

From Table 1.1, there will be serious shortages of qualified mathematics and physical sciences educators in the schools.

In a study by the Organization for Economic Co-operation and Development (OECD), July 2015, South Africa was ranked second from the bottom in the world in terms of mathematics and science achievement. A World Economic Forum report from 2014 placed South Africa last out of a list of 148 countries in terms of mathematics and science education, and the Centre for Development and Enterprise (CDE, 2015), Supply and Demand 2013-2025, attributed some of the sector's inability to attract qualified teachers to poor societal perceptions of the teaching professions.

According to the CDE, South Africa is in dire need of good, skilled teachers. This was summed up by Ann Bernstein, the founding director of CDE, as follows:

South Africa's education system is underperforming, especially in terms of mathematics and science results. When compared to many other developing countries, our expenditure on education is not matched by the results, and research shows decisively that good teaching is vital for better results. (Bauer, 2011).

The report illustrated the severe issues in education in teaching in South African government schools, as well as in the experiences and management of education

professionals. The report also pointed out that there should be an increase in the output of trained teachers by 15 000 annually to try to come close to meeting the requirement of 25 000 new teachers per year.

While acknowledging the importance of improving the performance of learners in science and mathematics in enhancing their chances of securing jobs after leaving school, as well as closing the country's skill gap, Segar (2012) describes the fact that 2 888 schools have a shortage of mathematics teachers and 2 669 need more teachers of physical science. Segar (2012) reports that 561 schools in KwaZulu-Natal need additional mathematics teachers, 557 need mathematics literacy teachers, and 508 schools have indicated a need for more science teachers.

The South African education department figures show that in 2010 there were 25 850 ordinary schools in South Africa, suggesting that more than 10 percent of schools have a teacher shortage in mathematics and science(CDE, Technical Report, March 2015). The Department of Basic Education statistics show that 5,139 teachers are either unqualified or under-qualified and that the majority of the set each in rural KwaZulu-Natal.

The Global Information Technology Report (WEF, 2014) uses a networked readiness index (NRI) to rank the state of countries' information and communication technology. South Africa is placed 70<sup>th</sup> out of 148 participating countries on the NRI, which is made up of 10 different sub-categories from which the overall NRI ranking is drawn.

The WEF's ranking did not reflect the ability of South Africa's school pupils, but an education system that needs urgent intervention. Quality education is a crucial necessity for creating more globally competitive young adults, much-needed jobs and entrepreneurs.

It is reasonable to argue that the paucity of adequately trained teachers, in both sufficient numbers and the depth of their MCK, has contributed significantly to the reported poor mathematical learner performance. With specific reference to learner

performance, Table 1.2 shows learner performance in functions and graphs in the National School Certificate (NSC) examinations of 2013 and 2016.

Table 1.2: Analysis of functions and graphs in NSC examinations in 2013-2016

Question Number	Content	2013	2014	2015	2016
4	Hyperbola	68%	49%	60%	42%
5	Exponential and Logarithmic	31.2%	44%	26%	27%
6	Parabola and Inverse of Parabola	40.1%	37%	47%	39%

From Table 1.2, these poor results could be as a result of poor teaching, a situation that goes back to the apartheid era during which time resourcing for African schools, as well as for colleges training African teachers, was very low. This resulted in a situation whereby African teachers were poorly trained, especially for mathematics and science, after having gone through a poorly resourced school system. There are still many of the teachers trained during that time in the education system. Adler (1994:103) contends that this situation has caused more “harm to mathematics education than to any other discipline.” In particular, mathematics was badly affected because apartheid policy restricted access to the subject by Africans. Accordingly, the subject was not taught in many schools for Africans. Rather, an African child was expected to study history and other social science subjects, ostensibly because black people did not have the intelligence needed to study mathematics (Graven, 2002).

Some of the weaknesses in the teaching and study of mathematics have emerged from South Africa’s participation in international assessment (Reddy, 2006; Van der Berg, 2015). Since the dawn of the democracy, South Africa has participated in three TIMSS (Trends in International Mathematics and Science Study) projects, and has turned up at the bottom of the hierarchy in both mathematics and science (Human Sciences

Research Council [HSRC], 2011; Reddy, Meyiwa, Juan, Nkondo, Chitiga-Mabugu, Sithole & Nyamnjoh, 2014; Simkins & Paterson, 2005). In the 2003 study, only 10% of South Africa's Grade 8 candidates scored 400 and above, where 400 and below means that the candidate only possesses a basic knowledge of mathematics (Reddy, 2006).

Shulman (1986 & 1987) has suggests that expertise in teaching be described and evaluated in terms of PCK. In this regard, the view is that educators who are low in PCK have still experience difficulties in designing appropriate learning tasks, presenting explanations and posing higher order questions, recognizing common performance errors, and providing appropriate feedback. Conversely, educators who are high in PCK are taken to have the ability and capacity to accommodate learners with diverse learning needs providing them with appropriate instructions (Griffery & Husner, 1991; Rovegno, 1994 & O'Sullivan 1996; Koehler & Mishra, 2009).

Hashweh (1987) reported that teachers who operated outside their area of expertise provided incorrect and misleading explanations, especially with regard to analogies and examples; that such analogies, examples and explanations tended to carry the teachers' own misconceptions. Similarly, Tobin, Tippins & Gallard (1994) reported that teachers who taught outside their area of expertise gave faulty explanations and analogies-that tended to reinforce learner's misconceptions. Magnusson, Krajcik, & Borko, (1999) argue that an educator's repertoire of teaching strategies depends on his/her understanding of the subject. However, be this as it may, subject matter knowledge does not necessarily guarantee that the teacher will successfully translate the curriculum in the most appropriate manner for each learner (Mishra & Koehler, 2006).

An experienced mathematics educator's knowledge of mathematics is organized from a teaching perspective and is used as a basis for helping learners to understand specific concepts (Cai, Kaiser, Perry & Wong, 2009; Cai & Ding, 2015). A mathematician's knowledge, in contrast, is organized from the 'structure of the discipline' and research perspectives, which are used as the basis for developing new knowledge in the field (Fennema & Franke, 1992; Richland, Stigler & Holyoak, 2012). In South Africa, the Curriculum Assessment Policy Statement (CAPS) aims to produce learners that are

able “to identify and solve problems and make decisions using critical and creative thinking; work effectively as individuals and with others as a member of a team; organize and manage themselves and their activities responsibly and effectively; collect, analyse, organize, and critically evaluate information; communicate effectively using visual, symbolic and /or language skills in various modes”(DBE, 2011a: 3). Accordingly, problem-solving and cognitive development are central to mathematics teaching in the country. Collaboration and working together are the recommended ways of learning. Learners need to spend time with each other, and be allowed to talk with one another in group work.

### **1.3 STATEMENT OF THE RESEARCH PROBLEM**

Over the years, there has been lack of agreement in the identification of factors associated with poor learner performance in mathematics in general, and in certain mathematical operations – such as exponential and logarithmic functions, in particular. Previous research on educators’ knowledge (Begle, 1979; General Accounting Office, 1984; Monk, 1994; Goldhaber & Brewer, 2001) has failed to establish a strong relationship between learners’ achievement in mathematics and educators’ knowledge of the subject. More specifically, exponential and logarithmic functions are pivotal mathematical concepts that play a central role in high school mathematics. Other related concepts such as calculus, algebra, differential equations and complex analysis are important to the overall understanding of exponential and logarithmic functions. This study has been motivated by the state of South African learners’ poor performance in mathematics, as evidenced in the TIMSS (1995, 1999, 2003, 2009), ICAS (2006), HSRC (2008), SAQMEQ (1999) assessment tests, DBE NSC Examinations results (2013, 2014, 2015, 2016) and DBE Technical reports(2013, 2014, 2015, 2016). Tables 1.3 and 1.4 show South African learners’ consistent poor performance in mathematics and physical science nationally and provincially, respectively.

Table 1.3: Learners performance in mathematics and physical science in NSC Examinations in South Africa

SUBJECTS	2013	2014	2015	2016
MATHEMATICS	59.1%	53.5%	49.1%	51.1%
PHYSICAL SCIENCE	67.4%	61.5%	58.6%	62.0%

Table 1.4: Learners performance in Mathematics and Physical Sciences in NSC Examinations in Province of KWAZULU-NATAL

SUBJECTS	2013	2014	2015	2016
MATHEMATICS	40.09%	40.7%	33.32%	37.9%
PHYSICAL SCIENCE	42.67%	55.8%	51.81%	57.8%

Table 1.5 shows learner performance in mathematics and physical sciences between 2012 and 2016 in the uMkhanyakude education district.

Table 1.5: District analysis of mathematics and physical science NSC results: **2012-2016**

SUBJECTS	2012	2013	2014	2015	2016
MATHEMATICS	40.10%	46.65%	37.43%	31.1%	37.01%
PHYSICAL SCIENCE	53.40%	64.02%	56.51%	66.32%	53.64%

From the Table 1.5, the low performance by South African learners in all mathematical domains indicates that there is a nationwide crisis in mathematics achievement. The data from the ICAS tests also evidences the learners' underperformance. Thus, this study has examined possible factors that may have contributed to learner's underachievement in exponential and logarithmic functions in the FET phase(Grade-12),focusing specifically on understanding and misunderstanding of content errors and misconceptions on the part of both educators and learners in the Ubombo Circuit, UMkhanyakude district, Kwazulu-Natal province.

## **1.4 AIM OF THE STUDY**

From the statement of the problem, the issue that requires a closer scrutiny is to examine the possible factors that contribute to learners' poor performance in exponential and logarithmic functions in the FET phase (GRADE-12). Thus, the aim of this study was to investigate the contributing factors to learner's underachievement in Exponential and Logarithmic functions in the Ubombo Circuit, UMkhanyakude, KwaZulu-Natal, and Republic of South Africa.

## **1.5 RESEARCH OBJECTIVES**

More specifically, this study sought to address the following research objectives:

- 1.5.1 Explore **Grade-12 FET** learners' understanding and misunderstanding in exponential and logarithmic functions.
- 1.5.2 Determine whether or not grade twelve teachers considered themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions.
- 1.5.3 Determine whether or not there is a relationship between educators' self-concept about their ability to teach exponential and logarithmic functions and the actual performance of their learners on these two mathematical constructs.
- 1.5.4 Find out whether or not educators' pedagogical content knowledge impacts learner achievement in exponential and logarithmic functions.

## **1.6 RESEARCH QUESTIONS**

This study leads to the following research questions:

- 1.6.1 What are the National Senior Certificate learners' main understandings and misunderstandings of exponential and logarithmic functions?
- 1.6.2 Do grade twelve teachers consider themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions?
- 1.6.3 Is there a relationship between educators' self-concept about their ability to teach exponential and logarithmic functions and the actual performance of their learners on these two mathematical constructs?

1.6.4 Does educators' pedagogical content knowledge impact learner achievement in exponential and logarithmic functions?

## **1.7 RESEARCH HYPOTHESES**

The first two research objectives / questions above are exploratory in nature, so they were not addressed through hypothesis testing. Following below are two hypotheses corresponding to research objectives/questions three and four:

### **Hypothesis 1**

H<sub>0</sub>: Learners taught by teachers who consider themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions will perform the same as learners taught by teachers who consider themselves not to be suitably qualified, knowledgeable and able to teach these two constructs.

H<sub>1</sub>: Learners whose teachers consider themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions will perform significantly higher than learners whose teachers consider themselves not to be suitably qualified, knowledgeable and able to teach these two constructs.

### **Hypothesis 2**

H<sub>0</sub>: There will be no significant difference in the performance of learners taught by teachers with high pedagogical content knowledge and those taught by teachers with low pedagogical content knowledge in exponential and logarithmic functions.

H<sub>1</sub>: Learners taught by teachers with high pedagogical content knowledge will perform higher than those taught by teachers with low pedagogical content knowledge in exponential and logarithmic functions.

## **1.8 CONCEPTUAL FRAMEWORK**

Looking at the aims and principles of CAPS, which are largely rested on Constructivism, the researcher surmised that for this study, the applicable theories which could be used as lenses to evaluate the contributing factors toward learners' underachievement in exponential and logarithms functions include Constructivism and Information Processing (Ernest, 2007; Dede, 2008; Yilmaz, 2011).

### 1.8.1 Constructivism

Constructivism is a direct product of the theory of cognitive development, and opines that learners arrive at meaning making by activity selecting and cumulatively constructing their own knowledge through both individual and social activities. Generally, researchers (e.g. Dede, 2008; Tobias and Duffy, 2009; Yilmaz, 2011; Kola, 2017) attribute constructivist thinking to the works of Vygotsky, Dewey, Piagét and Bruner. In particular, Kola (2017: 59) averred that “constructivist theories have their roots in Piaget and focus on the active character of the learner, interacting with the environment either singly or with others.” In a similar vein, Brown, Collins & Duguid (1989) earlier posited that knowledge was situated in the activity of the learner and was a product of that activity and the context and culture in which it occurred.

With regard to the assumptions made by constructivists, Jonassen (1991: 11) explained that “constructivism claims that reality is more in the mind of the knower, that the knower constructs a reality, or at least interprets it, based upon his or her apperceptions.” As such, constructivists believe that:

“... There is no real world, no objective reality that is independent of human mental activity ... our personal world is created by the mind, and therefore, no one world is any more real than any other. There is no single reality or any objective entity that can be described in any objective way; rather, the real world is a product of the mind that constructs that world. A less radical form of constructivism holds that the mind is instrumental and essential in interpreting events, objects, and perspectives on the real world, and that those interpretations comprise a knowledge base that is personal and individualistic. The mind filters input from the world in making those interpretations.” (Jonassen 1991: 11).

In concurrence, Dede contended as follows:

Constructivist theories of learning assume that meaning is imposed by the individual rather than existing in the world independently. People construct new knowledge and understandings based on what they already know and believe, which is shaped by their developmental level, their prior experiences, and their socio-cultural background and context.”Dede (2008, p 50)

According to Ernest (2007), constructivism is a theory of knowledge with roots in philosophy, psychology and cybernetics. Cybernetics can be defined as “science of communication”. Ernest also calls this theory as “new philosophy of mathematics”. According to Van de wall (2007: 22), the basic tenet of constructivism is simply that children construct their own knowledge; that children construct or give meaning to things they perceive or think.

Njisane(1992) stated that to ‘construct’ involves the work of mental structures known as schemata, which have to undergo change in the process of learning. As such, from the teaching point of view, constructivism requires a shift from the traditional approach of direct teaching to facilitation of learning by the mathematics educator. Mathematics learning by negotiation has to replace teaching by imposition.

From the mathematics and science education point of view, constructivism is explained more from the point of view of Piagét’s work than from the social constructivist viewpoint (Imenda, 2017).From this theoretical lens, learners learn mathematical functions through the two complementary processes of *assimilation* and *accommodation* – where the former refers to a process in which learners have the ability to notice similarities and match the new ideas to those they already possess (Niess, 2005; Woo & Reeves, 2007; Özdemir & Clark, 2007; Yang & Leung, 2011). Where assimilation is involved, there is no need to reframe existing cognitive structure because the new information easily tags onto existing ones. This theory will answer the research questions 1 and 4.

*Accommodation* is a process of altering or reframing existing cognitive structures or ideas that don’t fit into existing schemata. It is the process of reframing one’s mental representation to fit the new ideas/knowledge. Accommodation is facilitated by reflective thought and results in the changing or modification of existing schemata. The second research question will be addressed through the notion of accommodation.

Therefore, in constructivism the learner brings a wealth of personal knowledge and understanding from previous experiences, as well as interests, intentions, assumptions and motives to the learning and teaching situation at hand. The totality of all these experiences and dispositions will determine the course and quality of the learning that

may take place. Thus, as a perspective about learning, constructivism places a lot of emphasis on understanding, rather than memorization and regurgitation of information from previous lessons (Tyndale, 1999). Accordingly, the constructivist view of learning contrasts sharply with learning approaches based on behaviourism and constructivism which places greater emphasis on how well the teacher explains things to learners (Simon & Schaffer, 1993). In the constructivist perspective, knowledge cannot be transmitted because it is an activity which takes place in the mind of the learner. Accordingly, constructivist teaching is about helping learners to actively construct knowledge by assigning to the tasks that enhance this process. In this regard, Simon and Schaffer stated:

The educator's role in initiating and guiding mathematical negotiations is a highly complex activity that includes highlighting conflicts between alternative interpretations or solutions, helping learners to develop productive small-group collaborative relationships, facilitating mathematical dialogue between learners, implicitly legitimizing selected aspects of contributions to discussion in light of their potential fruitfulness for further mathematical constructions, redescribing learners' explanations in more sophisticated terms that are nonetheless comprehensible to learners, and guiding the development of taken-to-be shared interpretations when particular representational systems are established." Simon and Schaffer. (1993: 221).

Therefore, according to Wood (1995: 177), a constructivist educator must do the following:

An educator must provide instructional situations that elicit subject appropriate activities, which must view learners' conceptions from their (the learners') perspectives. Also an educator sees errors as reflecting their learners' current level of development and recognizes that substantive learning occurs in periods of conflict, surprise over periods of time, and through social interaction.

The above having been said, it is possible that using constructivism in teaching of exponential and logarithmic functions could go a long way towards resolving some of the difficulties which impede learners' understanding of this topic.

### **1.8.2 Information Processing Theory**

Through continuing developments in information processing, cognitive psychology has assisted people to have a better understanding of how thinking, reasoning and learning take place in humans (Forgas & George, 2001; Kühne & Schemer, 2015). The notion of information processing likens human cognitive functioning to the operations. Accordingly, it is believed that teaching and learning of mathematical functions in the classroom can be taught effectively by the use of information processing theory.

### **1.9 SIGNIFICANCE OF THE STUDY**

The importance and significance of this study was embedded within the notion of attempting to discover possible reasons behind learners' underachievement in exponential and logarithms functions, within the context of the content understanding and misunderstanding encountered in the learning process. It was thus envisaged that the study would contribute to the mathematics education literature by extending the notion of "understanding and misunderstanding" to the teaching and learning of exponential and logarithms functions. In so-doing, it was hoped that the study would open new possibilities for improving the teaching of exponential and logarithms functions – thereby enhancing learners' achievement in exponential and logarithms functions. Furthermore, it was envisaged that both the methodological approach used in this study and the findings arising from it would also be beneficial to both the community of researchers and educators in a number of ways, including guiding professional development of mathematics educators with reference to exponential and logarithms functions.

### **1.10 SCOPE AND LIMITATIONS OF THE STUDY**

The study locale consisted of four educational management wards under the Ubombo Circuit Management Cluster (CMC), namely Big Five False Bay, Jozini, Mkuze and Ntshongwe. Ubombo CMC total population comprises 32 public high schools, 72 mathematics educators teaching mathematics from grades ten to twelve and 4304 learners registered for mathematics in the FET school phase (Grade-12). However, the sample population will be selected randomly, but taking into consideration the

representation of each ward. Further, schools will be selected with high, low and average matric pass results to avoid the factors of bias in the collection of data.

## **1.11 RESEARCH METHODOLOGY**

In this section, the research paradigm, research design, sample size, data collection instrument, data analysis, instrument reliability and validity are outlined.

### **1.11.1 Research Paradigm**

This study was guided by a combination of both the Positivism and Interpretative paradigms. The former was used to test the research hypotheses, while the latter will be applied to the mathematics educators' interpretations of content knowledge (second research question), learners' conceptions and misconceptions related to exponential and logarithmic functions (first and third research questions), as well as to the analysis of the fourth research question dealing with the educators' pedagogical. Thus, this study used the mixed methods research approach.

### **1.11.2 Research Designs**

This study followed a descriptive survey research design to obtain quantitative and qualitative data to ascertain the existence of association relationships between / among a set of variables which this study addressed. The survey research design was deemed to be suitable for this study as it sought to identify factors contributing to learner poor performance in exponential and logarithm functions, as well as the perceptions of the teachers about the topic, from as wide a research sample as possible.

### **1.11.3 Sample Size, Selection and Justification**

This study was carried out in twelve (12) randomly selected high schools in the Ubombo Circuit, UMkhanyakude School District. From each participating high school, three grade 12 educators were randomly selected to participate in the study, and all mathematics learners in Grades 10 to 12 in the participating schools constituted the research sample. Altogether, the research sample consisted of 12 participating schools, 36 educators and 810 learners.

#### **1.11.4 Data Collection Instrument**

A researcher-designed test for learners was used for data collection from both the learners and their teachers. Whereas the learners were asked to solve the problems in the test, their teachers were asked to provide a marking key and indicate how their learners would have attempted the questions. The responses provided by the teachers were complemented by interviews. The test comprised items which gave learners an opportunity to think about their answers, so that the researcher would be able to establish the participants' (both learners and teachers) levels of understanding with respect to exponential and logarithmic functions and where possible tease out their conceptions and misconceptions. Many of the tasks in the tests involved the skill of "translation" that is, reasoning between graphical and algebraic representations. This enabled the researcher to address research questions 1 and 2.

As indicated above, one of the objectives of this study was to ascertain the extent to which educators could predict how their learners would will respond to the various questions in the test, as well as their reasoning. The reason for this was to investigate the teachers' understanding of the subject content and pedagogical content knowledge (PCK). This is a good approach in both in –service and pre-service teachers' training programmes because when educators think alone, or in groups, about possible learners' responses to a question, they tend to better understand learners' conceptions about the concept. Thus, the interviews were conducted to seek deeper reflections and explanations regarding the teachers' PCK as it related to exponential and logarithmic functions. This information was used to address the third and fourth research questions.

In addition to the test, documentary analysis was also conducted. The documents consulted included teachers' master portfolio records, planning and learners' portfolios containing various assessment tasks. This was part of the triangulation and an attempt to corroborate the evidence collected through interviews. It was felt that this additional information would add further insight to the information collected through tests and interviews and provide comprehensive answers to the third and fourth research questions.

### **1.11.5 Data Analysis**

A data analysis was carried out using quantitative data and qualitative analysis.

#### **1.11.5.1 Quantitative Data**

Quantitative data consisted of marks scored on the test on the part of both learners and teachers, against a marking memorandum prepared for this purpose. One mark was allocated for the intended answer and 0 for a “wrong’ answer. Both descriptive and inferential statistics were used in quantitative data analysis.

#### **1.11.5.2 Qualitative analysis**

Content analysis was used to enable the researcher to document learners’ reasons for their responses to each question-in the process also identify possible misconceptions in the exponential and logarithm functions. The MCK of the teachers was determined from their scores in the content part of the test. In each question, a teacher was allocated a score of 1 mark if he/she gave the intended response to the question, and 0 if the answer was mark. The total scores for the learners and teachers were recorded and used in the ensuing statistical descriptions and manipulations.

When regard to the mathematical knowledge of the educators the specific teaching strategies they used for teaching the concept of exponential and logarithmic functions, this was based on their responses in the interviews.

#### **1.11.6 Instrument Reliability and Validity**

The researcher designed the two instruments (test and interview schedule) and then requested a panel of experts (three experienced mathematics teachers and two university professors of mathematics) to edit them to ensure accuracy and appropriateness. Subsequently, the test and interview schedule were pilot-tested and the results analysed. From the analysis of the pilot study, involving ten learners and three teachers, the two instruments were adjusted to eliminate any ambiguity (Cohen, L., Manion, L., & Morrison, K.(2005). At the stage of data analysis, the reliability of the

test was calculated to ensure the respectability of the results. This is reported upon in chapter four, together with the results to the research questions.

## **1.12 EXPECTED CONTRIBUTIONS TO KNOWLEDGE**

It was expected that the findings of this study would contribute significantly to the body of knowledge in the following four distinctive ways: Firstly, it was envisaged that the study would contribute to the general literature in mathematics education, extending the concepts of PCK and MCK to the teaching and learning of exponential and logarithmic functions. This would open up new possibilities for improving the teaching of exponential and logarithmic functions and enhance learners' achievement in exponential and logarithmic functions. Secondly, one of the strategies used in the study was to ask the educators to predict both their learners' responses to question items and their reason(s) for their answers as a means of investigating the educators' PCK, and their understanding of their learners' thinking. This was a novel way of investigating both the educators' own understanding of exponential and logarithmic functions and their understanding of their learners. Thirdly, by using a set of questions to assess learners' achievements, this facilitated the assessment of higher order thinking and conceptual understanding. Fourthly, the tasks used in this research promoted deep mathematical understanding, which could be very useful in guiding professional development of mathematics educators with reference to exponential and logarithmic functions, in future.

## **1.13 DEFINITION OF TERMS**

For the purpose of this study, the following terms were understood as defined below:

### **1.13.1 Teacher Readiness**

Teachers' readiness was measured in terms of knowledge, skills and attitude (Wong, 2002), and how this was facilitated in terms of teachers' professional growth.

### **1.13.2 Pedagogical content knowledge**

The pedagogy of the content of the subject varies from educator to educator. In this study, pedagogy content knowledge (PCK) was constructed as “a blending of content and pedagogy in order to enhance the understanding of how particular topics, problems, or issues are organized, represented, and adapted to diverse interests and various levels of abilities of learners” (Kathirveloo , Puteh & Matematik, 2014: 2).

### **1.13.3 Mathematics content knowledge**

The knowledge of subject content, in this case the subject is mathematics i.e. the educators command on mathematical content, is important (Ball et al., 2001; Ma, 1999).

### **1.13.4 Assimilation:**

This refers to a cognitive process whereby new information is incorporated into a person’s existing knowledge structures (Piaget, 1983).

### **1.13.5 Accommodation:**

Accommodation refers to the process of reframing, changing or modifying one’s existing mental representation of a concept in order to fit the new ideas/knowledge (Piaget, 1983).

### **1.13.6 Misconception:**

A Misconception is an incorrect aspect of a learner’s understanding of a concept which is stable over time and may hinder his/her future learning (Leinhardt, et al., 1990; Alwyn, 1989). Misconceptions are therefore impediments to meaningful learning, more especially in the learning of mathematics. This is a natural stage of conceptual development.

## **1.14 ETHICAL MEASURES**

This research will be guided by the principles on ethical issues suggested by Fouke and Mantzorou (2011).

### **1.14.1 Informed Consent**

Permission and consent letters were sent to gain permission from the Department of Education for using the schools in the UMkhanyakude district. The school principals and their respondents were served with consent letters describing the research purpose stating that participation would be voluntary and the information gathered from them would be confidential.

### **1.14.2 Protection and Welfare of Respondents:**

Interviews were conducted at respondents' identified places. Confidentiality was observed by using pseudonyms in the presentation and discussion of results. To ensure privacy and anonymity of respondents, no names were recorded on the questionnaires (Fouke & Mantzourou, 2011).

### **1.14.3 Rights to Participate In the Study:**

Under no circumstances were respondents forced to participate or continue unwillingly with the study. By adhering to this principle, respondents were assured they could withdraw at any point in time they felt so, without being subjected to any form of discrediting.

## **1.15 ORGANISATION OF THE STUDY**

Chapter one of this research presents the introduction to the study, problem statement, research questions, and the aim of the study as well as the intended contribution of the study to the body of knowledge.

Chapter two contains relevant literature review, the hypothesis of the study and definition of terms of the research. The literature review is presented in three sections; with each section focusing on a particular aim of the study.

Chapter three discusses the methodology of the study. This includes the search design and the sample design, research instrument, its scoring, data analysis, and description

of procedure. Also, ethical consideration and the foreseen problems are discussed here. Chapter four contains the data presentation and their interpretation.

Chapter five discusses of the results of the research. The discussion links the findings to the literature review through the research questions.

## **CHAPTER TWO**

### **LITERATURE REVIEW AND THEORETICAL FRAMEWORK**

#### **2.1: INTRODUCTION**

This chapter presents a review of literature on the learning and teaching of exponential and logarithmic functions in line with the research questions, culminating in the construction of a conceptual framework for the study. More specifically, the presentation of the major topics will start with a reflection of the learners' understanding and misunderstanding of exponential and logarithmic functions, followed by a look at the issues and theories related to this study. These two topics are followed by an examination of the instructional strategies pertaining to the teaching of exponential and logarithmic functions and the role of educators' knowledge regarding learners' understanding of exponential and logarithmic functions. The literature review ends with a look at empirical findings pertaining to educators' pedagogical knowledge and learners' achievements in exponential and logarithmic functions.

#### **2.2 LEARNERS' UNDERSTANDING AND MISUNDERSTANDING IN EXPONENTIAL AND LOGARITHMIC FUNCTIONS**

##### **2.2.1 Understanding and misunderstanding in mathematics**

The South African Department of Education's Common Core Standards in mathematics stress the importance of conceptual and procedural understanding as a key component of mathematical expertise. Unfortunately, a mere publication of these standards does not necessarily offer sufficient guidance to most teachers (Wiggins, 2014). It seems many mathematics educators cannot differentiate between conceptual and procedural knowledge. Many teachers think that if learners know all the computations, definitions and rules, then they possess the requisite understanding thereof.

Here are few standards of understanding in mathematics: students understand connections between counting and addition and subtractions (e.g. adding two is the same as counting on two). They use properties of addition to add whole numbers and to

create and use increasing sophisticated strategies based on these properties to solve additions and subtraction problems. Learner's thinking in mathematics, to solve the mathematical problems in certain context, sometimes reveals misunderstanding. In mathematics, a learner misunderstands a mathematical concept when he/she captures incorrect solutions to solve mathematical problems from wrong and faulty thinking. Sometimes, learners use shortcuts that remove developmental mathematical concepts (Baumert Jurgen: 2010). Leinwand (2015) posts that effective educators see wonderful learning opportunities in the mistakes and confusion of their learners. As such, they pedagogically position themselves, in their lesson planning and implementation, so as to anticipate possible misconceptions that their learners might have. Accordingly, they arm themselves with the necessary array of strategies to address common misunderstandings, before they expand, solidify, and undermine confidence. Factors that are associated with learner achievement, progress and understanding include certain professional and personal characteristics educators (Imenda, 2017), the educational context, the socioeconomic class of the student's family and the teacher's own knowledge (Akiba; LeTendre; Scribner, 2007; OECD, 2009, 2010; Olfos, Goldrine, & Estrella, 2014; Del Sol, 2011).

### **2.2.2 Conceptual understanding in mathematics**

Teachers who possess the necessary conceptual understanding in mathematics tend to also have the ability to interact with learners in ways that are engaging and allow for meaningful discourse in the classroom. For this to happen, the educator should plan and select appropriate mathematical tasks which can trigger meaningful discussions about, inter alia, the meaning of mathematics and what doing mathematics entails (NCTM, 1991: 24).

### **2.2.3 Procedural understanding in mathematics**

The understanding of procedural knowledge in mathematics entails being able to relate ideas presented in different forms. Thus, a teacher who utilizes a variety of mathematical representations and ways of seeking solutions to mathematical problems is likely to enhance his/her learners' ability to acquire procedural knowledge of

mathematics. Furthermore, in the process of dealing with different transformations of a mathematical problem the learner begins to establish important relationships between and among different mathematical concepts and processes (Hanushek & Woessmann, 2009)

#### **2.2.4 Classification of errors**

In mathematics, errors result from very complex processes. Overall, errors committed by learners are a reflection of their lack of adequate understanding of mathematical concepts and processes. Understanding the concept of exponential function is important for students in mathematics. The National Council of Teachers of Mathematics' Principles and Standards NCTM, (2000), and the Common Core State Standards for Mathematics (2010) advocate for the inclusion of exponential functions in the curriculum in ways that emphasize real world contexts.

Research suggests that learners experience difficulty developing deep understandings of exponential functions (Kasmer & Kim, 2012, Melendy, 2008; Strom, 2007). In order to improve students' understanding, it is important that teachers are well prepared in the topic. Not only is it important that teachers have a solid understanding of the material they teach, it is also important for them to research the topic to find misconceptions and weaknesses before teaching their students. In addition to teachers knowing their learners' area of weakness, they also need to have resources to use in their daily lessons. Researchers have provided activities teachers can use for exponential modeling (Howard, 2010; Kennedy & Vasquez, 2003). For instance, Howard (2010, p. 1) presented a ready-to-implement laboratory that explored "the connection between exponential relationships and the depreciation of cars and trucks." From this, Howard (2010, p.1) identified the following six error types in high school mathematics: (i) misused data, (ii) misinterpreted language, (iii) logically invalid inferences, (iv) distorted theorems or definitions, (v) unverified solutions and, lastly (vi) technical errors.

Kennedy and Vasquez (2003) provided numerous examples teachers could use in their classroom to teach exponential equations. For example, they use population modeling for exponential growth and basketball deflating to show exponential decay. There is also

research focused on exponential laws, exponential modeling, and logarithmic equations. However, there is not much research specific to the underachievement of learners with exponential and logarithmic functions. It is for this reason that this study aimed to determine the factors associated with learners' underachievement in exponential and logarithmic functions.

Teachers need to be prepared to spend time on sharing common mistakes and misconceptions that are made by learners. When teaching a new topic to learners, teachers need to show them the common mistakes they have seen from past learners. In addition, teachers should research the topic before teaching it. This will allow them to discover common misunderstandings of the specific lesson they are about to teach. If learners are shown the mistakes others have made, they are more likely to not make the same mistakes. Furthermore, learners learn from their mistakes and misconceptions.

Many school curricula define a logarithm as an exponent, and a logarithmic function as the inverse of an exponential function. Not only do learners struggle with exponential laws and functions but research has shown that learners struggle with the topic of logarithmic functions (Berezovski, 2007; Chua, 2006; Gamble, 2005; Wood, 2005). In particular, Berezovski (2007) found that not only do learners lack conceptual understanding of exponential and logarithmic functions, but so do pre-service teachers. A teacher's mathematical knowledge has a strong impact on students' understanding and achievements, so it is important to try and mend this learning gap and find where learners are making mistakes when it comes to logarithms and adapt how teachers approach the topic.

### **2.2.5 Misconceptions and misunderstanding in mathematics**

Among factors influencing misconceptions and misunderstandings in mathematics are:

- 2.2.5.1 Difficulties in perception of mathematical language (Kane, 1967)
- 2.2.5.2 Insufficient subject matter knowledge (Mayer, 1992)
- 2.2.5.3 Problem posing (Butts, 1980)
- 2.2.5.4 Language deficiencies (Mestre, 1988)

2.2.5.5 Ineffective text processing (Nathan, Knitsch, & Young, 1992)

2.2.5.6 Lack of effective reading strategies in problem solving (Shuard & Rotthery, 1988)

There are many things that learners have to think about, such as formulas, the relevance of what is being done, boredom, and enjoyment, which form part of their attitudes and thinking about mathematics. Thus, it may not be enough merely to talk to them about misconceptions because conceptual change involves transforming cognitive structures through accommodation and assimilation; a learner's prior understanding must give way to a new way of seeing and understanding things.

### **2.2.6 Why study misconceptions?**

One major reason for studying misconceptions is that, once rooted in the student's memory, they are extremely hard to correct or erase. The situation is somewhat more complex, though. Researchers' interest in student concepts has been provoked by numerous studies. A study conducted by (Castillo-Magayon & Tan, 2018) indicated that:

- a) Before formal study, learners develop and hold on to their own systems of beliefs about mathematics.
- b) Typically, these belief systems differ from the content of the school curriculum.
- c) These belief systems are remarkably consistent across ages, abilities, and nationalities.
- d) The belief systems are deeply rooted and do not easily change through traditional instruction.

Past researchers suggest that repeating a lesson or making it clearer will not help students who base their reasoning on strongly held misconceptions (Bryant & Bryant, 2016; Fyte & Brown, 2017; Stylainides & Stylainides, 2018). Students tend to remain emotionally and intellectually attached to their misconceptions, partly because they have actively constructed them and partly because they give ready methodologies for solving various problems. However, the challenge to teachers is that these misconceptions tend to interfere with intended learning when learners apply them to

new experiences. This is why it is important to identify learners' mis – or alternative conceptions and to re-orientate them to use the right mathematical thinking patterns. To many people, mathematics is view as a basket of meaningless rigid rules and mystical procedures that are unrelated to each other, but which learners are required to master at all cost. For this reason, mathematics is seen and believed by many to be difficult and too demanding. Accordingly, many learners believe that it is socially acceptable to do poorly in mathematics. There is also a view that if mathematics is not required in one's job, then there is no need for one to take mathematics at school-yet, the knowledge of mathematics has become so pervasive in all facets of life nowadays.

Over the years, researchers, lecturers and teachers have developed and applied many instructional strategies in the classroom, with a view of enhancing the learning of mathematics. However, despite this, learners still experience many difficulties learning mathematics. One of the reasons advanced for this difficulty has been singled out as being 'the language of mathematics' itself. This comes from the difficulty exhibited by learner in interpreting mathematical concepts, manifested mainly in poor understanding of rule restrictions, over generalization, misinterpretation of concepts and incomplete application of rules, (Cuevas, 1984).

Another factor for poor performance in South African schools is the language of instruction. Subjects such as mathematics are taught to learners in English, while the majority of them have limited English proficiency and they struggle to understand mathematics lessons owing to poor everyday English vocabulary and a poor understanding of mathematical discourse.

Kiat (2005) and Wiggins (2014) opine that two ways in which learner's exhibit conceptual errors are in their failure (a) grasp the concepts in problem and (b) appreciate the relationships in a problem. From a cognitive standpoint, many conceptual errors are attributable to poor understanding of words and their uses while, according to Olivier (1989) interpretation errors, occur when students wrongly interpret a concept due to the over-generalization of the existing schema. To the exact, learners might know mathematical formula but they are unable to apply them appropriately. Alternatively,

linear extrapolation errors occur when students over-generalize the property  $f(a+b) = f(a) + f(b)$ , which applies only when  $f$  is a linear function, to the form  $f(a*b) = f(a) * f(b)$ , where  $f$  is any function and  $*$  any operation (Woessmann, 2010). In this regard, linear extrapolation errors are related to narratives as students fail to understand the restrictions of the rules. According to Higgins and Mcoah (2010), procedural errors occur when learners fail to carry out manipulations of algorithms, even when they understand the related concepts, while procedural errors pertain to routines whereby students fail to follow repetitive patterns in interlocutor actions. With regard to arbitrary errors, Baumert et al. (2010) opine that these occur when learners behave illogically and fail to take account of the constraints laid down in what was given. A Commognitive justification is where learners have poor visual mediators, as is the case of differentiation where learners do not know the appropriate formulae to be applied.

### **2.2.7 Formal and informal knowledge in mathematics**

There are three attributes that relate to formal learning, namely (a) it takes place in an organized, structured, educational institution, (b) it pertains to a system of interrelated definitions and proofs experiments and arguments, and (c) it is linked with written methods. In contrast, informal knowledge refers to more tentative intuitive conjectures and mental structures and is rooted in one's personal actions. A disjuncture often occurs between learners' formal and informal knowledge, and much of what characterizes mis-or alternative conceptions is attributable to this gap – the narrower the gap, the easier it is for the two types of knowledge to co-exist or even merge. There is also a corpus of research that has sought to connect misconceptions, as well contributions of acceptance of misconceptions about mathematics, mathematical self-concept, and arithmetic skills to mathematics anxiety. In a study involving ninety-two adult learners (16 males and 76 females) aged 18-57 years old, with median of 27, the results showed that the acceptance of misconceptions and mathematical self-concept were significantly related and arithmetic skills were significantly related to the participants' statistics course performance. Older learners returning to school after several years' absence were the ones most debilitated by negative attitudes toward mathematics. The researchers concluded that mathematics anxiety involved a

mechanistic, non-conceptual approach to mathematics, a low level of confidence and a tendency to give up easily when answers were not immediately apparent (Grimpe & Hussinger, 2013).

To get rid of misconceptions and misunderstandings teachers must help learners to reconstruct correct conceptions. Estrella and Del Sol (2011) described an effective inductive technique for these purposes. That technique can be categorized into three steps:

- 1) Probe for and determine qualitative understanding.
- 2) Probe for and determine quantitative understanding
- 3) Probe for and determine conceptual reasoning

### **2.2.8 Misconceptions in exponential and logarithmic**

Research has indicated that learners' misconceptions and misunderstandings in logarithmic and exponential functions are deeper than in algebra and algebraic functions. Learners lack in basic definitions and the use of fundamental properties of exponential and logarithm. A few of these misconceptions are

#### **2.2.8.1 Exponential Properties**

Learners are troubled by negative and fractional exponents and are unable to correctly simplify exponential equations e.g.

$$\frac{am}{a - n}$$

$$a^m a^{-n}$$

$$a^{-m} a^{-n}$$

#### **2.2.8.2 Exponential functions**

Learners' do not work well with exponents' e.g.

2.2.8.2.1 Linearizing exponential rules such as writing

$$e^{a+b} Ase^a + e^b$$

2.2.8.2.2 Similarly, we often see  $e^{ab}$  written as  $e^a e^b$

### 2.2.8.3 Logarithmic properties

2.2.8.3.1 Often learners will write

$$\text{Log}_x\text{-log}_y = \frac{\log x}{\log y} \text{ instead of the correct expression } \ln \frac{x}{y}$$

2.2.8.3.2 Learners will also linearize rules and produce such as log

$$\ln (a+b) + \ln (a) + \ln(b) \text{ and } \ln (2x) = 2\ln x$$

### 2.2.8.4 Logarithms solving equations

When solving logarithmic equations, learners tend to forget to check if the answers fall within the domain or, when they get two answers and the first one appears to be correct, they tend to automatically eliminate the second choice

2.2.8.4.1 Example

Solve  $\log_2(x - 4) = 3 - \log_2(x + 3)$

Solution  $\log_2(x - 4) = 3 - \log_2(x - 3)$

$$\log_2(x - 4) + \log_2(x - 3) = 3$$

$$\log_2(x - 4)(x - 3) = 3$$

$$(x - 4)(x - 3) = 3^2$$

$$(x - 4)(x - 3) = 8$$

$$x^2 - x - 12 = 8$$

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

$$x = 5 \text{ and } x = -4$$

The solution  $x = 5$  is valid, but the solution  $x = -4$  is not valid, learners often do not check this. This serves as the type of misconception or misunderstanding that if an algorithm is followed correctly, only correct answers will result.

#### 2.2.8.4.2 Example

Solve  $\log(x^2 - 7) = \log(x - 5)$

Solution  $\log(x^2 - 7) = \log(x - 6)$

$$\log(x^2 - 7) - \log(x - 6) = 0$$

$$\log\left(\frac{x^2-7}{x-6}\right) = 0$$

$$\left(\frac{x^2-7}{x-6}\right) = 1$$

$$(x^2 - 7) = (x - 6)$$

$$(x^2 - 7) - (x - 6) = 0$$

$$x^2 - x - 2 = 0$$

$$x = 2 \text{ and } x = -1$$

Note that when substituted into the original equation, both left and right sides are undefined

#### 2.2.8.5 Domain of rational functions

When a common factor is present in the numerator and denominator

##### 2.2.8.5.1 Example

What is the domain of?

$$F(x) = \frac{x-3}{x^2-4x+3} ?$$

Clearly, the domain is  $\{x \neq 3 \text{ and } x \neq 1\}$  students tend to cancel the common factor and work with what remains. Cancellation only works when the factor cancelled is non-zero.

### 2.2.8.6 Horizontal and vertical asymptotes of a function

Students often confuse vertical and horizontal asymptotes because of not properly understanding which one is vertical and which one is horizontal. In the same way, student will frequently count only the vertical ones when asked to count how many asymptotes a function has

Example: How many asymptotes does the rational function  $f(x) = \frac{x-3}{x^2-1}$  have?

Answer: the rational function  $f(x) = \frac{x-3}{x^2-1}$  has three asymptotes in which two are vertical and one is horizontal.

Vertical asymptotes,  $x = -1$  and  $x = 1$  and horizontal asymptotes,  $y = 0$

### 2.2.8.7 Translational misconceptions and misunderstandings

These misconceptions and misunderstandings arise due to non-standard problems. It can be elaborated by the following examples:

Write an equation using the variables C and S to represent the following statement: “At Mindy’s restaurant, for every four people who ordered cheese cake, there are five people who ordered strudel, “ let C represent the number of cheese cakes and S represent the number of strudels ordered.

The correct answer is  $5C = 4S$  and only 27% of the students answered it correctly. The typical wrong answer was  $4C = 5S$ . Students were also given the hint: “be careful. Some students put a number in the wrong place of the equation. “The hint improved correctness of this answer by only 6%. In this case, the correct conversion of the words to symbols was translational error. Clearly this problem is rather non-standard in appearance. One way to solve it is to consider the equation,  $\frac{S}{5} = \frac{C}{4}$  which equates groups of five strudel orders with groups of cheese cake orders.

There are two factors which may lead to mis- or alternative conceptions for students. The first reason is that they translate the words of the problem from left to right in the manner in which they normally read a sentence, and then they confuse variables and labels. Using a left –to-right strategy, students interpret the C and the S in the equation as labels for the terms “cheesecake” and “strudel” – thereby failing to consider variables as standing for numerical expressions. Thus, algebraic expressions are interpreted wrongly, such as when students are faced with the task of relating original and sales prices. Student often get confused with the way in which original and sales prices relate to one another. They may incorrectly calculate the original price from a sale price by applying the discount to the known sale price, rather than to an unknown original price. Consequently, although the proposed construct allows the study of the association between the teacher’s knowledge and the students’ understanding, it nonetheless requires the assessment of how this association is moderated by factors deriving from the educational context. The educational context is understood as the socioeconomic level of the student’s family and results obtained in national testing at the school at which the instructor works (Olfos, Goldrine, & Estrella, 2014: 918).

## **2.3 ISSUES ND THEORIES RELATING TO STUDY**

### **2.3.1 The content of exponential and logarithms functions pre- and post- CAPS**

In terms of mathematics in general, and exponential and logarithms functions in particular, not much has changed since 1994, post-apartheid, although there have been erratic changes in curriculum over the years. A careful study of the curriculum transformations, from Nated 550 to National Curriculum Statements (NCS) and Curriculum Assessment Policy Statement (CAPS), subsequently shows that not much has changed in terms of the content and pacing of exponential and logarithms functions. Reforms in terms of content specific to the curriculum transformation, vis-à-vis exponential and logarithms function in the area of policy clarity, gaps, resource constraints, and others, have not had much impact on how content must be taught. The curriculum changes have led to a shift in the position of the learner from being a participant in the learning process, as a negotiator of meaning, to a recipient of a body of predetermined knowledge (Grussendorff, 2014). In addition to that, CAPS has

removed the explicit recognition of the unequal status of languages and varieties and take little or no account of current realities for learners, parents and teachers, the state of language and culture, or the challenges posed by the economy (Lombard, 2008). The introduction of CAPS has not changed learners' perception about the structure and content of exponential and logarithm functions. It is viewed in the same light as before.

Exponential and logarithm functions are topics that are perceived as connected, and therefore may not be taught in isolation from other. Logarithmic functions may be viewed as the inverse of exponential functions (Makgakga & Sepeng, 2013). In figure 2.1, in the leftmost column, are the stages that students' progress through as they develop an understanding of exponential and logarithm function. In the middle column, is a description of observable skills that students with each level of understanding can exhibit? Both of these columns are described in more detail below. In the right column are proposed instructional techniques. Understanding exponentiation as actions and processes is very similar to Breidenbach, Dubinsky, Hawks and Nichols's (1992) analysis of how students view functions in general.

### **2.3.2 Exponentiation as an action**

Webber (2002) opines that exponential and logarithm functions are important concepts that play a fundamental role in mathematical courses, including calculus, differential equations, and complex analysis. The interpretation of functions seems to be a major challenge in the learning of exponential functions, yet, as an action exponentiation can be explained as a repeatable physical and mental transformation of objects that obtain other objects. In the case of exponents with powers that are specific positive integer coefficients, computing  $b^x$  involves repeatedly multiplying by  $b$ ,  $x$  times. It is therefore, imperative that educators teach these functions for understanding, as they are mostly used in real-world situations (NCTM, 2010; Mousel, 2006). Student limited to an action understanding of exponents will be able to evaluate exponential functions only in the

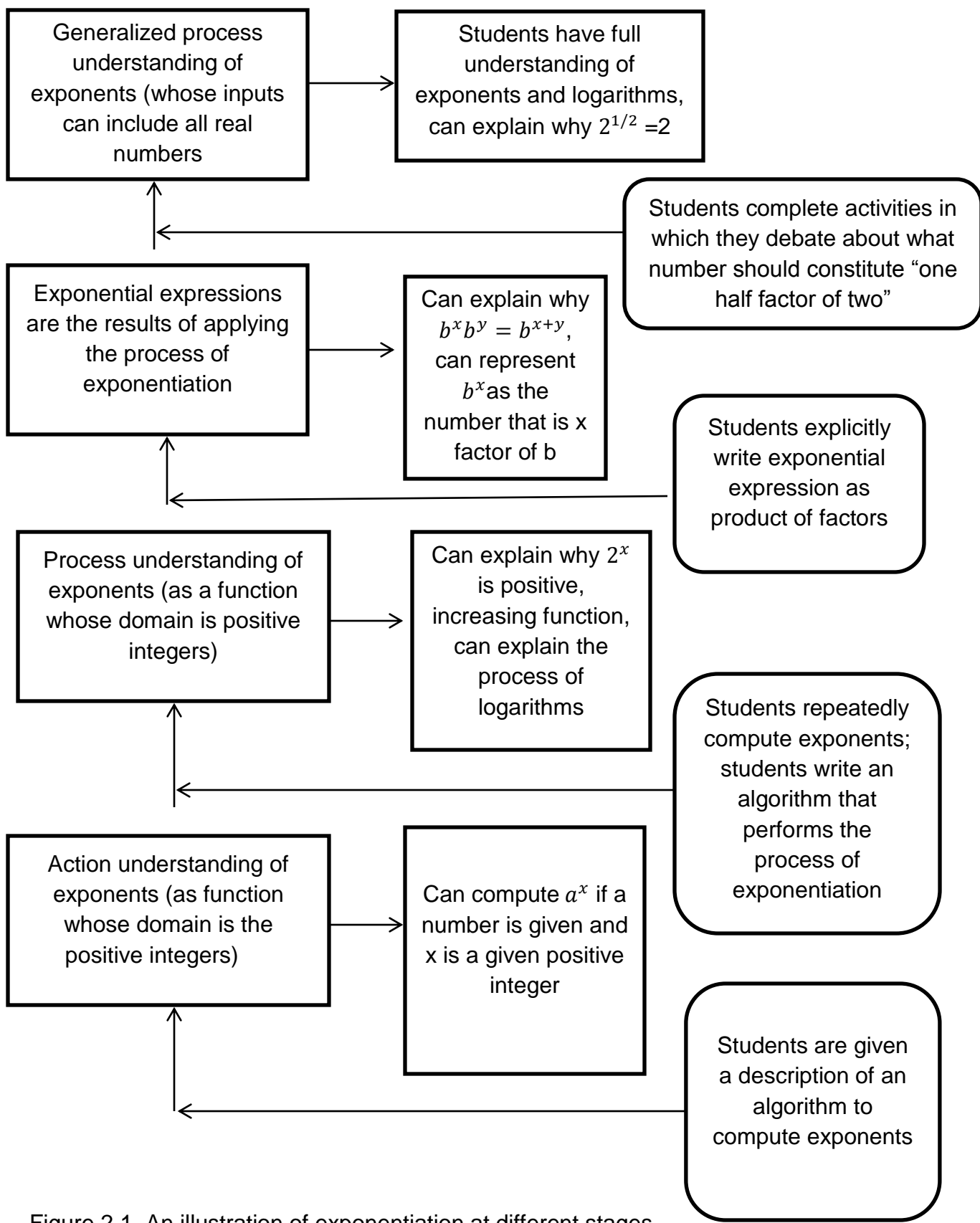


Figure 2.1. An illustration of exponentiation at different stages

cases when the power is given positive integer. This student will not be able to do much with exponents besides compute these values and manipulate formulas. Learners' understanding in exponential function and logarithmic functions as an action of exponentiation can be illustrated with the help of figure 2.1. In the diagram learners' understanding in exponential and logarithm functions has been displayed in different stages of students' understanding of exponents and the observable skills. The student can exhibit construction of a more advance stage of exponentiation.

### **2.3.3 Exponentiation as a process**

After a learner repeats an action and reflects upon it, he/she may internalize the action as a process. Students with a process understanding of a concept can imagine the results of a transformation without actually performing corresponding action, and can reverse the steps of the original transformation to obtain a new process (DBE, 2014). Students with a process understanding of exponentiation can view exponentiation as a function and reason about properties of this function (e.g.  $2^x$  will be a positive function since you start with the integer one and repeatedly multiply this by a positive number; it will be an increasing function since every time  $x$  increases by one,  $2^x$  doubles).they can also imagine the process obtained by reversing the steps of exponentiation to form the process of taking logarithms (Weber, 2012). As the result of a process term such as  $2^3$  exponential expressions can be viewed in two distinct ways. In one way, this can be interpreted as an external prompt for the student to compute two times two. However, this can also represent the output of applying exponentiation- that is,  $2^3$  represents the mathematical object that is the product of three factors of two. Research indicates that students are not capable of viewing exponentiation in this way Sfard (1991). In the same manner, representing  $b^x$  as the number that is the product of  $x$  factors of  $b$  is necessary, so that students can begin to think of computing  $\log_b x$  in order to understand laws of exponentiation such as  $b^x \times b^y = b^{x+y}$  (Victor, Pena, Klass, & Leung, 2004)

## 2.4 INSTRUCTIONAL STRATEGIES AND THE ROLE OF AN EDUCATOR

The role of educators has also been pointed out by researchers who attempt to correlate learners' achievements in mathematics, in general, with various aspects of teacher characteristics including the teacher's pedagogical content knowledge. According to Budayasa and Juniati (2018:1), "Pedagogical Content Knowledge (PCK) covers teacher's knowledge on subject matter, knowledge of pedagogy, and knowledge of students." In South Africa, the current mathematics teachers, particularly those who have recently emerged from teacher training institution, often exhibit common practices (Biyela, 2012; Dewantoro, Subandi & Fajaroh, 2018). In particular, Biyela (2012), opines that ideally a teacher's mathematics knowledge should transcend knowing the content of the textbook. Teachers should be able to contextualize mathematics concepts in order to facilitate their application even beyond the classroom. However, in this regard it seems that, like in some other countries, South African educators lack mathematics knowledge. In a study to determine the level of knowledge and understanding of mathematics involving 253 educators, using the same National Senior Certificate (NSC) questions given to high school learners, it was found that teachers scored an average mark of 57%, with half of them scoring below 61% and a quarter of the teachers scoring below 39% (Independentonline.co.za, 2016).

Mathematical functions, as concepts, form an important part of the school mathematics curriculum, and should be taught with understanding (DBE, 2012). Functions form a major part in school mathematics, and it is crucial that this topic is learned and taught effectively, notwithstanding that the topic is generally seen as one of the most challenging ones for teachers to teach (National Council of Teachers of Mathematics, 2010). It is very important for teachers to teach inverse as an extension of functions so that learners can understand what restrictions apply to functions, domains and ranges. The formal definition and understanding of a function must be emphasized within the context of mathematics by teachers in their classes.

### **2.4.1 Evolution of the function concept**

The evolution of the concept of mathematical functions goes back 4000 years (Kleiner, 1987; Chen, Liang & Hei, 2016; Sears, Tran, Lee, & Thomas, 2016). However, the notion of function in explicit form did not emerge until the beginning of the 18<sup>th</sup> century. the main reason for this relative late re-emergence of the function concept was a lack of motivation among ancient mathematicians who saw no need to define an abstract notion of 'function' in the absence of clear examples of this abstract concept. Furthermore, this retardation was also attributed to the lack if algebraic prerequisite concepts, the coming to terms with the continuum of real numbers, and the development of symbolic notations (Kleiner, 1987), Subsequently, however, “the importance of functions in school mathematics has grown tremendously within the past century in the process progressing from being scantily represented in school mathematics to being core mathematical topic” (Sears, et al., 2016: 14). Earlier, Markovitz, Eylor, and Bruckheimer (1986) eulogized the rise in the importance of this topic as follows:

The development of the function concept has revolutionized mathematics in much the same way as did the nearly simultaneous rise on non-Euclidean geometry. It has transformed mathematics from a pure natural science- the queen of the sciences- into something vastly large. It has established mathematics as the basis of all rigorous thinking- the logic of all possible relations. (Markovitz, et al., 1986: 18).

### **2.4.2 Subject content knowledge**

There are few studies of teacher quality in developing countries but those few available confirm a significant impact on student achievement. A study conducted in Peru, on a standardized mathematics test student’s achievement, showed an increase of about 9% of a standard deviation, taught by the educators with high achievement in mathematics (Metzler and Woessmann, 2012). In China, teachers of the highest professional rank affect rural students’ achievement more positively than teachers of lower rank (Chu et al., 2015). There may be a number of different ways to improve the quality of teaching

for rural students (for example, improving incentives for teachers (Loyalka et al., 2015; Muralidharan & Sundararaman, 2011; Muralidharan, 2012)).

In recent years, the Chinese government has invested heavily in teacher professional development (PD) programmes. In 2010, China's government launched the National Teacher Training Program (NTTP), the country's flagship teacher PD programme (MOE and MOF, 2010). 1 Beyond improving the quality of teaching, in general, one major goal of NTTP is to improve the quality of teaching and learning in rural areas (MOE and MOF, 2010). Given the high level of investment in the NTTP, and the given its ambitious goals, the programme is currently one of the key national government initiatives for improving the human capital of rural students and improving the quality of educational outcomes between rural and urban students in China.

In developing countries, and even in China, policymakers on teacher PD programmes invested billions of dollars( e.g. Yan, 2013; Government of Chile, 2003; Government of India, 2013) There is only a small hope about these programmes effectiveness (Bruns & Luque, 2014)

The quality of education in any country mainly depends on the quality of educators. An investigation of the national education policy recognized that:

Educators are primary agents in education; the development of a quality education corps is thus a primary condition for education transformation. This recognition asserts that no educational system can rise above the quality of its educators. In spite of how dedicated and efficient administrators, or even how enlightened the aims might be, or how up-to-date equipment, the educators are the key role players of the curriculum. (Rimillard, 1999: 235).

It is evident that the quality of mathematics education, in most countries could be greatly enhanced through effective teacher education programmes, both during pre- and in-service programmes. The learner's ability to achieve at the completion of a given task, measures the transformation. Solely the learner's achievement in mathematics depends on the educator's role to utilize their content knowledge in the subject. In general, there is a relationship between a student's achievement and teacher's content and

pedagogical knowledge. In particular, demand of cognitive levels and processes, with regard to mathematical domains, reveal and certain patterns (Tchoshanov, 2014; Salazar, 2008)

### **2.4.3 Knowledge of learners' conceptions and misconceptions**

Learners' mistakes can provide valuable insights into the state of their knowledge and understanding. Thus, from a learners' thinking and misconceptions, the educator should be able to plan effective instructions and interventions, where necessary. Accordingly, many researchers content that experienced teachers with a deep understanding of their subject matter but are unable to consider how their learners think about that content will almost invariably face difficulties in teaching that content (Hope & Townsend, 1983, Jong, 1992 in Halim & Meerah, 2002). It therefore follows that failure to consider learners' knowledge and understanding about a topic can be a major constraint in teacher's ability to teach effectively (Halim & Meerah, 2002). This suggests that good knowledge of the subject matter alone may not be sufficient for effective teaching, while a lack of knowledge of learners' misconceptions could be the result of ineffective teaching by some educators.

Educators who carry mis- and/or alternative conceptions similar to their learners are unlikely to identify the learner's misconceptions (Berg & Brouwer, 1991; Smith & Neale, 1991). Olivier (1992: 3) Distinguishes between "slips, errors and misconceptions' by pointing out that whereas both slips and errors will result in wrong answers, errors point to the possibility of mis- and/ or alternative conceptions. The same errors will be made regularly in similar contexts due to learners' misconceptions. In contrast, slips are not conceptually regarded as errors, but are careless, computational failures.

### **2.4.4 Educators' pedagogical content knowledge**

A successful relationship between conceptual and procedural knowledge relies heavily on a teacher's pedagogy and the ability to promote this knowledge for effective use. Shulman's (1986) ideas for effective teaching embody the ability of teachers to represent content in powerful and different ways and to frame it comprehensibly to all

learners in the same learning environment. Teachers who are traditional-transmission-information processing inclined (Barkastsas & Malone, 2005), or behavioristic, move quickly past the errors and learners are not given an opportunity to give feedback on their understanding of the error or an explanation of what was wrong. Brodie (2008) describes the frame of the reference of these teachers as one of only two categories – “right or wrong” (p.8). In contrast, constructivism, as a learning theory, is complemented by teaching practices such as challenging learners with problems in realistic context, engendering a classroom culture in which discussions are valued, analyzing and solving problems from different perspectives and promoting meta-learning and problem-solving (Borasi, 1996; Maree, 2004). To constructivists, learning involves actively constructing knowledge, facilitated through exploration and negotiation, while the learner is developing autonomy (Borasi, 1996; Ernest, 1998; 1991). Thus, teachers who believe that mathematics is “a practice of shared mathematizing” (Bakersfield, 1994, p.140) will employ teaching approaches (such as discussion) which encourage each individual learner to negotiate meaning.

Beyond the relevance of strong content knowledge, several researchers showed that a deep knowledge of professional development is a basic requirement to be successful mathematics educator in the class to teach the content of a branch of mathematical knowledge (Wilson, Floden, & Ferrini- Mundy, 2002; Olfos et al., 2014)

The characteristics of student’s progress and the socioeconomic background of their families in the context of education also have significant impact on educator’s performance on learner’s achievement in the class (Akiba; LeTendre; Scribner, 2007; OECD, 2009, 2010; Olfos, et al., 2014, Del Sol, 2011).

Although we may expect that state subsidiary policies for an educational system’s success assure an equitable education for all societies in a country but it is not implacable in heterogeneous countries in particular and their segmented societies at large. In educational inequities context, it affects the measure of achievement, both the students and educators.

A comparative study between mathematics and science educators, years of experience and subject content knowledge to know the influence on their student's achievement. There was a fundamental difference between these two disciplines. A constantly developing domain, a greater advantage to science educators in their teaching. On the other hand, in mathematics the teaching experience and the mathematical content knowledge is more important than the domain knowledge (Zuzovsky, 2003; Renll, 2013)

Research in Germany shows that the impact of teachers on learning depends on overall teacher quality in terms of the pedagogical approaches they use (Hanushek & Rivkin, 2010). Effective teachers are generally perceived as those whose role is that of facilitator and believing in personal construction of knowledge which, in turn, concurs with the problem-solving view of mathematics instruction (Ernest, 1988). Thus, teaching ability is seen as an important skill of PCK, to present content knowledge, to students in a way to understand and knowing the difficult concepts easily (Park, 2012).

The main idea of PCK lies in teaching ability, which includes knowing how to represent any content for others to understand and knowing what aspects of a certain concept make learning the content difficult (Jungeun, 2012). In various research, the researchers suggested that teachers should possess knowledge beyond mathematical content knowledge, although they are different from each other in terms of components of knowledge for teaching mathematics and levels of specificity in each component.

#### **2.4.5. Knowledge of instruction strategies**

A multiple set of instructional strategies, representations, and activities are in use to teach mathematics in the classroom, depending on the various socio-economic backgrounds. Knowledge of specific strategies involves the knowledge of how to represent specific concepts to facilitate learning. Representations include illustrations, examples, models and analogies. "Each analogy for instance has conceptual advantage and disadvantage with respect to others" (Treagust, 2007, p. 379). In this area, PCK encompasses understanding the relative strengths and weaknesses of particular representations, and which strategies and activities could be used to help learners understand specific concepts and/or relationships. In this regard, the teacher may make

use of well-considered laboratory activities, simulations and demonstrations. As pointed out above, for these approaches to be effective, the teacher must understand the learners' conceptions, misconceptions and alternative conceptions about the topic at hand. This way, the teacher will be able to anticipate and plan for possible difficulties which the learners could encounter with the topic. Representations must be clearly linked and the relationship among concepts must also be clear. Lu, Loyalka, Shi, Chang and Rozelle (2017) found that educators who taught outside their field of expertise provided incorrect and misleading explanations and representations in the form of analogies and examples which depicted their own misconceptions. Corkin and Ekmekci (2016) reported similar findings indicating that educators teaching outside their areas of expertise gave explanations and analogies which reinforced the misconceptions that learners had. Accordingly, Campbell, Nishio, and Smith (2014) surmised that teachers' knowledge of appropriate strategies depended on their subject matter knowledge about a particular concept. This may not always be true, however, as subject matter knowledge does not necessarily guarantee that such knowledge will be translated into instructional modalities which will be optimal for each learner to understand targeted concepts, or that educators will successfully and competently decide when it is pedagogically best to use particular strategies and instructional approaches

Table 2.1 shows South African Learners' performance in TIMSS 1998-1999, out of 38 countries. The top five and worst five countries are listed in Table 2.1

Table 2.1: Top five and worst 5 countries on TIMSS [Source: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999]

<b>Name of country</b>	<b>Average score (Std. error)</b>	<b>scale</b>	<b>Years of formal schooling</b>	<b>Average</b>
Chinese Taipei	569 (4.4)		8	14.2
Singapore	568 (8.0)		8	14.4
Hungary	552 (3.7)		8	14.4
Japan	550 (2.2)		8	14.4
Republic of Korea	549 (2.6)		8	14.4
International Average	488 (0.7)		8	14.4
Lowest five achievers				
Tunisia	430 (3.4)		8	14.8
Chile	420 (3.4)		8	14.4
Philippines	345 (7.5)		7	14.1
Morocco	323 (4.3)		7	14.2
South Africa	243 (7.8)		8	15.5

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Japan	550(2.2)	8	14.4
Republic of Korea	549(2.6)	8	14.4
International Average	488(0.7)	8	14.4
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Tunisia	430(3.4)	8	14.8
Chile	420(3.4)	8	14.4
Philippines	345(7.5)	7	14.1
Morocco	323(4.3)	7	14.2

South Africa	243(7.8)	8	15.5
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Table 2.1 presents TIMMS scores for 10 out of 38 participant countries. It is evident that South Africa is listed in the lower achievers. The South Africa is the last number in the worst performer's list. This performance illustrates the poor educators' content and pedagogical knowledge in the country. Moreover, Lobito et al. (2003) stated that construction of this instructional environment is not completely "under the teacher's control" (p.30). Another study on the topic "Teachers' Mathematical Knowledge for teaching instructional Practices, and Students' outcomes" reveals the same findings (Gencturk, 2012).

#### **2.4.6 Teacher Content Knowledge (TCK)**

Previous research shows that there is a logical premise that teacher knowledge affects classroom practice directly and positively and subsequently student's achievement (Smith & Esch, 2011). It is evident that TCK is positively correlated with student learning, and there are some theoretical assumptions that support this hypothesis. One such assumption is that teachers who are very strong in their subject matter will create better opportunities to enhance students' learning and interest and that they would be more capable of addressing diverse learning needs in their classrooms (Ball, 2000). Furthermore, it is envisaged that because teaching mathematics generally involves significant mathematical reasoning, teachers with strong content knowledge will tend to positively influence student outcomes (Ball, 2000)

An investigated study conducted in Germany, Baumert et al. (2010) to investigate grade 10 experienced mathematics educators, to have an empirical distinguish between MVK and PCK in an open ended-assessment. The results of this investigation revealed that there is a significant relationship between educators MCK and learner's achievement in mathematics as well as educators PCK and learner's achievements in mathematics also. It is also noted that an educator with strong PCK assessed learners understanding at the end of instructional unit with a high level of cognitive task. This investigation

identified that presuming educators adequate MCK, PCK is a stronger predictor of learners learning in mathematics.

Ladd (2008) avers that three main methodological issues arise in attempting to ascertain teacher effectiveness. Firstly, an educator's achievement does not only depend on his/her MCK as there are other contextual factors involved- thereby making it difficult to isolate and decisively pinpoint only the effect of teacher quality from all other inputs, especially those deriving from contextual factors. Second, it is not always possible to measure, or let alone control for all the intervening determinants and indicators of student achievement. Lastly, and most importantly, casual inferences in most cases, is compromised due to methodological limitations, such as random assignment of subjects to experimental conditions. Therefore, learner's attitude towards learning, their ability and motivation, have significant impact on educators' performance. Teacher effects may be confounded by unobservable attributes of students, such as their ability and motivation. If the non-random matching of students to teachers is neglected, the estimate of teacher effects will be biased upwards if more motivated or academically capable students are allocated to more effective teachers, or downwards, if the opposite is true. In order to solve this puzzle, value-added models (VAM) are employed to assess the effect of individual teachers on raising student achievement (Chetty, Friedman, & Rockoff, 2012; Ladd, 2008). Thus, the VAM highlights that there is no relationship between educator's performance effects on learner's achievement as learner's attitude, ability and motivation, or their socio-economic background within one year of learning. Intuitively, the vale-added approach attempts to isolate teacher effects from other factors such as prior student performance and socioeconomic status, by focusing on learning gains during one school year (Ladd, 2008; McCaffrey et al., 2004). Much pre-service and in-service teacher education rests on the premise that teacher content knowledge directly and positively affects both classroom practice and, ultimately, student learning. The majority of researchers and educationalist, across the globe, generally believe that educators with strong MCK and PCK play a vital role in effective teaching of mathematics and science in a classroom. Researchers and practitioners across the philosophical spectrum generally agree that, for effective teaching, it is necessary for teachers to process strong subject matter knowledge.

However, while this premise is logical, empirical support is thin, largely because of a lack of suitable measures. Studies investigating this relationship rely largely on proxy measures of teacher content knowledge.

At the same time, recent studies examining mathematics instructions noted that teachers seem to draw not only on their pedagogical and content when teaching but also on their beliefs (Beswick, 2007; Bray, 2011). Moreover, educators' content and pedagogical knowledge in mathematics, and learners' performance, also has an effect on their classroom environment (Philipp, 2011). On the other side, the awareness of the conditions of classroom and their interactions (Philipp, 2011).

Few studies use direct measures of teacher content knowledge. Teaching and learning in mathematics are complicated issues in our society, especially when educator's attitude and beliefs towards teaching are not understood well. Teaching is a complex social activity, which draws on many teacher competences and characteristics, including teacher knowledge beliefs, beliefs, and attitudes in ways that are not well understood. Although it is generally agreed that subject matter knowledge is important, empirical support for this perspective is thin. However, many people remain convinced that if teachers do not understand the content, unlikely to help students develop understanding of such content; that it is even possible that such teachers will do harm to the learners. Similarly, there is also widespread agreement on the importance of content-specific knowledge of how to teach the subject matter (Shulman, 1986).

#### **2.4.6.1 Hypothesized content-specific domains of teacher knowledge**

In learning, some content in the domain of teacher knowledge are taught in a way that learners learn to accumulate knowledge and later employ them during their learning, discussions on complex and interdisciplinary topics. Prior knowledge may facilitate recalling information on well structured, fact-based topics (Schmidt, Rothgangel & Grude, 2017). The hypothesized content-specific domain of teacher knowledge further can be described in two different categories.

#### **2.4.6.1.1 Knowledge of disciplinary content**

This knowledge refers strictly to disciplinary content, with no other elements of what a teacher would need to know in order to relate the contents to students.

#### **2.4.6.1.2 Knowledge that enables teacher to recognize alternative frameworks**

The realization and belief that alternative frameworks of thinking about disciplinary content exist make teachers recognize the need to have different ways to organize the content for learners of varying academic readiness. The ability to do so will make teachers focus on helping students understand the important ideas, without necessarily requiring them to understand the content in exactly the same way. Such understanding about the nature of knowledge enables teachers to present lessons differently for different learners.

#### **2.4.6.1.3 Knowledge of the relationships**

This refers to the relationships between singular concepts and the more encompassing rules, principles and theories. The realization of knowledge at these different levels of complexity enables teachers to present lessons at the correct levels of intended levels of complexity. The notion that big ideas subsume smaller ideas is a good understanding about the nature of knowledge.

#### **2.4.6.1.4 Knowledge of understanding students' thinking**

Concerning the content to help students understand content, it is important for teachers to know what ideas students bring with them in terms of prior knowledge, and where they are likely to face some difficulties. Already, there are some content areas where there is a rich repertoire of research on student preconception and misconceptions. Teachers must reach into this knowledge in preparing their lesson. The available research outcomes include some common mis- and alternative conceptions, some of which are resistant to normal teaching.

#### **2.4.6.1.5 Knowledge of strategies to diagnose students' ideas**

This concerns the thinking of particular groups of students. Teachers need to know how to discern what ideas students have about a content area, both prior to and during a unit of instruction.

#### **2.4.6.1.6 Knowledge of how to sequence instruction**

This type of tradition comes from 'information processing' in the cognitive psychology paradigm and highlights the important contribution that the notion of 'sequencing instruction' can make to the systematic presentation of content. Teachers need to think about content in terms of how students can most efficiently come to understand it. Teachers need to know which ideas are prerequisites for later ideas and to progress from less complex to more complex ideas.

#### **2.4.6.1.7 Knowledge of content-specific strategies**

The focus of this knowledge type is on the awareness that not all strategies will work equally well with all groups of students. This takes cognizance of possible student differences which need to be considered during both lesson preparation and implementation in order to appeal to different learning needs of the individual students. Some students may have academic learning barriers while others may carry physical impairments, such as visual and hearing impairments. Some peculiarities may relate to socio-geographical circumstances, such as rural versus urban and peri-urban learners – and learners from poorly resourced versus well-resourced schools. The knowledge of all these circumstances is important for teachers to prepare the most appropriate lesson for their students.

In essence, it means that if teachers don't have a command of a certain topic of the content, it affects the learner's achievement in that specific content. One indicator of the teachers' conceptual understanding of mathematics is an ability to engage students into understandable, simple and interesting discourse in the learning of mathematics in a classroom, through selecting topics and task, for instruction and assessment that embody learning objective (Shepard et al., 2005). The recent research on the successful

inclusion of these mathematical constructs, supported by educators understanding and acknowledgement of their learners' misunderstanding, as an indicator in the assessment of mathematics educators' teaching quality and have an impact on their learner's performance. (Stanford Centre for Assessment, Learning, and Equity, 2012).

## **2.5 EDUCATORS' KNOWLEDGE OF LEARNERS' UNDERSTANDING AND MISUNDERSTANDINGS AND THEIR GAINS IN EXPONENTIAL AND LOGARITHMIC FUNCTIONS**

Mathematical content knowledge (MCK) and mathematical representation knowledge are connected with each other. Similarly, learners' prior knowledge and pedagogical content knowledge (PCK) are inter-dependents (NCTM, 2014; Shulman, 1995). Researchers have long speculated that a teacher's knowledge of common students' conceptions and misconceptions could be important to student learning. Therefore, some researcher advocates the need for teachers to be familiar with common student misconceptions for the topics that they teach. This could be achieved through teachers interviewing or testing their students to reveal student preconceptions early on in the learning process. Prior research indicates that educators' innate ability to achieve goals in the mathematics teaching and educators' belief in mathematical study are related with their mathematical knowledge of teaching (MKT) (Corkin, Ekmekci, & Papakonstantinou, 2015)

The norms and Standards for Educators outlined the skills and competences that each teacher must exhibit (Department of Education, 2003), and these were set out as follows:

- a) Educator as a leader and administrator.
- b) Educator in a pastoral role.
- c) Educator as a learning program developer
- d) Educator as a researcher scholar and life-long learner.
- e) Educator as assessor
- f) Educator as a mediator of learning.
- g) A learning area specialist.

The NCS further envisages learners who are imbued with values and act in the best interest of society, based on respect, democracy, equality, human dignity, and social justice, as promoted in the constitution. It is important that teachers are able to understand and implement the NCS for the field of specialization effectively and efficiently (Msila 2007). Thus, teachers need to develop themselves along the three aspects of the educational field: professional attitude towards teaching, teaching approaches and the most important, content knowledge (Kriek & Grayson 2008:199). When educators possess a negative attitude, it reflects not only in their teaching but also in their learning environment. A conducive and vibrant learning atmosphere stimulates a learner to learn, especially in mathematics science (Sztajn, Marrongelle, & Smith, 2011).

Professional development of teachers on content-focused instruction has tremendous effect on student achievement. The study of Blank and de las Alas (2009) provided a scientifically based evidence for its positive effect. A significant performance difference has been noted among learners whose educators participate in professional development programs and those who do not participate in those activities organized by professional bodies.

In their study Porter, McMaken, Hwang & Yang (2011) found that teacher's MCK strongly impacted learner achievement. On his part, Babbie (2010) reported that learners of absentee teachers scored much lower in mathematics than those taught by teachers who consistently came to school. Thus, mathematics achievement can be improved by improving teacher's MCK, commitment in the profession and always engage in professional development. In teaching, the most difficult issue to handle is underachieving students. These students are discouraged and uninterested and lack the confidence, and motivation to learn. Research conducted by Bray (2011) suggested that to develop above average, higher achieving and motivated learner, the educators must focus on students' potential to learn, value challenge and learning are able to concentrate on effort and learning in the face of hardships, and are able to engage in processes that foster learning like task analysis and study skills.

## **2.6 EDUCATORS' PEDAGOGICAL KNOWLEDGE AND LEARNERS' ACHIEVEMENT IN EXPONENTIAL AND LOGARITHMIC FUNCTIONS**

One key feature of the knowledge component of the mathematics teaching cycle is that educators have in-depth knowledge of the content subject mathematics and the knowledge of learner's misconceptions about that specific content, and also hold knowledge of presentation of the content (Burke, 2013).

Zuze (2010) analyzed in his study that the independent and dependent variable are interchangeable, and explained the performance of learners from Botswana. Zuze (2010) shows in particular that there is uncertain evidence to suggest that students attending well-resourced schools are likely to perform better.

According to Shulman (1995: 130), pedagogical content knowledge include,

... the ways of representing and formulating the subjects that make it comprehensible to others... an understanding of what makes the learning of specific topic easy or difficult; the conceptions and preconceptions that students of different ages and background bring with them to the learning of those most frequently taught topics and lessons.

To promote and increase the learners' thinking their reasoning ability, the educators play a vital role by asking questions of them. The educators questioning strategy differs depending on the nature of their learners (Yasemin, 2012). Teachers' questioning strategies also play vital role in the quality of instruction student receive. Teachers can foster students' reasoning ability by asking questions that promote student thinking (Yasemin C G, 2012).

### **2.6.1. Components of Pedagogical Content Knowledge**

An, Kulm and Wu (2004) Describe pedagogical content knowledge as comprising the knowledge of subject matter content, knowledge of the curriculum and the knowledge of how to deliver instruction or teaching. The knowledge of teaching the curriculum and the knowledge of representation of the content have been accepted as the core component

of the pedagogical content knowledge (An et al., 2004). The interrelation of these components can be expressed by the following diagram (see Figure 2.2).

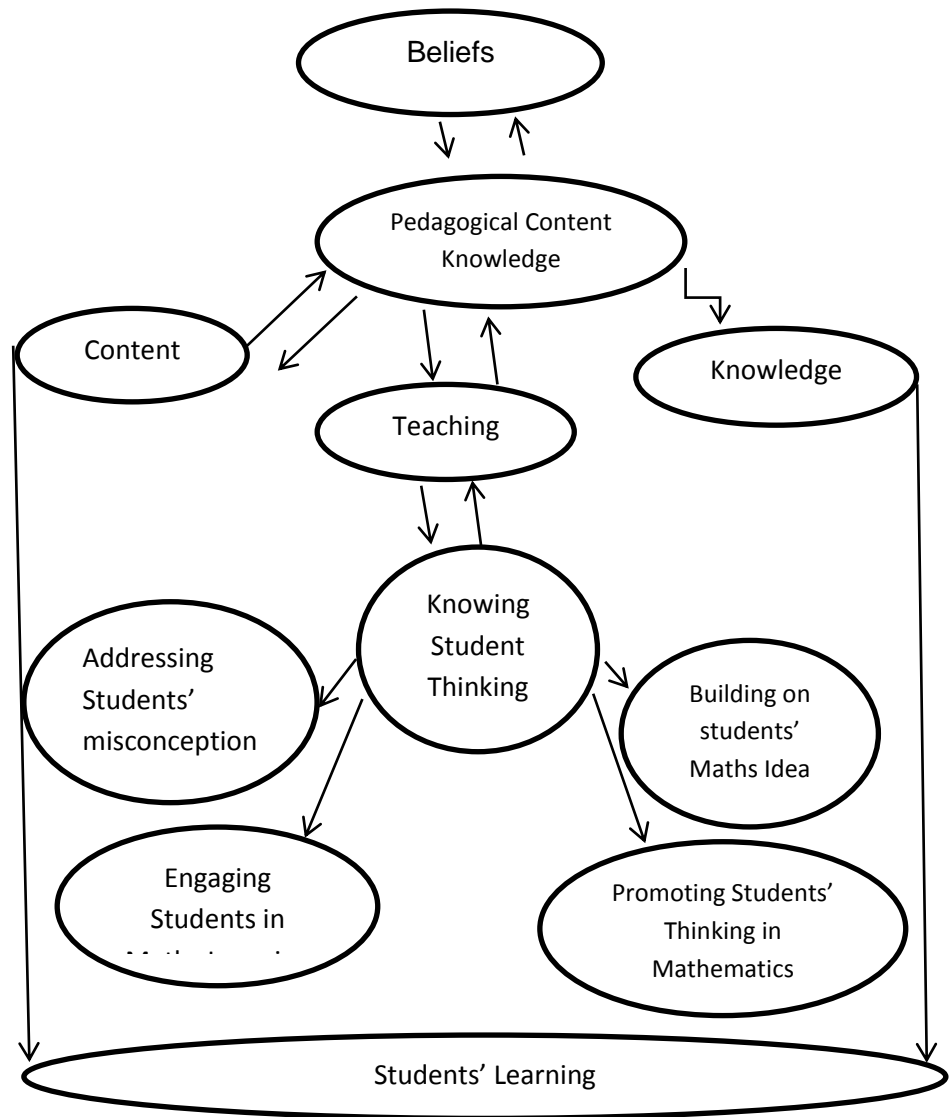


Figure 2.2: The network of pedagogical content knowledge

Figure 2.2 show that PCK breaks down into the content of what must be taught, the knowledge of various aspects of the field of education, as well as the skills and ability to teach. Under the ability to teach, are issues such as knowing and understanding learners' misconceptions, knowing how learners think and respecting the ideas learners

bring to the classroom. For effective teachers, this knowledge and understanding of the teaching/learning environment leads to interactive teaching approaches.

## **2.7 SUMMARY**

Factors that could affect the ability of the learner to succeed in understanding instruction related to exponential equation and logarithmic functions fell under two major categories, namely, learner and educator readiness. Under each of these two elements are variables which, taken together, constituted the conceptual framework of the study. Starting with learner variables, Figure 2.3 shows the nine variables which could have a bearing on the quality of instruction in exponential equations and logarithmic functions. These are learners' readiness to understand (a) concepts related to exponential equations and logarithmic functions, (b) procedural aspects of the operations related to exponential equations and logarithmic functions, (c) domains of rational functions, (d) the notion of horizontal and vertical asymptotes of a function, (e) underlying mathematical concepts, (f) the interaction of informal and formal knowledge, (g) the nature of their misconceptions about mathematics, generally, and exponential equations and logarithmic functions, in popular, and (h) possible error types committed by learners related to mathematics, generally, and exponential equations and logarithmic functions, in particular.

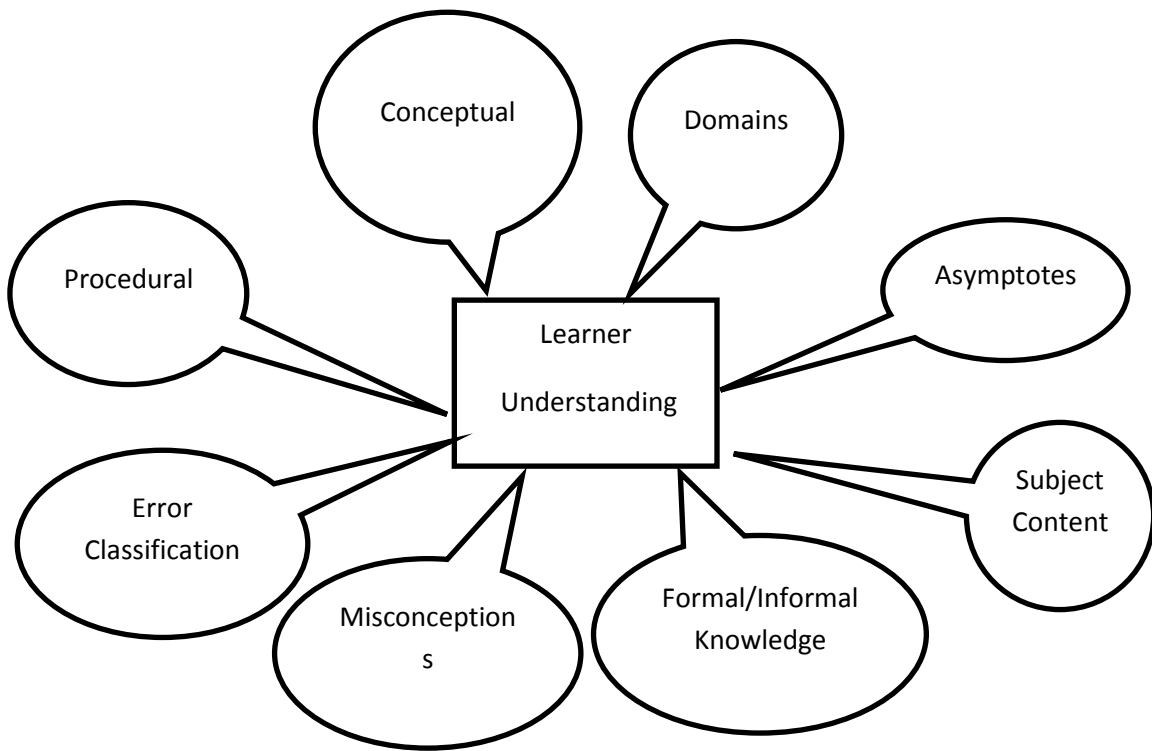


Figure 2.3: Learner variables which could affect learner performance in exponential equations and logarithmic functions

With regard to educator variable, these are summarized in figure 2.4. Briefly, the variables are: (a) educators' pedagogical knowledge (educator knowledge of curriculum, educator knowledge of learner academic needs, educator knowledge of instructional strategies, educator knowledge of social needs [pastoral care], and educator teaching ability), (b) educator content knowledge, and (c) the educator as leader and administrator.

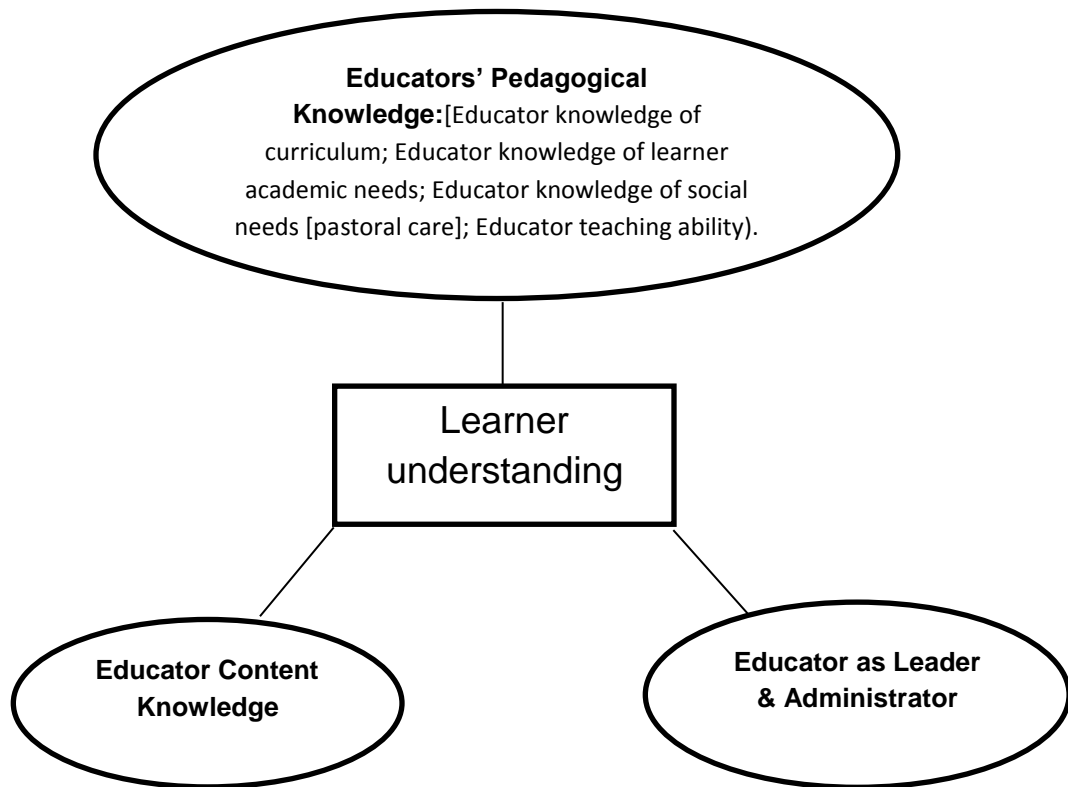


Figure 2.4: Educator variables that could affect learning

## 2.8 CONCLUSION

The literature reviewed in this chapter served to illuminate the study focus in various ways by examining factors associated with learners' performance in the NSC mathematics examinations on the topics of exponential and logarithmic functions. Figures 2.4 and 2.5 illustrate the literature review into the two the conceptual models. These two conceptual models were very important to guide the rest of the study. The research methods followed in the study are described in the next chapter.

## **CHAPTER THREE**

### **RESEARCH METHODOLOGY**

#### **3.1 INTRODUCTION**

This chapter focuses on the research methodology followed in conducting the empirical investigation of the factors contributing to learners' poor performance in exponential and logarithmic functions in grade twelve, in the Ubombo Circuit Management Cluster (CMC). The research methodology used in this study was informed by the purpose of the study, which was to investigate, inter alia, to what extent educators' pedagogical content knowledge affected learners' achievement in exponential and logarithmic functions, and the type of data needed to answer the research questions of the study, namely:

- 3.1.1 What are National Senior Certificate learners' understanding and misunderstandings in exponential and logarithmic functions?
- 3.1.2 Do grade twelve teachers consider themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions?
- 3.1.3 Is there a relationship between educators' self-concept about their ability to teach exponential and logarithmic functions and the actual performance of their learners on these two mathematical constructs?
- 3.1.4 Does educators' pedagogical content knowledge impact learner achievement in exponential and logarithmic functions?

Presented below under different sub-headings are various descriptions of the methodology followed in this study.

#### **3.2 RESEARCH PARADIGM**

This study used a mixed-methods research paradigm, as there was need to collect both quantitative and qualitative data in order to answer the above four research questions (Creswell, 2008).

### 3.3 RESEARCH DESIGN

A research design describes the major procedure to be followed in carrying out the research. It is a detailed description of the most sufficient and the enough actions taken to achieve the certain specific research ambition in a research (Bless & Higson-Smith, 1995). To Maree (2010), a research design refers to the degree of control the researcher manages to exert over his or her research environment. In contrast, Sibaya (2014), stated that a research design is a figurative sense to collect the answers to a research question: that it is a layout detailing what the researcher will do from writing the hypotheses if any and implications to the final analysis of the data. To Mathiyazhagan and Nandan, a survey is “a method of descriptive research used for collecting primary data based on verbal or written communication with a representative sample of individuals or respondents from the target population.”(Mathiyazhagan & Nandan, 2010, p 34). All these definitions agree that a research design is a specific plan for getting information which will address the research questions posed in a particular study. Thus, for this study, the survey research design was chosen.

This design was employed to enable the researcher to interpret and describe how the identified variables affected learners’ achievement in exponential and logarithmic functions. In this regard, both quantitative and qualitative data were collected simultaneously in which the researcher “further explored findings from one method by the use of the other” (Creswell, 2008, p. 557), thereby combining the “best” of both quantitative and qualitative research.

Quantitative data were gathered about teachers’ mathematical content knowledge, and learners’ achievement in exponential and logarithmic functions; qualitative data were gathered about learners’ conceptions and misconceptions in exponential and logarithmic functions. Quantitative and qualitative data were gathered about teachers’ knowledge of learners’ conceptions and misconceptions, and about their knowledge of strategies for teaching exponential and logarithmic functions.

Some of the cited advantages of survey research design are that it is (a) cost-effective and can be automated to provide real-time access, (b) time efficient, (c) convenient to

respondents, (d) offers design flexibility, (e) offers good conditions for anonymity, and (f) offers respondents an environment in which they may be more willing to share information (Mathiyazhagan & Nandan, 2010).

### 3.4 TARGET AND ACCESSIBLE POPULATIONS

The study locale was the province of Kwazulu- Natal, South Africa. Thus, the target population were all grade twelve learners registered for mathematics, and all grade twelve mathematics teachers. However, the province of Kwazulu- Natal is quite large, one school district was conveniently chosen for the purpose of this study – that is, the uMkhanyakude school district. The researcher is a mathematics teacher in the uMkhanyakude school district.

### 3.5 THE RESEARCH SAMPLE

For the purpose of this study, nine (9) high schools were conveniently selected to participate in the study because of their proximity to the researcher’s school. This convenient sample was used because the researcher was a fulltime teacher throughout the duration of this study. It would have been extremely difficult to involve far-away schools in the study due to the logistics of time and accessibility. From the participating schools nine grade twelve mathematics teachers were identified and invited to participate in the study. This gave the researcher access to nine classes of grade twelve mathematics learners. The nine educators who agreed to participate in the research, taught in nine different schools and wards.

With respect to the learner research sample, from each participating class, randomly selected learners were given to write the test, giving a total of 242 learners. Table 3.1.1 shows the total learners population selected to participate in the study, in each school.

**Table 3.1.1: Learner’s population selected randomly to participate in this study**

SCHOOLS:	A	B	C	D	E	F	G	H	I
Total Learners	37	70	91	55	7	37	75	79	48

Selected Learners	21	39	35	20	7	21	36	42	21
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The random sampling was used to avoid selection bias. The characteristics of the two research samples were as follows:

### 3.5.1 Gender Distribution of Teacher Participants

Table 3.1.2 gives a breakdown of the teacher participants by gender

**Table 3.1.2 Gender distribution of teacher sample (n = 09)**

GENDER	NUMBER OF EDUCATORS
Male	07
Female	02

Table 3.1.2 shows that there were more male mathematics teachers than female in the research sample. The researcher is not aware of the gender distribution of mathematics teachers in the district, so it cannot be determined whether or not this breakdown is representative of the gender distribution of the accessible population.

### 3.5.2 Teaching Experience

The distribution of the teacher participants by years of teaching experience is given in Table 3.1.3

**Table 3.3 Participants' years of teaching experience (n = 9)**

Experience (Years)	n
0 - 5	6
6 - 10	3

From the distribution in Table 3.1.3 it is clear that the teachers in the research sample had limited teaching experience.

### 3.5.3 The Teachers' Qualification Profile

The qualification profile of the teacher participants is presented in Table 3.1.4

Table 3.1.4 Teacher participants' qualification profile (n=9)

<b>Qualification</b>	<b>Number of Teachers</b>
Degree in mathematics education or mathematics	4
Diploma in mathematics education or mathematics	2
Certificate in mathematics education or mathematics	1
Any other qualifications	2
TOTAL	100

The highest qualification of the educators was a degree in mathematics or mathematics education, with four participants – making 44% of the research sample. Two of the respondents held diplomas in mathematics or mathematics education. In terms of the minimum requirements for registration with the South African Council for Educators, it may be said therefore that six out of the nine participating teachers (that is 67%) were duly qualified to register as teachers. However, there were three teachers (33%) who appeared not to be qualified to teach grade twelve mathematics.

The learners were African (Black) learners, predominantly coming from rural and low-income family backgrounds. One characteristic of rural schools is that they lack resources for mathematics teaching and learning; there were no graph boards, overhead and/or data projectors, and no computers which could be used to enhance the teaching and learning of mathematics in the seven schools. However, a few learners had calculators, and at least one mathematics textbook.

Demographically, the average age of the learners was 17 years. All the learners had completed the study of exponential and logarithmic functions prior to this investigation.

### **3.6 DATA COLLECTION PROCESS**

Having secured the permission from the provincial and local authorities, the researcher approached potential participants within the accessible population to request for their participation in the project. The purpose of the investigation was explained, as well as the nature and extent of what was needed from them. The School Governing Bodies (SGBs) of the participating schools were approached for permission to conduct the study on their premises (see Appendix iii). This was done through the principals of the schools. The researcher explained the purpose of the study and the level of participation needed from both the educators and the learners. Because the learners were minors, permission was sought from their parents (see Appendix ii).

The empirical investigation was conducted in phases as follows:

#### **3.6.1 Phase One**

A researcher-constructed test was used to collect data from learners (see Annexe 3), a questionnaire, also constructed by the researcher, was used to collect data from the educators (Annexe 4), followed by an interview schedule (Annexe 5).

#### **3.6.2 Phase Two**

A pilot study was conducted for the purpose of validating the instruments developed in phase 1. The reliability of the test was also established from the pilot study.

#### **3.6.3 Phase Three**

The learners' test was administered to the learners in the main study in order to establish their achievements in exponential and logarithmic functions, and identifying their conceptions and misconceptions. The test was administered by the class teachers to the learners in their respective schools between 7th and 21<sup>st</sup> September 2016 and lasted about one hour and thirty minutes. The researcher was present in each of the venues when the learners wrote the test and collected the scripts after the learners had completed it.

#### **3.6.4 Phase Four**

The educators' questionnaire was administered to the nine mathematics educators in order to establish (a) their mathematical content knowledge (MCK) in exponential and logarithmic functions, (b) how well they knew their learners' conceptions and (c) their own misconceptions about exponential and logarithmic functions. This was after the learners had written the test. Due to time constraints on the part of the educators, the researcher allowed them to complete the test at their convenience but not exceeding five days.

#### **3.6.5 Phase Five**

The educators' instructional strategies in the teaching of exponential and logarithmic functions were investigated by means of interviews. The interview questions were open ended ( see Annexure 5), and followed the following format: "if you were asked to teach the concept involved in question ' x'....., how would you teach it so that the learners understood the mathematical concept involved?". Follow-up questions were asked, seeking clarity where the researcher had not clearly understood the educator's response, or sought to get further information. The interviews lasted between thirty minutes and an hour. They were conducted on a one –to-one basis with each educator at his/her school.

#### **3.6.6 Phase Six**

On the basis of the educators' PCK data, the educators were categorized as either possessing an adequate PCK level or not. The scores obtained by learners who were taught by educators who were classified as possessing adequate PCK was compared against those who were taught by educators whose PCK scores were lower-and were therefore deemed not to possess adequate PCK. This was done in order to ascertain the relationship, if any, between the educators' PCK and their learners' performance in exponential and logarithmic functions. With these analyses, the role of the educators' PCK in the teaching of exponential and logarithmic functions was established.

### 3.7 Performance of Sample Schools in the Matric Examination

Table 3.1.5 presents statistics reflecting the performance of grade twelve National Senior Certificate from the participating schools in the year 2013 to 2015.

Table 3.1.5 Performance of NSC candidates from participating schools in mathematics: 2013-2015

SCHOOL	A	B	C	D	E	F	G	H	I	MEAN
2013	51	38	15	37	48	40	38	39	42	39
2014	68	31	61	71	43	42	53	37	39	49
2015	73	68	30	29	51	44	29	41	51	46

(Department of Education, Matric Result 2013, 2014 & 2015)

This table gives a picture of mathematics performance for the schools selected to participate in this study. From the table, it was only School A that consistently scored above 50% over the indicated period.

### 3.8 INSTRUMENTATION

The three instruments used for data collection in this study are described below:

#### 3.8.1 Learners' Test

A researcher-designed test was used to gather data about learners' achievements, and learners' conceptions and misconceptions about the concept of exponential and logarithmic functions. To ensure content validity, the researcher used South Africa's NCS, as expressed in the CAPS documents, concerning exponential and logarithmic functions. The following knowledge components, indicative of understanding of exponential and logarithmic functions, were used in constructing both the learners' test and the educators' test. Effects of varying the value of 'a' in an exponential and its inverse (logarithmic) function.

- i. The impact of the value of base “b”.
- ii. The Infinite nature of an exponential function and its relationship to the y–intercept.
- iii. Symmetrical properties of an exponential and its inverse function.
- iv. Effects of varying the value of ‘c’ on the line symmetry, and y–intercept.
- v. Horizontal shifting of the exponential and logarithmic function.

The learners’ test consisted of a variety of non-standard problems, most of which provided an opportunity for qualitative considerations and reasoning. The reason for this type of questioning was to enable the researcher to determine not only the learners’ achievement but also their understanding and possible misconceptions about exponential and logarithmic functions. Most tasks in the test required that the learner demonstrate translational skills to operate between graphical and algebraic representations of the given problems. The test was designed as follows:

- i. In order to avoid rote memorisation or computational routines, questions used in the test were taken from non-prescribed (unfamiliar) text books. The assumption was that when faced with unfamiliar tasks, the learners would fall back on their strategies to solve the problem rather than rely on previously practised algorithms.
- ii. A deliberate effort was made to reduce the amount of quantitative information to the barest minimum in order to ensure that the learners focused their attention on the qualitative properties of the questions.
- iii. Most tasks demanded that the learners exercise and apply logical reasoning.

Learners were required to work out each problem, give an answer and give a justification for the answer.

### **3.8.2 Educators’ questionnaire**

Although referred to as a ‘questionnaire’ the instrument used for the educators comprised the same mathematical tasks as contained in the learners’ test. This was done in order to establish the educators’ mathematical content knowledge (MCK)

related to exponential and logarithmic functions. In addition, the instrument contained questions to assess the educators' knowledge of their learners' conceptions and misconceptions in quadratic functions. The assumption underlying the construction of the educators' questionnaire was:

- i. That a more direct way to investigate educators' MCK was to ask them to work our mathematical tasks on the topics they were teaching (Rowland, Martyn & Barber, 2001).
- ii. That one way to ascertain educators' knowledge and understanding of the subject content was to ask them the same questions which have been given to their learners (Halim & Meerah, 2002; Viri, 2003). Thus, the educators' questionnaire used in this study consisted of the same ten tasks given to their learners. In each of the tasks, educators were required to (a) describe the answers they expected from their learners, (b) give reasons for their expected answers, and then (c) give your own answers to the questions.

The educators' ability to provide the correct answers to the questions demonstrated his/her level of subject matter knowledge in exponential and logarithmic functions, while his/her ability to predict their learners' reason(s) for their answers would demonstrate the educators' understanding of his/her learners' conceptions and misconceptions. The educators' questionnaire was also used to collect data their biographical information, such as qualifications, experience, gender, and about the nature of the in-service training on exponential and logarithmic functions, which they may have attended.

Since the tasks in the educators' questionnaire were the same as those in the learners' test, the process for the development of the educators' test included the processes and considerations for the development of the learners' test (see section 3.6.5). The process of validating the educators' test included giving the draft to a mathematics specialist at the KwaZulu-Natal department of Education who affirmed that any educator who possessed the knowledge demanded for answering the questions in the educators' questionnaire had sufficient MCK to effectively teach exponential and logarithmic functions in grade 12. The subject specialist also contended that the instrument also

adequately assessed the educators' understanding of their learners' conceptions and misconceptions in exponential and logarithmic functions.

### **3.8.3 The Educators' Interview Schedule**

To investigate the educators' knowledge of specific teaching strategies for teaching the concept of exponential and logarithmic functions, an interview schedule was constructed having lead questions to enter into the conversation with the respondents (see Annexe 5). The process for constructing and validating the interview schedule was as described above with respect to the educators' questionnaire.

### **3.8.4 Reliability of Instruments**

Instrument reliability refers to the degree to which independent administration of the same or similar instrument would consistently yield the same or similar results under comparable conditions (De Vos, 2002). Ways to establish instrument reliability include: split half method, or calculation of the Chronbach's alpha coefficient, alternative forms method, and the test-retest method (De Vos, 2002). As already stated, the learners' test was designed by the researcher and, thus, did not contain standardised items. Its reliability was achieved using the test-retest method involving ten Grade 12 learners. The questionnaire was pre-tested on five Grade 12 mathematics educators. For both groups, the retesting took place four weeks after the first testing. For the learners, the Pearson product-moment correlation coefficient was  $r = 0.94$ , while  $r = 0.91$  was recorded for the educators' questionnaire-which was the same test as for the learners. Both correlation coefficients were high and statistically significant. This made both instruments reliable and therefore appropriate for use in the study.

### **3.8.5 Validity of the instruments**

Instrument validity refers to its appropriateness and adequacy for measuring what it purports to measure (De Vos, 2002: 166). Four types of instrument validity have been described: content, face, criterion, and construct validity. In a research conducted by, Babbie (2001), described the four categories as a basic tool of a research validity. The validity of the content (Content validity), in this study means that a measure of a sample

representing the research subject in the questionnaire. Actually, Face validity has no measure to any instrument like a questionnaire or interview, but it seems to measure the relevance of the instruments. Criterion validity emphasizes the level of consistency of the instrument with the results of the independent and external factors. While, construct validity outlines the consistency between the hypothetical constructs and instruments related to the research objectives.

In validating the learner's test, the first draft was discussed with ten Grade 12 learners to determine if they understood the questions in terms of the language used, and also what was intended. The feedback was used in drafting the next version of the test, which was then given to three experienced heads of department (HOD) of mathematics in the FET phase to verify if it adequately covered the content of exponential and logarithmic functions, as reflected in the Grade 12 CAPS curriculum specifications, and if the questions were within the scope of Grade 12 learners. Each of the three heads of department affirmed that the knowledge required for the solution of the tasks in the test correctly reflected what was expected of Grade 12 learners. The test was then given to the mathematics subject specialist at the KwaZulu-Natal Department of Education (uMkhanyakude district) who also indicated that the questions were appropriate for probing Grade 12 learners' knowledge of exponential and logarithmic functions and exploring their conceptions and misconceptions. These reports affirmed the validity of the instrument in line with the assertion that evidence of content validity is "a matter of determining whether the samples are representative of the larger domain of tasks it is supposed to represent" (Gronlund, 1998: 202).

### **3.9 DATA ANALYSIS**

To determine the contributing factors towards learner underachievement in exponential and logarithmic functions, the analyses were carried out as described below. The educators' pedagogic content knowledge (PCK) was determined from their scores on the questionnaire. From these results, the educators were grouped according to the level of their PCK. Learners were then grouped according to the PCK level of their educators. The achievements of the groups of learners were then compared to determine the effects of educators' PCK on learner achievement.

### **3.9.1 Analyses of learners' questionnaire**

Two sets of data were collected from the learners' test– quantitative and qualitative.

#### **3.9.1.1 Quantitative analyses**

The quantitative data were used to measure the learners' achievement in exponential and logarithmic functions. In each question, a learner was scored 1 if he/she got the right answer and 0 if the answer was wrong. Descriptive statistics were used to present and interpret the data.

#### **3.9.1.2 Qualitative analyses**

Elo & Kyngäs (2007: 107) stated that

Content analysis of the learners' reasons for their responses in each question was made, from which the researcher abstracted the learners' misunderstanding in exponential and logarithmic functions. Content analysis is a method that may be used with either qualitative or quantitative data and in an inductive or deductive way, in order to build a model to describe the phenomenon in a conceptual form.

Accordingly, Elo and Kyngäs (2007) described two types of content analysis, namely inductive and deductive – which both involve three main phases: preparation, organizing and reporting. Explaining these two processes further, Elo & Kyngäs (2007) averred that “deductive content analysis is used when the structure of analysis is operationalized on the basis of previous knowledge” whereas “inductive content analysis is used in cases where there are no previous studies dealing with the phenomenon or when it is fragmented.” In this regard, the deductive approach is useful if the general aim is “to test a previous theory in a different situation or to compare categories at different time periods” (Elo & Kyngäs, 2007: 107).

In this study, content analysis took the form of an inductive approach because there was no pre-existing model against which the new information was to be matched, or compared.

## **3.9.2 Analyses of educators' questionnaire**

### **3.9.2.1 Quantitative analyses**

The educators' MCK levels were determined from their scores in the content part of the questionnaire. On each question, an educator was scored 1 if he/she gave the right response to the question and 0 if the answer was wrong. The total score for each educator was recorded (see Table 4.1). An educator's MCK in a question is deemed strong if the response to the question was correct but weak if the response was incorrect.

### **3.9.2.2 Qualitative analyses**

Content analysis of the educators' expectations of the reasons given by their learners were done to obtain data on educators' knowledge of their learners' conceptions and misconceptions in exponential and logarithmic functions. In the analysis, the researcher compared the educators' expectations of their learners' reasons with the reasons given by the learners themselves. This approach was used by Halim & Meerah (2002) and Viri (2003).

## **3.10 ETHICAL ISSUES CONSIDERED IN THE STUDY**

Ethical considerations involve a set of moral principles that should guide the behaviour of a researcher towards respondents and other researchers (De Vos, 2002). To obtain the participants' informed consent, the researcher provided all the participants with adequate and necessary information about the study, but also explained that the success of the project depended on their participation. The information provided to the potential participants included the purpose of the study, the extent of information required from each participant, and the credibility of the researcher. The participants willingly decided to participate. The educators' written consent was obtained. Because the learners were minors, their parents' written consents were obtained as well (see Annexures a & b).

The participants' rights to privacy and confidentiality of the participants were maintained throughout the research through coding and other measures. For instance, numbers were used to represent educators' names while letters of the alphabet were used to represent the names of the schools.

To maintain anonymity, the learners were not asked to write their names on the test; rather, they were asked to indicate their educators' names at the back of their questionnaire. Identifying the educator who taught particular learners was necessary for the analysis. Educators' responses to their questionnaire and interview sessions were kept confidential. The study did not expose the participants to harm of any kind as the learners and educators responded to the questionnaires at a time considered convenient to them.

The research findings were accurately and objectively reported to the best of the researcher's abilities. Participants were informed about the findings in an objective manner, and without offering any details that revealed the confidentiality of the respondents. The reason for sharing the research findings with the educators was to encourage them to change their practices in line with the findings of the study, and to maintain good relationships in case the researcher decides to conduct a follow-up study.

### **3.11 SUMMARY**

This chapter has presented and described the research paradigm used in this study, research design, the research sample, the instruments employed in collecting the data, the procedure followed in data collection, and the approaches followed in analysing the data. The ethical issues pertinent to the study were also presented.

## CHAPTER FOUR

### PRESENTATION AND INTERPRETATION OF RESULTS

#### 4.1 INTRODUCTION

In this chapter, data obtained from the investigation of the factors contributing to the underachievement of learners in exponential and logarithmic functions in the Ubombo Circuit Management Cluster (CMC), uMkhanyakude District, are presented. The data are presented to answer the following research questions of the study:

- a) What are the National Senior Certificate learners' main understandings and misunderstandings of exponential and logarithmic functions?
- b) Do grade twelve teachers consider themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions?
- c) Is there a relationship between educators' self-concept about their ability to teach exponential and logarithmic functions and the actual performance of their learners on these two mathematical constructs?
- d) Does educators' pedagogical content knowledge impact learner achievement in exponential and logarithmic functions?

The results are presented below according to each of these research questions, following the presentation of the participants' biographical characteristics.

#### 4.2 BIOGRAPHICAL CHARACTERISTICS OF LEARNERS

The biographical characteristics of the participants presented in this section are meant to give a sense of basic attributes of the participants used in this study. The participants were all grade 12 learners in Ubombo CMC, in the uMkhanyakude District.

##### 4.2.1 Gender Distribution

Table 4.1 shows the total number of participants, distributed according to their gender classification. Out of the 242 learners, 138 were female – representing 57% of the total research sample, while 104 participants were male, representing 43% of the research sample.

Table 4.1: Gender distribution of the participants (n = 242)

<b>Gender</b>	<b>%</b>
Male	43
Female	57
TOTAL	100

#### 4.2.2 Age Distribution of the Learner Participants

The age distribution of the learner participants is given in Table 4.2. It can be seen from the table that the learners who were 22 years of age and above constituted 6% of the total population, while the majority of learners fell between 16 and 19 years of age, making 74% of the total. The remaining 20% were composed of learners between 20 and 21 years of age.

Table 4.2: Age distribution of the learners (n = 242)

<b>Age</b>	<b>n</b>	<b>%</b>
16-17	85	35
18-19	95	39
20-21	49	20
22+	13	6
TOTAL	242	100

### 4.3 LEARNERS' UNDERSTANDING OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

The first research question concerned the learners' understanding of exponential and logarithmic functions. For this research question, the learners' understanding was measured through their demonstrated ability to correctly answer the questions in the test, while 'misunderstanding' referred to their inability to answer the questions correctly. The first measure of these concepts is given in Table 4.3 in reference to measures of central tendency and variability.

Table 4.3: Learner performance on the test (n = 242)

Mean	21.62 (54%)
Median	22.50 (56%)
Std. Dev	12.730
Variance	162.061
Range	40
Minimum	0
Maximum	40

The total score for this test (administered to test participants' basic understanding of exponential and logarithm functions) was 40 and the minimum was zero. The pass mark was 20, representing 50% of the total score. The mean score for this study was 21.62 (representing 54%). The median score was a pass mark of 22.50 (representing 56%). These results suggest that the participants had a basic understanding of the topics under investigation. It needs to be pointed out, however, that the number failing the test (46%) and who scored below the median mark (44%), was too large to ignore. This number represents those participants who did not demonstrate sufficient understanding of the concepts of exponential and logarithm functions. It may, therefore, be concluded that although there was a generally demonstrated level of competence in terms of the arithmetic mean and median, there were a number of participants who did not demonstrate sufficient understanding of these concepts.

Data to further explain whether participants understood the concepts of exponential equations and logarithm functions are presented here through the analysis of individual questions in the test instrument. Table 4.4 represents participants' performance on question 1 of the test. This item tested participants' understanding of the difference between an algebraic equation and an algebraic function.

Table 4.4: Participants' performance on Question 1 (n = 242)

**Difference between an equation and a function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	44	18.2	18.2	18.2
1 Mark	14	5.8	5.8	24.0
2 Marks	75	31.0	31.0	55.0
3 Marks	109	45.0	45.0	100.0
Total	242	100.0	100.0	

Table 4.4 shows that question 1 had a total of 3 marks. Participants scoring zero and 1 mark were judged not to have sufficiently mastered the concepts underlying the question. Thus, the table shows that, cumulatively, 24% of the participants did not demonstrate understanding of the underlying concepts, while 76% demonstrated understanding of the concepts in this item. It can be concluded that the majority of participants did understand the **difference between an equation and a function**.

On defining exponential and logarithm functions with examples, 75 learners out of a total of 242, were able to explain the concept with an example (see Table 4.5).

Table 4.5: Participants' performance on Question 2 (n = 242)

**Definition of exponential and logarithmic functions**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	22	9.1	9.1	9.1
1 Mark	17	7.0	7.0	16.1
2 Marks	35	14.5	14.5	30.6
3 Marks	93	38.4	38.4	69.0
4 Marks	32	13.2	13.2	82.2
5 Marks	43	17.8	17.8	100.0
Total	242	100.0	100.0	

However, 93 learners were not able to explain the concept with examples, suggesting that they experienced some level of difficulty part with this concept.

Table 4.6: Participants' performance on Question 3 (n = 242)

**Sketch the function  $f(x) = a \cdot b^x \forall a > 0; a \neq 1; b > 1$  and  $x \in \mathbb{R}$**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	106	43.8	43.8	43.8
2 Marks	8	3.3	3.3	47.1
3 Marks	128	52.9	52.9	100.0
Total	242	100.0	100.0	

In Table 4.6 participants were asked to sketch the basic exponential graph. The result showed that 114 participants (47.1%) did not show any understanding of how to sketch an exponential graph. On the other hand, 128 learners (52.9%) did show an understanding to the concept. Collectively participants are able to sketch the exponential graph. The number of 114 learners who failed is too large to ignore.

Table 4.7: Participants' performance on Question 4 (n = 242)

**Sketch the inverse function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	158	65.3	65.3	65.3
2 Marks	3	1.2	1.2	66.5
3 Marks	81	33.5	33.5	100.0
Total	242	100.0	100.0	

In Table 4.7, total marks of the question are 3, and participants had to score at least 2 marks to show their understanding on this concept. The result revealed that 158

participants (65.3%) scored a 0 mark, an indication of lack of understanding of the concept. Only 33.5% of learners were able to sketch the logarithm graph successfully, whereas only 1.2% had basic understanding of the question under review.

Table 4.8: Participants' performance on Question 5(A) (n = 242)

**The impact of Constant “a” on the sketch of f(X)**

	n	%	Valid %	Cumulative %
0 Marks	116	47.9	47.9	47.9
1 Marks	126	52.1	52.1	100.0
Total	242	100.0	100.0	

on the exponential function  $f(x) = a \cdot b^{x+p} + q \forall a > 1; b > 1$  and  $p, q$  and  $x \in R$ , Table 4.8 shows that 126 learners, representing 52.1%, were able to provide a correct answer concerning the possible impact of Constant “a” on the sketch of  $f(x)$ , while 116 learners, representing 47.9%, were not able to do so.

Table 4.9: Participants' performance on Question 5(B) (n=242)

**Effect of the Base “b” on the sketch of f(X)**

	n	%	Valid %	Cumulative %
0 Marks	70	28.9	28.9	28.9
1 Marks	172	71.1	71.1	100.0
Total	242	100.0	100.0	

In reference to exponential function  $f(x) = a \cdot b^{x+p} + q \forall a > 1; b > 1$  and  $p, q$  and  $x \in R$ , learners were asked to explain how Base ‘b’ would affect the graph of  $f(x)$ . Table 4.9 shows that 172 participants (71.1%) had an understanding to this concept. On the other hand, 70 participants (28.9%) did not understand the concept.

Table 4.10: Participants' performance on Question 6 (n = 242)

**Definition of logarithmic function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	123	50.8	50.8	50.8
1 Marks	7	2.9	2.9	53.7
2 Marks	112	46.30	46.30	100
Total	242	100.0	100.0	

Table 4.10 shows that a total of 2 marks were allocated to this question. The participant's needed to score at least 1 mark to demonstrate an understanding of this concept. 130 participant (53.70%) in the study failed to demonstrate an understanding, while 112 participants (46.30%) scored full marks and showed an understanding to the definition of logarithmic functions.

Table 4.11: Participants' performance on Question 7 (n = 242)

**Sketch the functions of a graph and indicate all the key points**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	89	36.8	36.8	36.8
3 Mark	12	5.0	5.0	41.7
4 Marks	38	15.7	15.7	57.4
5 Marks	1	0.4	.4	57.9
6 Marks	8	3.3	3.3	61.2
8 Marks	94	38.8	38.8	100.0
Total	242	100.0	100.0	

Table 4.11 shows that question 7 had a total of 8 marks. For participants to be deemed to have achieved in this question he/she had to score 4 marks or higher. On this basis, cumulatively, 141 (58%) of the participants demonstrated understanding of the

underlying concepts, while 42% demonstrated a lack or insufficient understanding of the concepts in this item. It can be concluded that although the majority of the participants demonstrated understanding of the concepts of s this item, quite a sizeable number (42%) did not.

In terms of the details, 94 learners, comprising 38.8% of the research sample, scored 100% marks; 89 learners, representing 36.8%, were unable to provide the correct answer to this question; 38 learners, which represented 15.7%, scored at the 50% performance level, which was taken as the cut-off point for acceptable learner performance.

Table 4.12: Participants' performance on Question 8 (n = 242)

**Sketching the inverse function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	110	45.5	45.5	45.5
3 Mark	18	7.4	7.4	52.9
4 Marks	25	10.3	10.3	63.2
7 Marks	2	.8	.8	64.0
7 Marks	1	.4	.4	64.5
8 Marks	86	35.5	35.5	100.0
Total	242	100.0	100.0	

Table 4.12 shows that question 8 had a total of 8 marks. For participant to be deemed to have achieved in this question they had to score 4 marks or more. Cumulatively, it can be seen from the table that 110 plus 18 learners scored below 4 marks, giving a total of 128 (53%), deemed not to have demonstrated understanding of the underlying concepts; 47% were deemed to have demonstrated understanding of the concepts in this item. Therefore, the majority of participants did not understand the concepts.

Table 4.13: Participants' performance on Question 9(A) (n = 242)

**Domain and Range of the given function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	66	27.3	27.3	27.3
1 Mark	28	11.6	11.6	38.8
2 Marks	148	61.2	61.2	100.0
Total	242	100.0	100.0	

Learners were asked to write down the domain of the function  $f(x)=2.5^{x-2}+1$ . The results showed that more than half of the learners (148), representing 61.2% of the research sample, obtained 100% marks, and 28 (12%) obtained 1 mark – giving a total of 176 (73%) deemed to have achieved on this item. This indicates that the majority of learners understood the concept in this question.

Table 4.14: Participants' performance on Question 9(B) (n = 242)

**Domain and Range of the given function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	61	25.2	25.2	25.2
1 Mark	56	23.1	23.1	48.3
2 Marks	125	51.7	51.7	100.0
Total	242	100.0	100.0	

In Table 4.14, the participants scored 2 marks represent 51.7% (125) of the research sample – thereby demonstrating an understanding of the concept. Fifty-six (56) learners obtained 1 mark on this item; plus 125 learners who got full marks; this means that 181 learners (75%) demonstrated an understanding of the concept tested in this question. It may then be concluded that the majority of the participants demonstrated an understanding to the concept of a domain and range in an exponential and logarithmic functions.

Table 4.15: Participants' performance on Question 10(A) (n = 242)

**Horizontal Asymptote of the function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	62	25.6	25.6	25.6
1 Mark	180	74.4	74.4	100
Total	242	100.0	100.0	

In Table 4.15, the results show that 180 (74%) participants answered this question correctly, while 62 (26%) learners were unable to provide correct answers to this question. It is therefore evident that the majority of the participants did understand the concept of 'horizontal asymptote in exponential and logarithmic functions'.

Table 4.16: Participants' performance on Question 10(B) (n = 242)

**Horizontal Asymptote of the function  $f(x) = 2\ln(x + 3) - 1$**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	58	24.0	24.0	24.0
1 Mark	184	76.0	76.0	100.0
Total	242	100.0	100.0	

In Table 4.16, results show that 184 participants (76%) scored the full mark, while 58 participants (24%) scored 0 marks. It can be concluded that the participants did understand the concept of horizontal asymptote of the function  $f(x) = 2\ln(x + 3) - 1$  in the study.

**4.3.1 Summary (Research question no 1)**

Table 4.17 gives a summary of the results described earlier. As reflected in Table 4.17, the analysis of the participants' scores pertaining to the first research question, on learners' understanding of the basic concepts in exponential equations and logarithmic functions, indicate that, overall, 38% experienced some difficulty with these topics. **On**

**the whole, however, close to two-thirds of the participants (i.e. 62%) demonstrated sufficient understanding of the concepts in exponential equations and logarithmic functions.** This, then, is the answer to the first research question.

Table 4.17 Summary response profile to the first research question (n = 242)

Question	Concept	Understand (%)	Did Not Understand (%)
1	Difference between an equation and a function	76	24
2	Definition of exponential and logarithmic functions	69	31
3	Sketching the function $f(x) = a \cdot b^x \forall a > 0; a \neq 1; b > 1$ and $x \in \mathbb{R}$	56	44
4	Sketching the inverse graph of an exponential function	35	65
5A	The impact of Constant "a" on the sketch of f(X)	52	48
5B	Effect of Base "b" on the sketch of f(X)	71	29
6	Definition of logarithmic function	49	51
7	Sketching the functions of a graph indicating all the key points	58	42
8	Sketching the inverse function	47	53
9A	Determining domain and range of a given exponential function	73	27
9B	Determining of domain and range of a given logarithmic function	75	25
10A	Horizontal asymptote of the function	74	26
10B	Horizontal asymptote of the function $f(x) = 2\ln(x + 3) - 1$	76	24
	<b>Average</b>	<b>62</b>	<b>38</b>

From qualitative analysis, the study revealed the following difficulties in the learners' understanding of the concepts related to exponential and logarithmic functions:

- 1) The participants misunderstood sketching an inverse function from a basic exponential function,  $f(x) = a \cdot b^{x \forall a > 0; a \neq 1; b > 1 \text{ and } x \in \mathbf{R}}$ . a question in the instrument, question no 4.
- 2) Learners had trouble defining logarithmic function, a question in the instrument, question no.6
- 3) The participants failed to sketch the different function on the same set of axis and to mark the key features, question no.7 on the instrument.
- 4) Learners found it difficult to sketch the inverse functions on the same set of axis and to indicate the key points ,which they were asked to sketch in question 7,and a question on the instrument ,question no.8
- 5) Although the learners showed an understanding and scored a 50% mark on question numbers3, 5a, and 9b, the number of learners who failed is too large to ignore.

In conclusion, the result to the research question 1 can be declared in the form of the average percentage of the concepts (understanding and misunderstanding) in exponential and logarithmic functions as 56.63% and 43.36% respectively.

#### **4.4 EDUCATORS' SELF-CONCEPT ABOUT THEIR READINESS AND ABILITY TO TEACH EXPONENTIAL AND LOGARITHMIC FUNCTIONS**

The second research question sought to establish whether or not the participating teachers considered themselves competent and knowledgeable to teach exponential and logarithmic functions. The question was “whether the educators are suitably qualified, knowledgeable and have an ability to teach the given content”. The data were based on the educators’ readiness to teach exponential and logarithmic functions in a self-reporting manner, and derived from their biographical information and responses given to a written questionnaire on a Likert scale completed by educators.

In answering this research question, the data were sub-divided into three parts, namely, suitably qualified, knowledgeable and ability to teach. Accordingly, the researcher used the educators’ questionnaire, Section A, educators’ biographical information items marked A4 (teaching experience) and A5 (teaching qualifications) as measures for

'suitably qualified'; Section C, results were used to determine whether or not the educators were 'knowledgeable'; and the last and final part, 'ability to teach', was answered by results obtained from Section B of the Educators' Questionnaire. In this regard, the answer to the research question relied on the participants' honesty in responding to the 'self-report' section of the questionnaire, and the ability to correctly answer the mathematical questions in the questionnaire. On the other hand, lack of understanding was shown by the participants' inability to solve the mathematical questions.

#### **4.4.1 Suitably Qualified**

The Educators' Employment Act of 1994, as amended, stipulates that it is compulsory for any person who teaches, educates or trains other persons or who provides professional educational services, including professional therapy and education psychological services at any public school, departmental offices or adult basic education centre and who is appointed in a post on any educator establishment under this Act to register with the South African Council for Educators (SACE). To be able to register with SACE, a person must meet the minimum requirements as set out in the South African Schools Act (SASA) of 1998, as amended, as follows:

- 1) Matric +3 years teaching diploma  
M+3 qualification in education  
NQF 13 (National Qualification Frame Work)
- 2) Matric + 4 years teaching degree (B.Ed/Hons)  
M+4 qualification in education  
NQF 14

A qualified educator can be regarded as suitably qualified when he or she complies with the minimum qualifications as stipulated above. The biographical data, also indicated these necessary qualifications, obtained from the participants are analysed and presented in Table 4.18.

Table 4.18: Biographical data of Educators

Name	Gender	Age	Teaching Grade	Teaching experience	Teaching Qualification
Edu.1	2	2	3	2	1
Edu.2	2	1	2	1	1
Edu.3	1	1	2	1	1
Edu.4	1	1	3	1	1
Edu.5	2	3	3	2	1
Edu.6	1	2	1	1	1
Edu.7	1	2	1	2	1
Edu.8	1	2	2	2	1
Edu.9	1	3	1	2	1

KEY:

Gender: Male=1, Female=2; Age: 20-24=1, 25-29=2, 30-34=3;

Teaching Grade: grade10/11/12=1, Grade11/12=2, Grade12=3;

Teaching Experience: 0-3year=1, 4-10 year=2, 11-20 year=3, 21≥yr=4;

Qualifications: professional Degree=1, Diploma=2, certificate=3, any other =4

Table 4.18 shows that there were three female teachers in the group, out of 9 teachers who participated in the study; three teachers including one female fell under the 20-24 year old age group, while four teachers fell under the 25-29 year age group; and two, including one female, fell under the 30-34 year age group.

The table further shows that three teachers, out of 9, were currently teaching all FET grades (10-12), while the other three teachers were teaching grades 11-12; and three teachers were teaching only grade 12. With regard to teaching experience, four teachers out of the nine had less than four years of teaching experience, while the rest of the teachers (5) had between five and ten years of teaching experience. All the participating teachers were suitably qualified in terms of the minimum requirements cited above; they were all in possession of a professional degree in teaching mathematics. Thus, it was concluded that the educators who participated in the study were suitably qualified.

#### 4.4.2 Mathematical Content Knowledge

The second part of this research question concerned the educators' competency, knowledge and ability to teach exponential and logarithmic functions. The first measure of these concepts is given in Table 4.19, which gives a summary in terms of measures of central tendency and variability.

Table 4.19: Educators' performance using as measure of central tendency (n = 9)

N	9
Above Median	5 (55.6%)
Below Median	4 (44.4%)
Minimum	10
Maximum	35
Mean	23.56
Median	22
Range	25
Standard deviation	10.2
Variance	103.3

In Table 4.19, the total possible score for this test (administered to test participants' mathematical content knowledge (MCK) in exponential and logarithmic functions) was 40. The pass mark was set at 20, representing 50% of the total score. The results yielded a mean score of 23.56 (representing 58.9 %). The median score was a pass mark of 22 (representing 55%). These results suggest that participants' MCK in the tested subject area was a little above average. It needs to be pointed out, however, that the number failing the test, 4 out of 9 educators (44.44 %), and falling below the median mark (44.44%), is too large to ignore. This number represents the educators who did not demonstrate understanding of some of the concepts behind exponential and logarithm functions. Table 4.19 shows that 5 educators (55.6%) performed above the median pass mark. However, the researcher felt that for the educators, this was

somewhat low, as teachers are usually expected to be much more knowledgeable about the content than their learners.

To further explain whether or not participants understood the concepts of exponential and logarithm functions, the educators' performance on each question in the questionnaire is presented and analysed in the tables below. Table 4.20 presents participants' performance on question 1. This item tested participants' MCK in differentiating between an algebraic equation and an algebraic function.

Table 4.20: Participants' performance on Question no 1 (n = 9)

**Difference between an algebraic equation and an algebraic function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	1	11.1	11.1	11.1
2 Marks	3	33.3	33.3	44.4
3 Marks	5	55.6	55.6	100.0
Total	9	100.0	100.0	

Table 4.20 shows that question 1 had a total of 3 marks. For a participant to be deemed to have achieved this question, he/she had to score more than 1 mark. One educator failed to achieve the required 1 mark out of a total of 3 marks. Three educators scored 2 marks, while five educators achieved full marks 3 out of a total of 3 marks. Eleven percent (11.1%) of the participants did not demonstrate understanding of the underlying concepts, while 33.3% demonstrated understanding of the concepts above average on this item. On the other hand, 55.6% participants showed a command on the concept. Out of a total of nine participants, eight achieved above the 66.66% mark. It can be concluded that the majority of the participants demonstrated the understanding of algebraic equations and algebraic functions.

Table 4.21: Participants' performance on the definition of exponential and logarithmic functions (n = 9)

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
2 Marks	2	22.2	22.2	22.2
3 Marks	2	22.2	22.2	44.4
4 Marks	2	22.2	22.2	66.7
5 Marks	3	33.3	33.3	100.0
Total	9	100.0	100.0	

This question carried a total of 5 marks, of which 3 marks out of five was taken as the acceptable level of a performance. Thus, with regard to defining exponential and logarithm functions with an example, Table 4.21 shows that 2 educators (20%) fell below the criterion mark of 3 marks, while the rest of the participants (77%) participants demonstrated understanding of these two concepts. The next question sought to establish whether or not the participants could sketch the function  $f(x) = a \cdot b^x \forall a > 0; a \neq 1; b > 1 \text{ and } x \in \mathbb{R}$ . The results are presented in Table 4.22.

Table 4.22: Participants' performance on sketching the function  $f(x) = a \cdot b^x \forall a > 0; a \neq 1; b > 1 \text{ and } x \in \mathbb{R}$  (n = 9)

**Sketching the function  $f(x) = a \cdot b^x \forall a > 0; a \neq 1; b > 1 \text{ and } x \in \mathbb{R}$**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	4	44.4	44.4	44.4
2 Marks	2	22.2	22.2	66.7
3 Marks	3	33.3	33.3	100.0
Total	9	100.0	100.0	

Question 3 had a total of 3 marks. For a participant to be deemed to have achieved on this question, he/she had to score more than 1 mark. In Table 4.22; 4 (44%) participants scored 0 marks out of total 3 marks, which showed that they did not have an

understanding of the concept of sketching this function. The remaining five educators (56%) achieved the required passing mark. It can be concluded that the majority of the educators did have mathematical content knowledge of the concept, although 4 did not. The next task required the participants to sketch the inverse graph of the exponential function, and the results are displayed in Table 4.23.

Table 4.23: Participants' performance on sketching the inverse graph of the exponential function (Question 4) (n = 9)

**Sketching the inverse function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	4	44.4	44.4	44.4
2 Marks	2	22.2	22.2	66.7
3 Marks	3	33.3	33.3	100.0
Total	9	100.0	100.0	

Table 4.23 also shows the results of sketching the inverse graph of the basic exponential function,  $f(x) = a \cdot b^x \forall a > 0; a \neq 1; b > 1 \text{ and } x \in \mathbb{R}$ . Much like in the preceding question, the table shows that 5 (56%) Participants, out of 9, demonstrated the ability to sketch the inverse function of the basic exponential function, while 4 (44%) were not able to do so, suggesting that they lacked the knowledge of how to sketch an inverse function of an exponential function.

Table 4.24: Participants' performance on the impact of constant "a" (Question 5A) (n=9)

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
1 Mark	9	100.0	100.0	100.0
Total	9	100.0	100.0	

On this question, the participants were asked to explain the impact of constant "a" on the sketch of  $f(x)$  in the exponential function  $f(x) = a \cdot b^{x+p} + q \forall a > 1; b >$

1 and  $p, q$  and  $x \in R$ . Table 4.24 shows that all the participants scored full marks on this question. Although the weighting for this sub-question was only one mark, all 9 educators, representing 100%, were able to provide the correct answer to this question, suggesting that they possessed the necessary mathematical content knowledge (MCK) on this aspect of the content. On the participants' ability to explain the effect of base "b" on the graph of  $f(x)$ , Table 4.25 shows that 7 participants (77.80%) scored full marks on this sub-question.

Table 4.25: Participants' performance on the effect of base "b" on the graph of  $f(x)$  (Question 5B) (n=9)

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	2	22.2	22.2	22.2
1 Mark	7	77.8	77.8	100.0
Total	9	100.0	100.0	

The results in Table 4.25 indicate that the majority of the educators understood the concept; only 2 participants were unable to provide the intended response to the question.

Table 4.26: Participants' performance on defining logarithmic function (Question 6) (n=9)

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	5	55.6	55.6	55.6
2 Marks	4	44.4	44.4	100.0
Total	9	100.0	100.0	

On the sixth question, 5 participants (55.6%) scored 0 marks, while 4 participants (44.4%) scored full marks – thereby showing that more than half of the participants failed to demonstrate understanding of this concept (see Table 4.26).

Question number 7 tested the educators on their ability to graph and label functions on the same set of axis. The results presented in Table 4.27. On this question, 8 participants (88.88%) were able to graph two functions on the same graph paper – indicating all the key points; one participant was not able to do so.

Table 4.27: Participants' performance on graphing and labelling the functions on the same set of axis (Question 7) n=9)

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	1	11.1	11.1	11.1
6 Marks	6	66.7	66.7	77.8
8 Marks	2	22.2	22.2	100.0
Total	9	100.0	100.0	

With regard to the educators' ability to sketch inverse functions on the same set of axis, Table 4.28 summarises the results.

Table 4.28: Participants' performance on sketching the inverse functions on the same set of axis (Question 8) (n=9)

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	5	55.6	55.6	55.6
6 Marks	2	22.2	22.2	77.8
8 Marks	2	22.2	22.2	100.0
Total	9	100.0	100.0	

On this question, there was an almost equal split between those who were able to do so (44%) and those who failed to demonstrate this ability (56%). For teachers, this was not a good result as it shows that the majority of the teachers did not have this ability.

Table 4.29 shows that question 9(A) had a total of 2 marks, and only 2 participants out of the nine obtained full marks, three obtained one mark on the question – and four

participants (44%) demonstrated a complete lack of understanding of the 'domain and range of exponential and logarithmic functions.

Table 4.29: Participants' performance on the Domain and Range of the exponential and logarithmic function (Question 9A) (n=9)

**Domain and Range of exponential and logarithmic functions**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	4	44.4	44.4	44.4
1 Marks	3	33.3	33.3	77.8
2 Marks	2	22.2	22.2	100.0
<b>Total</b>	<b>9</b>	<b>100.0</b>	<b>100.0</b>	

It is of concern that a clear majority did not emerge for understanding this concept. This again casts doubt that if the teachers do not demonstrate adequate understanding of this concept, if they could they teach their learners.

Table 4.30 shows that question 9(b) had a total of 2 marks, testing participants' understanding of the Domain and Range of an exponential and logarithmic function.

Table 4.30: Participants' performance on Domain and Range of an exponential and logarithmic function (Question 9B) (n=9)

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	3	33.3	33.3	33.3
1 Marks	4	44.4	44.4	77.8
2 Marks	2	22.2	22.2	100.0
<b>Total</b>	<b>9</b>	<b>100.0</b>	<b>100.0</b>	

To be deemed to have achieved on this question, the participant had to score at least 1 mark. On this basis, six participants (66%) were deemed to have demonstrated understanding of this question.

The participants were asked to write the horizontal asymptote of the function  $f(x) = e^{3x}$ . This question had a total of 1 mark. The table shows that 8 participants (88.9%) scored full marks, which shows that the majority of the participants had the ability to write the horizontal asymptote of a function. This was a good result, although concern still remained with the one teacher who was not able to provide the correct answer.

Table 4.31: Participants' understanding of a horizontal asymptote of a function (Question 10A) (n=9)

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	1	11.1	11.1	11.1
1 Marks	8	88.9	88.9	100.0
Total	9	100.0	100.0	

Table 4.32 shows that the 5 participants (55.6%) scored full marks on the exponential function  $f(x) = 2\ln(x + 3) - 1$ , while 4 participants (44.4%) failed to provide the correct answer to this question.

Table 4.32: Participants' understanding of a horizontal asymptote of a function  $f(x) = 2\ln(x + 3) - 1$  (Question 10B) (n=9)

**Horizontal asymptote of the function**

	<b>n</b>	<b>%</b>	<b>Valid %</b>	<b>Cumulative %</b>
0 Marks	4	44.4	44.4	44.4
1 Marks	5	55.6	55.6	100.0
Total	9	100.0	100.0	

#### 4.4.3 Abilities to Teach Exponential and Logarithmic Functions

Two of the most important elements which any teacher needs are (a) a good command of his/her subject and (b) a repertoire of pedagogical skills to mediate learning (i.e. teaching strategies and techniques to facilitate learning). Indeed, it may be said that

without subject knowledge, one has nothing to teach, and without the ability to mediate between the subject matter and the learner, learning may be impossible. Without teaching skills, one will not be able to get the learner to understand the subject matter, and the curriculum goals will not be achieved.

In this study, the researcher used the data obtained from the Likert scale (Annexure 5) to assess the participants' teaching ability and competence. In this regard, the participants were asked to indicate by means of a tick (√) how they assessed themselves on the various teaching skills, abilities and competencies. There were 10 questions, each with 4 options and having a minimum of one mark and a maximum of four marks, giving a total of forty maximum marks and ten minimum marks.

Rather than using the 50% mark of 20 marks out of 40, the researcher raised the criterion mark for Educator Competence to 25 marks out of 40, which came to 63%. Thus, the score of 25 or 63% was used to represent the participants' professed ability to teach exponential and logarithmic functions in this study. Accordingly, the participant had to score at least twenty-five marks to be deemed to have demonstrated teaching competence.

Table 4.33 presents the data pertaining to the participants' performance on the Likert Scale on their readiness and ability to teach exponential and logarithmic functions.

Table 4.33: Educators' data to show an ability to teach exponential and logarithmic functions (n=9)

<b>N</b>	<b>Range</b>	<b>Minimum</b>	<b>Maximum</b>	<b>Sum</b>	<b>Mean</b>	<b>Std. Dev.</b>	<b>Variance</b>
9	7	20	27	217	24.11	2.204	4.861

Table 4.33 shows, the participants' mean score was 24.11, suggesting that they did not consider themselves to be suitably qualified, nor to have the ability to teach exponential and logarithmic functions. The range of marks was 7, the minimum mark

being 20 and the maximum 27. The total sum of marks was 217 on the data, out of a possible total of 360 marks – representing 62.3% of the total possible marks.

Table 4.34 presents individual scores obtained from the Likert Scale on the analysis of the individual participants’ ability to teach the content of exponential and logarithmic functions.

Table 4.34: The educators’ individual scores on the Likert scale (n=9)

Educators’	1	2	3	4	5	6	7	8	9
Score	25	23	23	25	27	24	27	23	20

From Table 4.34, participants 1,4,5 and 7 obtained 25 marks or more, demonstrating the ability to teach the content of exponential equations and logarithmic functions, while the rest of the participants did not meet this criterion mark – thus, failing to demonstrate ability to teach the content of exponential and logarithmic functions. Accordingly, because 5 out of 9 participants (55.55%) believed not to be able to teach the content of exponential and logarithmic functions, it is conclusive that the majority of the participants in this study did not believe that they had an ability to teach this content.

#### 4.4.4 Summary (Research Question 2)

The results to this research question showed that all the participants in this study were suitably qualified as they all possessed professional bachelor teaching degrees to teach mathematics. Furthermore, the majority of the participants demonstrated possession of an acceptable level of the required mathematical content knowledge (MCK), an indication that they were sufficiently knowledgeable in the subject. However, the 50% criterion mark used to determine knowledgeability was too low for the educators – especially since the same criterion mark was used for the learners. On the third aspect of ‘ability to teach’, although five participants (55.55%) did not believe that they could adequately teach exponential and logarithmic functions, it is important to remember that all the participants were suitably qualified to teach the subject. However, it appears that the specific content area of exponential and logarithmic functions posed some measure

of difficulty for them, as educators – as demonstrated by the inability of some of them to correctly solve some of the problems. In particular, the study revealed the following educator difficulties in answering some concepts in exponential and logarithmic functions.

- 1) The participants (educators) did not seem to understand the concept of ‘inverse function’.
- 2) The concepts of ‘domain and range’ also appeared to be a difficult area for the participants.
- 3) Although the participants satisfactorily answered the question on horizontal asymptote, which was an exponential function, in 10(a), they experienced difficulty answering question 10(B), focusing on the concept of ‘logarithmic function’.

In conclusion, the result to the second research question is expressed in the form of a table, representing the average percentages (refer to Table 4.35). It may be concluded from Table 4.35, the answer to Research Question 2 is that although all the participants were suitably qualified, some were knowledgeable and able to teach the content of exponential and logarithmic functions, while others were less so.

**Table 4.35: The answer as to whether or not the educators were suitably qualified, knowledgeable and able to teach exponential and logarithmic functions**

Suitably Qualified	Knowledgeable	Ability to teach	Result Average
100%	69.20%	44.44%	71.21%

#### **4.5 THE RELATIONSHIP BETWEEN EDUCATORS’ SELF-CONCEPT AND LEARNERS’ PERFORMANCE**

The third research objective of this study was to determine whether or not there was a relationship between educators’ readiness to teach exponential and logarithmic functions and the actual performance of their learners in exponential and logarithmic functions. To address this question, the researcher used data obtained from the learners’ test (n=242) divided in to two sets. One set of data represented the learners

who were taught by the educators with High Self-Concept (EHSC) about their ability to teach exponential and logarithmic functions, while the other set of data represented the results of learners who were taught by educators with Low Self-Concept (ELSC) about teaching exponential and logarithmic functions. Table 4.36 shows the mean scores of the learners in the two groups on the test on exponential and logarithmic functions.

**Table 4.36: Learner’s performance in the test (n1 = 84; n2 = 158)**

	EHSC	ELSC
n	84	158
Range	80	100
Minimum	0	0
Maximum	80	100
Mean	14.39	25.45
Standard deviation	18.30	23.07

Table 4.36: Learner’s performance in the test (n1=84 & n2=158)

Learners	n	Range	Minimum	Maximum	Mean	Standard Deviation
EHSC	84	80	0	80	14.39	18.30
ELSC	158	100	0	100	25.45	23.07

In Table 4.36, mean scores of the two data sets are 14.39 (n=84) for learners taught by EHSC and 25.45 (n=158) for learners taught by ELSC. Thus, the short preliminary answer to the third research question was therefore that learners taught by ELSC performed better than learners taught by EHSC. This preliminary result was put to a statistical test using the ‘t’ test statistic as follows:

### Hypothesis

H<sub>0</sub>: Learners taught by teachers who consider themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions will perform the

same as learners taught by teachers who consider they are not to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions.

H<sub>1</sub>: Learners whose teachers consider themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions will perform significantly higher than learners whose teachers consider themselves not to be suitably qualified, knowledgeable and not able to teach exponential and logarithmic functions.

The *t*-test statistic was used to test the hypothesis for two independent samples. The *t*-test statistic is the most powerful of the related sample tests. This is because the *t*-test makes full use of the data while, for instance, the WILCOXON test considers only the ranking of the pairs. The result is displayed in Table 4.37.

Table 4.37: Results of a “t” test on learner’s performance with EHSC and ELSC.

Learners	$\bar{x}$	$\sum D$	$\sum D^2$	n	df	$\alpha$	$t_o$	$t_c$	Results
EHSC	14.39	0	335.18	84	240	0.05	-42.34	1.62	significant
ELSC	25.45	0.03	532.29	158					

The ‘t’-test result in Table 4.37 shows that learners whose teachers considered themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions performed significantly lower than learners whose teachers considered themselves not to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions. Thus, the null hypothesis is rejected and the converse of the alternative hypothesis is accepted. This result was not predicted because the researcher expected that learners who were taught by EHSC would perform significantly higher than learners who were taught by ELSC. The meaning and implications of this result need to be interrogated closely, and the research replicated before far-reaching conclusions can be drawn.

#### 4.6 THE IMPACT OF EDUCATORS' PEDAGOGICAL CONTENT KNOWLEDGE (PCK) ON LEARNER PERFORMANCE

This research question sought to establish whether or not there was a significant relationship between educators' pedagogical content knowledge (PCK) and learners' achievement in exponential and logarithmic functions. To investigate this question, the researcher used data obtained from the learners' test and the educators' questionnaire. Learners' average marks per school are given in Table 4.3.

Table 4.38: Learners' performance (average marks) per school

School Name	A	B	C	D	E	F	G	H	I
School average mark (out of 40)	23.15	8.17	29.75	5.52	18.27	34.52	10.31	32.26	35.90
Performance (%)	58	20	74	14	46	86	26	81	90

The best performing school was School I with an average mark of 35.9(90%), followed by School F with 34.5 (86%). The worst performing school was School D with an average performance of 14%, followed by School B with an average of 20%.

Educators' scores obtained from the Likert scale represent their PCK. A summary of the data, in terms of educators' average scores per school, is given in Table 4.39.

Table 4.39: Educators' Likert scale score per school

School name	A	B	C	D	E	F	G	H	I
Score	23	23	25	25	27	24	27	23	20
Performance	57.5%	57.5%	62.5%	62.5%	67.5%	60%	67.5%	57.5%	50%

For the purpose of answering the fourth research question, the following hypothesis was tested:

## Hypothesis

H<sub>0</sub>: There will be no statistically significant difference between the performance of learners taught by teachers with higher pedagogical content knowledge (PCK) and learners taught by teachers with lower PCK.

H<sub>1</sub>: Learners taught by teachers with higher pedagogical content knowledge (PCK) will perform significantly higher than learners taught by teachers with lower PCK.

The 't'-test statistic for two independent samples was used to test the above null hypothesis. To do this, learners taught by teachers who obtained 60% or higher marks in the PCK questionnaire formed Group A and those taught by teachers who obtained lower than 60% in the questionnaire formed Group B. Accordingly, learners from schools C, D, E, F and G formed Group A (HPCK), while learners from schools A, B, H, and I constituted Group B (LPCK). Thus, to test the hypothesis, the means of these two groups were compared to answer Research Question 4. Table 4.40 presents the mean scores of the two groups.

**Table 4.40: Learner's performance in the test (n<sub>1</sub>=105; n<sub>2</sub>=137)**

Learners	n	Range	Minimum	Maximum	Mean	Standard Deviation
HPCK	105	80	0	80	18.41	12.81
LPCK	137	100	0	100	24.07	12.05

In Table 4.40, mean scores of the two data sets are 18.41 (n<sub>1</sub>=105) for the HPCK and 24.07 (n<sub>2</sub>=137) for the LPCK. Much like the case for the third research question, the 'eyeball' means score comparison shows that learners taught by educators with HPCK performed lower than learners taught by educators with LPCK. A statistical comparison (Table 4.41) confirms this finding.

Table 4.41: Results of a “t” test on learner’s performance between HPCK and LPCK

Learners	$\bar{x}$	$\Sigma D$	$\Sigma D^2$	n	df	$\alpha t_o t_c$	Results
HPCK	18.41	0.95	17237.57	105	240	0.05	-3.51 1.62 significant
LPCK	24.07	0.41	19919.27	137			

As in the case of the third research question, the null hypothesis is also rejected in this case, and the converse of the alternative hypothesis is accepted. This result was not predicted because the researcher expected that learners who were taught by educators with HPCK would perform significantly higher than learners who were taught by educators with LPCK. The meaning and implications of this result need to be examined closely, and the research replicated before far-reaching conclusions can be drawn.

#### 4.7 SUMMARY OF THE FINDINGS AND DISCUSSION

Many school curricula define a logarithm as an exponent, and a logarithmic function as the inverse of an exponential function. Not only do learners struggle with exponential laws and functions, but research has shown that learners struggle with the topic of logarithmic functions (Gamble, 2005; Wood, 2005; Chua, 2006; Berezovski, 2007). In particular, Berezovski (2007) found that not only did learners lack conceptual understanding of exponential and logarithms functions but so did pre-service teachers – thereby suggesting that teacher’s mathematical knowledge could have a strong impact on his or her students’ understanding and achievements. So, it is important to try and mend this learning gap and find where learners are making mistakes when it comes to logarithms, and appropriately adapt how teachers approach the topic.

Logarithmic functions may be viewed as the inverses of exponential functions. Learners appear to experience difficulties in understanding the two types of functions used in this study. Some learners struggle to express the exponential equation  $y = ax$  into a logarithmic equation,  $y = \log_{ax}$  (Bogley and Robson, 1999). Mathematical functions encompass concepts which form important aspects of school mathematics curricula and

should be taught with understanding (Department of Education, 2012). Webber (2002) makes a similar observation by stating that exponential equations and logarithmic functions are important concepts that play a fundamental role in mathematical courses, including calculus, differential equations, and complex analysis. Therefore, there is a need for teachers to teach these functions for understanding, as they are mostly used in many real-world situations. Typically, a student limited in his or her understanding of exponents will be able to evaluate exponential functions only in the cases where the power given is a positive integer.

Most of the difficulties learners exhibit come from previous learning, as Khanyile (2016 ) points out, since learners' "previous knowledge contains errors and misconceptions, the construction of new knowledge results in errors." Overall, her findings suggested that "teaching intervention is a necessary pedagogical technique and needs to be employed when addressing learners' errors and misconceptions in mathematics." Khanyile (2016, p.2).

The analysis of the results to the first research question revealed that learners demonstrated some understanding of exponential equations and logarithmic functions (Table 4.17), although they were not comfortable with the concepts of inverse function, domain and range, horizontal and vertical asymptotes. These results concur with those reported in literature from previous empirical studies (Gamble, 2005; Wood, 2005; Chua, 2006; Berezovski, 2007; Hanushek and Woessmann, 2009; Ludger Woessmann, 2010; Higgins & Mcoah, 2010; Wiggins, 2014).

In addressing the difficulties that learners (and some teachers) appear to experience, it may be necessary to explore the benefits of 'spatial' teaching since this involves using concrete objects, and therefore advances learning from a concrete operational angle (Younger, 2018). For instance, Younger's findings suggested that "incorporating spatial skill activities into mathematics lessons had a positive impact on both the teachers' reflective practice and the students' learning skills" (Younger, 2018: ii). As Younger (2018: 6) further affirmed, "moments of insight are more likely to occur through actively engaging – physically, intellectually and emotionally – with the process of learning,

rather than through the passive acceptance and reproduction of a prescribed set of algorithms to solve an abstract and possibly irrelevant problem” Younger(2018,p.6). As a case-in-point, it would be very important for teachers to teach inverses as an extension of functions so that learners can understand what restrictions apply to functions, domains and ranges. The formal definition and understanding of a function must be emphasized by mathematics teachers in their classes.

The answer to the second research question, which was specific to the teachers’ readiness to teaching exponential equations and logarithmic functions, was mixed. All the participating teachers were suitably qualified in terms of the minimum requirements needed for registration with the South African Council for Educators (SACE). All the participating educators were in possession of a professional degree in teaching mathematics. Thus, it was concluded that the educators who participated in the study were suitably qualified. However, with specific reference to their ability to teach exponential equations and logarithmic functions, although the educators were suitably qualified for registration with SACE, on being knowledgeable on the topic, their views were mixed. Indeed, there were questions which some educators failed to solve in the same test that had been given to their learners – much like some of the learners. Overall, on the content of exponential and logarithmic functions, the mean score of the educators (23.56 out of 40 = 59%) was quite close to the mean score of their learners on the same tasks (21.62 out of 40 = 54%). Certainly, one would have expected the teachers to have scored much higher than their learners. The difference in performance between the two groups was really marginal and did not give one much confidence on the side of the educators.

The results to the second research questions were put to a practical test in answering the third research question, where a statistical test was conducted to determine how the learners taught by educators, who rated themselves lower on being suitably qualified, knowledgeable and able to teach exponential and logarithmic functions, relative to those taught by educators, who rated themselves higher. The statistical test led to the rejection of the null hypothesis, which opined that there would no statistically significant difference in the performance of learners taught by educators who rated themselves

higher on being suitably qualified, knowledgeable and able to teach exponential and logarithmic functions and those who were taught by educators who rated themselves lower. However, the alternative hypothesis was also rejected, in favour of its converse. This finding was not predicted because the researcher expected that learners who were taught by EHSC would perform significantly higher than learners who were taught by ELSC. The meaning and implications of this result need to be interrogated closely, and the research replicated before far-reaching conclusions can be drawn.

The results to the second and third research questions were based solely on the teachers' self-concept vis-à-vis their readiness to teach exponential and logarithmic functions. So, it is probably possible that those who rated themselves **not to be** suitably qualified, knowledgeable and able to teach this topic were more aware of their limitations than those who rated themselves higher. Therefore, the former group probably worked harder than the latter group, to compensate for their felt shortcomings – thereby producing better results.

On the fourth research question, the results showed that learners taught by educators with HPCK performed statistically lower than learners taught by educators with LPCK. Accordingly, the null hypothesis, which predicted that there would be no statistically significant difference in the performance of learners taught by teachers with higher pedagogical content knowledge (PCK) and learners taught by teachers with lower PCK, was rejected. Instead, the results showed that learners taught by teachers with lower PCK performed significantly higher than those taught by teachers with higher PCK, contrary to the prediction of the alternative hypothesis. This result was not predicted because the researcher expected that learners who were taught by educators with HPCK would perform significantly higher than learners who were taught by educators with LPCK. This is a rare result where both the null hypothesis and its alternative are rejected, in this case, in favour of the converse of the latter. The meaning and implications of this result need to be examined closely, and the research replicated before far-reaching conclusions can be drawn.

The results obtained in the previous researches and in this study are coinciding. The literature supported these results. The learners to understand the specific concepts, an experienced mathematic educator with a deep knowledge of mathematics teaching is invertible (e.g. Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball, 2008; Cai, et al. Kaiser, Perry & Wong, 2009; Cai & Ding, 2015).

From these studies, there is a suggestion that strong educator knowledge could yield benefits for classroom instruction and learners' achievement (Hill, et al., 2008). Indeed, the quality of education in any country, to a large extent, depends on the quality of its educators. This view is affirmed by South Africa's Department of Education (2006: 5) in stating that "teachers are the essential drivers of a good quality education system." This recognition of the role of teachers suggests that no educational system can rise above the quality of its educators., and that educators are the key role players insofar as the interpretation and implementation of the school curriculum is concerned (Rimillard; 1999). There is evidence that countries which have strong mathematics education can transform and industrialise faster than countries that are weak in mathematical knowledge (OECD, 2010).

There is an assumed connection between teacher knowledge and student achievement in general, and there are revealing patterns in the connection with regard to specific mathematical domains, processes and levels of cognitive demand in particular (Tchoshanov, Cruz, Huereca, Shakirova, Shakirova & Ibragimova, 2017). The results reported by Tchoshanov, et al. (2017: 683) showed a statistically significant correlation between teacher content knowledge and student performance, thereby suggesting "that teachers' content knowledge plays an important role in student performance at the lower secondary school." Olfos, et al. Goldrine & Estrella (2014: 913) also reported that "the constructivist-oriented subcomponent of the teachers' content knowledge showed a significant association with student learning, although it is less significant than the association with the teacher's experience.

There are few studies on teacher quality in developing countries, but those available concur that differences in teacher quality can significantly impact student achievement.

One such study found that in Peru, teachers with high achievement in mathematics increased student achievement on standardised mathematics tests by about 9 percent of a standard deviation (Metzler & Woessmann, 2012). In China, teachers of the highest professional rank more positively affected rural students' achievement than teachers of lower rank (Chu et al., 2015).

It should, furthermore, be pointed out that there may be a number of different ways to improve the quality of teaching of rural students, such as improving incentives for teachers (Loyalka et al., 2015; Muralidharan & Sundararaman, 2011; Muralidharan, 2012).

In contrast, the findings reported in this paper contradict the findings of the research conducted by Lu, et al. (2017).

Corkin & Ekmekci (2016) report that, when they teach outside their areas of specialization, educators give explanations and analogies which reinforce the misconceptions. Campbell, Nishio & Smith (2014) argue that the knowledge of teaching strategies is dependent on the educator's subject matter knowledge about that particular concept. On their part, Jacob, Hill & Corey (2017) report that there is limited evidence of positive impacts of teachers' MCK in teaching and no effects on the quality of instructional practice or student outcomes. Thus, it may not necessarily be the case that a teacher's subject matter knowledge results in better learner performance and understanding in targeted concepts.

It has been observed that teachers may be low in their PCK scores but could be exceptional in their "knowledge of students' knowledge, regarding both their knowledge of students' challenges and frequent mistakes and the difficulties of and errors made by the students in their specific classes" – resulting in better performance by their learners (Olfos, et al., 2014: 928). Although teachers' knowledge of their students was not measured in this study, the above point highlights the possibility that there could be other "factors that affect the student gain and achievement in mathematics" (Olfos, et al., 2014: 930) beyond the ones investigated in this study. In their study, Olfos, et al., found that teacher PCK was more closely correlated with teacher mathematical content

knowledge than with learner gains or performance. The contribution of PCK to learner performance, therefore, appears to be questionable. On the basis of the results, it believes that teacher education should be a requirement for registration with teacher professional bodies.

The chapter that follows presents a summary of the whole study, conclusion and some recommendations.

## CHAPTER FIVE

### SUMMARY, CONCLUSION, RECOMMENDATIONS AND LIMITATIONS

#### 5.1 INTRODUCTION

This chapter presents the summary of the whole study, including the findings in relation to the research questions, conclusion, recommendations and limitations of the study.

#### 5.2 SUMMARY OF THE STUDY

The Summary is presented under several sub-headings to answer the research questions.

##### 5.2.1 Aims of the study

This study investigated factors contributing to learner underachievement in exponential equations and logarithmic functions. The study was carried out within the South African context which, like other developing countries, is burdened with the problem of under-qualified and unqualified mathematics educators.

More specifically, this study sought to address the following research questions:

- a) What are the National Senior Certificate learners' main understandings and misunderstandings of exponential and logarithmic functions?
- b) Do grade twelve teachers consider themselves to be suitably qualified, knowledgeable and able to teach exponential and logarithmic functions?
- c) Is there a relationship between educators' self-concept about their ability to teach exponential and logarithmic functions and the actual performance of their learners on these two mathematical constructs?
- d) Does educators' pedagogical content knowledge impact learner achievement in exponential and logarithmic functions?

##### 5.2.2 The Literature Review and Conceptual Framework

The conceptual framework for the study took the form of two models depicting **learner** and **educator** readiness. These models illustrated factors that could possibly affect the

ability of the learner to succeed in understanding instruction related to exponential equations and logarithmic functions, as well as those that would prevent educators from delivering optimum instruction to learners. In this regard, nine learner variables which could have had a bearing on the quality of instruction in exponential equations and logarithmic functions were (a) concepts related to exponential equations and logarithmic functions, (b) procedural aspects of the operations related to exponential equations and logarithmic functions, (c) domains of rational functions, (d) the notion of horizontal and vertical asymptotes of a function, (e) underlying mathematical concepts, (f) the interaction of informal and formal knowledge, (g) the nature of their misconceptions about mathematics, generally, and exponential equations and logarithmic functions, in particular, and (h) possible error types committed by learners related to mathematics, generally, and exponential equations and logarithmic functions, in particular. The educator variables were (a) educators' pedagogical knowledge (educator knowledge of curriculum, educator knowledge of instructional strategies, educator knowledge of learners' academic needs, educator knowledge of learners' social needs [pastoral care], educator teaching ability), (b) Educator content knowledge, (c) Educator as leader and administrator.

### **5.2.3 Methodology**

This study used a mixed-methods research paradigm, as there was need to collect both quantitative and qualitative data in order to adequately answer the four research questions. A research design is a specific plan for getting information which will address the research questions posed in a particular study. Thus, for this study, the survey research design was chosen as it enabled the researcher to interpret and describe how the identified variables affected learners' achievements in exponential and logarithmic functions.

For data collection, a researcher-designed test was used to gather data about learners' understanding and misunderstanding about the concept of exponential and logarithmic functions. A questionnaire for educators was also developed by the researcher for the purpose of collecting data on mathematics educators' mathematical content knowledge

(MCK) on the topics of exponential equations and logarithmic functions – as well as the knowledge of their learners and their pedagogic content knowledge (PCK).

The research sample was drawn from a target population of high school learners in the uMkhanyakude education district, in terms of which the Ubombo CMC was the accessible population. More specifically, the research sample consisted of 242 learners based in nine randomly selected schools. The nine 9 educators teaching high school mathematics participated in the study. After securing permission to carry out the research, the principals and teachers who consented to the research were given the questionnaire to complete. Analysis was done using the SPSS version 23 software programme.

#### **5.2.4 Results of the study**

The results revealed that learners had basic understanding of exponential and logarithmic functions in most aspects of the topic, although their performance was borderline at 54%, with 46% of them falling below the 50% mark. On the second research question, although all the educators who participated in the study were suitably qualified in terms of the minimum requirements for registration with the South African Council for Educators (SACE), their performance on the same test which their learners wrote (which was a measure of their level of MCK) was **not** much above the performance of their learners, at 59%. To some extent, this level of performance was explained by the educators' response to the question about their ability to teach exponential equations and logarithmic functions, whereby their views were mixed. Indeed, there were questions which some educators failed to solve in the same test that had been given to their learners.

On the relationship between educators' self-concept about their ability to teach exponential and logarithmic functions and their learners' performance, the result showed that learners whose teachers considered themselves to be suitably qualified, knowledgeable, and able to teach exponential and logarithmic functions performed significantly lower than learners whose teachers considered themselves not to be suitably qualified, knowledgeable, and able to teach exponential and logarithmic

functions. This led to the rejection of the null hypothesis; the alternative hypothesis was also rejected because it hypothesized that the learners taught by the high self-concept educators would out-perform those taught by the low- self-concept educators (on their ability to teach these topics).

The last research question sought to establish the impact of educators' pedagogical content knowledge (PCK) on learner performance in exponential and logarithmic functions. The null hypothesis tested was that there would be no statistically significant difference between the performance of learners taught by teachers with higher pedagogical content knowledge (PCK) and those taught by teachers with lower PCK. Again, like in the case of the fourth research question, the null hypothesis, here, was also rejected, as well as the alternative, which opined that learners who were taught by educators with HPCK would perform significantly higher than learners who were taught by educators with LPCK. That was not the case.

### **5.3 CONCLUSION**

This study has highlighted the factors contributing to underachievement of learners in exponential and logarithmic functions in the FET school phase. Certainly, looking at the performance level of both the teachers and their learners on the topics of exponential equations and logarithmic functions, a lot needs to be done to improve the situation. Furthermore, some unexpected results appeared in this study, which cast some shadows on two big assumptions which undergird, particularly, pre-service teacher education – namely that both mathematical content knowledge and pedagogical content knowledge are important for teachers to do a good job. From the results of this study, this is not the case. So, quite clearly, more research needs to be conducted to investigate these matters. Overall, however, the researcher is satisfied that the purpose of this research project has been satisfactorily served, and the results have an important contribution to this field of research.

### **5.4 RECOMMENDATIONS**

This study was conducted after the topics of exponential equations and logarithmic functions had been taught. Accordingly, learner performance at 54% suggests that there

were as many aspects which were understood, as those that were not understood, in the teaching / learning of these topics. The finding that the educators themselves performed only marginally above the performance of their learners, points to the fact that the teachers themselves need to be assisted to better understand these topics. So, the first recommendation is that in-service intervention is needed for teachers of mathematics, specifically targeting the topics of exponential equations and logarithmic functions.

On the question of MCK, it may be said that its contribution to learner performance remains inconclusive. It would be too presumptuous to run to the conclusion that teachers need not be well-versed in the subject matter content for them to teach the subject well. Certainly, looking at the levels of performance of both the learners and their teachers on the same test (as reported in this study), it is tempting to surmise that there is a relationship between how much teachers know and understand about the specific content of a subject and how well their learners will perform on that content. Flowing from this, it may then be recommended that the first recommendation above be re-enforced – that is, that an in-service intervention be implemented to assist the teachers better understand the specific subject matter. Furthermore, in order to improve educators' MCK with respect to exponential and logarithmic functions, mathematics subject specialists, advisers and school-based mathematics Heads of Department (HOD's) should have continuous support programmes for newly appointed educators, as well as educators who are teaching mathematics, but whose area of specialisation is not mathematics.

From the results of this study, it is important to recommend that the MCK interventions emphasize the graphical feature of exponential equations and logarithmic functions, shape of the curve, size of the curve, domain of the exponential and its inverse functions, range of the exponential function, range of the logarithmic function, defining the difference between the exponential and logarithmic functions and an equation, as well as giving analogies that help differentiate between exponential equations and logarithmic functions.

However, the interventions organized for mathematics educators should not only deal with issues of educators' MCK, but also address educators' PCK, with respect to specific topics in mathematics. Thus, the same recommendation is hereby made with respect to PCK. Beyond the good command of the subject matter content, it appears reasonable (although not supported by the findings of this study) to posit that an intervention focusing on how to teach exponential equations and logarithmic functions could be helpful to the educators. Accordingly, educators with good knowledge and strategies for teaching exponential and logarithmic functions should be identified so that they can mentor other educators. Such mentors could help other educators to come up with conceptual explanations, analogies and powerful representational models relevant to the teaching of the specific concepts in exponential and logarithmic functions. In line with these suggestions, the mentors could be given time-off so that they can use these periods for meetings. In these meetings, other educators can seek clarifications on concepts that they find challenging.

More specifically, the results of this study suggest that teachers should must emphasize the concept of asymptote in exponential and logarithmic functions as the learners found it overly difficult. The results also showed that learners exhibited much weakness in visual reasoning and translating between graphical and symbolic representations of exponential equations and logarithmic functions. The best ways to teach these concepts need to be explored as part of the suggested professional teacher development initiatives.

In the meantime, however, learners must not be left outside the 'direct intervention loop'. To have a critical mass of teachers, who are well-qualified, knowledgeable and able to teach exponential equations and logarithmic functions to an acceptable level of competence, will take some time? It is a systemic challenge whose timelines cannot be guaranteed. Accordingly, a concerted effort should be made to design learning support materials and mount interventions directed at the learners, to assist them cope with the current challenges they face in understanding exponential equations and logarithmic functions.

Thus, this two-pronged effort (targeting both educators and learners) will go a long way towards alleviating the difficulties experienced by both learners and educators in this part of the FET mathematics curriculum. Certainly, empowering educators by increasing their repertoire of instructional strategies, which they could use to teach exponential equations and logarithmic functions, could make a significant contribution to their day-to-day operations. In turn, this could have a significant benefit for their learners. Therefore, there is a huge need for well-articulated support and professional development programmes for FET phase mathematics educators.

Lesson observations are also recommended as part of professional teacher development, particularly for newly appointed educators. Lesson observations and assigning mentors to educators has been part of very successful professional development programmes in Japan– a country reputed and well known for excellence in mathematics achievement. Direct classroom observations could also help reveal other areas requiring additional resources and support, as well as identify some strengths, which serve as building blocks for a successful career in mathematics teaching. For instance, some newly employed educators may be good at integrating information communication technologies (ICT's) in their lessons or may be very good at using context-based questions to enhance learners' achievement. These capabilities must not be lost to old traditions of teaching which the new recruits may find in the schools, but must be harnessed and nurtured to enhance the instructional repertoire of the school as a whole.

Membership of professional bodies, such as the Association for Mathematics Educators of South Africa (AMESA) and the Southern African Association for Research in Mathematics, Science and Technology Education (SAMSTE) is recommended for all mathematics educators. Through these organizations educators can keep abreast with current research and other developments that are relevant to their levels of practice.

Finally, the Department of Basic Education is hereby requested to provide modern technologies for use in the teaching and learning of exponential and logarithmic functions, in particular, and mathematics, in general. The use of these technologies

(computers, graphing programmes, and calculators) would help learners to make conjectures easily and reduce the incidence of learning modalities, which would lead to misconceptions, while at the same time correcting some alternative conceptions with respect to exponential equations and logarithmic functions.

## **5.5 LIMITATIONS OF THE STUDY**

Five limitations may be cited with regard to this study:

- a) The sample size used in this study in respect to the educators was small, thus, care must be taken about the extent to which one generalizes the results.
- b) Financial constraints and the researcher's commitments (no time-off from teaching) restricted the geographical scope of the study which, in turn, restricted the sample size of participants.
- c) The study was conducted on the assumption that the educators' MCK and PCK may affect what transpired in the classrooms, however, there were no direct observations made to verify this assumption. Such direct classroom observations could have revealed additional and use full information which could have enriched the study. However, as stated above, the study was conducted after the topics of exponential equations and logarithmic functions had been taught.
- d) The questionnaires for the educators were left with the respondents to complete in their own time. Although the researcher emphasized the need for the educators to attend to the questionnaire independently, this remains a limitation of the study.
- e) In question 2, there was an incomplete instructional sentence, which may have confused learners and educators alike. In question 3, there was a printing mistake " $x \in R$ ", and there was a repetition in question 4. In question 9, there was a typing error. All these mistakes could have adversely affected the study in one way or another. However, it is not clear how these limitations could have affected the specific and overall results of the study.

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# ANNEXURES

## ANNEXURE 1

### CONSENT LETTER TO PARENTS/GUARDIAN

School

Igugu Lesizwe secondary

P.O. Box 69

Jozini

3969

30/06/2016

Dear parent/guardian

Your ward Master/Miss ..... has been selected to participate in a study aimed at shedding light on the impact of evaluation of factors contributing to underachievement in exponential and logarithmic functions in further education and training(FET) phase. The research will be used for academic purposes only. However, your consent is required since your ward is a minor. Your ward will only be required to respond to a questionnaire containing questions about his/her understanding /conceptions about quadratic functions. This questionnaire will take your ward about one hour to complete. Arrangements will be made to ensure that this does not disturb his/her school work.

Rest assured that the information supplied by your ward will be treated confidentially.

Thank you for your cooperation.

Yours truly,

Mohammad Javed Khizer (Mr.)

(Please complete the acceptance form that follows to confirm your approval)

CONSENT FORM

I..... the parent/guardian of  
Master/Miss..... hereby give my approval for  
His/her participation in the study described above.

Signature..... Date.....

**ANNEXURE 2**

**CONSENT LETTER TO SCHOOL GOVERNING BOARD**

Igugulesizwe secondary School  
P.O. Box 69  
Jozini  
3969  
30/06/2016

The Secretary,

.....  
.....

Governing Board

Sir/madam,

Permission to conduct a research

I hereby request your permission to allow me to conduct a study in your school. The research is for academic purposes only. It is aimed at shedding light on the impact of the Evaluation of factors contributing to exponential and logarithmic functions in further educations and training (FET) phase. The information obtained can be useful in improving the teaching and learning of mathematics, specifically the teaching and learning of exponential and logarithmic functions. The study will require the participation of grade-12 twenty learners and your mathematics educators. The educators will respond to a questionnaire, and at a later stage will be interviewed about their strategies in helping their learners to understand the exponential and logarithmic functions. The learners will also respond to a questionnaire which will last for about one hour. Meanwhile, arrangements will be made to ensure that the process does not disturb any

academic activities in your school. It might be important to inform you that I have already negotiated with the mathematics educators and they showed interest in participating in the research if you approve of it.

Your cooperation in this regard will be highly appreciated.

Yours truly,

Mohammad Javed Khizer (Mr.)

(Please sign the consent form to confirm your acceptance)

### CONSENT FORM

We the School Governing Board of .....,

Hereby approve your request to carry out the study described above.

.....

Chairman

.....

Secretary

.....

Principal

School stamp
--------------

## **ANNEXURE 3**

### **LEARNERS' QUESTIONNAIRE**

Igugulesizwe secondary School

P.O. Box 69

Jozini

3969

30/06/2016

Dear Respondent,

Thank you for accepting to participate in this study. The information supplied by you will be used for academic purposes only. Specifically, the information will be used in research work for a M.Ed. degree in mathematics education. The research is aimed at to evaluate the factors contributes to underperform the learners in exponential and logarithmic functions in further education and training (FET) phase. It is my belief that the information obtained from this research will be profitable in improving the teaching and learning of mathematics, and specifically, the teaching and learning of exponential and logarithmic functions. Rest assured that the information supplied by you in this questionnaire will be treated confidentially.

May i thank you once more for the sacrifice you have made in completing this questionnaire despite numerous other engagements.

Yours,

Mohammad Javed Khizer (Mr.)

(Researcher).

(Please complete the acceptance form that follows to confirm your approval)

## CONSENT FORM

I undersigned Surname.....Name(s)..... the  
gender..... of an age.....years learning mathematics in grade  
12 , hereby give my approval to participate in the study described above.

Signature..... Date.....

## **ANNEXURE 4**

### **CONSENT LETTER TO THE EDUCATORS**

Igugu Lesizwe secondary School  
P.O. Box 69  
Jozini  
3969  
30/06/20

Dear colleague,

Following our oral discussion, I hereby write to confirm your acceptance to participate in this study. As I informed you in our discussion, the investigation will be used for academic purposes only. The topic of my study is “Evaluation of factors contributing to underachievement in exponential and logarithmic functions in further education and training (FET) phase. It is my belief that the information gathered in this research will be useful in improving the teaching and learning of mathematics and specifically, the teaching and learning of exponential and logarithmic functions. Rest assured that the information supplied by you will be treated with the confidentiality it demands. The research is structured such that you will be required to respond to a questionnaire containing questions about exponential and logarithmic functions, and your experience and understanding about your learners’ thinking and conceptions about exponential and logarithmic functions. The questionnaire will take you about 1 hours to complete. At a later stage, you will be interviewed about the strategies that you adopt to ensure that your learners understand the concept of exponential and logarithmic function very well. Your Learners will also respond to a questionnaire which will be used to collect data about their achievement and understanding of different components of exponential and logarithmic functions. Once more thank you for accepting to be part of this study

Yours truly,

Mohammad, JavedKhizer (Mr.)

(Please, fill the consent form below to confirm your acceptance)

CONSENT FORM

I, undersigned ..... hereby confirm

That I have accepted to participate in the study as explained above.

Signature ....., Date .....

**ANNEXURE 5**

**UNIVERSITY OF ZULULAND**

**FACULTY OF EDUCATION**

**DEPARTMENT OF MATHEMATICS SCIENCE AND TECHNOLOGY**

**RESEARCH INSTRUMENT**

A STUDY OF FACTORS CONTRIBUTING TO UNDERACHIEVEMENT  
INEXPONENTIAL AND LOGARITHMIC FUNCTIONS IN THE FURTHER EDUCATION  
AND TRAINING SCHOOL PHASE

**Instructions for Teachers:**

Indicate by means of a tick ( $\checkmark$ ) your choice in the appropriate space..

**SECTION A**

**BIOGRAPHICAL INFORMATION - yourself**

**GENDER:**

Male	
Female	

**AGE:**

20 – 24	
25 – 29	
30 – 34	
35 – 39	
40 – 44	
45 – 49	
50 $\geq$	

**TEACHING GRADE:**

Grade 10	
Grade 11	
Grade 12	

**TEACHING EXPERIENCE:**

GENERAL	√	MATHEMATICAL FUNCTIONS	√
0-3		0-3	
4-10		4-10	
11-20		11-20	
21≥		21≥	

**TEACHING QUALIFICATIONS**

	DEGREE IN MATHEMATICS EDUCATION OR MATHEMATICS	√	×
1			
2	DEPLOMA IN MATHEMATICS OR MATHEMATICS EDUCATION		
3	CERTIFIACTES IN MATHEMATICS OR MATHEMATICS EDUCATION		
4	ANY OTHER QUALIFICATION		

## SECTION B

Indicate by means of a tick ( $\checkmark$ ) your choice in the appropriate space.

ITEM	strongly agree	agree	disagree	strongly disagree
1. I have an understanding of exponential and logarithmic functions.				
2. I am competent and knowledgeable to teach exponential and logarithm functions.				
3. I appreciate learners understanding of exponential and logarithmic functions.				
4. I find a relationship between teachers' readiness to teach exponential and logarithmic functions and the actual performance of learners.				
5. I find a relationship between educators' pedagogical knowledge content and learner achievement in exponential and logarithmic functions.				
6. I can differentiate between an algebraic equation and an algebraic function.				
7. I can define an exponential and logarithmic function with an example.				
8. I can sketch a general exponential function and indicate all the key points.				
9. I can explain and sketch an inverse function on graph paper with all the key points.				
10. I can write down the horizontal asymptote of a given function.				

Grand Total = 10 questions

## ANNEXURE 6

### LEARNER'S QUESTIONNAIRE

Marks: 40

Time: 45 min

#### INSTRUCTIONS

This questionnaire consists of two sections (A&B). Please, complete the two sections.

- 1) Section A is for general information
- 2) Section B contains 10 questions. Please, answer all the questions in the spaces where provided.
- 3) Graph sheets are included as per question needs.
- 4) Show all you are working.
- 5) Do all your best.

#### SECTION A

NAME OF CIRCUIT ...UBOMBO.....

WARD.....

AGE.....

GENDER.....

SCHOOL.....

GRADE.....

#### SECTION B

##### QUESTION NO 1

Differentiate between an algebraic equation and an algebraic function? (03)

.....

.....

.....

.....

QUESTION NO 2

Define an exponential and logarithmic function giving at least one example. (05)

.....

.....

.....

.....

QUESTION NO 3

Sketch the general exponential function  $f(x) = a \cdot b^x \quad \forall a > 0 ; a \neq 1 ; b > 1$  and  $x \in R$  (03)  
where R is a Real number. On the graph paper provided indicate all the key points

QUESTION NO 4

Sketch the inverse function of the function in QUESTION NO3 and name it. (03)

QUESTION NO 5

In the exponential function  $f(x) = a \cdot b^{x+p} + q \quad \forall a > 1 ; b > 1$  and  $p, q$  and  $x \in R$

(a) What is the impact of constant a on the sketch of f(x)? (01)

.....

.....

.....

(b) How does the base “b” affect the graph of f(X) (01)

.....

.....

.....

QUESTION NO 6

According to the definition of logarithmic function; (02)

$\log_b a = x, \forall a > 1 \text{ and } x > 0$  If and only if

.....  
.....

QUESTION NO 7

Graph the following functions on the same graph paper provided and indicate all the key points.

$f(x) = \left(\frac{6}{5}\right)x - 1$  and  $f(x) = \frac{5}{6}x + 1 \forall x \in R$  (08)

QUESTION NO 8

Sketch the inverse functions of the functions given above in QUESTION NO 7 on the same graph paper provided with all key points. (08)

QUESTION NO 9

Write the domain and range of the following functions

(a)  $f(x) = 2.5^{x-2} + 1$  (02)

.....  
.....  
.....

(b)  $f(x) = -1\left(\frac{2}{3}\right)^{x+1} - 2$  (02)

.....  
.....

QUESTION NO 10

Write down the horizontal asymptote of the following functions

(a)  $f(x) = e^{3x}$  (01)

.....  
.....  
.....

(b)  $f(x) = 2\ln(x + 3) - 1$  (01)

.....  
.....  
.....

## ANNEXURE 7

### Educators' questionnaire

Marks: 40

Time: 45 min

#### INSTRUCTIONS

This questionnaire consists of two sections (A&B). Please, complete the two sections.

- 6) Section A is for general information
- 7) Section B contains 10 questions. Please, answer all the questions in the spaces where provided.
- 8) Graph sheets are included as per question needs.
- 9) Show all you are working.
- 10) Do all your best.

#### SECTION A

NAME OF CIRCUIT ...UBOMBO.....

WARD.....

AGE.....

GENDER.....

SCHOOL.....

GRADE.....

**SECTION B**

**QUESTION NO 1**

Your learners' were given the question below. What is your answer to the question?

Differentiate between an algebraic equation and an algebraic function? (03)

.....

.....

.....

.....

**QUESTION NO 2**

Your learners' were asked to define the following functions. How will you define?

Define an exponential and logarithmic function giving at least one example. (05)

.....

.....

.....

.....

**QUESTION NO 3**

Your learners' were given the general equation of an exponential function to sketch on the graph paper. Use a graph paper provided to sketch the following function.

Sketch the general exponential function  $f(x) = a \cdot b^x \quad \forall a > 0 ; a \neq 1 ; b > 1$  and  $x \in \mathbb{R}$  (03) where  $\mathbb{R}$  is a Real number. On the graph paper provided indicate all the key points

QUESTION NO 4

Your learners' were asked to sketch the inverse function and named it. Sketch the inverse function on the same graph paper provided in question no.3 and name it.

Sketch the inverse function of the function in QUESTION NO3 and name it. (03)

QUESTION NO 5

Your learners' were asked to explain the impact of "a" and "b". Give your answer with an explanation of an impact on the following graph.

In the exponential function  $f(x)=a.b^{x+p} + q \forall a > 1; b > 1$  and  $p, q$  and  $x \in R$

(b) What is the impact of constanta on the sketch of f(x)? (01)

.....  
.....  
.....

(b) How does the base "b" affect the graph of f(X) (01)

.....  
.....  
.....

QUESTION NO6

Your learners' were asked to define the following question. Provide your answer.

According to the definition of logarithmic function; (02)

$\log_b a = x, \forall a > 1$  and  $x > 0$  if and only if

.....  
.....  
QUESTION NO 7

Your learners' were asked to sketch the graphs on a graph paper. Sketch the following functions on a graph paper with all key points.

Graph the following functions on the same graph paper provided and indicate all the key points.

$$f(x) = \left(\frac{6}{5}\right)x - 1 \text{ and } f(x) = \frac{5}{6}x + 1 \forall x \in R \quad (08)$$

QUESTION NO 8

Your learners' were sketching the inverse functions. You are asked to sketch the inverse functions of the functions given above in question no.7 on the graph paper provided.

Sketch the inverse functions of the functions given above in QUESTION NO 7 on the same graph paper provided with all key points. (08)

QUESTION NO 9

Your learners' were asked to write down the Domain and Range of the functions given below. Answer the question according to your understanding to the subject.

Write the domain and range of the following functions

(a)  $f(x) = 2.5^{x-2} + 1$  (02)

.....  
.....  
.....

(b)  $f(x) = -1\left(\frac{2}{3}\right)^{x+1} - 2$  (02)

.....  
.....  
.....

QUESTION NO 10

Your learners' were asked to write down the asymptote of the functions given. You are requested to answer the following functions.

Write down the horizontal asymptote of the following functions

(a)  $f(x) = e^{3x}$  (01)

.....  
.....  
.....

(b)  $f(x) = 2\ln(x + 3) - 1$  (01)

.....  
.....  
.....

## **ANNEXURE 8**

### **EDUCATORS' LIKERT SCALE**

**UNIVERSITY OF ZULULAND**

FACULTY OF EDUCATION

DEPARTMENT OF MATHEMATICS SCIENCE AND TECHNOLOGY

### **RESEARCH INSTRUMENT**

A STUDY OF FACTORS CONTRIBUTING TO UNDERACHIEVEMENT  
INEXPONENTIAL AND LOGARITHMIC FUNCTIONS IN THE FURTHER EDUCATION  
AND TRAINING SCHOOL PHASE

#### **Instructions for Teachers:**

Indicate by means of a tick (✓) your choice in the appropriate space..

#### **SECTION A:**

#### **BIOGRAPHICAL INFORMATION - yourself**

#### **GENDER:**

Male	<input type="checkbox"/>
Female	<input type="checkbox"/>

**AGE:**

20 – 24	
25 – 29	
30 – 34	
35 – 39	
40 – 44	
45 – 49	
50 ≥	

**TEACHING GRADE:**

Grade 10	
Grade 11	
Grade 12	

**TEACHING EXPERIENCE:**

GENERAL	✓	MATHEMATICAL FUNCTIONS	✓
0-3		0-3	
4-10		4-10	
11-20		11-20	
21≥		21≥	

## SECTION B

Indicate by means of a tick (✓) your choice in the appropriate space.

ITEM	strongly agree	agree	disagree	strongly disagree
1. I have an understanding of exponential and logarithmic functions.				
2. I am competent and knowledgeable to teach exponential and logarithm functions.				
3. I appreciate learners understanding of exponential and logarithmic functions.				
4. I find a relationship between teachers' readiness to teach exponential and logarithmic functions and the actual performance of learners.				
5. I find a relationship between educators' pedagogical knowledge content and learner achievement in exponential and logarithmic functions.				
6. I can differentiate between an algebraic equation and an algebraic function.				
7. I can define an exponential and logarithmic function with an example.				
8. I can sketch a general exponential function and indicate all the key points.				
9. I can explain and sketch an inverse function on graph paper with all the key points.				
10. I can write down the horizontal asymptote of a				

given function.				
-----------------	--	--	--	--

Grand Total = 10 questions

# ANNEXURE 9

## PERMISSION LETTER TO CONDUCT RESEARCH IN SCHOOLS



education

Department:  
Education  
PROVINCE OF KWAZULU-NATAL

Enquiries: Phindile Duma

Tel: 033 392 1004

Ref: 2016/870

Mr JK Mohammad  
PO Box 69  
3<sup>rd</sup> Street  
H/No 55  
Jozini  
3969

Dear Mr Mohammad

### PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: "EVALUATION OF FACTORS CONTRIBUTING TO UNDERACHIEVEMENT IN EXPONENTIAL AND LOGARITHMIC FUNCTIONS IN FET PHASE", in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 25 July 2016 to 01 December 2017.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Connie Kehologile at the contact numbers below
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report / dissertation / thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

(See List of Schools Attached)

Adv. MB Masuku  
Acting Head of Department: Education  
Date: 02 August 2016

KWAZULU-NATAL DEPARTMENT OF EDUCATION

POSTAL: Private Bag X 9137, Pietermaritzburg, 3200, KwaZulu-Natal, Republic of South Africa ...dedicated to service and performance  
PHYSICAL: 247 Burger Street, Anton Lembede House, Pietermaritzburg, 3201. Tel. 033 392 1004 beyond the call of duty  
EMAIL ADDRESS: [kehologile.connie@kzndoe.gov.za](mailto:kehologile.connie@kzndoe.gov.za) / [Phindile.Duma@kzndoe.gov.za](mailto:Phindile.Duma@kzndoe.gov.za)  
CALL CENTRE: 0860 596 363; Fax: 033 392 1203 WEBSITE: [www.kzndoe.gov.za](http://www.kzndoe.gov.za)

# ANNEXURE 10

## ETHICAL CLEARANCE LETTER FROM UNIZULU

**UNIVERSITY OF ZULULAND  
RESEARCH ETHICS COMMITTEE**  
(Reg No: UZREC 171110-030)



### RESEARCH & INNOVATION

Website: <http://www.unizulu.ac.za>  
Private Bag X1001  
KwaDlangezwa 3886  
Tel: 035 902 6887  
Fax: 035 902 6222  
Email: [ManqeleS@unizulu.ac.za](mailto:ManqeleS@unizulu.ac.za)

### ETHICAL CLEARANCE CERTIFICATE

Certificate Number	UZREC 171110-030 PGM 2017/366					
Project Title	A study of factors contributing to underachievement in exponential and logarithms functions in the further education and training school phase					
Principal Researcher/ Investigator	JK Mohammand					
Supervisor and Co- supervisor	Prof SN Imenda					
Department	Mathematics, Science and Technology Education					
Faculty	Education					
Type of Risk	High – Data collection from people					
Nature of Project	Honours/4 <sup>th</sup> Year	Master's	x	Doctoral		Departmental

The University of Zululand's Research Ethics Committee (UZREC) hereby gives ethical approval in respect of the undertakings contained in the above-mentioned project. The Researcher may therefore commence with data collection as from the date of this Certificate, using the certificate number indicated above.

- Special conditions:
- (1) This certificate is valid for 2 years from the date of issue.
  - (2) Principal researcher must provide an annual report to the UZREC in the prescribed format [due date-30 April 2018]
  - (3) Principal researcher must submit a report at the end of project in respect of ethical compliance.
  - (4) The UZREC must be informed immediately of any material change in the conditions or undertakings mentioned in the documents that were presented to the meeting.

The UZREC wishes the researcher well in conducting research.

  
Professor Gideon De Wet  
Chairperson: University Research Ethics Committee  
Deputy Vice-Chancellor: Research & Innovation  
15 May 2017



## **ANNEXURE 11**

### **TRANSE SCRIPT ISIZULU TRANSLATION**

#### **Isithasiselo sesishiyagalombili (8)**

#### **Incwadi yokuvunyelwa eyakubazali/ababheki**

Igugulesizwe Secondary School

P.O.Box 69

Jozini

3969

30 Kunhlangulana 2016

Sawubonamzali /mbheki

Ingcithabuchopho yesiyingi .....  
isiyeyakhethwa ukuba ihlanganyele kwezemfundo eziyoletha umehluko ophawulekayo esifundweni sezibalo kusukela ebangeni leshumi kuya ebangeni leshuminambili. Lolucwaningo luzosetshenziswa ngenjongo yokufunda kuphela. Ngakoke imvumo yakho iyadingeka njengoba sazi ukuthi isiyingisethu sincane. Lesisiyinki sakho kuzodingeka siphendule imibuzo eyobe ibuzwa nguyena ngokuqonda kwakhe okuphathelene nolwazi analolwezibalo ukusebenzakwalo. Le mibuzo izothatha cishe ihora lonke ukuze iphenduleke. Izinhlelo kumele zenziwe ukuze kuqinisekise ukuthi lolucwaningo aluphazamisi umsebenzi wakhe wesikole.

Siyaqinisekisa ukuthi ulwazi oluyonikezwa esiyingini luyoba imfihlo

Ngiyabonga ngokubambisana

Yimina ozithobayo

Mohammad, Javed Khizer (mr)



## **Isithathelo sesishiyagalolunye(9)**

### **Incwadi yokuvunyelwa eya emkhandlwini omelele isikole**

Igugulesizwe Secondary School

P.O.Box 69

Jozini

3969

30 Kunhlangulana 2016

Nobhala

Mkhandluwesikole

Mnumzane/ Nkosikazi

### **Imvumoyokwenzaucwaningo**

Ngiyazithoba ngicela imvumo kuni yokwenza ucwaningo esikoleni senu. Lolucwaningo lumayelana nezinhliso zokufunda kuphela. Inhliso yalo ukuletha ukukhanya okumayelana nomthelela noma izimbangela ezithinta ubuxhakaxhaka bokubala ebangeni leshumi kuya ebangeni leshuminambili. Lolulwazi oluyotholakala luzosetshenziswa ekuthuthukiseni ukufunda nokufundiswa kwezibalo, Ikakhulukazi umakuyiwa ezinxenyeni ezibucayize zibalo.

Lolucwaningo luzodinga ukuhlanganyela kwabafundi bebanga leshumi nambili kanye nothisha bezibalo. Othisha kuzodingeka baphendule imibuzo, bese kuthi kamuva kuzoba nezinxoxo eziphathelene namasu abawasebenzisayo ekusizeni abafundi baqonde izinxenye ezibucayi zezibalo. Nabafundi ngokufanayo bazobuzwa nemibuzo edinga ukuba babonise ukuthi baziqondakanjani lezinxenye ezibucayi zezibalo okungenani ihora lonke, ngakhoke izinhlelo ziyokwenziwa ukuze kungabibikho iziphazamiso ekufundeni kwabafundi esikoleni. Kungokubalulekile ukunazisa ukuthi

kakade sengibonisene nothisha bezibalo babonisa uthando ekuhlanganyeleni kulolucwaningo umanizongigunyaza.

Ukubambisana kuloludaba kuyonconywa kakhulu

Yiminaozithobayo

Mohammad, Javed Khizer (mr)

### **Ifomu lokugunya**

Thina njengomkhandlu omelele isikole.....siyasigunyaza isicelo sakho, sokuqhubeka nezifundo ezibalulwe ngenhla

.....

Usihlalo

Unobhala.....

isitembu sesikoleUthishanhloko

## **Isithasiselo seshumi (10)**

### **Imibuzo yabafundi**

Igugulesizwe Secondary School

P.O.Box 69

Jozini

3969

30 Kunhlangulana 2016

#### **Mfundi**

Ngiyabonga ngokuhlanganyela kulesisifundo. Ulwazi ozolunikeza luzosetshenziselwa ezinhlosweni zokufundakuphela. Ngokuqondile lolulwazi luyosetshenziswa ekucwaningeni ngomsebenzi we-(M.ED degree yezibalo) Lolucwaningo inhloso yalo ukuhlola izinto eziwumthelela ezenza abafundi bangenzi kahle esifundweni sezibalo ebangenileshumikuyaebangenileshuminambili. Ngiyakholelwa ukuthi ulwazi oluzotholakala kulolucwaningo luyoba oluzuzisayo, luzothuthukisa ukufunda nokufundisa ngezibalo, ngiyaqinisekisa ukuthi ulwazi olunikeza yoluyoba yimfihlo.

Ngithanda ukuphinde ngikubonge ngokuzidela kwakho ekuphenduleni imibuzo, kungakhathaliseki imisebenzi eminingi oyenzayo.

#### **Yimina**

Mohammed, Javed Khizer (mr)

(umcwaningi)

## Ifomu eligunyazayo

Mina ngiyazibophezela, isibongo ..... amagama  
..... ubulili ..... iminyaka  
..... inani leminyaka ngifunda ibangaleshumi.....  
ngiyavuma ukubangihlanganyele kulolucwaningo oluchazwengenhla.

Isiginesha.....usuku.....  
.....

Imiklomelo: amashumi amane (40)  
amane nanhlanu (45)

Isikhathi:imzuzu engamashumi

Imiyalelo

Le mibuzo iqukethe izingxenye ezimbili (A& B) ucelwa ukuba ugcwalise lezingxenye ezimbili.

1. Ingxenye A imayelana nolwazi olujwayelekile
2. Ingxenye B iqukethe imibuzo eyishumi, ucelwa ukuba uphendule yonke imibuzo kulezizikhala ezinikeziwe.
3. Amaphepha awamagilafu ahlanganiswe emibuzweni edingekayo.
4. Bonisa yonke imisebenzi oyenzile.
5. Yenza konke ongakwenza.

#### Ingxenye A

Igama lesekethi.....  
Isiyingi.....  
Iminyaka.....  
Ubulili.....  
Isikole.....  
Ibanga.....

Ingxenye ka-B

Umbuzo wokuqala (01)

Veza umehluko phakathi kwezingxenye ezihlukahlukene eziminxa emibili kanye nezingxenye ezihlukahlukene ezihlanganiswe ndawonye ukusebenza kwazo?  
(03)

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

Umbuzo wesibili (02)

Chaza ngengxenye ephethelene nokuphindaphindeka kwezinombolo kanye nengxenye ephathelene nokuhlanganisiswe nokuncishisiwe, unikeze okungenani isibonelo esisodwa.

.....  
.....  
.....  
.....  
.....  
.....  
.....

.....(05)

Umbuzo wesithathu (03)

Dweba ngalengxenye ephindaphindekayo ukusebenza kwayo  $F(X)=a.b^x$ ,  $a>0$ ,  $a=1$ ,  $b>1$ ,

X E. Uma-R eyinamba ephilayo. Ephepheni legilafu esilnikiwe bonisa zonke izinto noma amaphuzu ayinhloko. (03)

Umbuzo wesine (04)

Dweba ngokusebenza ngokuphikisana nokusebenza embuzweni wesithathu (3) bese siyakusho.

(03)

Umbuzo wesihlanu (05)

Ngokusebenza kwalengxenywe ephindaphindayo  $F(X)=a.b^x+P+a$ ,  $a>1$ ,  $b>1$ , futhi P.Q kanye no

X E R.

a) Uyini umphumela wokungaguquki ka-A emdwebeni oku-F(X)? (01)

.....  
.....  
.....  
.....  
.....

b) Kukanjani lengxenywe engaphansi ka-B ithinteka kulegilafu eku-F(X) (01)

.....  
.....  
.....  
.....  
.....

Umbuzo wesithupha (06)

Ngokwenzazelo yokusebenza kwalengxenywe ephathelene nokuhlenganiswa nokuncishiswa ukusebenza kwayo.

(02)

$\log_b a = X$ ,  $a > 1$  futhi  $X > 0$  uma kuwukuthi kuphela.

.....  
.....  
.....  
.....

Umbuzo wesikhombisa (07)

Dweba lemisebenzi elandelayo ephepheni elifanayo olinikeziwe futhi ubonise wonke amaphuzu abalulekile.

$F(x) = \frac{6}{5}x - 1$  futhi  $F(x) = \frac{5}{6}x + X$  E R  
(08)

Umbuzo wesishagalombili (08)

Dweba ngokusebenza kwalengxenywe ephepheni elifanayo onikezwe yona ngenhla embuzweni wesikhombisa (7) ephepheni elifanayo legilafu olinikeziwe ngawo wonke amaphuzu abalulekile.

(08)

Umbuzo wesishagalolunye (09)

Bhala izinga lalomsebenzi olandelayo

a)  $F(x) = 2.5^{(x-2)}$  +1  
(02)

.....  
.....  
.....  
.....

b)  $F(X)=-1(2/3)X$  +1-2  
(02)

c) .....  
.....  
.....  
.....  
.....

Umbuzo weshumi (10)

Bhala phansi ngomsebenzi onqumisile olandelayo

a)  $F(X)=e^3x$   
(01)  
.....  
.....  
.....  
.....

b)  $F(X)=2\ln(X+3)-1$   
(01)  
.....  
.....  
.....  
.....  
.....

## ANNEXURE 12

### Frequency table representing marks obtained by learners

#### Total Score

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	3	1.2	1.2	1.2
	1	2	.8	.8	2.1
	2	1	.4	.4	2.5
	3	13	5.4	5.4	7.9
	4	11	4.5	4.5	12.4
	5	3	1.2	1.2	13.6
	6	6	2.5	2.5	16.1
	7	8	3.3	3.3	19.4
	8	8	3.3	3.3	22.7
	9	5	2.1	2.1	24.8
	10	3	1.2	1.2	26.0
	11	8	3.3	3.3	29.3
	12	6	2.5	2.5	31.8
	13	6	2.5	2.5	34.3
	14	5	2.1	2.1	36.4
	15	5	2.1	2.1	38.4
	16	6	2.5	2.5	40.9
	17	3	1.2	1.2	42.1
	18	5	2.1	2.1	44.2
	19	5	2.1	2.1	46.3
	20	3	1.2	1.2	47.5
	21	1	.4	.4	47.9
	22	5	2.1	2.1	50.0
	23	5	2.1	2.1	52.1

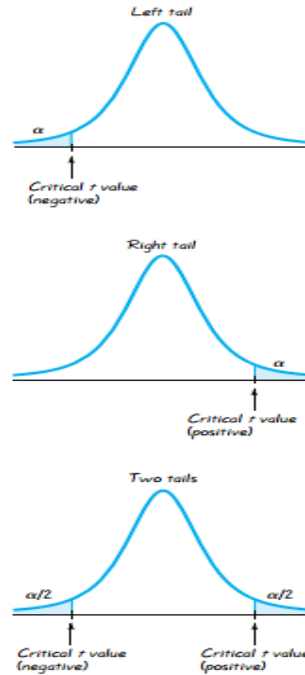
24	5	2.1	2.1	54.1
25	5	2.1	2.1	56.2
26	5	2.1	2.1	58.3
27	12	5.0	5.0	63.2
28	2	.8	.8	64.0
29	1	.4	.4	64.5
30	5	2.1	2.1	66.5
31	2	.8	.8	67.4
32	6	2.5	2.5	69.8
33	3	1.2	1.2	71.1
34	8	3.3	3.3	74.4
35	2	.8	.8	75.2
36	19	7.9	7.9	83.1
37	5	2.1	2.1	85.1
38	25	10.3	10.3	95.5
39	9	3.7	3.7	99.2
40	2	.8	.8	100.0
Total	242	100.0	100.0	

The cumulative frequency indicates that 95.5 % of learners scored marks 38 and And below, whiles only 2 learners out of a total of 242 surveyed scored 100% mark.

# ANNEXURE 13

## T-TEST TABLE

TABLE A-3 t Distribution: Critical t Values						
Degrees of Freedom	0.005	0.01	Area in One Tail 0.025		0.05	0.10
	0.01	0.02	Area in Two Tails 0.05			0.20
1	63.657	31.821	12.706	6.314	3.078	
2	9.925	6.965	4.303	2.920	1.886	
3	5.841	4.541	3.182	2.353	1.638	
4	4.604	3.747	2.776	2.132	1.533	
5	4.032	3.365	2.571	2.015	1.476	
6	3.707	3.143	2.447	1.943	1.440	
7	3.499	2.998	2.365	1.895	1.415	
8	3.355	2.896	2.306	1.860	1.397	
9	3.250	2.821	2.262	1.833	1.383	
10	3.169	2.764	2.228	1.812	1.372	
11	3.106	2.718	2.201	1.796	1.363	
12	3.055	2.681	2.179	1.782	1.356	
13	3.012	2.650	2.160	1.771	1.350	
14	2.977	2.624	2.145	1.761	1.345	
15	2.947	2.602	2.131	1.753	1.341	
16	2.921	2.583	2.120	1.746	1.337	
17	2.898	2.567	2.110	1.740	1.333	
18	2.878	2.552	2.101	1.734	1.330	
19	2.861	2.539	2.093	1.729	1.328	
20	2.845	2.528	2.086	1.725	1.325	
21	2.831	2.518	2.080	1.721	1.323	
22	2.819	2.508	2.074	1.717	1.321	
23	2.807	2.500	2.069	1.714	1.319	
24	2.797	2.492	2.064	1.711	1.318	
25	2.787	2.485	2.060	1.708	1.316	
26	2.779	2.479	2.056	1.706	1.315	
27	2.771	2.473	2.052	1.703	1.314	
28	2.763	2.467	2.048	1.701	1.313	
29	2.756	2.462	2.045	1.699	1.311	
30	2.750	2.457	2.042	1.697	1.310	
31	2.744	2.453	2.040	1.696	1.309	
32	2.738	2.449	2.037	1.694	1.309	
34	2.728	2.441	2.032	1.691	1.307	
36	2.719	2.434	2.028	1.688	1.306	
38	2.712	2.429	2.024	1.686	1.304	
40	2.704	2.423	2.021	1.684	1.303	
45	2.690	2.412	2.014	1.679	1.301	
50	2.678	2.403	2.009	1.676	1.299	
55	2.668	2.396	2.004	1.673	1.297	
60	2.660	2.390	2.000	1.671	1.296	
65	2.654	2.385	1.997	1.669	1.295	
70	2.648	2.381	1.994	1.667	1.294	
75	2.643	2.377	1.992	1.665	1.293	
80	2.639	2.374	1.990	1.664	1.292	
90	2.632	2.368	1.987	1.662	1.291	
100	2.626	2.364	1.984	1.660	1.290	
200	2.601	2.345	1.972	1.653	1.286	
300	2.592	2.339	1.968	1.650	1.284	
400	2.588	2.336	1.966	1.649	1.284	
500	2.586	2.334	1.965	1.648	1.283	
750	2.582	2.331	1.963	1.647	1.283	
1000	2.581	2.330	1.962	1.646	1.282	
2000	2.578	2.328	1.961	1.646	1.282	
Large	2.576	2.326	1.960	1.645	1.282	



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