UNIVERSITY OF ZULULAND

PRE-SERVICE PRIMARY TEACHERS' COGNITION OF MKT FOR EUCLIDEAN GEOMETRIC PATTERNS AND ITS EFFECTS IN CLASSROOM PRACTICE

by

SIMON N DLAMINI

A Thesis Submitted to the Faculty of Education

Department of Mathematics, Science and Technology Education

Of

University of Zululand

In fulfilment of the requirement for the

DOCTOR OF EDUCATION

(Mathematics Education)

Supervisor: Dr T.W. Chinaka

(University of Zululand)

Co-supervisor: Prof. G. Moyo

(University of Zululand)

2021

ORIGINALITY DECLARATION

Full Names and Surname	Simon Mmeli N Dlamini
Student Number	201974612
Title of dissertation/thesis	Pre-service primary teachers' cognition of MKT for Euclidean geometric patterns and its effects in classroom practice

I acknowledge that I have read and understood the University's policies and rules applicable to postgraduate research, and I certify that I have complied with their requirements to the best of my knowledge and belief. In particular, I confirm that I had obtained an ethical clearance certificate for my research (Certificate Number UZREC 171110-030 PGD 2019/53) and that I have complied with the conditions set out in that certificate. I further certify that this thesis is original and that the material has not been published elsewhere, or submitted, either in whole or in part, for a degree at this or any other university. I declare that this thesis is the product of my work and effort, save for the supervisory guidance received. To the best of my knowledge and belief, I have complied with the University's Plagiarism Policy and acknowledged all sources of information in line with normal academic conventions. I have subjected the document to the University's text-matching and/or similarity checking procedures.

Candidate's signature	....
Date	December 2021

ABSTRACT

This study investigates the mathematical knowledge for teaching Euclidean geometric patterns held by pre-service primary school mathematics teachers during their final year of study. This investigation was intended to establish the cognition of the pre-service teachers on MKT for geometric patterns and its effect in classroom practice. The literature on teacher knowledge alludes to the fact that many teachers exit university with limited knowledge for teaching mathematics. This study seeks to determine the pre-service teachers' mathematical knowledge for teaching (MKT) geometric patterns. Using a mixed methods design, the study investigated the applicability of the LMT measures in Eswatini, the cognition of pre-service teachers on geometric patterns and the effect of this cognition on classroom practice. The MKT framework underpinned the study in data collection and data analysis. Data analysed to respond to the three research questions was collected through a pilot study, pen-and-paper cognition test and lesson observations. The study was conducted in two phases. The first phase investigates the applicability of the LMT measures in Eswatini, through a pilot study, and determines the cognition of the pre-service teachers on geometric patterns. The pilot study involved 34 pre-service primary school mathematics teachers who were given the LMT measures in the form of a pen-and-paper cognition test. The data from the pilot study was statistically analysed to adapt the measures to the local context and to establish their applicability in Eswatini. The results indicated that the difficulty index of the measures was higher for the Eswatini pre-service teachers than it was reported for the USA teachers. Hence, the measures are only valid in identifying teachers with high levels of MKT in Eswatini. To determine the cognition of the pre-service teachers on geometric patterns, the adapted measures were administered to a sample of 94 (n=94) final year pre-service primary school mathematics teachers registered for a Primary Teachers Diploma programme. The study revealed that the pre-service teachers' cognition of Euclidean geometric patterns was varied, with most teachers having low levels of mathematical knowledge for teaching (Mean = 45%, SD = 17). The second phase of the study investigates the effect of the pre-service teachers' cognition on geometric patterns in classroom practice. Data was collected through classroom observations, which were video-recorded, of lessons on geometric patterns conducted by three stratified sampled teachers. The classroom observation videos were analysed using the Assessing Quality of Instruction in Primary Mathematics (AQIPM) framework. The results showed that there was a positive correlation between the pre-service teachers' level of mathematical knowledge for teaching and their quality of mathematics lessons ($r = .99$, $p < .05$). These results indicate that teacher education institutions need to review their teaching approach so that they promote the

development of mathematical knowledge for teaching in pre-service teachers. Further study is recommended in this area to help develop a teaching model that will facilitate the acquisition of mathematical knowledge for teaching in teacher education institutions.

ACKNOWLEDGEMENTS

First of all, my thanks go to God almighty for His sustaining grace, protection and guidance during the time of my studies. I would also like to thank Dr T. Chinaka, my supervisor and Prof. G. Moyo, my co-supervisor, for their professional guidance when reviewing my thesis. Prof. Bayaga played a pivotal role in preparing the research proposal and obtaining the ethical clearance certificate.

I am very grateful to my supervisor, DR T. Chinaka, for his guidance, advice and encouragement that led to my completion of this thesis. My sincere thanks also go to the Director of Education and the Principals of Colleges in Eswatini, who permitted me to research their institutions. I would also like to thank the pre-service teachers who took part in this research project. Without them, this study would not have been a success. I thank Dr Tsikati from UNESWA, who also gave me professional guidance in adapting the instrument to the Eswatini context.

I am forever grateful to my lovely wife for her endless emotional and financial support that made this study a success. I also appreciate the support from my mother, brothers, sister and work colleagues who provided me with unending inspiration.

DEDICATION

This study is dedicated to my mother, Christinah, my wife, Phindile and my three children, Nkosinathi, Sphesihle and Snenkosi.

TABLE OF CONTENTS

ACRONYMS

AQIPM	Assessing Quality of Instruction in Primary Mathematics
CCK	Common Content Knowledge
CK	Content Knowledge
ECESWA	Examinations Council of Eswatini
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
LCP	Learner-Centred Pedagogy
LMT	Learning mathematics for teaching
MKT	Mathematical knowledge for teaching
MoET	Ministry of Education and Training
NCTM	National Council of Teachers of Mathematics
PCK	Pedagogical Content Knowledge
PTD	Primary Teachers Diploma
QMI	Quality of Mathematics Instruction
SAARMSTE	Southern Africa Association for Research in Mathematics, Science and Technology Education
SACMEQ	The Southern and Eastern Africa Consortium for Monitoring Educational Quality
SCK	Subject Content Knowledge
SMK	Subject Matter Knowledge
USA	United States of America

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1
INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

Teachers significantly affect learner achievement in mathematics (Bardach & Klassen, 2020). Studies have shown that one of the factors influencing the effectiveness of teachers on student achievement is cognition of mathematics knowledge for teaching (Bardach & Klassen, 2020; Hill, Blunk, et al., 2008). Even though teacher cognition in mathematics profoundly affects learner achievement, there is not enough empirical research to determine how teacher cognition is used in teaching (Hill, Blunk, et al., 2008; Hoover, Mosvold, Ball, & Lai, 2016).

In a systematic review of the literature between 2006 and 2013, Hoover et al. (2016) identified 190 peer-reviewed articles that report on research in mathematical knowledge for teaching. The results of this systematic review of the literature revealed that very few studies on mathematical knowledge for teaching had been carried out in Africa. Most studies have been done in North America, Asia and Europe. Hence, there is a need to conduct studies on mathematical knowledge in Africa to inform policy and practice in primary mathematics teaching and primary teacher education.

The development of conceptual knowledge is the main objective of mathematics teaching. By conceptual knowledge, we refer to an understanding of the interrelations “between concepts and principles including the existing schema behind concepts”(Sahidin, Budiarto, & Fuad, 2019, p. 302). These interrelations are formed when new knowledge in mathematics is linked to pre-existing related concepts in one’s mind. These interrelations form concept maps that give meaning to the new knowledge.

Sahidin et al. (2019) assert that “conceptual knowledge is the knowledge that binds information that was once fragmented into a relatively complete net” (p.302). For teachers to teach conceptual understanding, they need to have special knowledge for teaching, referred to as Mathematical Knowledge for Teaching (MKT) by Ball et al. (2008). There are several reasons why understanding this kind of knowledge for teaching mathematics is essential.

One of the reasons is that understanding this MKT will enable teacher training institutions to structure their courses to develop this knowledge in pre-service teachers. The development of MKT at teacher training will ultimately improve the teaching/learning of mathematics, resulting in improved learner achievement. As a teacher educator, the researcher is interested in finding out teacher knowledge that positively affects learner achievement. Many scholars assert that teacher knowledge is an imperative factor in teaching that results in high learner gains. However, there is an on-going debate on the kind of knowledge teachers need to conduct effective lessons.

The ground-breaking assertion of Shulman (1986) that there is special knowledge that teachers need sparked interest among researchers to investigate and understand more about this special knowledge. Mathematics scholars have proposed different frameworks to conceptualise this knowledge. These frameworks highlight two main categories of this knowledge, namely content knowledge and knowledge that is used in teaching (Charalambous, Hill, Chin, & McGinn, 2020). Among the proposed frameworks of teacher, knowledge is the mathematical knowledge for teaching (MKT) developed by Ball and colleagues (Ball, Thames, & Phelps, 2008). This framework has two categories; subject content knowledge and pedagogical content knowledge.

To better understand this knowledge and how it is used in teaching, it is important to develop measures to test its existence in teachers. The Learning Mathematics for Teaching (LMT) project has developed benchmarks for MKT. However, these measures were developed in the United States of America (USA). It is thus imperative to determine if these measures are applicable in contexts outside the USA. This research aims to determine the applicability of the LMT measures in the Eswatini context and to find out how MKT is used in teaching.

The answers to these questions will help determine the validity of the LMT measures in Eswatini, as no study has been carried out in Eswatini to assess the adaptability of these measures in Eswatini. Understanding how MKT is used in teaching is principal to help teacher educators structure their courses to facilitate the development of MKT at pre-service education. In an endeavour to determine the level of MKT for teaching geometric patterns, the present study explores the cognition of pre-service primary teachers on geometric patterns. The researcher sought to investigate how this cognition affects classroom practice.

This study was carried out in Eswatini when the country was implementing a new curriculum framework, in primary schools, based on competency-based education. Competency-based education is a curriculum that is focused on developing observable competencies in learners. By introducing competency-based education, the ministry of education and training aimed to equip learners with knowledge and skills to make them competent in a technological and knowledge-driven society. An investigation of the teachers' mathematical cognition at the time of implementing the new curriculum framework was deemed necessary by the researcher.

The examinations council of Eswatini's reports on the performance of Grade 7 learners at the end of year national examinations shows that many learners have difficulties in mathematics. For example, in the 2019 Eswatini Grade 7 results, mathematics was the least passed subject in a curriculum of 10 subjects. The mathematics examiners' report highlights that most learners had difficulties with geometry and problem-solving questions (ECESWA, 2019). The researcher believes that geometric pattern recognition can improve learners' performance, not only in geometry but also in mathematics.

According to Pi et al. (2008), pattern recognition is the basic process of human cognition that facilitates the process of memory and reasoning in the human mind (Barkman, 2018). It is a problem-solving strategy that facilitates information processing in the mind and enables learners to acquire and retrieve information from long-term memory (Barkman, 2018; Lachman, Lachman, & Butterfield, 2015). In this research, the cognition of pre-service teachers on geometric pattern recognition was determined through the use of a cognitive test developed by Hill, Schilling and Ball (2004) during the Learning Mathematics for Teaching (LMT) project at the University of Michigan.

The researcher also intended to investigate how pre-service teachers' cognition of geometric pattern generalisation can be developed in teacher education. This research objective is in response to the findings of Jank vist and Mosvold (2020), which allude to the fact that there is not much research on how to facilitate the development of the knowledge needed for teaching. The results of Hoover et al. (2016) systematic review of literature on mathematical knowledge for teaching also revealed a need for studies on developing mathematical knowledge for teaching during pre-service teacher training.

Hence, in this study, we asked, 'how can cognition of MKT for geometric patterns be developed at pre-service teacher training?' From the articles reviewed, Hoover et al. (2016) also report that most of the studies in this area "investigated the nature and composition of mathematical knowledge for teaching and developing teachers' knowledge, fewer studies have investigated the impact such knowledge has on teaching and learning" (Hoover et al., 2016, p. 16). According to Hill et al. (2008), the main question is for researchers to investigate how mathematical knowledge for teaching is used in instruction to improve student achievement.

The study on geometry is necessitated because learners have difficulties answering geometry questions in the Grade 7 national examinations in Eswatini (ECESWA, 2019). An improvement in the teaching/learning of geometry would improve learners' performance in mathematics since geometry affects the learning of other concepts in mathematics. According to Andila and Musdi (2020), geometry enhances high-level thinking processes. It also enhances learners' understanding of other mathematics topics (Andila & Musdi, 2020; Sunzuma & Maharaj, 2019).

Sunzuma and Maharaj (2019) allude to the fact that there is not much research on the cognition of geometry. According to the NCTM (2018), understanding geometry enables learners to understand abstract mathematical concepts (NCTM, as cited in Tsao, 2018). Therefore, learners must have a solid foundation in elementary geometry to understand more advanced and abstract concepts in mathematics. The current research was carried out on pre-service primary school teachers since primary school teachers lay a foundation for future work in mathematics (Stylianides & Delaney, 2018).

The importance of researching pre-service teachers' cognition was also in response to the contentions of Santagata and Lee (2019) that pre-service teachers lack experience in teaching.

Hence they are perfect candidates for learning how the knowledge acquired in pre-service training is used in teaching mathematics (L. Shulman, 1987).

1.2 BACKGROUND

The researcher intended to determine pre-service primary teachers' cognition of Euclidean geometric patterns in this study. The study was carried out in Eswatini, a country with a population of about 1 million people. In Eswatini, two government-supported universities and two government-owned teacher training colleges are responsible for training primary school teachers. The colleges offer a three-year Primary Teachers' Diploma (PTD) programme. The current study was conducted in one teacher training college with a three-year primary teaching diploma.

This study investigates claims that MKT has an impact on the teaching/learning of mathematics. A number of studies have reported that teacher knowledge influences instruction (e.g., Borko et al.; Fennema & Franke; Leinhardt & Smith; Putnam et al.; Sowder et al., all cited in Hill et al., 2008). The findings of these studies show that teachers with stronger MKT have high quality classroom instruction, resulting in high student achievement. These studies fall under the 'affordances' category (Hill et al., 2008). Affordances studies focus on the richness of classroom instruction brought about by a teacher's strong MKT. On the other hand, 'deficit' studies highlight "a teacher's lack of mathematical understanding and patterns in her mathematics instruction" (Hill et al., 2008, p.433). In deficit studies, the authors highlight mathematical errors that are observed in mathematics instruction due to a teacher's lack of adequate MKT. Even though affordances studies assert that teachers with strong MKT provide high quality mathematics instruction, there is still a need for an in-depth study of the relationship between MKT and classroom instruction.

In this study, the researcher seeks to investigate how the different aspects of MKT affect instruction. This investigation makes use of an instrument called *Assessing Quality of Instruction in Primary Mathematics* (AQIPM) to understand the relationship between MKT and instruction. Through the use of the AQIPM instrument, the researcher shall quantitatively establish the relation between each aspect of MKT and instruction. The aspects of MKT that shall be considered in this research are the; Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), Common Content Knowledge (CCK), and the Specialised Content Knowledge

(SCK). This study shall investigate both the affordances and deficits in mathematics instructions in relation to high MKT and low MKT. Hence, the researcher shall observe lessons of a teacher who obtained a high MKT score in a pen-and-paper test and lessons of one who obtained a low MKT score in a pen-and-paper test. The researcher's interest is to find out how each aspect of MKT affects classroom instruction.

There are four levels in the education system of Eswatini, namely, pre-school, primary, secondary and tertiary education. The primary school has seven levels starting from Grade 1 to Grade 7. A national assessment examination in Grade 7 determines the learners' progression to secondary school. The primary school curriculum consists of five core subjects: English Language, Mathematics, Science, Religious Education, and siSwati. There are also five optional subjects; Agriculture, Home Economics, French, Practical Arts and Physical Education.

The goal of primary education is to equip learners with "general knowledge, independent thinking skills + organizational skills developed in preparation for successful progression in the education system" (MoET, 2018, p. xx). Hence, the main aim of primary school is to lay a foundation of knowledge and skills that will be useful in secondary and tertiary education. Eswatini implemented the free primary education programme in 2010. This programme allowed all children to get primary education, regardless of their family's economic status.

In 2011, the education and sector policy was crafted to improve the quality of education in the country. One of the guidelines given in the education sector policy was that teachers should use learner-centred pedagogy in teaching mathematics (MBABANE, 2011). The performance of learners in mathematics, however, has not improved. A revised teaching syllabus was introduced in 2013, focusing on improving mathematics teaching. The revised syllabus emphasised the need for teaching through the problem-solving approach to develop critical thinking skills in learners.

The examination council of Eswatini's reports on Grade 7 national exams reveals that learners have difficulties answering geometry questions during the mathematics exams. In 2019, the Ministry of Education and Training (MoET) introduced competency-based education in primary schools in Eswatini.

Even though research findings have established that teachers' cognition on mathematics teaching positively affects instruction and, consequently, on learner achievement, not much research has been done to establish teachers' awareness of teaching geometry in Eswatini (Ndlandla, 2017).

The following operational definitions of terms are adopted in this study: Pre-service teachers, also called student teachers, are prospective teachers enrolled in a university or College of Education to be trained as teachers. The term cognition in this study refers to the information processing in mind (Woolcott, 2020). It involves activating higher mental processes to attain knowledge and understanding through perception, interpretation, and storage in long-term memory (Woolcott, 2020). Euclidean geometry refers to the study of planes and solid figures (Borsuk, 2018).

Euclidean geometry axioms and theorems are attributed to Euclid, a Greek mathematician. Euclidean geometry is commonly taught in primary and high schools, involving properties of points, lines, and shapes. According to Pi, Liao, Liu and Lu (2008), the meaning of pattern recognition is the basic process of human intelligence or cognition. It is a process of matching any new information with knowledge already exists in the person's mind. For example, face recognition and determining the specific formula for calculating the area of a figure are actions enhanced by pattern recognition. Pattern recognition occurs when information from the environment is received and entered into short-term memory, causing automatic activation of specific content of long-term memory (Rau, Sen, & Zhu, 2019).

Presently, there is growing research in pre-service teacher education in mathematics to address the issue of mathematical knowledge for teaching Euclidean geometry (Browning, Edson, Kimani, & Aslan-Tutak, 2014). Elementary geometry is important because teachers lay a foundation for more advanced work in the topic. It has been widely reported that learners regard geometry as a problematic topic in elementary mathematics. For instance, studies that have been conducted so far include but not exclusively; Koeno et al. (2016) , Ramdhani et al. (2017) , Sunzuma and Maharaj (2019) , Naidoo (2013) .

As suggested by Ball et al. (2005), studies before the studies mentioned above indicate that learners' achievement in mathematics is directly related to teachers' knowledge of mathematics. However, Jankvist and Mosvold (2020) note that most studies focus only on whether teachers have mathematical knowledge for teaching or not. The authors above report that very few studies have investigated how this mathematical knowledge for teaching is used in instruction. Hence, as Hill et al. (2008) articulated, the missing link is a need to fully understand how mathematical teacher cognition affects classroom instruction and, consequently, learner achievement.

Nevertheless, Ng (2011) argues that "studies on the factors that may contribute to the development of such knowledge [mathematical knowledge for teaching] required for effective teaching are thus deemed necessary to inform policymakers, teacher preparation programs, and professional development providers" (p.152). The current research sought to understand how this mathematical knowledge for teaching is used in the teaching of geometry in Eswatini. This information would help design appropriate pre-service courses to develop mathematical knowledge for teaching mathematics.

Shulman's (1986) ground-breaking assertion that teachers in mathematics require special knowledge, research studies in teacher cognition of mathematical knowledge for teaching. Shulman called this special knowledge pedagogical content knowledge (PCK). Shulman's assertion sparked interest among researchers in this area of teacher knowledge. As a result of Shulman's view, several studies have been carried out to investigate mathematical teacher knowledge. Some of these studies are discussed in the following paragraph.

Shulman's (1986) pedagogical content knowledge and Hill et al. (2008) mathematical knowledge for teaching models were developed from general mathematics teaching and learning studies. Hill et al. (2008) carried out a survey to determine how mathematical knowledge is used in teaching. The study of Hill et al. (2008) formulated the mathematical knowledge for teaching (MKT) model. However, the MKT model developed by Hill et al. (2008) addresses the teaching of mathematics in general. It is not specific to any topic in the mathematics curriculum.

Steele (2013) narrowed the research on mathematical knowledge to investigate teacher cognition in geometry. Steele's (2013) study led to the development of the mathematics for teaching geometry (MKT-G) model.

The present study builds on these studies on teacher cognition in mathematics teaching. In the present study, the researcher sought to measure the mathematical knowledge for teaching geometric patterns and their effect in classroom practice. It is important to fully understand the elements of teacher cognition necessary to improve classroom instruction and consequently learner achievement (Hoover et al., 2016). Understanding the elements of teacher cognition that are necessary to improve classroom instruction will enable pre-service teacher education to provide a curriculum that will enhance this teacher cognition (Hoover et al., 2016).

The results of Hoover's et al. (2016) systematic review of literature in mathematical knowledge for teaching revealed the following statistics on the regions where studies of mathematics teacher knowledge have been undertaken.

Table 1. Research studies undertaken on mathematical knowledge for teaching around the world

Region	Number of articles	Percentage
Africa	7	3.7%
Asia	27	14.2%
Europe	22	11.6%
Latin America	3	1.6%
North America	112	58.9%
Oceania	15	7.9%
Across Regions	4	2.1%

Adapted from Hoover et al. (2016, p. 7)

The information on the table shows that many studies on mathematical knowledge for teaching have been done in Asia, Europe and North America. Few (3.7%) studies have been done in Africa, including Eswatini. The results of the studies done abroad may not apply in Africa due to differences in context. There is, therefore, a need to carry out studies on mathematical knowledge for teaching in Eswatini (Africa) to inform policy on education and teacher training programmes for teacher development.

1.3 PROBLEM STATEMENT

Zakharov et al. (2016) assert that if teachers have sufficient content and pedagogical content knowledge, learners will learn meaningfully. Other studies affirm this assertion (Bardach & Klassen, 2020; Hill, Rowan, & Ball, 2005). However, very few studies have studied this relationship between teachers' cognition and instruction. Hence, Hoover et al. (2016) noted that there is "a need for more studies that unpack relationships among mathematical knowledge for teaching, teaching practice, and student learning" (p.20).

According to Hoover et al. (2016, p.20), such studies should determine the effects of "specific mathematical knowledge" on specific areas of mathematics to expound how the mathematical knowledge is used in teaching and learning. Hence, the 'specific mathematical knowledge' investigated in this study is geometric patterns in the 'area' of geometry. According to Mashingaidze (as cited in Sunzuma et al., 2019), teachers usually avoid geometry, apparently due to the teachers' low cognition in geometry.

Learners' performance in mathematics in primary schools in Eswatini is not improving. The Examinations Council of Eswatini (ECESWA) report shows that learner performance in Grade 7 mathematics is lower than their performance in other subjects. In the 2018 examinations, learners' performance in mathematics was the bottommost in a curriculum with 11 subjects. In its examination report, ECESWA (2018) reported that "most candidates did not perform well in this paper; as the raw score average was 45…question 9 was the most challenging in this paper" (p.18). Question 9 in this examination was on geometry.

This shows that learners in Eswatini have difficulties in geometry. Statistics for 2017 examinations also show that learners' performance in mathematics was not good. This shows a challenge in teaching/learning mathematics in the primary schools in Eswatini (ECESWA, 2017). A research study in geometry is deemed necessary by the claims that an understanding of geometry is needed to understand other areas of mathematics (Sunzuma & Maharaj, 2019).

The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) on mathematics proficiency level at the 6th grade in Eswatini found that 5.4466% of learners were mathematically proficient in 2007.

On concrete problem solving, the 6th-grade learners were rated at level 7 with 0.3304%. In its concluding remarks, the reporters say, “The worrying picture was in mathematics, as the study indicated that 22 percent of the Grade 6 pupils were performing at the bottom two “non-numerate” levels” (Murimba, 2005, p. 229).

Studies on the teaching/learning of geometry in the primary schools in Eswatini include one by Ndlandla (2017). Ndlandla (2017) study on the primary school in-service teachers’ understanding of geometry through the problem-solving approach found out that the teachers had limited knowledge of geometry through the problem-solving process. This study found out that the teachers used the traditional instructional teaching methods instead of problem-solving.

These findings show a need to improve the teaching/learning of geometry in primary schools in the country. The current study investigated pre-service teachers’ mathematical knowledge for teaching geometric patterns. The study also sought to understand how the pre-service teachers’ cognition of geometric patterns could be improved during pre-service training.

1.4 PURPOSE AND OBJECTIVES OF THE STUDY

The purpose of this study was to investigate the pre-service primary teachers’ cognition of mathematical knowledge for teaching geometric patterns in Eswatini. The study also sought to determine if the teachers’ MKT on geometric patterns affects their choice of strategy for teaching geometric patterns.

Objectives of the study are as follows:

1. To determine the adaptability of the LMT measures of MKT in the Eswatini context.
2. To determine the pre-service primary teachers’ cognition of mathematical knowledge for teaching geometric patterns in Eswatini.
3. To establish how the pre-service primary teachers’ mathematical knowledge of geometric patterns affects their lesson planning and presentation.

1.5 RESEARCH QUESTIONS

The following questions were posed and will be addressed in the study:

1. What is the adaptability of the LMT measures of MKT to the Eswatini context?
2. What is the pre-service primary teachers' cognition of mathematical knowledge for teaching geometric patterns in Eswatini?
3. How much does the pre-service primary teachers' cognition of mathematical knowledge for teaching geometric patterns to influence their lesson planning and presentation?

1.6 SCOPE OF THE STUDY

This study sought to determine pre-service primary school teachers' mathematical knowledge for teaching geometric patterns and their effect on mathematics instruction. This study responded to the poor performance of primary school learners in mathematics in Eswatini. The scope of this study is on pre-service teachers' cognition of mathematical knowledge for teaching geometric patterns. The study also sought to understand the effect of this cognition in mathematics instruction. This study was also interested in developing a teaching model that would improve pre-service teachers' cognition of mathematical knowledge for teaching geometric patterns, however due to the COVID-19 pandemic outbreak this objective could not be achieved.

The researcher first adapted the LMT measures for Eswatini. Through psychometric analysis, items that were not well-performing in the Eswatini context were removed from the instrument used in determining mathematical knowledge for teaching geometric patterns in Eswatini. The researcher used the adapted instrument with LMT measures to determine pre-service teachers' mathematical teaching of geometric patterns. Classroom observations were conducted to determine the quality of instruction of the pre-service teachers.

Participants involved in the study were pre-service teachers enrolled for a mathematics teaching qualification in one college of education in Eswatini. They were in their final year of study. Hence, they had already done some mathematics and pedagogy courses at the college during the first and the second year of study.

1.7 LIMITATIONS

The following are the limitations of the study:

1. Due to constraints of time and resources, the study will be restricted to one college of education and limited to 80 participants.
2. Due to time constraints, the classroom observations shall be limited to only three pre-service teachers.
3. The outbreak of the COVID-19 pandemic restricted the investigation with an intent to develop a teaching model that can be used to develop MKT for teaching geometric patterns at pre-service education. The outbreak of COVID-19 resulted in classes being held only virtually. Hence the researcher could not engage with the pre-service teachers to develop the teaching model since this required a face-to-face interaction.

1.8 INTENDED CONTRIBUTION TO THE BODY OF KNOWLEDGE

The main aim of this study was to determine the cognition of pre-service primary teachers on geometric patterns, which is a new perspective of the mathematics knowledge for teaching (MKT) model. So far, studies on MKT have considered it in the teaching of mathematics in general. In this study, the researcher investigated the cognition of mathematical knowledge for teaching geometric patterns in mathematics teaching. This research makes valuable contributions to the existing knowledge in the areas stated below.

Theoretical Contribution

The findings of this study will add to the knowledge base of the mathematical knowledge for the teaching model. Understanding the effects of the cognition of Euclidean geometric patterns is essential to inform programme review at teacher training institutions to address the development of mathematical knowledge for teaching geometric patterns at pre-service teacher education. Another theoretical contribution is that this will be the first study to use the LMT items in Eswatini.

Therefore, this study will produce an instrument to be used in Eswatini to measure teachers' cognition in geometric patterns.

Since most previous studies have focused on in-service teachers' awareness of mathematical knowledge for teaching, this study will provide an insight into pre-service primary teachers' cognition of mathematical knowledge for teaching geometric patterns.

Methodological Contribution

The data will be analysed using the partial least squares structural equation modelling to determine the relationship between pre-service teachers' cognition of mathematical knowledge for teaching geometric patterns and classroom practice. The use of pls-sem will result in a model representing the relationship between mathematical knowledge for teaching geometric patterns and classroom practice. This model will serve as a framework for future studies in research on geometry.

Practical Contribution

The findings of this study add to the knowledge base of the mathematical knowledge for the teaching model. Understanding the effects of the cognition of mathematical knowledge for teaching geometric patterns is essential to inform programme review at teacher training institutions to enhance the development of this cognition at pre-service training.

Policy Contribution

This study will reveal the pre-service primary teachers' cognition of mathematical knowledge for teaching Euclidean geometric patterns in Eswatini. This information will inform policymakers on what needs to be done to address the issue of geometric patterns in teacher education.

Chapter Outline

The thesis is divided into the following chapters:

Chapter One introduces the research study. It presents the background of the study, objectives, research questions, Problem statement, Scope of the study, and the intended contribution to the body of knowledge.

Chapter Two gives an analysis of related literature that discusses the research topic. The literature review discusses studies related to the research topic. This chapter intends to highlight why research on geometry is necessary, the rationale for research on pre-service teachers, and the importance of teacher cognition. Through this review, gaps in the literature on this area of research are identified and noted.
Chapter Three discusses the Theoretical framework that underpins the study. It outlines the methodology, data collection, and analysis framework that guides the study.

Chapter Four outlines the methodology used in conducting the study. It gives details of the research design, sampling methods, data collection procedures, and data analysis.

Chapter Five contains a presentation of the research findings and an analysis of the data.

Chapter Six presents a discussion of the results, conclusions and recommendations.

Table 2. Summary Table showing gaps in themes of knowledge

Gaps	Sources	Research objective
1. Few studies on mathematical knowledge for teaching were carried out in Africa.	Hoover et al. (2016)	To determine the cognition of pre-service primary teachers' cognition of geometric patterns
2. Not much research on cognition of geometry teaching.	Sunzuma and Maharaj (2019)	
3. No research on teachers' cognition of mathematical knowledge for teaching in Eswatini before implementing competency-based education	Nhlengethwa (2019)	
4. No research on causes of poor performance of primary school learners in geometry questions in	ECESWA (2019)	

Gaps	Sources	Research objective
Eswatini		
5. Not much research on how to facilitate the development of mathematical knowledge for teaching	Jankvist and Mosvold (2020); Hoover et al. (2016)	To investigate how teacher educators can enhance the pre-service primary teachers' cognition of geometric patterns
6. Few studies have investigated the effect of mathematical knowledge for teaching on teaching/learning	Hoover et al. (2016) Hill et al. (2008)	To establish how the pre-service primary teachers' cognition of geometric patterns affects lesson planning and lesson presentation
7. Need for studies to understand how mathematical knowledge for teaching is used in instruction.	Hill et al. 2008	
8. Not enough empirical research on how the cognition of mathematical knowledge for teaching is used in teaching.	Hoover et al. (2016)	

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Educators and researchers' claims are to the effect that the ability for teachers to produce learners who are competent in mathematics is based on the teachers' competence in mathematics (Kohen & Kramarski, 2018). Hence, producing capable learners in geometric patterns depends on the teachers' cognition of mathematical knowledge for teaching geometric patterns. This claim implies that the teachers must have sufficient mathematical knowledge to teach geometric patterns to help their learners understand them.

The starting point of this process of learning through geometric patterns is pre-service primary mathematics teachers. The pre-service teachers are the future mathematics teachers who lay a foundation in teaching geometry (Kohen & Kramarski, 2018; Stylianides & Delaney, 2018). Hence, the objectives of the present study are (i) determining the applicability of the LMT measures in the Eswatini context, (ii) determining the pre-service teachers' cognition of mathematical knowledge for teaching geometric patterns, and (iii) investigating the effect that mathematical knowledge for teaching geometric patterns has on classroom practice.

A geometric pattern refers to a sequence of numbers that have a clear rule of calculating each term, either from its predecessor or using its position in the sequence. Such a pattern is called a geometric sequence since the numbers relate to "geometric figures in which each figure is derived from the previous figure by some well-defined procedure" (Bishop, 2000, p. 110). The study of geometric patterns to make generalisations involves pattern recognition and the use of inductive reasoning, i.e. use of particular cases to arrive at a general relation that is true for all cases and can be used to predict other terms in a sequence (Friel & Markworth, 2009).

In this study, the researcher shall investigate the cognition of pre-service primary teachers by assessing their competence in making generalisations of Euclidean geometric patterns using pattern recognition.

In this chapter, the researcher shall conduct a review of literature on the teaching of geometric patterns, focusing on the use of mathematical knowledge for teaching geometric patterns.

2.2 THE IMPORTANCE OF RESEARCH ON PRE-SERVICE PRIMARY TEACHERS

Pre-service teachers, also called student teachers, are prospective teachers enrolled in a University or College of Education to be trained as teachers. According to Santagata and Lee (2019), the importance of research pre-service teachers' cognition is that these teachers lack experience in teaching. Hence, they are perfect candidates to learn how teachers use their mathematical knowledge to teach Euclidean geometric patterns in teaching geometry.

Shulman (1987) also calls for research on pre-service teachers since a study of how new/inexperienced teachers conduct their lessons gives excellent insights into “the complex bodies of knowledge and skill needed to function effectively as a teacher” (L. Shulman, 1987, p. 4). Similar insights (studies of developing babies) enabled Piaget to develop the ‘Stage approach to cognitive development in psychology. Information on how primary teachers use their mathematical knowledge to teach Euclidean geometric patterns in classroom practice is useful for designing teacher preparation programmes.

This research intends to investigate the impact of pre-service teachers’ cognition of mathematical knowledge for teaching geometric patterns and how this cognition affects classroom instruction. Understanding how the cognition of mathematical knowledge for teaching Euclidean geometric patterns affects teaching will inform policymakers on the necessary ability teachers should acquire during pre-service training.

The importance of research on pre-service primary teachers is also derived from the fact that primary teachers lay a foundation for future work in geometry (Stylianides & Delaney, 2018). Studies by Koeno et al. (2016) and Ramdhani et al. (2017) revealed that most learners find mathematics challenging to understand, and Euclidean geometry is one of the topics that learners have difficulty understanding. Moreover, it has been indicated in the literature that the problem of learners not doing well in geometry may be due to inadequate preparation of teachers at pre-service training

(Martinovic & Manizade, 2018). However, Steele (2013) decries that literature on pre-service teachers' cognition for teaching geometry contains very few studies. This study responds to this need for pre-service primary teachers' cognition on Euclidean geometric patterns.

Aydogdu and Kesan (2014) research findings that pre-service primary teachers used guessing and testing strategies when solving geometry problems raise questions about the teachers' cognition in geometry teaching. Consequently, Aydogdu and Kesan (2014) recommended that research be carried out to ascertain the pre-service primary teachers' cognition of geometry. The focus of this study on pre-service primary teachers is also in response to Bowie, Venkat and Askew's (2019) assertion that most research on teacher cognition in South Africa has focused chiefly on in-service primary teachers.

For example, the typical findings of studies by Carnoy and Chisholm (2008); Taylor (2011), Venkat and Spaull (2015) is that there are significant gaps in the primary teachers' mathematical knowledge. These findings have implications for a need for pre-service primary teacher education research. The focus on pre-service teacher education is assumed to play a vital role in equipping teachers with mathematical knowledge for teaching (Bowie, Venkat, & Askew, 2019).

Table 3. Summary Table showing gaps in themes of knowledge

Gaps	Sources	Research objective
1. Few studies on mathematical knowledge for teaching carried out in Africa.	Hoover et al. (2016)	
2. Not much research on cognition of geometry teaching.	Sunzuma and Maharaj (2019)	
3. No research on teachers' cognition of mathematical knowledge for teaching in Eswatini before implementing competency-based education	Nhlengethwa (2019)	To determine the cognition of pre-service primary teachers' cognition of geometric patterns.
4. No research on causes of poor performance of primary school	ECESWA (2019); Martinovic and	

Gaps	Sources	Research objective
learners in geometry questions in Eswatini and globally.	Manizade (2018)	
5. Very few studies on pre-service teachers’ cognition for teaching geometry	Steele (2013)	
6. Why do teachers use guessing and testing strategies when solving geometry problems.	Aydoğdu and Keşan (2014)	
7. Most research on teacher cognition in South Africa focuses on in-service teacher	Bowie et al. (2019)	
8. Not much research on how to facilitate the development of mathematical knowledge for teaching	Jankvist and Mosvold (2020); Hoover et al. (2016)	To investigate how teacher educators can enhance the pre-service primary teachers’ cognition of geometric patterns
9. Few studies have investigated the effect of mathematical knowledge for teaching on teaching/learning	Hoover et al. (2016) Hill et al. (2008)	To establish how the pre-service primary teachers’ cognition of geometric patterns affects lesson planning and lesson presentation
10. Need for studies to understand how mathematical knowledge for teaching is used in instruction.	Hill et al. 2008	
11. Not enough empirical research on how the cognition of mathematical knowledge for teaching is used in teaching.	Hoover et al. (2016)	
12. There are significant gaps in the primary teachers’ mathematical knowledge of geometry	Carnoy and Chisholm (2008); Taylor (2011); Venkat and Spaull (2015)	

2.3 EUCLIDEAN GEOMETRIC PATTERN RECOGNITION

According to Pi, Liao, Liu and Lu (2008), pattern recognition is a process that identifies an input stimulus and matches it with already existing knowledge in memory. Pi et al. (2008) consider pattern recognition as the basic process of human cognition. It is a facilitator in the process of memory and reasoning in the human mind (Barkman, 2018). As a fundamental process of human cognition, it is pertinent that educators study how it can be developed in learners. Kurzweil (as cited in Barkman, 2018) asserts that pattern recognition has the highest correlation with general intelligence.

According to cognitive psychology, Pi et al. (2008) discuss three pattern recognition models. Research on these models is better for understanding pattern recognition (Barkman, 2018). These models are template matching, prototype matching, and feature analysis. Each model describes how information is stored in long term memory to facilitate pattern recognition. This knowledge is focal for pre-service primary teachers since it will enable the teachers to present lessons in geometry in a manner that will allow learners to use pattern recognition in learning the concepts taught.

Geometric pattern recognition in teaching geometry implies that fundamental concepts are essential since the basic concepts form templates for future learning in geometry. This claim is based on the discussion of the pattern recognition theory of learning. The template matching model holds that a long-term memory template represents each previously encountered stimulus. "Pattern recognition, then, occurs by matching input with a specific, perfect specimen stored in memory" (Lutz & Huitt, 2003, p. 4). This implies that there would be too many templates existing in long term memory, representing the many stimuli that one has encountered before, making pattern recognition "less flexible and stiffer" (Pi, Liao, Liu, & Lu, 2008, p. 435).

This shortfall is addressed in the prototype model. This model holds that memory stores prototypes instead of templates of patterns earlier formed in memory. Hence, the prototype model appears "more flexible and more elastic" than the template model (Pi et al., 2008, p. 435). However, the drawback of both the template and the prototype models is that they only have one-way processing, i.e. top-down processing. The feature theory considers that the human mind stores only pattern instead of a template or a prototype. In this model, an input stimulus is identified by its characteristics.

These characteristics enable the mind to classify it under a specific category. Hence, the feature model is a "bottom-up processing model" (Pi et al., 2008, p. 436). According to Arroyo (2014), cognition is based on retrieving concepts from long-term memory when required. Pi et al. (2008) contend that pattern recognition, a neurological construct, enables the retrieval of information from long-term memory through identifying similarities. This makes geometric pattern recognition a critical process in teaching and learning geometry. Research in pattern recognition is also vital since pattern recognition is considered the basic process of human intelligence or awareness (Barkman, 2018; Pi et al., 2008).

Lachman (2015) says that pattern recognition is key for our daily interaction with the world around us. It is important in "reading, understanding speech, and distinguishing the familiar from the unfamiliar (Lachman et al., 2015, p. 489). So pattern recognition is not simply a matter of matching new information with existing information but an information-processing process that can be explained by understanding neurophysiology (Lachman et al., 2015).

It is worrying that even though pattern recognition is such an imperative aspect in understanding the world around us, not much research has been carried out to ascertain how it can be used in the teaching and learning process (Philip J Kellman & Massey, 2013). Its application has only been in machine artificial intelligence manufacturing. There is a need for research to understand how pattern recognition can be developed and used in teaching and learning geometry.

The study of pattern recognition is based on the field of psychology. Goldberg, a neuropsychologist (cited in McCluskey, Mitchelmore, & Mulligan, 2013), contends that cognition is developed by identifying patterns in data and recalling it through the process of pattern recognition. Even though pattern recognition is a concept in psychology, a clear understanding of this construct and how it is used in learning is beneficial in teaching and learning geometry (McCluskey et al., 2013).

Pattern recognition is a problem-solving strategy that enables learners to acquire and retrieve information from long-term memory (Barkman, 2018). Leaners in Eswatini have difficulties in geometry (ECESWA, 2019). According to Kellman et al. (2010), one method of developing pattern recognition in learners is to use perceptual learning models. Kellman and Massey (2013) note that the past decade has seen a growing interest in research on the application of perceptual learning (PL) technology in mathematics and science learning (Kellman et al., 2010; Massey et al., 2011; Silva & Kellman, 1999; Wise et al., 2000, all cited in Kellman & Massey, 2013).

The authors note that studies in the learning of mathematics and science have always focused on declarative and procedural knowledge, ignoring the fact that these subjects “rely substantially on pattern recognition and fluent processing of structure” (Philip J Kellman & Massey, 2013, p. 151). Furthermore, Kellman et al. (2013) note that “few instructional activities directly address these aspects of learning, and a variety of indicators suggest that they may be disproportionately responsible for students’ difficulties in learning” (p.151). The researcher has found it of utmost importance to investigate the effect of geometric pattern generalisation in the teaching and learning of geometry. In the next section, the researcher shall discuss the importance of the cognition of MKT for geometric patterns in the teaching and learning of geometry.

2.4 COGNITION OF MKT FOR GEOMETRIC PATTERNS

Sugesti, Rukmini, Faridi, and Fitriati (2020) assert that teachers’ cognition in mathematics differentiates their classroom practice from the other. The teaching strategies, materials used in instruction and the teacher’s interaction with learners are affected by their cognition of mathematics teaching. Cognition embraces other constructs, e.g. teacher beliefs and perceptions on how to teach mathematics. These beliefs and perceptions guide the teaching practice of a teacher. Hence, cognition is an unobservable construct that manifests in the teacher’s classroom practice.

According to Lutz and Huitt (2003), the study on cognition, for mathematics educators, is to understand how people learn. Mathematics educators want to know how geometry information is acquired, retained and stored in long-term memory. Hence, a study on cognition involves a “study of the mind holistically” (Lutz & Huitt, 2003, p. 1). The study of how geometry knowledge is processed in mind for understanding will inform mathematics educators on selecting appropriate methods of teaching/learning that will improve student understanding. Nonetheless, researchers report that teachers lack this information, as discussed in the following paragraphs.

The lack of research on teachers' cognition in geometry was noted by Shulman (1986) as he asserts that most research carried out in education investigates teacher behaviours that result in improved academic performance by learners. Shulman (1986) avers that there is not enough research on the cognition of teachers on the subject matter and calls this scarcity of research on this area a "missing paradigm" problem (L. S. Shulman, 1986, p. 6). Shulman referred to this knowledge required by teachers as Pedagogical Content Knowledge (PCK).

Shulman (1986) asked the question, "What are the sources of teacher knowledge? What does a teacher know, and when did they come to know it? How is new knowledge acquired, old knowledge retrieved, and both combined to form a new knowledge base?" (p.8). The general assessment offered by Shulman (1986), in part, serves as the motivation for the current study. The other motive for the recent research is positioned upon the fact that no study on teacher cognition on geometric patterns has been found in Eswatini, where currently the researcher resides.

Additionally, there have been studies in other countries investigating the teaching of geometry in primary schools, for example; Susilawti et al. (2017) in Indonesia; Douaire and Emprin (2017) in France; Hock et al. (2015) in Malaysia; and Steele (2013) in the USA. The evidence is that teaching geometry in these countries is not up to the required standards. For instance, the teachers were found to lack adequate mathematical knowledge for teaching (MKT) geometry.

It is of the essence to recognise that the following studies were conducted in Eswatini on teaching mathematics in primary school; Dlamini (2017) revealed that primary school mathematics teachers did not use the learner-centred approach in their teaching. The in-service teachers were found to lack pedagogical content knowledge (PCK) for learner-centred pedagogy (LCP). Another study by Ndlandla (2017) on the primary school in-service teachers' understanding of teaching geometry through the problem-solving approach found out that the teachers had limited knowledge of teaching geometry through the problem-solving process.

It was revealed in this study that teachers used the traditional instructional methods of teaching instead of using the problem-solving approach. However, both studies involved in-service teachers, focusing on pedagogy in mathematics.

Contrary to the studies mentioned above, no traceable research has been found that reports on pre-service primary teachers' cognition of mathematical knowledge for teaching geometric patterns in Eswatini. This study seeks to address this gap.

Mathematical knowledge for teaching geometric patterns is significant in the teaching and learning of geometry since Scheiner (2018) contends that teaching and learning is a complex process that involves an interaction between teacher, learner and subject matter. Scheiner (2018) presents this interaction in the form of a triangle, called the didactic triangle, whose vertices represent teacher, student and subject matter (see Fig. 1).

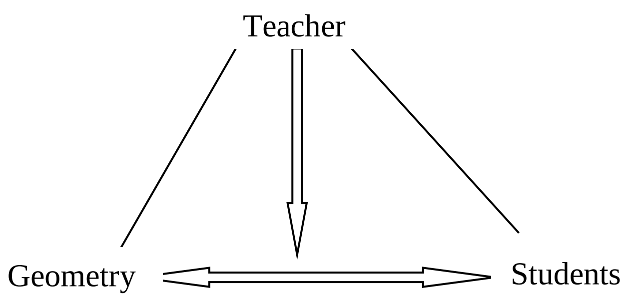


Figure 1. The didactic triangle of teaching and learning (Scheiner, 2018, p.1)

The didactic triangle represents the multidimensional process of mathematics education. The relation between students and subject matter gives mathematical cognition in mathematics education. The connection between teachers and students and teachers and subject matter encompasses teacher cognition in mathematics education research. This relation is the prime focus of this research project. Teacher cognition focuses on teacher interaction with the subject matter and the learner. Mathematical knowledge for teaching geometric patterns is important for many reasons, some of which are;

i. Presenting geometry to students in a simple way and understanding how the students process the knowledge for understanding.
ii. To boost the teachers' self-efficacy in teaching mathematics, especially geometric patterns.

2.4.1 The importance of mathematical knowledge for teaching geometric patterns for student achievement

The study of teachers' content knowledge has been an essential issue in mathematics education research since Shulman presented the need to study teacher knowledge required in teaching (Matthews, Rech, & Grandgenett, 2010). Shulman's presentation highlighted that there is knowledge, besides subject content knowledge, that teachers require. The research on cognition of mathematical knowledge for teaching geometric patterns is also because studies have shown that teachers' content knowledge (CK) is directly related to learners' achievement in mathematics (Ball et al., 2005; Hill et al., 2005; Hine, 2015; Matthews et al., 2010).

Research findings have also established that mathematical knowledge for teaching also affects classroom practice (Cifarelli, Pugalee, & Higbie, 2019; Fennema & Franke, 1992; Mapolelo & Akinsola, 2015). Ribeiro and Carrillo (2012) contend that there is a need to investigate if possession of mathematical knowledge for teaching means that a teacher will provide meaningful mathematical instruction that promotes effective learners' understanding. Previous research studies have investigated the effect of mathematical knowledge for teaching in student achievement.

No study has been found that has investigated the impact of mathematical knowledge for teaching geometric patterns on student achievement in Eswatini.

Pattern recognition is a learner-centred strategy that promotes an understanding of concepts by learners through conceptual mapping with already existing knowledge. Understanding is defined by Van de Walle (2006) as measuring the number and quality of connections that a concept has with other concepts in the already existing knowledge. This definition aligns with Shield and Galbraith's (1998) assertion that understanding is accepted to involve linking the information being learnt with prior knowledge in mathematics learning.

Skemp (1976) discusses two types of understanding: instrumental and relational. An accumulation of rules characterises instrumental understanding without comprehending why those rules work. Knowledge acquired through instrumental understanding does not apply to novel situations. On the other hand, relational understanding refers to a grasp of 'the process and the reason' (Korn, 2014).

In relational understanding, a concept is linked to many other related concepts in one's mind. Hence, if a learner understands a new concept well, it will have many strong connections with the already existing knowledge, forming a network of ideas. On the contrary, a concept that has been acquired through instrumental understanding will not have any connections with the already existing knowledge. Pi et al. (2008) claim that new information is related to existing knowledge through template matching, prototype matching, or feature analysis through pattern recognition. Therefore, the assessment of Pi et al. (2008) leads to the proposition that learning geometry through geometric pattern recognition results in relational understanding since the new knowledge is linked with the already existing knowledge.

Van de Walle (2014) articulates the understanding of any concept as falling on a

Figure 2. Continuum of understanding (From Van de Walle, 2014)

Therefore, when we teach for understanding, we want the learner to connect the new information with his already existing knowledge, as Skemp (1976) articulated. Learning with understanding enables a learner to 'apply' the learnt knowledge in problem-solving. Choosing appropriate facts and procedures for any situation is called "meaningful learning and creative problem solving", and this is achieved through pattern recognition (Philip J Kellman & Massey, 2013, p. 149).

Teaching for understanding is attained when the teacher uses the learner-centred approach (Mutilifa & Kapenda, 2017; Yap, Neo, & Neo, 2016). However, Ndlandla (2017) study revealed that in-service teachers in Eswatini use teacher-centred methods to teach geometry to primary school learners. The study findings above imply that the learners in Eswatini learn geometry by rote memorization.

This could be why they have difficulties with geometry questions in the national examination given by the examinations council of Eswatini. The question then is 'what is the teachers' cognition in teaching geometry'?

Drageset (2010) investigated the relationship between teachers' mathematical knowledge and their beliefs about mathematics. In his research, Drageset (2010) tested 356 elementary and middle school teachers on their cognition of mathematics content knowledge and pedagogy on mathematics using the LMT items. The teachers' beliefs on mathematics were also tested. Two beliefs constructs were investigated. The first construct was based on the view that instrumental understanding (Skemp, 1976) is the key to understanding mathematics.

The other construct was based on the belief that mathematics is a dynamic subject that should be learnt through learner-centred approaches. The results revealed that teachers with high cognition scores on mathematics content knowledge did not believe in the importance of instrumental understanding in mathematics. These findings imply that for teachers to teach towards relational understanding through learner-centred approaches, they must have high cognition of mathematics content knowledge and pedagogy for teaching mathematics.

The researcher believes that enhancing geometric pattern recognition to pre-service primary teachers can improve geometry teaching in Eswatini. However, such a claim must be based on empirical results, hence the current study.

2.4.2 Cognition of geometric patterns and teachers' self-efficacy

According to Depaepe and König (2018), developing pre-service teachers' professional competence is one of the main of teacher education. The authors, as mentioned earlier, assert that teachers' professional competence includes cognitive and affective aspects. The affective aspect is in developing self-efficacy beliefs, i.e. beliefs about one's competence in teaching. Research studies have shown that teachers' cognition on mathematical knowledge for teaching and content knowledge affects a teacher's self-efficacy beliefs, i.e. "beliefs about one's capacities to teach" (Depaepe & König, 2018, p. 178).

Conversely, studies of teachers’ self-efficacy beliefs in teaching reveal that self-efficacy beliefs in mathematics teaching are influenced by teachers’ cognition in mathematics (Catalano, Asselta, & Durkin, 2019). Cifarelli et al. (2019) note that teachers’ cognition on content knowledge in mathematics directly influences their beliefs about teaching and consequently affects their classroom practice.

Consequently, other researchers have established a positive correlation between teachers’ self-efficacy beliefs and instructional practices (De Laat & Watters ; Holzberger et al., as cited in Depaepe & König, 2018). Drageset (2010) found out that teachers’ cognition on mathematical content knowledge and self-efficacy beliefs influence teaching, and consequently, learner achievement in mathematics. For example, in a study involving 58 Greek in-service teachers, Poulou, Reddy, and Dudek (2019) found out that there was a high correlation between the teachers’ self-efficacy beliefs and effective classroom instruction. These findings believe that high teacher cognition in geometric patterns will result in high-quality geometry teaching and, consequently, improved student achievement (Hill, Blunk, et al., 2008).

Hence, according to Catalano et al. (2019), teachers’ self-efficacy beliefs occur during their preparation for the teaching profession. Teachers who are well empowered with teaching skills, e.g. pattern recognition, have been found to have high self-efficacy beliefs for teaching (Catalano et al., 2019; Cifarelli et al., 2019). The study's recommendation by Catalano et al. (2019) emphasises the importance of enriching teachers’ cognition in content knowledge, increasing their self-efficacy beliefs in mathematics teaching.

The preceding sections highlight the importance of teacher cognition for teaching geometry, i.e., boosting teachers’ self-efficacy and improving student achievement. Despite the importance of teacher cognition, no study has been found in Eswatini investigating teachers’ cognition in geometry. The need for an in-depth analysis of primary teachers’ cognition in mathematics has been raised in several studies (Ball et al., 2008; Catalano et al., 2019; Matthews et al., 2010).

Ball et al. (2008) contend that there is a lack of robust research on mathematical knowledge for teaching to understand better what it entails and how it affects teaching/learning.

A clearer understanding of teacher knowledge necessary for teaching informs teacher preparation programmes on what needs to be taught to prospective teachers.

The lack of research on teacher cognition in mathematics is also raised by Hill et al. (2008) as they say that there is not enough research on the relation between Pedagogical Content Knowledge (PCK) and learner achievement. The authors say this gap results from not enough studies measuring teachers' mathematical knowledge for teaching (MKT). The authors say educational studies that have been done in this area lack empirical evidence collected from teachers' lessons to deduce the correlation between classroom practice and the level of teacher knowledge.

They say this "lack of specifics regarding how knowledge affects instruction leave critical gaps in our theoretical knowledge, and significant holes in recommendations for both policy and teacher education"(Hill, Blunk, et al., 2008, p. 431). However, a study was carried out by Zakharov, Tsheko, and Carnoy (2016) in South Africa, Swaziland and Kenya to investigate the effect of teacher quality and classroom resources on learner achievement.

The study found that increasing teacher subject knowledge contributed to higher learner achievement. Hill et al. (2008) note that even though there are such established results of the relationship between teacher cognition and the quality of mathematics instruction, there is still a lack of an in-depth understanding of "how teacher knowledge affects classroom instruction and student achievement" (p.431). Such an in-depth understanding of what aspects of teacher instruction are exhibited by teachers with high cognition can help design appropriate teaching programmes in pre-service teacher education. Hence, in this study, the researcher intends to make classroom observations to establish how the cognition of MKT for geometric patterns affects classroom practice.

Table 4. Summary Table showing gaps in themes of knowledge

Gaps	Sources	Research objectives
Not much research on how pattern recognition is used in the teaching/learning process	Kellman and Massey (2013); (McCluskey et al., 2013)	How cognition of geometric patterns affects lesson planning and lesson presentation
In-service teachers have a limited understanding of teaching geometry through the problem-solving approach	Ndlandla (2017)	
Lack of empirical evidence collected from teachers' lessons to deduce the correlation between classroom practice and the level of teachers' cognition of geometric pattern recognition	Hill et al. (2008); Hoover et al. (2016)	
Scarcity of literature on cognition of teachers on subject matter knowledge	Shulman (1986)	To determine teachers' cognition of MKT for geometric patterns
No study on teacher cognition of MKT for geometric patterns has been found in Eswatini		
Teachers in Eswatini do not fully use the learner-centred approach in teaching mathematics	Dlamini (2017)	To determine teachers' cognition of MKT for geometric patterns

2.5 MEASURES FOR TEACHERS' COGNITION OF MKT FOR GEOMETRIC PATTERNS AND ITS EFFECTS IN CLASSROOM PRACTICE

Since Shulman's (1986) assertion that there is the knowledge required for teaching, different conceptualizations of teacher knowledge have been proposed, for an example, the mathematical knowledge for teaching (MKT) and the mathematical knowledge for teaching geometry (MKT-G) (Ball, Thames & Phelps, 2008; Herbst & Kosko, 2014). Despite the different categorizations of teacher knowledge, there is agreement among researchers that teacher knowledge for teaching is important for student achievement (Ball et al., 2008; Kazima, Jakobsen, & Kasoka, 2016).

Since the researcher intends to determine pre-service teachers' cognition of MKT for geometric patterns in the current study, this study falls under understudies that measure teachers' mathematical knowledge for teaching. A wide variety of instruments for measuring knowledge required for teaching have been developed. Some of the mechanisms that have been developed include the COACTIV, DTMAS, LMT, and the TEDS-M (Hoover et al., 2016). The COACTIV measures were designed to measure secondary teachers' competency in the teaching profession.

They were used to "understand issues of practice and policy related to the mathematical education of teachers" (Hoover et al., 2016, p. 10). The Diagnostic Teacher Assessment in Mathematics and Science (DTMAS) instrument was produced to measure in-service middle school teachers' content knowledge and pedagogical knowledge. Likewise, as the COACTIV measures, the DTMAS instrument measures general and overall teachers' mathematical knowledge (Hoover et al., 2016). On the other hand, Hoover et al. (2016) report that the TEDS-M was a study on teacher education programs for pre-service mathematics teachers across seventeen countries.

The TEDS-M study investigated policy, practice and pre-service primary and secondary teachers' readiness to teach mathematics. It is also an overall assessment of mathematics knowledge for teachers rather than a specific topic assessment. All the measures above are not suitable for the current study.

In the present study, the researcher is investigating the effect of a particular topic, i.e. cognition of mathematical knowledge, for teaching Euclidean geometric patterns on the teaching of mathematics. Hence, the researcher will use the LMT measures.

The LMT measures were developed at the University of Michigan as the Learning Mathematics for Teaching (LMT) project (Hill, Ball, & Schilling, 2008). They measure mathematical knowledge for teaching mathematics at the primary and secondary levels. The LMT items were developed in geometry, number concepts and operations, patterns, functions and algebra. Hence, they can be adapted for use in specific areas of mathematics.

Since the LMT items were developed in the American context, Hill et al. (2008) caution that the usability of the items outside the U.S. context is not guaranteed; however, the LMT items have been adapted for use in contexts outside the United States, e.g. in Ireland (Delaney, 2012); Indonesia (Ng, 2012); Ghana (Cole, 2012); and in Malawi (Kazima et al., 2016). However, the researchers report that the items were adapted for each context.

In this study, the researcher intends to use the LMT items in the Eswatini context to determine the pre-service primary teachers' cognition of MKT for Euclidean geometric patterns. Since no study has used the LMT items in Eswatini, their usability to measure teachers' cognition has to be investigated (Hill, Ball, et al., 2008). Before the LMT items were used in Eswatini, they had to be adapted to the Eswatini context. The adjusted items then serve as a framework for other studies measuring mathematical knowledge for teaching geometry in Eswatini.

Martinovic and Manizade (2018) allude that assessing knowledge for teaching mathematics is not an easy task. Hence, researchers have used different methods to measure teachers' knowledge for teaching mathematics. Herbst and Kosko (2014) research used closed-ended survey questions that provided quantitative data subjected to statistical analysis. Their study identified mathematical knowledge for Teaching Geometry at the high school level (MKT-G). They developed items that assessed understanding of plane figures, solid figures and coordinate geometry. However, the results of this study do not give an insight into how the teachers arrived at their answers.

Steele (2013) designed an instrument to assess teachers' level of common content knowledge (CCK) and specialized content knowledge (SCK) in geometry teaching. In his study, Steele (2013) designed open-ended items to assess CCK and SCK on polygons' length, perimeter, and area. The reason for developing open-ended items was that the large-scale multiple-choice measures that had been used in previous research provided "little detail on the teacher's thinking in arriving at their solution" (Steele, 2013, p. 248). Steele (2013), therefore, used open-ended items to get rich data, providing insights into teachers' thought processes in teaching. Gaining a fuller understanding of teachers' thought processes is vital in designing professional development programmes and pre-service courses that inculcate the necessary knowledge for teaching mathematics (Hoover et al., 2016).

The discussion above points to the fact that no one form of assessment can overall evaluate teachers' knowledge for teaching. Hence, Martinovic and Manizade (2018) stress the importance of triangulation in research. Therefore, in their study, Martinovic and Manizade (2018) used five different approaches to develop measures of teachers' PCK in finding the area of a trapezoid. They also used both qualitative and quantitative data collecting methods. Six other instruments were used for collecting data. Different research approaches were used in their study; hence, different data analysis methods were employed. The importance of triangulation, according to Martinovic et al. (2018), is to "decrease, negate, and counterbalance the deficiency of a single method approach" (p.624).

Another critical issue in measuring teachers' mathematical knowledge for teaching is using multiple-choice or open-ended questions. Multiple-choice questions take less time to answer and are easy to grade. On the other hand, open-ended questions require participants to think before answering, taking more time to answer, and the data collected is challenging to analyse. Even though data from multiple-choice items are easy to grade and analyse, its shortfall is that it does not give reasons for a teacher's response, whilst open-ended questions provide rich data on teachers' mathematical knowledge for teaching on a topic of study (Steele, 2013). In their conclusion, Martinovic et al. (2018) recommend that research studies attempting to measure teachers' mathematical knowledge for teaching should use probes on specified topics in mathematics.

These probes should be used together with classroom observations, analysis of teachers' lesson plans, and reflections on lessons to better understand the teacher's pedagogical content knowledge. The current study shall use qualitative and quantitative measures to collect this background data. The researcher believes that triangulation of data collection will provide rich and reliable findings to inform teacher education practice. The quantitative measures used in this study were the LMT items, consisting of multiple-choice items.

The researcher collected data through classroom observations using the Mathematical Quality of Instruction (MQI) scale for the qualitative research phase. Researchers have used different instruments to measure teachers' cognition. These instruments include "self-reports, verbal commentaries, observations, and pencil-and-paper tests" (Sugesti et al., 2020, p. 563). Out of these instruments, Sugesti et al. (2020) assert that observations of the teachers' classroom practices are vital in identifying factors that lead to the development of teachers' cognition.

A teacher's cognition in mathematics is expressed in how they react to learners' responses and the teaching strategies they use in instruction. Therefore, a teacher's classroom practise becomes a window to the teacher's mind about their cognition on teaching mathematics. For this reason, the researcher used classroom observations to get a deeper understanding of the pre-service teachers' cognition of geometric pattern generalisations.

The MQI framework examines "content-focused aspects of teaching mathematics" (Charalambous & Litke, 2018, p. 435). The MQI instrument was developed by Hill et al. (2008) under the MKT framework. Participant teachers were given a paper and pencil test to establish this instrument to determine their MKT level. The teachers' scores on this test were correlated with scores on assessing their mathematical quality of instruction. The literature explains that the MQI instrument consists of "deficits" and "affordances". These are expressed in the elements of mathematics quality of instruction tool shown:

Elements of Mathematical Quality of Instruction Tool

- Mathematics errors—the presence of computational, linguistic, representational, or other mathematical errors in instruction; ◦Contains subcategory specifically for errors with mathematical language
- Responding to students inappropriately—the degree to which teacher either misinterprets or, in the case of student misunderstanding, fails to respond to student utterance;
- Connecting classroom practice to mathematics—the degree to which classroom practice is linked to essential and worthwhile mathematical ideas and procedures as opposed to either non-mathematical focus, such as classroom management, or activities that do not require mathematical thinking, such as students following directions to cut, colour, and paste, but with no apparent connections between these activities and mathematical meaning(s);
- The richness of the mathematics—the use of multiple representations, linking among representations, mathematical explanation and justification, and explicitness around mathematical practices such as proof and reasoning;
- Responding to students appropriately—the degree to which teacher can correctly interpret students' mathematical utterances and address student misunderstandings;
- Mathematical language—the density of accurate mathematical language in instruction, the use of language to convey mathematical ideas, and any explicit discussion of the use of mathematical language.

The first three themes indicate the deficiencies in one's teaching, while the last three themes indicate affordances in one's teaching. It is expected that teachers with strong cognition of MKT for Euclidean geometric patterns will have fewer deficiencies and more affordances. Likewise, teachers with weak cognition of MKT for Euclidean geometric patterns will likely have less affordance and more deficiencies. This framework has four main dimensions, namely; "Richness of the Mathematics, Errors and Imprecision, Working with students and Mathematics, and Common Core-aligned student practices" (Charalambous & Litke, 2018, p. 448). The researcher used the MQI framework to develop a framework for analysing the data from the classroom observations.

2.6 THE IMPORTANCE OF RESEARCH ON GEOMETRY

The researcher has chosen geometry because of its importance in teaching/learning mathematics. Researchers argue that geometry connects the mathematics curriculum by facilitating an understanding of other topics in mathematics (Andila & Musdi, 2020; Tsao, 2018). According to the NCTM, understanding geometry enables learners to understand abstract mathematics concepts NCTM (as cited in Tsao, 2018). Other branches of mathematics are based on Euclidean geometry, e.g. topology and algebra (Ro, 2019).

Andila and Musdi (2020) contend that geometry is the only branch of mathematics that connects mathematics with the "physical form in the real world" (p.1). The geometry provides a valuable means of communication in mathematics and everyday interaction by using geometric terminology to describe objects' shapes. Therefore, literacy in geometry is essential in mathematics and daily life interaction(Hwang, Purba, Liu, Zhang, & Chen, 2018). Pasani (2019) claims that geometry teaching/learning can enhance learners' critical thinking skills. Moreover, Ramdhani, Usodo and Subanti (2017) assert that the study of mathematics, especially Euclidean geometry, is important in the education of learners since it inculcates high-level thinking skills in learners such as critical thinking, spatial reasoning, and problem-solving skills.

However, research studies have revealed that most learners find mathematics challenging to understand, and Euclidean geometry is one of the topics that learners have difficulty understanding (Koeno et al., 2016; Ramdhani et al., 2017). The importance of research studies in geometry is also since the teaching/learning of geometry results in learners attaining critical thinking skills (Ramdhani et al., 2017). According to French (as cited in Alex & Mammen, 2018), the general understanding of learners in mathematics is closely related to their geometric understanding. This is because a knowledge of geometry inculcates the development of problem-solving skills, which is an essential skill in mathematics.

A study by Alam, Das & Das (2018) revealed that one of the factors causing learners' difficulty in mathematics was the poor quality of mathematics teaching. By the poor quality of teaching, the authors mean that the teaching results in rote memorization of facts, with no links of new knowledge with appropriate prior understanding (Richard R. Skemp, 2006; Van de Walle, Bay-Williams, Lovin, & Karp, 2014).

The poor quality of mathematics teaching at primary school harms future learning of mathematics. This is because achievement in mathematics " depends on systematic, cumulative learning, and each new skill needs to be built on a solid foundation laid at earlier stages" (Alam, Das, & Das, 2018, p. 46).

New concepts in mathematics are learned through association with prior knowledge. When learners fail to associate further information with pre-requisite knowledge in memory, they know it through rote memorization instead of relational understanding. Therefore, a lack of the appropriate cognition on geometry by primary teachers results in learners entering secondary school with inadequate knowledge of geometry, "which would result to rote memorization of geometry without meaningful learning" (Aslan-Tutak, 2009, p. 65).

According to Steele (2013), there is a need to investigate pre-service primary school teachers' mathematical knowledge for teaching geometry as geometry lays a foundation for more advanced studies in mathematics. Steele (2013) says one of the reasons for the poor performance of learners in geometry is limited teachers' cognition on geometry. However, the author says that studies on teachers' cognition of geometry "have been nearly non-existent in the research literature" (p.246). In support of this assertion, Tutak and Adams (2017) recommend investigating the pre-service primary school teachers' cognition of geometry.

A study carried out by Ndlandla (2017) in Eswatini to determine in-service primary teachers' knowledge on teaching geometry through the problem-solving approach revealed that the participants' knowledge on teaching geometry through the problem-solving approach was limited. The teachers' lack of knowledge on the teaching of geometry is attributed to inadequate preparation of teachers at pre-service to teach geometry by some researchers (Martinovic & Manizade, 2018). Little is known about pre-service teachers' cognition in Eswatini (Ndlandla, 2017). However, in the research literature, Steele (2013) decries very few research studies on pre-service mathematical knowledge for teaching geometry globally.

The assessment of the studies mentioned above has motivated carrying out this research study in the teaching/learning of geometry in Eswatini.

2.7 CRITICISM OF THEORIES AND MODELS USED FOR PRE-SERVICE TEACHERS COGNITION OF MKT FOR EUCLIDEAN GEOMETRIC PATTERNS AND ITS EFFECTS ON CLASSROOM PRACTICE

The study of teacher cognition in mathematics is a vast and dynamic area. A teacher's mathematical knowledge goes beyond the scores the teacher gets in tests and examinations, but there is a need to understand how the teacher uses this knowledge in lesson planning and delivery. Due to the complexity of this area of research, some theoretical frameworks and models have been developed. These include Shulman's PCK model and Ball et al. MKT model (Hoover et al., 2016).

This study aims to determine pre-service teachers' cognition of MKT for Euclidean geometric patterns and their effects in classroom practice. Teachers' cognition in MKT for Euclidean geometric patterns is mathematical knowledge required for teachers to teach geometric patterns (Philip J Kellman & Massey, 2013). It is therefore embedded in the mathematical knowledge for teaching model. In the mid-twentieth century, studies on teacher education were primarily focused on general pedagogy, independent of subject matter (Hoover et al., 2016).

The lack of attention on the subject matter in educational research was noted by Shulman (1986). Shulman introduced the concept of pedagogical content knowledge (PCK) as a ramification for what he called a 'missing paradigm' in educational research (L. S. Shulman, 1986).

2.7.1 Shulman's PCK Model

Shulman's introduction of PCK was to draw the attention of researchers to the critical role of subject matter in teaching and teacher education (Depaepe, Verschaffel, & Kelchtermans, 2013). Shulman's PCK model had seven categories of teachers' knowledge, contributing to the unique knowledge required in teaching/learning. These categories are "content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes, and values, and their philosophical and historical grounds"(L. Shulman, 1987, p. 8).

Pedagogical content knowledge was of particular interest because it distinguished the mathematics teacher from the mathematics specialist. This type of knowledge enables a teacher to provide appropriate support to learners with learning difficulties in mathematics. This involves transforming the content knowledge to be learnt into representations that are in line with the learning abilities of learners. In Shulman's model, PCK has three categories; namely subject matter content knowledge, pedagogical content knowledge, and curricular knowledge (Cole, 2012). Even though Shulman's (1986) proposition of PCK was ground-breaking in educational research, there has been criticism of the theory in literature. The first criticism pertains to Shulman's model lacking an empirical basis establishing PCK as a separate category in teachers' knowledge base (Ball et al., 2008).

Another criticism of Shulman's model was its assumption that PCK consists of knowledge about teaching that can be acquired outside the classroom context. Those opposing this view of PCK held a "dynamic view on PCK", believing that it develops during the act of teaching and is context-specific (Depaepe et al., 2013, p. 13). Other scholars have also questioned the rationale of separating PCK from content knowledge by Shulman. This criticism is because, in the dynamic view of PCK, it is meaningless to separate content knowledge from pedagogical content knowledge.

Shulman's PCK has also been criticized for its narrow conceptualization of PCK only to include two aspects, i.e. knowledge of learners and their characteristics and pedagogical content knowledge. Other scholars felt more elements of teaching need to be included in PCK, e.g. "curriculum knowledge, beliefs/emotions" (Depaepe et al., 2013, p. 13). As a result of these criticisms of Shulman's PCK model, other knowledge models for teachers have been developed. One of the developed models is the mathematical knowledge for teaching (MKT) representation of learning for teachers.

2.7.2 The MKT Model

The MKT model was developed by Ball et al. (2008). The MKT model has been the most useful conceptualization of knowledge for mathematics teachers over the past decade. The model mentioned above encompasses both content knowledge and pedagogical content knowledge. It is subject-specific as it focuses on identifying knowledge required by teachers to teach mathematics. MKT is a refined version of Shulman's PCK.

The MKT model was developed due to empirical results from a Mathematics Teaching and Learning to Teach project (MTLT) that was carried out at the University of Michigan (Cole, 2012).

Through a study of experienced teachers' records of teaching, Ball et al. (2008) developed hypotheses for the categories of MKT. Hence, MKT was developed through empirical research on the knowledge required to teach mathematics. In this regard, MKT "provides an empirical grounding to PCK" (Depaepe et al., 2013, p. 14). The refinement of the MKT model from Shulman's PCK resulted in the fact that the MKT model provides empirical evidence showing a positive relation of teachers' mathematical knowledge for teaching on learner achievement.

According to Depaepe et al. (2013), even though the MKT representation of knowledge was developed through a rigorous research process, it has been criticized for several reasons. One of its criticisms is that it does not consider teachers' beliefs about mathematics teaching. Another concern is that there is no clear distinction between the categories of the MKT model. However, the MKT model is still the most accepted representation of teacher knowledge required for teaching.

Since the MKT model has developed measures on specific topics of the mathematics curriculum, the researcher has found it suitable for the current study on geometry teaching/learning. The MKT model will be considered from the geometry perspective for this study. Hence, to determine the cognition of the pre-service teachers on MKT for geometric patterns, the geometry measures developed for the MKT model will be used. Another criticism of the MKT model is that it considers that the testing of teacher knowledge can be done independent of the teaching context. This concern implies that MKT has to be observed during teaching. For that reason, in this study, the researcher shall follow up the cognition test with classroom observations of the teachers to understand how MKT for geometric patterns is used in teaching.

Table 5. Summary Table showing gaps in themes of knowledge

Gaps	Sources	Research objective
No standard measures to measure teachers' cognition in geometry teaching in Eswatini	Hoover et al. (2016)	To determine the primary teachers' cognition of MKT for geometric patterns
No research to determine why learners find Euclidean geometry difficult to understand	Koeno et al. (2016); Ramdhani et al. (2017); Adams (2017)	To establish how teachers' cognition of MKT for geometrical patterns affects their lesson planning and lesson presentation
Teachers beliefs about mathematics are not considered in the MKT model of Ball et al. (2008)	Depaepe et al. (2013)	
Ball et al. (2008) MKT model assumes teacher knowledge can be tested independent of the teaching context	Depaepe et al. (2013)	To establish how teachers' cognition of MKT for geometric patterns affects their lesson planning and lesson presentation

2.8 SUMMARY

The previous discussion of the literature review highlights the need for research in geometry since research studies and examinations reports have shown that learners have challenges understanding geometry concepts in Eswatini. Learners' difficulty in geometry is attributed to the teachers' lack of mathematical knowledge. However, no study has been undertaken in Eswatini to determine the teachers' cognition of mathematical knowledge for teaching geometry.

There is also a lack of empirical studies to establish how mathematical knowledge for teaching geometry can be developed in pre-service teacher education. Hence, the need for research on the teachers' cognition of MKT for geometric patterns at teacher education and the effects of this cognition in classroom practice. This research will inform teacher education on the required knowledge to be acquired by pre-service teachers for effective geometry teaching.

The MKT model will guide the research on pre-service teachers' cognition of MKT for geometrical patterns and classroom observations. Table 6 highlights gaps in the literature identified through this literature review.

Table 6. Summary of gaps in the literature

Theme	Major gaps in the literature	Sources
Cognition of mathematical knowledge for teaching	-No studies determining the cognition of MKT for geometric patterns -Scarcity of studies in the area of cognition of subject matter knowledge - Few studies done in Africa - Why teachers fail to teach geometry for understanding - Few research studies on pre-service teachers. -No standard measures for teachers' cognition of geometry teaching	-Hashemia, Abua, Kashefia & Rahimib (2013) -Shulman (1986) - Hoover et al. (2016) - Sunzuma and Maharaj (2019) - Martinovic and Manizade (2018) - Steele (2013) - Aydogdu and Kesan (2014) - Bowie and Askew (2019) - Ndlandla (2017)
How mathematics knowledge for teaching influences classroom practice	-There is a lack of explanations on how teachers' cognition affects classroom practice -Reasons for rote learning of geometry concepts by primary learners -How MKT for geometric patterns can be used in teaching mathematics - Effect of teachers' beliefs about mathematics in teaching geometry	- Hill, et al. (2008) -Aslan-Tutak (2009) -McCluskey, Mitchelmore & Mulligan (2013 - Depaepe (2013) - Hoover et al. (2016) - Kellman and Massey (2013) - McCluskey et al. (2013) - Koeno (2016) - Ramdhani (2017) - Adams (2017)
Pre-service primary teachers mathematical knowledge for teaching	- Studies in other countries have shown there is a lack of MKT on geometry in in-service teachers. No study in Eswatini on cognition of pre-service teachers. -Reasons for the poor	Susilawati, Suryadi & Dahlan (2017); Douaire & Emprin (2017); Hock, Tarmizi, Yunus & Ayub (2015); Steele (2013); Marchis (2012); Dlamini (2017); Ndlandla (2017); Tutak & Adams (2017).

Theme	Major gaps in the literature	Sources
	performance of learners in mathematics at National assessments	ECESWA (2019) Murimba (2005)
Developing MKT for geometric patterns	- Few studies investigating how to facilitate the development of MKT for geometric patterns - Gaps in mathematical knowledge for primary teachers in geometry	- Jankvist and Mosvold (2020) - Hoover et al. (2016) - Venkat and Spaull (2015)

CHAPTER 3
THEORETICAL FRAMEWORK

3.1 INTRODUCTION

This thesis believes that to understand a complex process like teaching and learning; there is a need to view it from different perspectives. In this thesis, different theoretical frameworks will be blended to provide new insights into pre-service teachers' cognition of MKT for geometric patterns that would not be possible when using a single theoretical perspective in isolation. According to Scheiner (2018), the genesis of frameworks for mathematical knowledge construction includes Bruner's enactive-iconic-symbolic modes, Piaget's assimilation and accommodation theory, Van Hiele's levels of learning geometry. The latest framework is by Hershkowitz, Hudas, Dreyfus, and Schwartz (2007), which elaborates on recognizing, building-with, constructing, and consolidation (also called the RBC+C model) (Scheiner, 2018).

This model is grounded in the belief that new knowledge is constructed by relating it to existing knowledge in the mind. The researcher shall use the MKT model, Radford's architecture of algebraic generalisation and the Information Processing Theory of Learning to understand the pre-service teachers' cognition of geometric pattern generalisations and the use in lesson planning and lesson presentation.

3.2 MATHEMATICAL KNOWLEDGE FOR TEACHING MKT MODEL

The study of mathematical knowledge necessary for teaching has been a topic of discussion over the past three decades after Shulman's proposal of PCK. Shulman's PCK can be broadly described as the ability to represent and formulate subject matter knowledge to a format that makes it understandable by learners (Shulman, as cited in Scheiner, 2018). This teacher knowledge is classified as the knowledge that "embodies the aspects of content most germane to its teachability" (L. S. Shulman, 1986, p. 9). This involves transforming the content knowledge that the teacher has acquired into representations that are in line with ways of learning by children. This transformation is described by Grossman et al. (as cited in Scheiner, 2018) as "translating the knowledge of subject matter into instructional representations" (p.45).

The works of Ball (1990), Ma (1999), Hill et al. (2008) and others have played an important role in the study of the effects of teacher knowledge on classroom instruction. Based on Shulman's (1986) assertion that there is 'special knowledge' required for teaching, these researchers have helped in conceptualising this special knowledge and in the formulation of its framework. In the teaching of mathematics, this knowledge is called the mathematical knowledge for teaching (MKT). It was developed by Ball and her colleagues to answer the question, ''what mathematical knowledge is needed to help students learn mathematics?'' (Hill et al, as cited in Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012, p. 388).

Ball et al. (2008) also developed multiple-choice items, called the Learning Mathematics for Teaching (LMT) measures, to measure teachers' MKT on different topics. According to Cole (2012), while Shulman's seminal work is vital in teacher knowledge, it is not specific to the discipline of mathematics and does not provide measures for this teacher knowledge. On the other hand, the MKT framework is focused on primary mathematics. It is grounded in teaching and provides measures for teacher knowledge. Since the current study was carried out with pre-service primary school teachers in mathematics, the MKT framework was more suitable for understanding how MKT is used in instruction.

The researcher believes that the MKT framework is useful in studying teachers' cognition of MKT for geometric patterns. The LMT measures were developed to assess teachers' knowledge of teaching specific topics in mathematics. Fauskanger et al. (2012) say "the items were intended to represent the knowledge that is vital for teaching elementary mathematics" (p.389). Hence, this study will use these items to determine the pre-service primary teachers' cognition of MKT for geometric patterns.

Ball, Thames and Phephs (2008) discuss the mathematical knowledge for teaching (MKT) framework. According to Sahidin et al. (2019), MKT is dependent on a teacher's understanding of the content to be taught and mastery of pedagogy. Hence, the MKT model consists of two main types of teacher knowledge, namely pedagogical content knowledge (PCK) and subject matter knowledge (SMK), as one inseparable unit.

Each type of knowledge consists of three components, as shown in Figure 3.

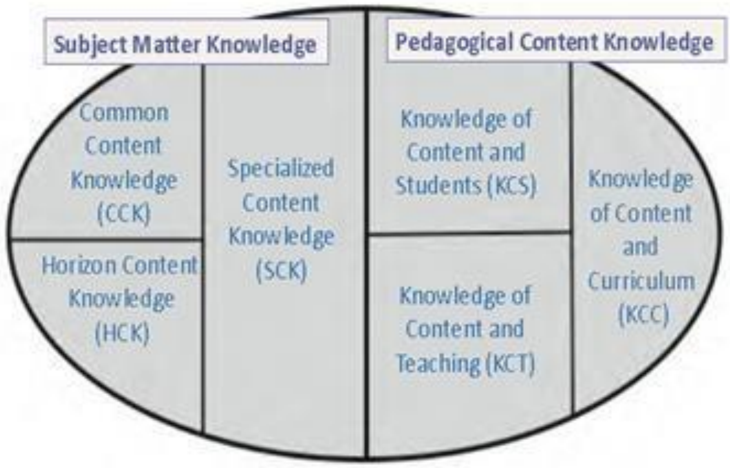


Figure 3. Representation of the MKT Framework (Zambak & Tyminski, 2017)

Subject Matter Knowledge (SMK) consists of Common Content Knowledge (CCK), Horizon Content Knowledge (HCK) and Specialised Content Knowledge (SCK). CCK consists of knowledge and skills that are not unique to teachers. It is knowledge used in many areas, even outside teaching. HCK is knowledge of how topics in mathematics relate to each other. This is the teacher's knowledge of the content that the learners have covered in earlier work and content that the learners are still to cover in future work in that topic. The last component of SMK is specialised content knowledge (SCK). This component refers to mathematical knowledge and skills that are used in teaching.

In the MKT framework, PCK consists of three components; namely knowledge of content and teaching (KCT), knowledge of content and students (KCS) and knowledge of content and curriculum (KCC). KCT is the knowledge of teaching that enables the teacher to use appropriate teaching strategies to enhance learners' comprehension of a concept. The problem-solving strategy of geometric patterns generalisations falls in this category of PCK. KCS involves knowledge of mathematics and learners' learning strategies.

This knowledge also enables teachers to anticipate difficulties and misconceptions learners might have about a particular concept. Hence, this knowledge is important for teachers in lesson preparation. The researcher is interested in determining the pre-service teachers' level of these two types of PCK.

According to Evens, Elen and Depaepe (2015), sources contributing to the development of PCK include teaching experience, PCK courses targeting an improvement of teachers' knowledge on how to teach a subject/topic, CK, teachers' past experiences on learning a subject, etc.

Evens et al. (2015) note that even though scholars have identified sources that lead to PCK development, "there is still little empirical evidence on how education can contribute to PCK development" (p.2). According to Sahidin et al. (2019), teachers lack adequate MKT in many topics in the mathematics syllabus. Hence, the researcher intends to determine pre-service teachers' cognition in teaching geometric pattern generalisations in this research.

This research is interested in 3 aspects of MKT, namely;

1. Measurement of MKT for geometry held by pre-service primary school teachers.
2. Influence of MKT for geometry in lesson planning and lesson presentation.
3. How teacher educators can raise the level of MKT for geometry held by pre-service teachers.

Only four categories of MKT were considered helpful in this study. These are the standard content knowledge, specialised content knowledge, content and students, and knowledge of content and teaching. The tasks of teaching that exhibit each MKT category are summarised in Table 7.

Table 7. Aspects of mathematical knowledge for teaching (Van den Kieboom, 2013)

Mathematical Knowledge for Teaching	
1. Common Content Knowledge (CCK) **Tasks of teaching:** -Know the mathematics being taught -Use terms and notations correctly -Identify student errors -Identify inaccurate textbook definitions -Understand mathematics in the	**2. Specialized Content Knowledge (SCK)** **Tasks of teaching:** -Present mathematical ideas -Recognize what is involved in a representation -Select representations for particular purposes -Give mathematical explanations

curriculum	-Choose and develop useable definitions -Link representations to underlying ideas
3. Knowledge of Content and Students (KCS) **Tasks of teaching:** -Anticipate what students are likely to think -Predict what students will find interesting -Anticipate what students might do with a task -Listen to and interpret students' thinking -Know common conceptions and misconceptions -Anticipate the difficulty level of a task	**4. Knowledge of Content and Teaching (KCT)** **Tasks of teaching:** -Know how to design instruction -Know how to sequence instruction -Choose tasks for instructional purposes -Identify what different methods afford instruction -Evaluate the advantages and disadvantages of using specific representations

Charalambous and Pitta-Pantuzi (2015) allude that measuring the quality of instruction is more challenging than measuring learning. The authors contend that the difficulty in developing instruments for measuring the quality of teaching is due to “relatively less attention than teaching has received, compared to learning” (Charalambous & Pitta-Pantazi, 2015, p. 32). In research on mathematics teaching, a framework focused on mathematics teaching was developed by Hill et al. (2011). This framework called the Mathematical Quality of Instruction (MQI), is based on the MKT model by Ball et al. (2008). MQI examines aspects of teaching mathematics in four main dimensions.

The aspects of teaching measured under MQI are: “Richness of the Mathematics, Errors and Imprecision, working with students and Mathematics, and Common Core-aligned student practices” (Charalambous & Litke, 2018, p. 446). According to Lutz and Hitt (2003), instruction to learners should be presented to enhance the processing of new information to long-term memory. This requires that information be presented to be linked to already existing knowledge.

For a more rigorous elaboration of information, Lutz et al. (2003) recommend that instruction should engage learners in almost all of Bloom's taxonomy of cognitive domain. The involvement of learners in these domains constitutes the quality of the mathematics instruction delivered by a teacher.

Charalambous et al. (2018) elaborate that mathematics richness and mathematical errors, and imprecision of the MQI model are measures of the relationship between teacher and content. Common core aligned student practices indicate the interaction between the students and content. Working with students and mathematics suggests the relationship between the teacher and students, i.e. how the teacher facilitates students' interactions with the mathematics; as such, it focuses on how the teacher hears and responds to students' thinking and contributions in the context of the lesson (p.446).

Because of the complexity of MKT, Charalambous and Pitta-Pantunzi (2015) recommend that the measurement of instructional quality should be done through a variety of methods, including classroom observations, teacher or student ratings, debriefings with teachers about their lessons. Each of these methods has its strengths and weaknesses. Hence, the authors above propose that researchers on instructional quality should use mixed data collection methods to capture all aspects of a lesson to understand the quality of instruction better. Hence, in this research, classroom observation using the MQI framework shall be used to collect data. This method is called "the gold standard of studying instructional quality" by Charalambous and Pitta-Pantanzi (Charalambous & Pitta-Pantazi, 2015, p. 32).

3.3 RADFORD'S ARCHITECTURE OF ALGEBRAIC GENERALISATION

This study draws input from Radford's (2008) architecture of algebraic generalisation of patterns. According to Radford (2008), the importance of generalisations was recognised by Vygotsky, who argued that all concepts come from generalisations. We would deal with particulars; $a,b,c,$ … were $a \neq b$ and $b \neq c$. Generalisations enable us to organize knowledge, using similarities to form concepts. Organizing information into concepts enables us to make sense of the world around us.

Radford (2008) differentiates between three types of generalisations. These generalisations are the algebraic, arithmetic and generalisations from naïve induction. The algebraic generalisation of number patterns involves three stages;

1. Identify a commonality, C, of some particular cases.
2. Generalising this commonality to all terms of the number pattern.
3. Expressing a rule in an algebraic form enables one to find any sequence term directly.

At the first stage, Radford describes abduction as noticing a commonality of particular terms (p_1, p_2, $p_3 \ldots p_k$), expressed as C in Figure 5. In the second stage, the commonality is transformed and extended to all the following terms after the particulars (p_{k+1}, p_{k+2}, $p_{k+3} \ldots$). In the final stage, a general algebraic expression for the general term p_n is deduced from the commonality, C (see Fig. 4).

Particulars p_1, p_2,p_3,	Noticing a commonality,	Making C a hypothesis	Producing the expression of

$\overrightarrow{Abduction}$

$\overrightarrow{Transforming\ the\ abduction}$ $\overrightarrow{Deducing\ p_n\ from\ C}$

Figure 4. The architecture of algebraic pattern generalisations (Radford, 2008)

In the case of an arithmetic generalisation, a commonality is identified and extended to subsequent terms, but there is no generalisation for finding any term in the sequence. The (n^{th}) or general term cannot be found directly without calculating it from the previous term of the sequence.

Naïve induction is a generalisation that is reached through guessing until one gets the expression that works for all the terms of the sequence. The general expression is deduced through 'low level' induction. Educators need to note how learners deduce their generalisations to determine algebraic, arithmetic, or naïve induction.

Teachers should encourage learners to form algebraic generalisations. Hence, teachers should have MKT strategies to involve learners in algebraic pattern generalisations. Radford (2008) notes that learners usually resort to using naïve induction when generalising a pattern.

An example of a teacher's lesson engaging learners for an algebraic generalisation is discussed in Radford (2008, p.86). A sequence of figures was presented to Grade 7 learners in this lesson, as shown in Figure 5.

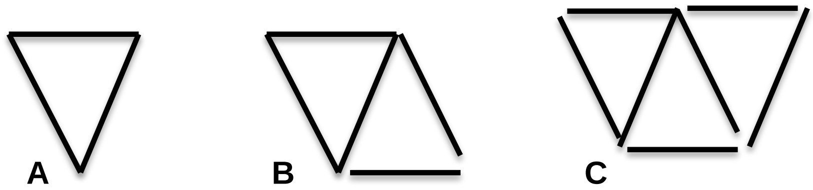


Figure 5. Geometric figures constructions using sticks

Each figure is made by arranging sticks in the patterns shown in Fig. 5. The teacher asked the learners to find the number of sticks needed to make the 100th figure in this pattern. In the first step, the students determined that there were 3, 5, and 7 sticks in Figs A, B, and C. Through a discussion, the teacher then led the learners to observe that two sticks are added each time to get the next term. This leads to the algebraic generalisation that in the n^{th} figure, the number of sticks is $1+2n$.

It calls for a teacher with adequate knowledge of content and teaching (KCT) and content and students (KCS) to engage learners in a learner-centred approach instead of traditional teacher-centred methods. The researcher shall investigate how pre-service teachers generalise geometric patterns in this study.

3.4 INFORMATION PROCESSING THEORY OF LEARNING

According to the information processing theory, the researcher intends to understand the teachers' lesson planning and presentation. The information processing theory (IPT) of learning was developed by Miller (1956). Miller (1956) presented two concepts of the IPT theory, namely chunking and using the computer as a model for human learning (Larsen & Eargle, 2015).

Considering the second concept, Miller considers the process of information processing in 3 stages. It is therefore called the stage model of information processing.

The Stage Model of Information Processing

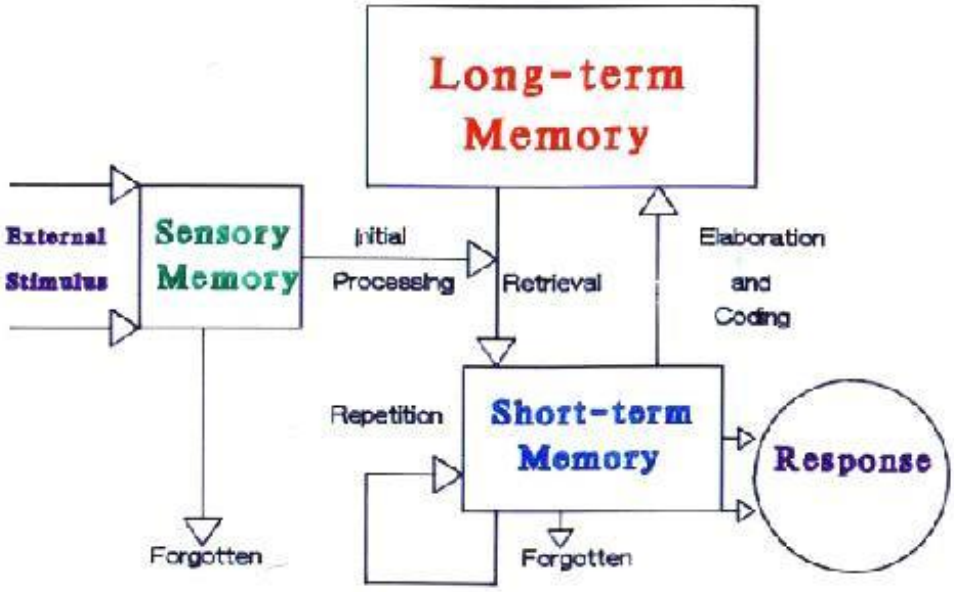


Figure 6. The stage model of information processing theory (Lutz & Huitt, 2003)

The first stage consists of Sensory registers, the second stage is the short term memory (STM), and the third stage is the long term memory (LTM) (Lutz & Huitt, 2003).

Sensory Registers/Sensory Memory

The sensory memory is the input portal for all information perceived through the senses. The sensory registers receive information through the human senses. This information is transduced to electrical energy that the brain can understand. The information at this stage is kept for a short period. If it is not appealing to the mind, it will be discarded. The transfer of the information to working memory is facilitated by attention and automaticity. Attention involves focusing on one stimulus, ignoring the others. One of the factors that enhances concentration to anyone stimulus is "the meaningfulness of the new stimulus to the learner" (Lutz & Huitt, 2003, p. 3).

On the other hand, Automaticity occurs when the incoming information is familiar to the person, requiring minimum attention. In this case, the incoming information is transferred with minimum attention, while more attention is directed to other details. This facilitates the ability to attend to more information in a short space of time.

This stage speaks to the introduction of mathematics lessons and geometric patterns' generalisations. The information presented must be appealing to learners to be processed and passed to the short term memory. Hence, teachers need to have KCS of the MKT model to present information to learners in various ways to ensure that it is appealing to learners of different learning styles so that the sensory registers capture it.

Pattern recognition facilitates the presentation of information in a meaningful way. Identifying patterns in the new information triggers the automatic process of transfer. Through pattern recognition, the attachment of meaning to the incoming information can be explained through anyone of three models. According to Driscoll (as cited in Lutz et al., 2003), the three models are the template matching, the prototype, and the feature analysis model.

Short term memory

Short term or working memory is also called conscious/active memory. Short term memory is where information is held in conscious awareness for a short period while it is processed to be transferred to long term memory. It is kept in working memory by the process of maintenance rehearsal. Lutz and Huitt (2003) say learners are more likely to pay attention to information that activates a known pattern through pattern recognition, where new information is compared to already existing knowledge. Hence, when presenting new information, the teacher should relate it to existing information in the learners' minds.

The information is further manipulated to be transferred to long-term memory. There are three encoding models for information stored in long-term memory (Lutz & Huitt, 2003). In the first case, if the incoming information matches an existing structure in memory, it would be added to that structure. If the new information does not match precisely with the existing structure, the structure would be modified to accommodate the latest information. In the third case, where the new information does not match any existing structure, a new structure will be created in memory.

However, the new structure will be linked to related structures in memory. This implies teaching that new knowledge must be presented to the learner in a meaningful way if it is to be learnt with understanding. Maintenance rehearsal without elaboration of information results in rote learning of that information.

Long term memory

Long term memory is where all knowledge previously encountered by the learner is stored. The stored information is meaningfully understood if it can be related to previously learned information. Skemp (2006) says the level of understanding of new information is determined by the number of its linkages to already existing knowledge. Hence, Skemp (2006) discusses two types of understanding, instrumental and relational understanding. He says instrumental understanding happens when knowledge is not linked to already existing knowledge, and relational understanding happens when knowledge has been closely linked to already existing knowledge. Hence, mathematics teachers need to promote relational understanding of knowledge by presenting it linked to already existing knowledge.

Researchers believe that a teacher with high-quality MKT will better assist learners to comprehend concepts than a teacher with low-quality MKT (Chinnappan & Lawson, 2005). By high-quality MKT, Chinnappan and Lawson (2005) say that it relates to the internal and external connectedness of knowledge in a teacher's mind. Internal connectedness refers to the strength and quality of connections in the knowledge that form a schema. On the other hand, external connectedness refers to the strength and quality of connections between new knowledge and already existing knowledge in one's mind.

"For example, a teacher might be expected to relate a schema for proportion with schemas for ratio or fraction"(Chinnappan & Lawson, 2005, p. 201). Anderson (2000, in Chinnappan & Lawson, 2005) contends that concepts/knowledge that is characterised by strong internal and external connectedness is very useful in problem-solving. Hence, a teacher's success in teaching a new concept depends on their ability to present the concept such that learners develop strong internal and external connectedness of the concept.

Ball et al. (2008) argue that research in teacher knowledge for teaching must focus on how a teacher uses their expertise in education. Hence, in this research, the researcher sought to find out how teachers with a high level of MKT differ in teaching from teachers with a low level of MKT. It is expected that a teacher with a high level of MKT will present high-quality mathematics lessons than a teacher with a low level of MKT. The researcher shall use this theory as a blueprint to analyse the teachers' lesson preparation and presentation. This analysis will determine how the teacher's cognition of MKT for geometric patterns affects their classroom practice, as articulated in objective 2 of the research study.

3.5 ASSESSING QUALITY OF INSTRUCTION IN PRIMARY MATHEMATICS (AQIPM)

According to Klette, Blikstad-Balas, and Roe (2017), research on classroom practices is important to understand the aspects of lessons that result in student achievement (affordances) and what is lacking in lessons where learners achievement is low (deficiencies). This is what Klette et al. (2017) call opening "the black-box of instruction" (p.3). Many authors have tried to conceptualise quality in a mathematics lesson. The emerging consensuses from the various research projects on this area identify key quality elements in mathematics instruction.

These elements are; "instructional clarity, cognitive activation, discourse features and supportive climate" (Klette et al., 2017, p.4). Based on these elements, the Assessing Quality of Instruction in Primary Mathematics (AQIPM) instrument was developed to assess the quality of mathematical instruction of the pre-service primary school teachers. The researcher adopted this instrument from the EQIPM 4.0 instrument (Mesa et al., 2019). The AQIPM instrument has three categories: Quality of Student-Content Interaction, Quality of Instructor-Content Interaction, and Quality of Instructor-Student Interaction. An underlying indicator for the quality of these interactions is mathematical errors and inaccuracies in content and language. This is presented in Table 8.

Table 8. Dimensions and codes for the AQIPM instrument (Mesa et al., 2019)

Assessing Quality of Instruction in Primary Mathematics			
Qual. Student-Content Interaction	**Qual. Instructor-Content Interaction**	**Qual. Instructor-Student Interaction**	**General/underlying Quality of Instruction**
Mathematical Reasoning and Sense-Making **Connecting content to related concepts** **Quality of student questions and responses**	Making Sense of Mathematics Situating the Mathematics Organization in the Presentation Mathematical explanations Making links among related content	Instructor-Student Continuum of Instruction Classroom Environment Inquiry/ Exploration	Appropriate Mathematical language Remediation of Student Errors and Difficulties

The AQIPM was developed based on the MKT framework. Each category of the AQIPM instrument was based on a particular aspect of the MKT framework. The relation is shown in Figure 7.

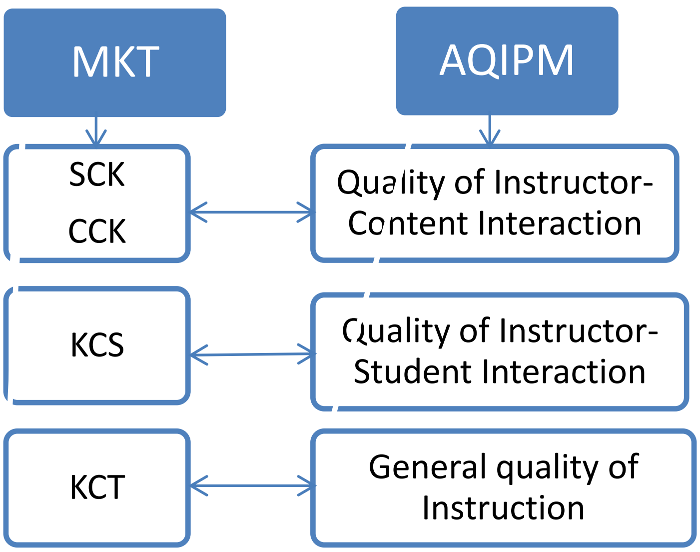


Figure 7. The relation between the MKT framework and the AQIPM instrument

The quality of instruction measured by the AQIPM instrument also reflects the development of Higher Order Thinking Skills (HOTS). According to Tanujaya, Mumu, and Margono (2017), Higher Order Thinking Skills are related to student achievement since they involve the last three levels of Bloom's taxonomy of the cognitive domain, namely analysis, evaluation, and creativity. Hence, HOTS affect learning as they develop the skills of analysis, critical thinking, visualisation, and problem-solving skills.

This instrument assesses if instruction in geometric pattern generalisation involves learners in all skills associated with HOTS. These are visualisation skills through visual representations of the patterns, either by making models or drawings. The analysis skill is developed as learners investigate the pattern to determine how it grows or decreases. Critical thinking and problem-solving are involved as learners determine the general rule for any pattern term.

3.6 SUMMARY

The following themes and gaps arise from the stated theoretical framework: lack of a model for developing MKT for geometric patterns, lack of specifics on how knowledge affects instruction.

Table 9. Summary of gaps in Theories for pattern recognition

Theme	Major gaps in the theories	Sources for the theories
Theory and practice	-Lack of specifics on how knowledge affects instruction	Hill et al. (2008)
Framework for teaching	-Lack of model for developing MKT for geometric pattern generalisation	Ball et al. (2008)
Measurement of mathematical knowledge for teaching	-The developed LMT items for measuring mathematical knowledge for teaching are culturally biased for the American context. There are no universal measures that can be used across all cultures	Fauskanger et al., (2012)

CHAPTER 4
METHODOLOGY

4.1 INTRODUCTION

This research aimed to determine the cognition of pre-service primary teachers on MKT for geometric patterns. The previous chapter has interrogated related literature to the issue of teacher knowledge and classroom practice. This review enabled the researcher to identify a framework to underpin the study and instruments used for data collection. In this chapter, the researcher shall outline the research methodology that guided this study.

4.2 RESEARCH PHILOSOPHY (ONTOLOGICAL, EPISTEMOLOGICAL AND AXIOLOGICAL STANCE)

Based on the discussions of Creswell (2014); Dawadi, Shrestha, and Giri (2021), the research philosophy of a study entails the ontological, epistemological and axiological views underpinning the study. It describes the methodological process of scientific knowledge development that guides the analysis. Ontology examines the nature of reality. The ontological theory answers the question, ‘what is reality?’. There are three ontological views; the first is that there is a single reality in the world, the second is multiple realities, and the third is that reality is constantly negotiated, debated or interpreted.

Epistemology is the belief of how reality can be discovered. The first belief is that knowledge can be measured through reliable tools, the second belief states that reality needs to be interpreted to discover the underlying meaning, and the third belief is that reality should be examined using the best tools to solve the problem. Hence, ontology is the belief like knowledge, whilst epistemology believes in examining reality. The first ontological and the first epistemological beliefs are aligned to the Positivist worldview.

The second ontological and epistemological beliefs are inclined to the Constructivist worldview. Consequently, the third ontological and epistemological beliefs translate to the Pragmatic worldview(Dawadi et al., 2021). The philosophical worldview that guided the present study is the Pragmatic Worldview. According to Dawadi et al. (2021), pragmatism holds that various approaches may be used to understand a research problem.

Pragmatists are concerned with applications and solutions to problems (Patton, as cited in Creswell, 2014; Dawadi et al., 2021; Schoonenboom & Johnson, 2017). Hence, pragmatism promotes qualitative, quantitative, or mixed techniques as long as the chosen technique(s) will better understand the research problem. In Creswell's (2014) view, pragmatism allows a researcher to research mixed methods, resulting in collecting different forms of data and different forms of data analysis to have a comprehensive analysis of the research problem.

For the current research, pragmatism as a research paradigm using the mixed methods research as research techniques were employed to guide the research process of determining the pre-service primary school teachers' cognition of MKT for geometric patterns. This knowledge will help understand how the cognition of MKT for geometric patterns influences the teachers' practice in lesson planning and presentation.

The epistemological position of the researcher in this study is that credible, reliable and relevant data can be collected through the use of a cognition test and classroom observations. Pragmatists believe that no single point of view can fully understand a problem, but using different methods clarifies the different aspects of the problem (Saunders, Lewis, Thornhill, & Bristow, 2015). Ball et al. (2008) assert that it is not easy to determine teachers' mathematical knowledge for teaching.

Hence, to fully understand this phenomenon, the researcher used a mixed-methods design following the 'Explanatory Sequential design' (Schoonenboom & Johnson, 2017) where the quantitative stage preceded the qualitative stage. According to Clark (2019), "this design allows researchers to dive deeper into understanding the nuances and mechanisms that explain the quantitative results" (p.108). The researcher first tested the LMT measures to ascertain the pre-service teachers' cognition of geometric patterns. Then through purposive sampling, participants for the classroom observations were selected. According to Cresswell (2014), purposive sampling is mainly used in qualitative research. It involves selecting participants that can provide rich information on the research phenomenon using minimum research resources.

In research, axiology is a study of how one's values affect the way they carry out a study (Maarouf, 2019). It is also a stance of what we value in the results of a survey. The researcher respects the dignity of other people. Hence, all participants' perspectives were valued. Axiology also includes how a researcher values research. The researcher believed that this research has great value in informing policy and changing mathematics teaching practices in this study.

4.3 RESEARCH DESIGN

This study sought to examine the pre-service primary teachers' cognition of MKT for geometric patterns. Orrill et al. (as cited in Martinovic & Manizade, 2018) assert that it is not easy to examine teachers' knowledge since no single concept characterises SCK for teaching. Hence using one approach may not give comprehensive data to understand the research problem. Therefore, the researcher used an explanatory sequential mixed methods design (Figure 8 (Creswell, 2014).

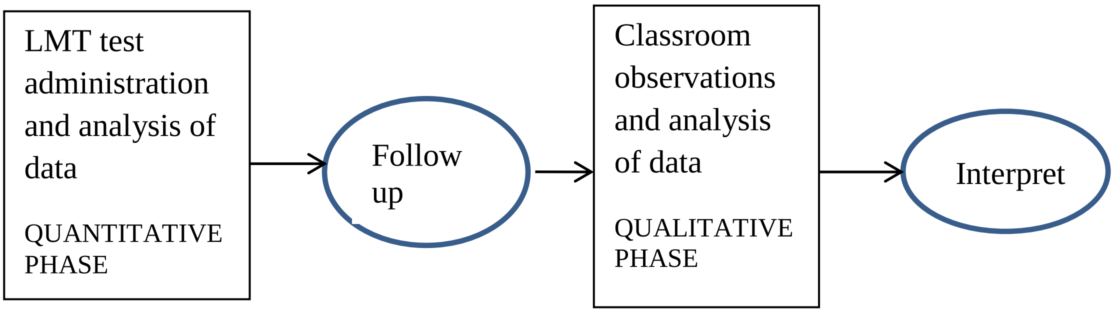


Figure 8. Explanatory sequential mixed methods (Adapted from Creswell, 2014)

Dawadi et al. (2021) believe that a mixed methods design is a unique research methodology that supports "the collection and analysis of data from multiple sources in a single study" (p.27). This approach offers breadth (through the quantitative approach) and depth (through the qualitative approach) to fully understand a phenomenon (Dawadi et al., 2021). In this study, the results of the quantitative data collected through the administering of the LMT test will be explained with an analysis of qualitative follow-up data through classroom observations (Schoonenboom & Johnson, 2017). Hence, the researcher undertook to measure the teachers' cognition "quantitatively and describe the factors that impinge on it qualitatively" (Hill, Blunk, et al., 2008, p. 435).

Therefore, data collection was in two phases. In the first phase, quantitative data was collected by administering the LMT test to the pre-service primary teachers. The data was analysed using descriptive statistics—the second phase (qualitative phase) built on the results of the first phase. This was therefore a developmental and complementary study, according to Schoonenboom and Johnson (2017). For the developmental purpose, the results of the quantitative study helped in the sampling of the pre-service teachers to be involved in the qualitative phase. Complementarity seeks to explain, enhance or clarify the results from one method with the results from the other method. In this research, the results of the qualitative study were to help understand how the cognition of MKT for Euclidean geometric patterns is used by a teacher in the teaching of geometric patterns.

Hill et al. (2008, p.431) assert that there is a need to investigate "the practice of higher-knowledge and lower-knowledge teachers". The researcher, therefore, categorised the participants into low-knowledge, average-knowledge and high-knowledge according to their scores on the first phase of the study. To determine teachers' classroom practice from each category, participants for this phase were purposely selected to be involved in classroom observations. The research design is summarized in Table 10.

Table 10. Main components of the research design

Item	**First phase**		**Second phase**
Research Question	What is the adaptability of the LMT measures of MKT to the Eswatini context?	What is the pre-service primary teachers' cognition of MKT for geometric patterns?	To what extent does the pre-service primary teachers' cognition of MKT for geometric patterns influence their lesson planning and lesson presentation?
Instrument	Pilot test of	MKT for	Classroom observation

	the instrument	geometric patterns' test	
Data Type	Quantitative	Quantitative	Qualitative
Analysis	Quantitative methods	Quantitative methods	Thematic analysis using the AQIPM tool
Output		Article 1	Article 2

4.4 CONTEXT

The research project was conducted at a government-owned teacher training college in Eswatini. The diploma programme at this college produces teachers who teach all subjects in primary school. However, the pre-service teachers must choose an area of specialisation in their final year of study. The areas of specialisation are Sciences (Mathematics, Science and ICT), Social Studies, Applied Sciences (Agriculture and Consumer Science), and Languages (English, Siswati and French).

The first two years of the diploma programme are focused on developing the pre-service teachers' subject matter knowledge and Pedagogical content knowledge according to the needs of primary school teaching in Eswatini in the constructivist framework of teaching and learning. The third and final year of study is mainly to develop the pre-service teachers' common content knowledge on the subjects in their area of specialisation. This is done so that the pre-service teachers develop a deeper understanding of the subjects.

Since the pedagogical content knowledge acquired at the college cannot only be assessed through paper-and-pencil assessments, the pre-service teachers are required to go for teaching practice in the government schools around the country. Each pre- service teacher is allocated a mentor/supervisor during the teaching practice period. The mentor/supervisor guides and assesses the pre-service teacher on their classroom teaching. Hence, each pre-service teacher obtains a score reflecting their competence in classroom instruction.

The respondents for this study were PTD pre-service teachers who took mathematics and science as their specialisation area. These teachers are expected to be more competent in teaching mathematics than their counterparts who did not specialise in mathematics.

4.5 SAMPLE POPULATION AND SAMPLE SIZE

The sampling frame for this research consisted of pre-service teachers in their final year of study at the college, specialising in mathematics. One hundred and ten pre-service teachers specialised in the Science stream at the third-year level. Ninety four of these pre-service teachers were specialising in mathematics. All these ninety four (n=94) pre-service teachers were involved in the first phase of the research. This sample size was more than 70% of the total pre-service teachers specialising in Science at the college at a confidence level of 95% with a 5% marginal error (Gill et al., as cited in Taherdoost, 2017; Cohen & Manion, 2013).

Sample size in qualitative research is guided by the information power of the sample (Malterud, Siersma, & Guassora, 2016). According to Malterud et al. (2016), items that determine the information power of a sample include; “(a) study aim, (b) sample specificity, (c) use of established theory, (d) quality of dialogue, and (e) analysis strategy” (p.2). The aim of the present study was not too broad since it is to investigate the pre-service primary school teachers’ cognition of MKT for geometric patterns in Eswatini. The participants in this study were all pre-service teachers. Hence, the sample specificity is dense. According to Malterud et al. (2016), the study sample has a high information power; thus, few participants are required.

For the qualitative phase of the research, stratified sampling was used to select participants for classroom observations. According to Mello (2021), stratified sampling involves dividing the population into a number of meaningful subsets. From each subset a sample is then selected. Stratified sampling ensures that each subset of the population is represented in the sample. Hence, after analysing the pre-service teachers’ LMT test, the pre-service teachers were divided into three groups, according to their scores; the high achievers, middle achievers and low achievers. The researcher then purposively selected 3 participants from each stratum to be followed up with classroom observations to help understand their use of MKT in teaching.

The researcher selected participants that stayed close to the college to minimise travelling costs for the researcher.

4.6 DATA COLLECTION

Data collection is essential in research as it provides the researcher with empirical evidence to draw sound conclusions for the research objectives. There are different ways in which it can be collected, depending on the research design. The use of different data collection techniques in the same research is encouraged to better understand the research problem. In this research, data collection was in two different phases since the research adopted the explanatory sequential mixed methods design (Creswell, 2014; Schoonenboom & Johnson, 2017). The first phase was quantitative in nature. In this phase, data was collected through the use of a cognition pen-and-paper test. In the second phase, classroom observations were conducted to investigate the influence of the pre-service teachers' MKT level on geometric patterns in classroom practice.

This study used two instruments to collect data. The Learning Mathematics for Teaching (LMT) measures and the AQIPM video coding instrument were used. The Elementary Patterns Functions and Algebra (ELPFA) 2006 A form of the LMT measures were selected for this study. This form consists of 21 questions. The content of these questions is based on the material taught in Grades 5 to Grade 7 of the Eswatini primary school. These questions assessed the pre-service teachers' subject content knowledge (SCK). An examination of the items matches the competencies stated in the Eswatini Primary school syllabus. According to the syllabus competencies, the learners should be able to:

- Make generalisations from data
- Draw a missing figure in a pattern
- Find a pattern unit
- Find a pattern rule
- Make a generalisation
- Continue a given number pattern
- Describe number patterns (Ministry of Education and Training, 2013)

Careful consideration of the Eswatini Primary school syllabus competencies shows that the items in the LMT measures assess the SCK that Eswatini primary school teachers should have.

4.6.1 The Pilot Study

Objective 1 – To determine the adaptability of the LMT measures of MKT to the Eswatini context.

A pilot study is a miniature research that is carried out before the main research project. It is carried out for various reasons. Malmqvist, Hellberg, Möllås, Rose, and Shevlin (2019) give the two main reasons as being (1) to determine the feasibility of the study in terms of the practicalities of carrying out the research, (2) to assess the performance of a research instrument. A pilot study helps to increase the research quality (Malmqvist et al., 2019). In this research, the pilot study was conducted to assess the performance of the LMT test in Eswatini, to answer research question 1. It was anticipated that the pilot study would enable the researcher to modify the instrument to the Eswatini context in order to obtain rich data (Malmqvist et al., 2019).

Research Instrument

The researcher used a measurement tool developed through a rigorous process to ensure reliability and validity (Ball et al., 2008). This measurement tool consisted of the LMT measures developed by Ball et al. (2008) in the United States of America (USA). These measures were to capture the cognition (MKT) of the pre-service teachers on geometric patterns. The two elements of MKT that were under study were the teachers' common content knowledge and the specialised content knowledge.

The measures were sought to check the pre-service teachers' knowledge of geometric pattern generalisations. The generalisation of number patterns is covered in Grade 7 of the primary school in Eswatini as an introduction to functions and algebra. Therefore, primary school mathematics pre-service teachers should guide learners on making generalisations of number patterns. However, this study focused on measuring the pre-service teachers' cognition of geometric patterns.

The LMT measures, developed by Ball and colleagues (2008), have a high practice-based approach in the United States of America (USA). The authors advise that they should be used with great caution in other cultures.

However, they have been adopted for use in some contexts outside the USA, e.g. in Ireland (Delaney, 2012), Indonesia (Ng, 2012), Ghana (Cole, 2012), and Malawi (Kazima, Jakobsen and Kasoka, 2016). In all these studies conducted outside the USA, the researchers emphasise the importance of adapting the measures before they are used in a context outside the USA.

The researcher requested and got permission to use the LMT test items from the authors who developed them in this research. To use the items in Eswatini, the researcher first determined their adaptability to the Eswatini context through a pilot test of the measures. Before piloting, the measures were adapted for use in Eswatini.

The first stage in the adaptation process involved selecting the most appropriate measures. Due to the research objectives of this study, the researcher selected measures in the ‘geometry 2006B’ category. Twenty-one items were selected in this category. After a careful study of the items, the researcher believed that there was no need to translate the items from the English language to the local Siswati language since the official language of instruction in the schools in Eswatini is English. Hence, the pre-service teachers would not have a challenge in understanding the items. However, the researcher adapted the items by changing the names of people and places used and replacing them with locally familiar names. The changed names included Mrs Teva, which was replaced by Mrs Tfwala, Mr Jones replaced by Mr Manana. The currency in the items was also changed and replaced with the currency used in Eswatini, i.e., the US dollar ($) was replaced by the Eswatini currency, Emalangeni (E).

The S.I units replaced the measurements of length, e.g., the stride was replaced by the metre. Likewise, inches were replaced by centimetres. These changes did not affect the difficulty level of each item. Instead, they familiarized the scenarios to the Eswatini pre-service teachers. After these changes, the items were discussed for relevance with a researcher who is a PhD holder and a lecturer at a university in Eswatini. The lecturer, familiar with the Eswatini education system and a teacher educator, gave his feedback, which helped modify the items for applicability in Eswatini. The modified items were then piloted with 33 pre-service primary school mathematics teachers in the faculty of education of a university in Eswatini.

Pilot Testing

Approval to collect data from the tertiary institutions of Eswatini was obtained from the Director of Education at the Ministry of Education and Training (MoET). The pre-service teachers involved in the pilot testing were in their third year of study at the university. The pre-service teachers were informed of the study's objectives and their right to participate or not participate in the study. All 33 pre-service teachers volunteered to participate in the study by answering the items. The piloted instrument had 21 items. The teachers were given as much time as needed to complete the test. They took 20 – 30 minutes to finish answering the items. The data obtained from the pilot testing was used to analyse the items using the SPSS version 20 software. This item analysis determined the item descriptives, including reliability, inter-item correlation, Cronbach's alpha if item deleted, difficulty index and discrimination index.

This analysis aimed to identify items that were not relevant to measuring the teachers' cognition of MKT for geometric patterns in Eswatini. The Cronbach's alpha, which determines the items' internal consistency, was found to be 0.72—examining the 'Cronbach's alpha if it deleted' revealed that the internal consistency of the items could be increased by deleting item 21 of the instrument. Deleting this item increased the Cronbach's alpha to 0.74 with 20 items. By Rule of thumb, a reliability index of 0.70 is the minimum acceptable index for any cognitive test (Quaigrain & Archin, 2017; Bichi & Talib, 2018).

Further analysis of the remaining 20 items was done by finding their difficulty index and discriminating index. The difficulty index of an item indicates the number of respondents answering the item correctly. In the context of a cognition test, the difficulty index may indicate that an item is not well framed, hence causing difficulty in answering it.

According to Okoye and Ndubuzor (2021), the difficulty index (p) is interpreted as follows:

$p > 0.71$ = Easy

$0.30 \geq p \leq 0.70$ = Good

$p < 0.29$ = Difficult.

The discrimination index indicates the ability of an item to discriminate between students with a high cognition and those with a low cognition. A well discriminating item has a high probability of being answered correctly by students with a high cognition but a low probability of being answered correctly by students with a low cognition. This index varies between -1 and 1.

Table 11. Interpretation of the item's discrimination index

Discrimination Index (DI)	**Interpretation**
>0.35	Excellent
0.25-0.34	Good
0.15-0.24	Marginal
<0.15	Poor

Sourced from (Iinnette D'Sa, Alharbi, & Iiza Visbal-Dionaldo, 2018)

The difficulty index and the discrimination index of the remaining 20 items that were used in the pilot testing are as shown in Table 12.

Table 12. Item analysis indices

Item	Difficulty index	Interpretation	Discrimination index	Interpretation	Decision
1	*0.45*	Good	0.33	Good	Retain
2	*0.21*	Difficult	0.33	Good	Retain
3	*0.42*	Good	0.22	Marginal	Retain
4	*0.39*	Good	0.56	Excellent	Retain
5	*0.33*	Good	0.56	Excellent	Retain
6	*0.36*	Good	0.44	Excellent	Retain
7	*0.39*	Good	0.33	Good	Retain
8	*0.27*	Good	0.56	Excellent	Retain
9	*0.06*	Difficult	0.22	Marginal	Retain
10	*0.15*	Difficult	0.11	Poor	Remove
11	*0.09*	Difficult	0.11	Poor	Remove
12	*0.18*	Difficult	0.44	Excellent	Retain
13	*0.27*	Good	0.78	Excellent	Retain
14	*0.27*	Good	0.33	Good	Retain
15	*0.27*	Good	0.78	Excellent	Retain
16	*0.30*	Good	0.89	Excellent	Retain
17	*0.27*	Good	0.56	Excellent	Retain
18	*0.27*	Good	0.33	Good	Retain
19	*0.24*	Difficult	0.67	Excellent	Retain
20	*0.39*	Good	0.56	Excellent	Retain

From this analysis, it can be seen that item 10 and item 11 were difficult and had poor discrimination. These items were therefore removed from the scale used in this study. Therefore, the scale used to determine the cognition of the pre-service teachers in Eswatini had 18 items with an internal consistency, measured by Cronbach's alpha of 0.76. The researcher believes that the scale was suitable to measure the pre-service teachers' cognition of MKT for geometric patterns in Eswatini.

Objective 2 – To determine the cognition of the pre-service teachers' mathematical knowledge for teaching (MKT) geometric patterns.

To determine the pre-service teachers' MKT level for geometric patterns, they were given a test using the adapted LMT items for Eswatini. This test measures the pre-service teachers' knowledge of geometric patterns and learners' learning strategies. It also tests the teachers' cognition of pattern generalisation. The reliability and validity of these items have been reported in the literature. The LMT items were developed for use by Hill, Blunk, et al. (2008) at the University of Michigan in the USA. The authors caution that the items should be used in other countries outside the United States.

The findings of studies outside the U.S that have used the LMT items are that there is a need to adapt them to the context in which they will be used (Cole, 2012; Delaney, Ball, Hill, Schilling, & Zopf, 2008; Fauskanger et al., 2012). The items were adapted and piloted to ensure well-performing in the Eswatini context. The instrument to determine the pre-service primary mathematics teachers' cognition of geometric patterns, research question 2, contained 18 close-ended questions.

The questions were answered using pencil and paper by 94 pre-service primary mathematics teachers who were doing their final year in one of the colleges in Eswatini. All the participants had taken mathematics as their major subject in the final year of their study. Permission to collect data from the college was obtained from the Director of education in Eswatini and the college's Principal. The research objectives were explained to the pre-service teachers, and the researcher sought their consent to participate in the study. All 94 pre-service teachers volunteered to participate in the study.

They wrote the test during the day when it was time for their mathematics lecture. The researcher administered the test.

Their mathematics lecturer agreed that we use his teaching time for the test. The participants were given as much time as needed to complete the test. They took not more than 30 minutes to complete the test. The researcher supervised the participants while they wrote the test. This was to ensure that they did not collaborate in answering the questions so that what they wrote was their conception of geometric patterns generalisation.

Responses to the test from the pre-service teachers were coded and entered into SPSS 20 software. The coding was such that a one (1) was recorded for a correct response, and a zero (0) was recorded for an incorrect response. A zero (0) was recorded for any item that was not attempted by a participant. A total score out of 18 was calculated for each participant. A calculation of the test descriptives enabled the researcher to determine the pre-service teachers' cognition of MKT for geometric patterns in Eswatini. This information is a response to research question 2 of the research.

The test scores also enabled the researcher to categorise the participants into three groups; those with a high MKT score, medium MKT, and low MKT score. One participant from each achievement group was selected to be involved in the next research phase through stratified sampling.

4.6.2 Phase 2 (Qualitative Phase)

Objective 3 – To determine the influence of the level of a teachers' MKT level in geometric patterns on classroom practice.

There was a need to examine the teaching practice of the pre-service teachers. The question under study was 'how do teachers use their cognition of MKT for geometric patterns in the classroom?' The researcher wanted to understand how the pre-service teachers' MKT affects their mathematics instruction. Therefore, the researcher conducted recorded classroom observations for three teachers selected through stratified sampling after scoring their LMT test in phase 1 of the study. The selected teachers' consent to observe and record their mathematics lessons was sought from them. All the teachers agreed to participate in the study.

The pre-service primary mathematics teachers are usually engaged in six weeks of teaching practice each academic year.

Teaching practice is conducted in the public schools in the country. This is part of their Diploma requirement at the college. The researcher had planned to conduct the classroom observations during the pre-service teachers' teaching practice. However, due to the COVID-19 pandemic, the pre-service teachers did not go for their teaching practice.

The number of learners in the schools was limited to only 20 in a classroom; hence the teaching practice exercise was cancelled. The researcher requested the selected participants to conduct lessons for the purposes of collecting data for the research. The participants agreed to conduct the lessons. All COVID-19 protocols were observed during the lessons, including social distancing and the use of hand sanitisers before and at the end of each lesson. The issue of social distancing limited the participants' use of group work during the teaching/learning process. In the cases where learners were organised for group work, there was a need to ensure social distancing and minimum sharing of teaching aids by the learners.

The COVID-19 protocols affected the pre-service teachers' classroom organisation. Due to issues of social distancing it was not easy for the pre-service teachers to organise group work for the learners. Even those who organised the learners for group work had to do this cautiously to avoid physical contact between the learners. The learners also had to keep distance from each other, making communication among the group members to be a challenge. It was also not easy for the pre-service teachers to check/mark learners' work because they had to keep social distancing.

Two lessons were video recorded for each of the three teachers involved in the study. According to Klette et al. (2017), videos provide a more helpful image of what goes on in the classroom better than any other means as they simultaneously capture students' and teachers' activities for a closer analysis (Fischer and Neumann, quoted in Klette et al., 2017). Video analysis is also useful because it leads to a deeper understanding of classroom processes and enables an "integration of qualitative and quantitative analyses" (Klette et al., 2017, p. 7). Therefore, the video recordings were useful in this study as it used a mixed-methods design. An analysis of the lessons was based on the day-to-day tasks of mathematics teaching (Ball et al., 2008) and the required cognition of MKT for Euclidean geometric patterns to carry out these tasks. Ball et al. (2008) identified 16 mathematical teaching tasks that are part of the day-to-day work of a teacher. These mathematical teaching tasks of a teacher are listed in Table 13.

Table 13. Mathematical Tasks of Teaching (Ball et al., 2008, p. 400)

• Presenting mathematical ideas
• Responding to students' "why" questions
• Finding an example to make a specific mathematical point
• Recognizing what is involved in using a particular representation
• Linking representations to underlying ideas and to other representations
• Connecting a topic being taught to topics from prior or future years
• Explaining mathematical goals and purposes to parents
• Appraising and adapting the mathematical content of textbooks
• Modifying tasks to be either easier or harder
• Evaluating the plausibility of students' claims (often quickly)
• Giving or evaluating mathematical explanations
• Choosing and developing useable definitions
• Using mathematical notation and language and critiquing its use
• Asking productive mathematical questions
• Selecting representations for particular purposes
• Inspecting equivalencies

These tasks of teaching were used to develop the Mathematical Quality of Instruction (MQI) instrument used to measure the specialised content knowledge (SCK) of the MKT model. It should be noted that the MQI instrument was developed in the U.S. context, without a consideration of contexts outside the U.S. Hence, to use the instrument in Eswatini, the researcher had to determine if the 16 tasks of mathematical teaching identified by Ball and colleagues (2008) were similar to tasks the Eswatini mathematics teachers do as part of their day-to-day work. Hence, the researcher conducted interviews with a group of 15 mathematics teachers from different schools in the Shiselweni region. These teachers were attending an in-service mathematics workshop. The researcher requested the 15 mathematics teachers to volunteer to be part of those interviewed.

Hence, the 15 teachers volunteered to be interviewed by the researcher. During the interview, each teacher was requested to identify the tasks that he/she found to be part of his/her daily work as a mathematics teacher.

The practising teachers responded to the positive for 15 tasks except for task 7, i.e. 'explaining mathematical goals and purposes to parents'. The practising teachers said this was not a common task in Eswatini. Even when they met with parents during 'open days', they only discussed the learners' performance with the parents. Therefore, the quality of the pre-service teachers' lessons was guided by these teaching mathematics tasks. A teacher needs specialised mathematical knowledge called MKT to execute these tasks of teaching mathematics successfully.

An observation-based instrument called the Assessing Quality of Instruction in Primary Mathematics (AQIPM) instrument was developed based on the EQIPM 4.0 instrument (Mesa, Duranczyk, & Watkins, 2019). The codes were developed based on the Mathematical Quality of Instruction (MQI) instrument (Hill et al., 2008). The MQI elements are shown in Table 14.

Table 14. Elements of Mathematical Quality of Instruction (Hill et al., 2008)

• Mathematics errors—the presence of computational, linguistic, representational, or other mathematical errors in instruction;

• Contains subcategory specifically for errors with mathematical language
• Responding to students inappropriately—the degree to which teacher either misinterprets or, in the case of student misunderstanding, fails to respond to student utterance;
• Connecting classroom practice to mathematics—the degree to which classroom practice is connected to important and worthwhile mathematical ideas and procedures as opposed to either non-mathematical focus, such as classroom management, or activities that do not require mathematical thinking, such as students following directions to cut, colour, and paste, but with no obvious connections between these activities and mathematical meaning(s);
• Richness of the mathematics—the use of multiple representations, linking among representations, mathematical explanation and justification, and explicitness around mathematical practices such as proof and reasoning;

• Responding to students appropriately—the degree to which teacher can correctly interpret students' mathematical utterances and address student misunderstandings;

• Mathematical language—the density of accurate mathematical language in instruction, the use of language to convey mathematical ideas, as well as any explicit discussion of the use of mathematical language.

The coding of the mathematics lessons was done in the same way as in Hill et al. (2008). In this coding scheme, each lesson was segmented into 5-minute sections. Each segment was then analysed using the codes outlined in the AQIPM instrument according to the following categories:

- Quality of Student-Content Interaction
- Quality of Instructor-Content Interaction
- Quality of Instructor-Student Interaction
- General/underlying Quality of instruction

The codes developed for each category are stated below:

Quality of Student-Content Interaction

- Assessment of the learners' involvement in the lesson
- Connecting content to related concepts
- Student questions and responses show comprehension of the content.

Quality of Instructor-Content Interaction

- Making Mathematics meaningful to learners
- Using relevant and familiar examples and teaching materials to explain concepts
- Presentation of content in a logical manner, e.g. from the known to the unknown
- Mathematical explanations are clear and correct
- Makes links among related content
- Use of learner-centred methods of teaching

Quality of Instructor-Student interaction

- Asks questions that invoke mathematical thinking
- Teacher's efforts to support learners' explanations
- How the teacher responds to learners' questions
- Teacher's feedback to learners' responses
- Involvement of learners in activities that will enable them to construct knowledge
- Provision of remedial activities to help learners overcome errors and difficulties
- Use of formative assessment to check learners' knowledge

General/underlying Quality of instruction

- Use of appropriate mathematical language
- Responding to errors or misconceptions that arise during the teaching/learning process.
- Use of appropriate mathematical symbols.
- Classroom arrangement to enhance student-student interaction.

The codes used in each category were adapted from existing rubrics, namely the MQI instrument (Hill et al., 2008) and the EQIPM 4.0 instrument (Mesa et al., 2019). These instruments were subjected to rigorous instrument development procedures to ensure validity and reliability. Each code was scored on a scale of 1 to 5, with 1 representing poor quality and 5 representing the high quality of the construct measured. The scoring of the codes for each lesson was done independently by two people, the researcher and a colleague. The colleague was also a mathematics educator; hence it was easy for him to understand the coding and scoring of the lesson. In cases where there were differences in scores, the two would discuss and reach a consensus. However, there were significantly few such instances; in the majority of cases (about 80%), there were no differences in the scoring of the codes.

According to Hill et al. (2008), teachers with high MKT levels are expected to present lessons with high-quality instruction in primary mathematics. Conversely, teachers with low MKT levels have a low quality of instruction in primary mathematics. To determine the extent to which the teachers' cognition of geometric pattern generalisation influences their lesson planning and presentation, the researcher calculated the correlation between each teacher's score on the geometric pattern generalisation paper-and-pencil test and their AQIPM score determined from the teachers' lesson videos.

4.7 DATA ANALYSIS

The results from the LMT test were analysed through statistical methods to answer research question 2. The analysis included finding the mean, variance and standard deviation of the measured MKT score for the pre-service mathematics teachers' paper-and-pencil test. The teachers' lessons were video recorded. The table of themes for AQIPM (see Table 20) was used to determine each teacher's mathematical quality of instruction in primary mathematics.

The MKT level score obtained in phase 1 of the study was correlated with the MQI score, using the Spearman rank-order correlation to establish the relationship between the two scores and answer research question 3 (Cohen, Manion, & Morrison, 2013). Hence, the correlation between the teachers' MKT level score and the MQI score determined the extent to which the teachers' cognition of MKT for geometric patterns influences their classroom practice.

4.8 RELIABILITY AND VALIDITY

According to Cresswell (2014), validity refers to the accuracy of the findings of a research study. Reliability is a measure of the consistency and stability of a study. Although threats to the validity of a study cannot be eliminated, researchers can take some steps to minimise them and hence, increase the validity of the research. One measure, according to Cresswell (2014), to enhance validity is to use multiple approaches to enable the researcher to "assess the accuracy of findings as well as convince readers of that accuracy" (Cresswell, 2014. p.251). In this study, the researcher has adopted a mixed-methods design to fully understand the influence of teachers' cognition on classroom practice.

Triangulation of data collection methods is another measure to ensure the validity of a study. The researcher shall collect qualitative data through classroom observations in this study.

The researcher shall also use peer debriefing to enhance the study's validity. Cresswell (2014) defines peer debriefing as presenting the study to another person to critic and bring to the researcher issues that have not been adequately addressed or need to be clarified in the study. The researcher is a member of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE). This association consists of researchers and promotes research in Mathematics, Science and Technology Education. They hold mini-conferences where members present their studies for input before publishing research papers. The researcher will present the study in a SAARMSTE mini-conference for criticism and input from other members.

4.9 ETHICAL CONSIDERATION

Cresswell (2013) advises researchers to consider ethical issues during their studies. Some of the ethical issues to be considered include; examining the institution's code of conduct, plagiarism, falsifying data and findings, pressuring participants into signing consent forms, and gaining permission into research sites (Cresswell, 2013). The researcher familiarised himself with the university's ethical policy document to address these ethical issues. Having read this document, the researcher undertook to comply with all the ethical issues raised. Hence, the researcher has acknowledged other people's work to avoid plagiarism.

The identity of all participants of the research has been kept confidential. Pseudonyms have been used in data presentation. The collected data has been kept secure from unauthorised users. Consent was sought from all participants before the start of the research. The researcher explained the research aims and procedures to be used. The participants' rights to participate or not participate and to withdraw from the research at any stage were explained to them before they signed the consent form. Permission to research the schools was sought from all relevant gatekeepers, i.e., the Director of education in Eswatini, Principals of the Colleges and schools where the research took place.

The issue of objectivity is of paramount importance when conducting research. Objectivity may be affected by the relationships between the researcher and the participants during the research process. There is an on-going debate on the so-called *insider-outsider* dilemma.

An insider is a description of a researcher who belongs to the group or organisation they are studying, while an outsider is a researcher who does not belong to the group or organisation under study (Flores, 2018 ; The Open University, 2016; Khanna et al., 2021). The debate is that an insider is usually not objective in data collection and analysis. The researcher was an insider in this research as he was a lecturer at the institution where the study took place.

The researcher did not experience any dilemma in researching the institution he worked. On the contrary, being an insider enabled the researcher to get rich data as the participants were already familiar with him. Participants would not easily open up to a 'stranger'; hence, it is recommended that researchers first familiarise themselves with the research environment before beginning their research (Khanna et al., 2021).

Some scholars believe in the superiority of an insider over an outsider because an insider has a better understanding of the research context. In this case, the research context includes the subject, participants and programme of study. A better understanding of insiders results in a clear perspective of matters, which may not be clear to an outsider, leading to bias in data collection and data analysis (The Open University, 2016). The insider is believed to be privy to inside knowledge that an outsider does not have.

Hence, "if you research your organisation, institution or profession you will have access to inside knowledge that an outsider will not be able to gain" (The Open University, 2016, p. 10). In this research, the researcher knew the content covered by the participants during their second year of study on geometric patterns. This information was helpful in the analysis of the pre-service teachers' lesson videos. Hence, the researcher believes that being an insider had affordances for an outsider's research.

CHAPTER 5

PRESENTATION AND ANALYSIS OF RESULTS

5.1 INTRODUCTION

In the previous chapter, the research methodology was discussed. The process of data collection was also described. In this chapter, the data collected shall be presented and discussed. The presentation of data shall be according to the research questions. The research questions for this study are:

1. What is the adaptability of the LMT measures of MKT to the Eswatini context?
2. What is the pre-service primary teachers' cognition of MKT for geometric patterns?
3. To what extent does the pre-service primary teachers' cognition of MKT for geometric patterns influence their lesson planning and lesson presentation?

5.2 RESEARCH QUESTION 1: ADAPTABILITY OF THE LMT MEASURES OF MKT TO THE ESWATINI CONTEXT

5.2.1 The Pilot Study

The pilot study's purpose was to address Research Question 1. According to Carpenter (2018), a pilot test prepares the survey in actual field conditions. The data collected in a pilot test helps determine how the data collection instrument can measure the latent quantity under study. The quantitative data collected in a pilot test enables the researcher to edit the data collection instrument to be well prepared for use in the actual research.

The participants for the pilot study were final year pre-service teachers in one of the universities in Eswatini. These pre-service teachers were pursuing the primary teachers' diploma (PTD). They had chosen Mathematics as their area of specialisation. Hence, these pre-service teachers had the same characteristics as the sample population for this study. Their biographical information is presented in Table 15.

Table 15. Biographic characteristics of the participants

Biographic characteristics	Description	Number of participants	Percent
Gender	Female	16	48
	Male	17	52
Age (years)	18 – 20	0	0
	21 – 25	18	55
	26 and above	15	45
Symbol obtained in mathematics at high school	A	5	15.2
	B	12	36.4
	C	13	39.3
	D	2	6.1
	E	1	3.0

Table 15 shows that the participants consisted of nearly an equal number of males and females, with most of them having obtained a C grade or better in mathematics in their high school results. Therefore, these pre-service teachers are competent in mathematics according to their high school results in mathematics. The pilot study was carried out to determine the applicability of the LMT measures in the Eswatini context. In the pilot study, the researcher administered a test consisting of 21 items to 33 pre-service teachers. A psychometric analysis was carried out using the data from the pilot study. The test descriptives were calculated using the SPSS software. The descriptives calculated were the reliability index (represented by Cronbach's alpha), the difficulty index, and the discrimination index of the items.

The decision to analyse the items by finding these descriptives was based on the first research question of the study (to determine the adaptability of the LMT measures of MKT to the Eswatini context). The descriptives were calculated to determine well-performing items and those that were not well-performing in the Eswatini context.

This exercise eliminated three items from the instrument due to their poor performance in the Eswatini context, as discussed in section 4.5.1.1. The remaining 18 items were of acceptable reliability (α = 0.76).

Table 16. Item statistics of the LMT measures for the Eswatini pre-service teachers.

Item Statistics

	Mean	Std. Deviation	N
Q1	.8485	.36411	33
Q2	.4545	.50565	33
Q3	.7879	.41515	33
Q4	.7576	.43519	33
Q5	.6970	.46669	33
Q6	.7879	.41515	33
Q7	.7879	.41515	33
Q8	.4848	.50752	33
Q9	.0909	.29194	33
Q10	.3939	.49620	33
Q11	.4242	.50189	33
Q12	.5152	.50752	33
Q13	.5152	.50752	33
Q14	.4848	.50752	33
Q15	.5152	.50752	33
Q16	.4848	.50752	33
Q17	.4545	.50565	33
Q18	.6970	.46669	33

The results enable the researcher to answer Research Question 1 as follows:

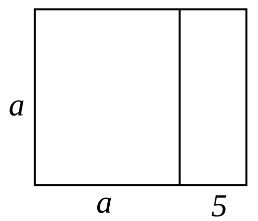
The LMT measures were constructed in the United States of America (USA), based on 16 core tasks of teaching mathematics (Kazima et al., 2016). With regards to those tasks of teaching, it was found that Eswatini and the USA had similar teaching contexts. This is because out of the 16 tasks of teaching mathematics identified in the USA, 14 tasks were also applied in Eswatini. Hence, teachers in Eswatini are trained to perform the same tasks as teachers in the USA. The LMT measures are thus applicable to measure the MKT level of the Eswatini mathematics teachers.

The pilot test of the LMT measures provided data that was analysed using the SPSS 20 software to determine the test's psychometric properties.

The test was reliable, with a reliability index (Cronbach's alpha = .76). This is an acceptable reliability index for a test (Bichi & Talib, 2018). A calculation of the difficulty and the discrimination index of the test resulted in an elimination of only two items. An analysis of these two items revealed that the stems of these items contained wording that was not common in the Eswatini context. Question 10 contained the wording 'baseball cards'.

It is noted that baseball cards are not standard in Eswatini. Question 11 asked about the properties of 'tetrominoes'. The researcher also found out that the term 'tetrominoes' was not common in Eswatini. Hence, the items with a high difficulty index for the Eswatini pre-service mathematics teachers were asked using American words unfamiliar in the Eswatini context. Therefore, it is evident that the LMT measures need to be carefully adapted to any context outside the USA before they are used in that context. This is because differences in culture, language and teaching practices can change the difficulty level of the items. Another difficulty of the items in the Eswatini context was brought about by the use of the table item format. The table item format are questions of the form shown in Figure 9.

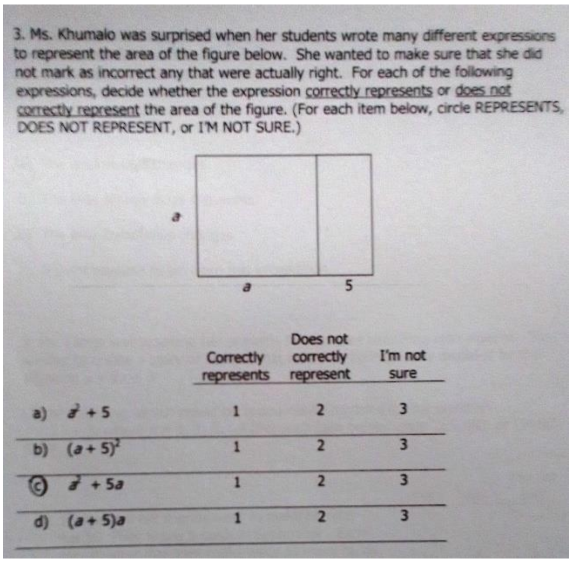
Ms Khumalo was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark incorrect any that were right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (For each item below, circle REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE.)



	Correctly represents	Does not Correctly represent	I'm not Sure
a) $a^2 + 5$	1	2	3
b) $(a + 5)^2$	1	2	3
c) $a^2 + 5a$	1	2	3
d) $(a + 5)a$	1	2	3

Figure 9. Sample LMT released item (Hill et al., 2008)

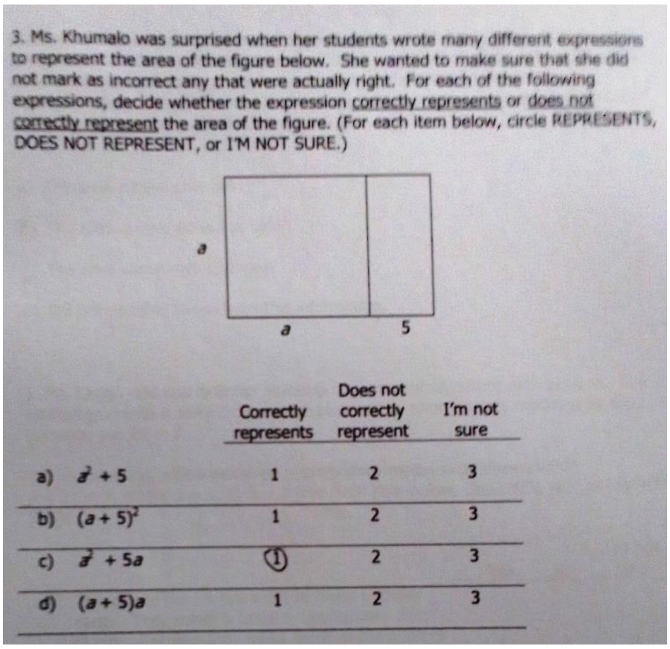
It seems like most pre-service teachers were not used to this type of item format. Hence, they had difficulty in answering items that were presented in this format. Some pre-service teachers would circle only the letter of an option, without indicating whether the option correctly represents or does not correctly represent the area of the figure.


3. Ms. Khumalo was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (For each item below, circle REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE.)

	Correctly represents	Does not correctly represent	I'm not sure
a) $a^2 + 5$	1	2	3
b) $(a + 5)^2$	1	2	3
c) $a^2 + 5a$	1	2	3
d) $(a + 5)a$	1	2	3

It was left to the decision of the researcher to believe that the pre-service teacher circled option (c) to indicate that it correctly represents the area of the figure. The participants who made their selection in this way confused the table item format with a multiple choice item. In this case, the participants did not realise that there are two correct options in this item (option c and option d). Hence, the participant would get only 1 point and lose 3 points for not indicating whether the other options were correct or not correct.

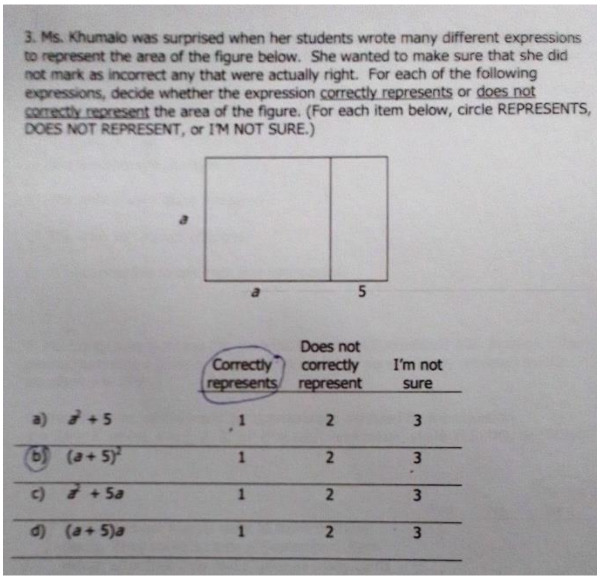
Still there were some participants who only circled number 1 to show that the option correctly represents the area. These participantss failed to indicate what they had decided about the other options.



3. Ms. Khumalo was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (For each item below, circle REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE.)

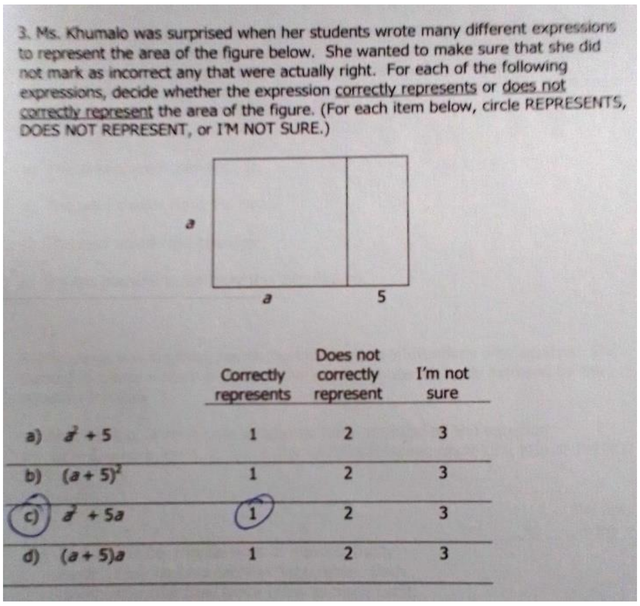
	Correctly represents	Does not correctly represent	I'm not sure
a) $a^2 + 5$	1	2	3
b) $(a + 5)^2$	1	2	3
c) $a^2 + 5a$	1	2	3
d) $(a + 5)a$	1	2	3

This case is similar to the first one, the participant considered this item as a multiple choice item. Other participants circled the letter for an option and number 1, or the letter for an option and 'correctly represents'.



3. Ms. Khumalo was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (For each item below, circle REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE.)

	Correctly represents	Does not correctly represent	I'm not sure
a) $a^2 + 5$	1	2	3
b) $(a + 5)^2$	1	2	3
c) $a^2 + 5a$	1	2	3
d) $(a + 5)a$	1	2	3


3. Ms. Khumalo was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (For each item below, circle REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE.)

	Correctly represents	Does not correctly represent	I'm not sure
a) $a^2 + 5$	1	2	3
b) $(a + 5)^2$	1	2	3
c) $a^2 + 5a$	1	2	3
d) $(a + 5)a$	1	2	3

All these cases are an indication that the participants were not familiar with the table item format. They, therefore, had difficulty in answering items of this format correctly. They used their knowledge of the multiple choice item format which has only one correct response.

The researcher concludes that the LMT measures are applicable in Eswatini, but they must be carefully adapted to the Eswatini context before they are used.

5.3 RESEARCH QUESTION 2: PRE-SERVICE PRIMARY TEACHERS COGNITION OF MKT FOR GEOMETRIC PATTERNS

5.3.1 The LMT Test

To answer Research Question 2, i.e. What is the pre-service primary teachers' cognition of geometric patterns?, the researcher used the adapted LMT instrument to measure the mathematical knowledge for teaching geometric patterns of a group of 94 pre-service primary school teachers. The pre-service teachers responded to the items using pencil and paper. The researcher administered the test during a mathematics lesson for the pre-service teachers. Hence, the participants were not in a rush to go home. The participants' responses were marked, and a score out of 18 was determined for each participant.

5.3.2 Biographic characteristics of the participants

The biographic characteristics of the participants for the study are shown in Table 17.

Table 17. Biographic characteristics of the research participants

Biographic characteristic	Description	Number of participants	Percentage
Gender	Male	52	55.3
	Female	42	44.7
Age (years)	15 – 20	2	2.1
	21 – 25	65	69.1
	26 and above	27	28.7
Symbol obtained in mathematics at high school	A	16	17.0
	B	45	47.9
	C	32	34.0
	D	1	1.1

To determine the pre-service teachers' cognition of mathematical knowledge for teaching geometric patterns in Eswatini, 94 pre-service teachers were assessed using the LMT measures that were adapted for the Eswatini context. The biographic information of the participants is shown in Table 16. The response characteristics are similar to those of the participants in the pilot test. There were more male participants than females. The age range of most of the participants was 21 – 25 years. Nearly all the participants (98.9%) got a C grade or better in mathematics in their high school results. Table 18 shows the test descriptives.

Table 18. Cognition Test descriptives

	N	Minimum	Maximum	Mean		Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
Percentage	94	11.00	89.00	45.8085	1.83420	17.78321	316.243
Valid N (listwise)	94						

The mean score for the pre-service teachers' test was 45.8%. This information shows that the pre-service primary teachers' cognition on mathematical knowledge for teaching geometric patterns is below average. However, the standard deviation (SD = 17.7) shows that the teachers' scores were widely dispersed, with the minimum score being 11% and a maximum score of 89%. The median score was 44.0%, less than the mean score. The scores are moderate positively skewed with a skew coefficient of 0.55. So a majority of the pre-service teachers had low scores on MKT for geometric patterns. The scores were also unevenly spread, with two peaks at 39% and 50%. The test performance of the 94 pre-service teachers is illustrated in Figure 10 and Table 19.

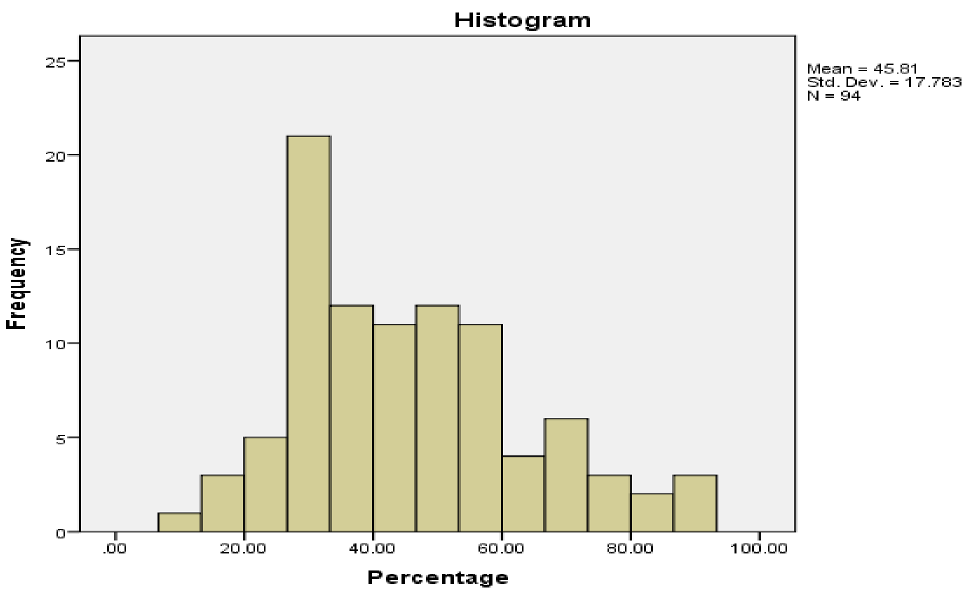


Figure 10. Distribution of scores for the LMT test

Table 19. Frequencies of scores by pre-service teachers

Score	Frequency	Percent	Cumulative Percent
11.00	1	1.1	1.1
17.00	3	3.2	4.3
22.00	5	5.3	9.6
28.00	11	11.7	21.3
33.00	10	10.6	31.9
39.00	12	12.8	44.7
44.00	11	11.7	56.4
50.00	12	12.8	69.1
56.00	11	11.7	80.9
61.00	4	4.3	85.1
67.00	3	3.2	88.3
72.00	3	3.2	91.5
78.00	3	3.2	94.7
83.00	2	2.1	96.8
89.00	3	3.2	100.0
Total	94	100.0	

The histogram (Fig. 9) shows that the pre-service teachers' scores were widely dispersed. Table 18 shows that a majority (56.4%) of the participantss had scored less than 44%. Hence, the general cognition of MKT for geometric patterns for these pre-service teachers was low. This low cognition could be due to Tchoshanov (2011) contentions, who opine that teachers' subject matter knowledge, including specialised content knowledge, grows with teaching experience.

5.4 RESEARCH QUESTION 3: TO WHAT EXTENT DOES THE PRE-SERVICE PRIMARY TEACHERS COGNITION OF MKT FOR GEOMETRIC PATTERNS INFLUENCE THEIR LESSON PLANNING AND PRESENTATION

5.4.1 Qualitative analysis of the lesson videos

Three pre-service teachers were sampled for classroom observations. The pre-service teachers were observed while teaching geometric pattern generalisations. The three pre-service teachers sampled for this phase of the research were Sipho, David and Mpho (not their real names). Each of these teachers was sampled from a category of the teachers' performance in the LMT test.

Sipho was selected from the high achievers' category, David was selected from the medium achievers' category, and Mpho was selected from the low achievers' category. These pre-service teachers were teaching in different schools when their lessons were recorded. Due to the COVID 19 pandemic, the pre-service teachers did not go to the schools for their teaching practice as was anticipated by the researcher. Therefore, the participants were requested to conduct the lessons for research purposes. They were requested to teach in a school close to their places of abode to avoid travelling costs.

These pre-service teachers were not involved in any workshop on conducting their lessons on pattern generalisations. The researcher wanted the pre-service teachers to use the knowledge they acquired from their college training course without other influence. The pre-service teachers were video recorded using a cellular phone in two lessons on pattern generalisations. The researcher was a non-participant observer in these lessons. Using a cellular phone minimised distraction in the lesson by the recording process.

The lessons were qualitatively analysed to understand further the relationship between the pre-service teachers' MKT in geometric pattern generalisations and their classroom practice. This analysis investigated how MKT in geometric pattern generalisation was used in teaching geometric pattern generalisations. The researcher thus watched each video in its entirety and summarised each lesson. The summary included the Grade level taught, the lesson topic and the main teaching approach used in the lesson. The researcher also noted the general affordances and deficiencies observed.

After watching the videos, the researcher re-visited the literature on teaching geometric patterns, especially pattern recognition. Pattern recognition states that learners can construct meaningful knowledge by relating the new information to existing knowledge. According to the information processing theory of learning, it must first appeal to the learners' minds for information to be processed to long-term memory. Hence, in the lesson introduction, the teacher should present new knowledge that catches the learners' attention. Activating appropriate prior knowledge enhances pattern recognition, triggering the automatic transfer process to long-term memory to form a schema.

According to Radford (2008), the three different geometric pattern generalisations are arithmetic, algebraic and generalisations from naïve induction. It is expected that teachers with high mathematical knowledge for teaching should develop algebraic-geometric pattern generalisations. In this type of generalisation, i.e. the algebraic generalisation, the learners first identify a commonality of given terms. For example, in the geometric pattern; 4, 7, 10, 13, … the commonality is that there is a difference of 3 between successive terms of the pattern. This commonality can be used to find the 5^{th}, 6^{th}, $seven^{th}$ terms. The distinguishing feature of an algebraic generalisation is that a general algebraic expression for the general term is deduced from the commonality. Hence, in this case, the general algebraic expression can be deduced based on the position (n) of the term in the pattern.

The general expression which enables one to find a term based on its position in the pattern is 3n+1. In the case of an arithmetic generalisation, each term is calculated from the previous term of the pattern. There is no direct calculation of a term based on its position. Naïve induction involves getting the pattern generalisation through trial and error, with no clear method for its deduction.

The literature also outlines affordances and deficiencies in mathematics lessons. The AQIPM instrument integrates an investigation of both the affordances and deficiencies in a lesson at the same time. The researcher then re-viewed the videos with this theoretical framework in mind. This led to a general evaluation of each lesson.

The Research Context

Sipho and David were teaching in an urban school. Sipho was teaching a Grade 4 class which had 24 learners. David taught a Grade 6 class with 22 learners. Mpho taught Grade 6. His class had 20 learners and was in a semi-urban area. The classes had a small number of learners due to the COVID-19 guidelines. One of the guidelines was that the number of learners in each class was limited to only about 20 learners. Hence, the big classes were split to form smaller classes of about 20 learners.

The Case of Sipho

Sipho had a high score on the pencil-and-paper LMT test. He was in the 90^{th} percentile rank of the pre-service teachers' LMT test performance. His score was 88%. Sipho's lesson was in Grade 4 on "Geometric patterns".

The desks in the classroom were combined such that a group of learners sat around facing each other. This sitting arrangement promoted collaboration among the learners. There was space between the learners, adhering to the COVID-19 guidelines.

An excerpt from one of Sipho's lessons is given below:

The pre-service teacher started this lesson by projecting the definition of geometric patterns on a whiteboard. The learners were asked to read out this definition together as a group. Before discussing the concept, the pre-service teacher found it important for the learners to understand geometric patterns. In this lesson, the main focus was on geometric patterns that 'grow'. Hence, the pre-service teacher explained that a geometric pattern is generated by addition or multiplication. The pre-service teacher then displayed a chart showing diagrams of the first three triangular numbers. The diagram for each triangular number was represented by 'dots'. Sipho defined a triangular number as a number that can be represented by dots that can be arranged to form a triangle. Sipho's definition was incomplete as it didn't specify that the dots should form a regular triangle.

Sipho first asked the learners to state the shape formed in each arrangement of dots. A learner responds that the shape formed is a triangle. By relating the geometric pattern of triangular numbers with the shape of a triangle, the pre-service teacher related the new concept with an already existing concept, 'triangle'. He then asked the learners to count the number of dots that had been used in forming each triangular number:

> *Teacher*: In the first term, how many dots do we have?
>
> *Learners*: 1
>
> *Teacher*: Lovely. Now let's count the number of dots in the second term. Let's count together.
>
> *Learners*: one, two, three.
>
> *Teacher*: So in term 1 we've got 1 dot, in term 2 we added how many?
>
> *Learners*: 2

Sipho now guides the learners to the fact that they have to add a certain number to get the next term. This step was important to enable the learners to realise that there was a certain number to be added to find the next term.

> *Teacher*: In term 2 we added 2 to get 3. Now in term 3, how many dots do we have? Let's count; one, two, three. four, five, six. Okay we have 6. So in term 3, we added how many?
>
> *Learners*: 3

By counting the number of dots, the learners found out that the first three triangular numbers are 1, 3, 6, … The pre-service teacher then asked the learners to form the 4th triangular number using small stones. He then moved around, checking the learners' progress as they formed the 4th triangular number. After all the learners had successfully arranged the stones to form the 4th triangular number, the pre-service teacher asked them to count the stones used. They responded that there were ten stones.

> *Teacher*: To move from term 3 to term 4, how many dots did we add?
>
> *Learners*: 4
>
> *Teacher*: Can you tell me how many stones are needed to form the 5th triangular number?

After pausing, the teacher realised that the learners could not figure out the answer. He then simplified the question and asked;

> *Teacher*: How many stones should I add to the ones used in term 4?
>
> *Learner*: 5
>
> *Teacher*: Lovely. Can you explain how you figured out that we should add 5 stones?
>
> *Learner*: In the 2nd term we added 2, in the 3rd term we added 3, in the 4th term we added 4. So in the 5th term we should add 5.

The pre-service teacher now led the learners to find the rule for generating triangular numbers. He led them to find the commonality between successive terms of the geometric pattern. The commonality is that you add the number equal to the position of the triangular number to the previous number. Hence, the 5th triangular number is 10 + 5 = 15. However, the pre-service teacher noted that this was an arithmetic generalisation of the pattern. He then asked the learners to deduce an algebraic generalisation:

> *Teacher*: So, to find the 12th triangular number, I have to first determine the 11th number before adding 12. Can you find a way of finding a triangular number without using the previous number? Please discuss with your group members and give me an answer.

The learners took about 5 minutes to discuss in their groups. One group noticed that to get any triangular number, and you add successive counting numbers up to the position of the triangular number:

> *Learner:* Teacher, the 2nd triangular number is 1 + 2; the 3rd triangular number is 1 + 2 + 3; the 4th triangular number is 1 + 2 + 3 + 4 and so on.
> *Teacher:* That's wonderful Group 3, Very good.

The learners deduced the general rule for finding any triangular number by adding successive counting numbers. The pre-service teacher was satisfied with the learners' generalisation. When asked by the researcher after the class why he settled for this generalisation, he stated that this was appropriate at the learners' level (i.e. Grade 4). The more formal one, i.e. $n(n+1)/2$, would be difficult to understand. The pre-service teacher's actions fit Bremmer's (2021) aspects of learner-centred education. According to Bremner (2021), learner-centred education is characterised by active participation of learners in a lesson through problem-solving activities. In this lesson, the learners were involved in problem-solving when the pre-service teacher asked them to find the rule for generating the triangular numbers. There was also learner interaction during the lesson as they discussed, in groups, how to calculate any triangular number.

An analysis of the lesson videos according to the AQIPM framework:

Quality of Student-Content Interaction

Learner involvement in the lesson was good. The pre-service teacher successfully applied the problem solving method in this lesson with guided discovery learning (Simamora & Saragih, 2019). He was able to identify the affordances of this method of teaching in this lesson, as outlined by Simamora and Saragih (2019). Simamora et al. (2019) assert that guided discovery learning promotes learners' problem solving skills, creativity, independent thinking, learners' construction of their own knowledge and learners' mathematical self-efficacy. The learners' responses showed that they were actively engaged in the lesson.

Quality of Instructor-Content Interaction

The quality of instructor and content interaction was an analysis of the extent to which the pre-service teacher was able to make mathematics meaningful to learners, e.g. through the use of relevant and familiar examples and teaching materials to explain concepts. In this regard, Sipho made the lesson meaningful to the learners through the use of a pattern that formed a geometric shape familiar to the learners (a triangle). Another aspect of the quality of instructor and content interaction pertained to the extent of the presentation of content in a logical manner. In the case of Sipho, he was able to guide the learners in finding the commonality which they later used to generalise the pattern. The quality of instructor and content interaction also concerned the method of teaching used by the pre-service teacher. Sipho showed a good application of specialised content knowledge for teaching by using a learner-centred approach. The pre-service teacher was able to use the question and answer method to actively engage the learners in the lesson.

Quality of Instructor-Student Interaction

He encouraged learners' mathematical reflections by asking questions like, 'Do you agree?', 'Is this answer correct?', 'Why do you think so?'. Since the learners were learning about geometric patterns, he correctly used 'terms' instead of just saying 'the next number'. Such appropriate terminology added to the richness of mathematics in his lessons. According to Van den Kieboom (2013), knowledge of content and students enables a teacher to anticipate the difficulty level. Sipho could tell what his learners knew about number patterns and what kind of content would be more challenging for them to understand. Hence, he was okay with the learners' arithmetic generalisation.

General/underlying Quality of Instruction

Sipho's lessons contained rich mathematics content knowledge. He used appropriate mathematical language. In his lessons, he intellectually involved his learners in the lesson. He also provided learners with the opportunity for collaborative learning when asked to find solutions to problems through group discussions. Such actions promote mathematical reasoning (Cengiz et al., 2011). Sipho's teaching strategies also encouraged learners' mathematical reflection. After group work, the learners were invited to present their solutions to the whole class. Mathematical reflection is an important step in the problem solving model of teaching/learning (Simamora & Saragih, 2019).

Sipho's lesson was mainly learner-centred. The lesson promoted collaborative learning, active participation of learners and problem-solving skills. These characteristics describe a learner-centred lesson according to Patel-Junankar (2017).

David's Lesson

David taught Grade 6. His performance in the paper-and-pencil test was average. His score was 52%. The recorded lesson was when he taught geometric patterns. This was a 60-minute lesson. An excerpt of David's lesson is presented below:

In the introduction, David described a geometric pattern as a sequence in which geometric shapes or numbers are repeated predictably. He then displayed a chart with different patterns of geometric shapes and asked the learners to state the next three terms in each pattern.
The displayed patterns were as follows:

1. △, ⬭, □, △, ⬭, □, △, __,___,___,
2. P Q R P Q R S P Q R P Q R S,___,___,___,
3. 2, 4, 6, 8,__, __, __,

The following class discussion then took place;

Teacher: In pattern 1, what will be the next 3 figures?

Learner: circle, then a square

Teacher: Good, what will follow after the square?

Learner: A triangle.

Teacher: Very good, the pattern will start with a triangle again. Now let us look at pattern 2. Can you predict what will follow after the letter S? ……Anyone with an idea?

Learner: P Q R.

Teacher: Okay, the next letters will be P, Q, R. So you have predicted the next letters since you already have a pattern of how the letters follow each other. Now let's look at the pattern of numbers. What would be the next number after 8?

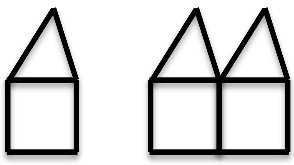
Learner: 10

Teacher: Yes 10, how did you realise that it would be 10? Can you explain to the class?

Learner: Because we add 2 each time. So 8 plus 2 is 10.

Teacher: Very good, so we get 10 by adding 2 to the previous term. Now I want you to write or draw your own pattern. Remember that it must have a rule to help us predict the next term.

The learners were given 5 minutes to think of their patterns individually. The pre-service teacher went around checking the learners' patterns. The pre-service teacher then gave the learners matchsticks (in groups). The following pattern was displayed on a chart:



The learners were told to form the next two shapes in the sequence, using the matchsticks. The learners worked in groups to form the next two shapes. The learners enjoyed the activity such that some groups made even the fifth shape in the pattern using the matchsticks. The learners were then asked to write the number of matchsticks in each shape, hence generating the sequence:

$$6, 11, 16, 21, \ldots$$

The pre-service teacher then guided the learners to find the rule of generating this pattern as follows:

Teacher: Let us see how to get the terms in this sequence. From 6 to 11, what do we add?

Learners: 5

Teacher: From 11 to 16:

Learners: add 5

> *Teacher*: So we add 5 each time to get the next term. Each new diagram uses 5 more matchsticks than the previous diagram. So the rule for this pattern is add 5 to get the next term.

Quality of Student-Content Interaction

David's lesson was found to have many affordances. The pre-service teacher showed that there are different types of patterns, that is, we do not only consider patterns of numbers. The pre-service teacher's questioning skills were also good. He asked the 'why' questions which required the learners to give explanations to support their answers. The lesson was also activity based. For example, David brought matchsticks for the learners to make shapes that represented number patterns. This activity of representing the number patterns using physical objects made mathematics meaningful to the learners. The pre-service teacher involved the learners in mathematically thought-provoking activities, e.g. when they were asked to write their patterns.

Quality of Instructor-Content Interaction

The pre-service teacher used the question-and-answer method in this lesson. However, the learners were not given a chance to use the problem solving method to find the pattern generalisation on their own. The pre-service teacher was quick to offer explanations before the learners could try to work out the solutions on their own. The lesson was mainly teacher dominated rendering it more as a teacher-centred lesson.

Quality of Instructor-Student Interaction

David lacked KCS on geometric patterns in that he did not realise that the learners could find an algebraic generalisation for the number pattern. He ended the lesson with students only finding an arithmetic generalisation. The pre-service teacher failed to engage the learners in group discussions. Such discussions would lead the learners to better understanding of the process of pattern generalisations.

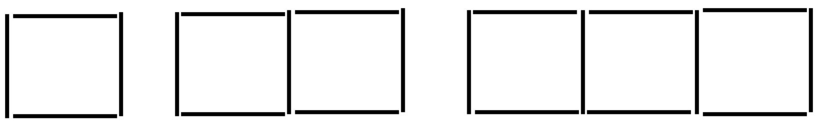
General/underlying Quality of Instruction

Some deficiencies were noted in this lesson. One of them is that the learners could not present their number patterns to the class. This could have promoted mathematical reflection by evaluating each other's number patterns. The pre-service teachers' presentation did not provide a basis for the development of the learners' problem solving skills. The lack of involvement of the learners in group work made it difficult for learners with difficulties in the lesson to catch up with those who understood the concept of pattern generalisation. David only involved the learners in group work when they formed the shapes in the pattern. However, there was no active involvement of the learners in pattern generalisation. According to Du Plessis (2020), learner- centred pedagogy results in learners constructing their own knowledge. In this lesson, there was no knowledge construction by the learners. Hence, the lesson was mainly teacher-centred.

The Case of Mpho

Mpho's score on the pencil-and-paper LMT test was less than average. His score was 28%. He taught Grade 6 Mathematics. The arrangement of desks in the classroom was such that the learners sat in groups. This seating arrangement suggests that the teacher believed in group work during mathematics teaching/learning. The lesson topic for the recorded lesson was geometric patterns. The following is an excerpt from the lesson;

Mpho started his lesson by drawing a pattern of shapes on the chalkboard. The pattern involved squares as shown in the diagram;



After drawing the pattern for the first three numbers, the teacher asked the following questions:

> *Teacher: So in the first diagram how many sticks are used to form the shape?*
>
> *Learners: 4*
>
> *Teacher: Count the number of sticks in the second diagram.*
>
> *Learners: 7*
>
> *Teacher: Now count the number of sticks in the third diagram.*
>
> *Learners: 10*
>
> *Teacher: So this pattern has the numbers 4, 7, 10,….Can you say what is the rule in this pattern?................. I mean how do you calculate the numbers. Yes Sakhile.*

Learner: we add 3.

Teacher: Yes, the rule is that we add 3 to get the numbers. So what are the next numbers ...class? It will be 13, 16, 19 and so on. Yes, that is how the numbers are calculated. Now I want you to find the rule of calculating the numbers in each one of the patterns given in Exercise 2 in your workbook.

The learners were then given an exercise to work out. In this exercise, the learners had to determine the next terms in a given pattern. The pre-service teacher went around marking a few of the learners' work. It was not easy to mark all the learners' work due to COVID-19 guidelines that discouraged a physical contact with the learners' workbooks.

Quality of Student-Content Interaction

There was shallow elaboration or explanations given by the pre-service teacher. The questions asked by the pre-service teacher in the lesson were not thought provoking and failed to engage the learners in meaningful mathematical thought. The learners were not given enough time to think after the pre-service teacher had asked a question. Instead, the teacher rushed to give solutions on the board. There was not enough discussion on how to make pattern generalisations, instead the pre-service teacher rushed to give the learners an exercise. The learners did not discuss the solutions of the exercise that they were given by the pre-service teacher, instead they worked individually.

Quality of Instructor-Content Interaction

There were no teaching/learning materials provided in the lesson. The pre-service teacher only used the chalk-and-talk method. Learners' participation in this lesson was minimal. The pre-service teacher failed to use the question-and-answer method effectively. The lesson was mainly teacher-centred

Quality of Instructor-Student Interaction

One of the weaknesses of the pre-service teacher in the lesson was failing to ask the learners thought provoking questions. Instead, the pre-service teacher would ask a question and then lead the learners in a 'chorus' response. The learners were not brought to a level where they could connect the geometric patterns with the mathematical process of deducing their generalisations. The level of the pre-service teacher's questions was not cognitively engaging the learners.

General/underlying Quality of Instruction

Mpho's lesson was mainly teacher-centred. However, one affordance of the lesson was the involvement of the learners in group work. This activity enabled some learners to share their understanding of the lesson, even though others preferred to work individually. There was also a lack of use of the correct mathematical language, e.g. he kept on referring to the terms of the pattern as 'numbers'. Failing to introduce the correct mathematical terminology robbed the learners of acquiring the correct terminology for the terms of a geometric pattern. This lesson falls under the deficit studies explained in Hill et al. (2008).

5.4.2 Quantitative analysis of the lesson videos

The codes listed under each category of the AQIPM instrument guided the evaluation of the quality of each recorded lesson. The presence and quality of each code were determined by the teacher/learner actions listed in the second column of Table 20.

Table 20. Quality of lesson evaluation framework

Evaluation code	Teacher/learner action
Qual. Student-Content Interaction Mathematical reasoning and sense making Connecting content to related concepts	Assessment of the learners' involvement in the lesson. Connecting content to related concepts. Learners ask questions and give responses that show comprehension of the content.
Qual. Instructor-Content Interaction Making sense of mathematics. Situating the mathematics. Organisation in the presentation. Mathematical explanations. Making links among related content.	Making mathematics meaningful to learners. Using relevant and familiar examples and teaching materials to explain concepts. Presentation of content in a logical manner. Mathematical explanations are clear and correct. Makes links among related content. Use of learner-centred methods of teaching.

Qual. Instructor-Student Interaction Instructor-Student Continuum of Instruction. Classroom environment. Inquiry / Exploration	Asks questions that invoke mathematical thinking. Teacher's efforts to support learners' explanations. How the teacher responds to learners' questions. Teachers' feedback to learners' responses. Involvement of learners in activities that will enable them to construct knowledge. Provision of remedial activities to help learners overcome errors and difficulties. Use of formative assessment to check learners' knowledge.
General/underlying Qual. of instruction Appropriate mathematical language. Remediation of student errors and difficulties.	Use of appropriate mathematical language. Responding to errors or misconceptions that arise during the teaching/learning process. Use of appropriate mathematical symbols. Classroom arrangement enhances student-student interaction.

The coding of the video lessons involved subdividing the lessons into 7-minute segments, as was done in Hill et al. (2008), Litke (2015), and Mesa et al. (2019). The codes used to analyse the lessons were adapted from existing rubrics (Hill et al., 2008; Mesa et al., 2019). Each code was graded on a scale of 1 to 5, with 1 representing poor presentation of the construct measured by the code and 5 representing an adequate presentation. For example, the code 'use of appropriate mathematical language' was graded 1 for poor use of mathematical language, 2 for fair use, 3 for acceptable use, 4 for good use, and 5 for very good use of mathematical language in that segment.

The grading of each code on a lesson segment provided a numerical score representing the quality of mathematics in that segment. This coding system rated the level of each aspect in a lesson rather than grading whether each aspect was present or not present. The scores for all the codes in each segment were totalled to determine the quality of mathematics teaching in that segment. The total score for each lesson segment represented the quality of instruction for that segment. To determine the overall quality of instruction for the whole lesson, the scores for each segment were summed up. The video analysis gave an insight into how mathematical knowledge for teaching is expressed in teaching. The AQIPM instrument has four aspects that determine the quality of a lesson. Each teacher's lessons were coded and scored on each of these aspects.

The scores were averaged for the two lessons to get an overall score for the teacher's two lessons. Figure 10 represents an analysis of the pre-service teachers' lessons using the AQIPM instrument. The analysis is presented according to each aspect of the instrument, i.e. Quality of Student-Content Interaction, Quality of Instructor-Content Interaction, Quality of Instructor-Student Interaction, and General/underlying Quality of instruction.

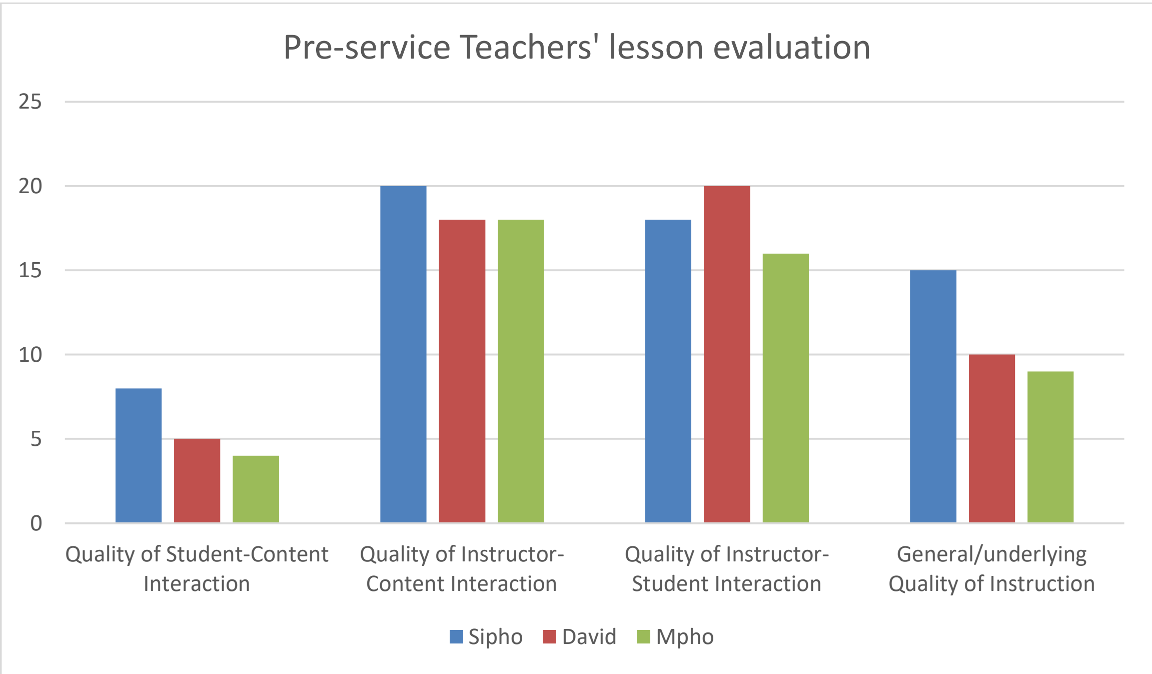


Figure 11. Representation of the pre-service teachers' quality of instruction

To determine the effect of MKT on classroom practice, the researcher first investigated any relation between the two variables. This relation was found by calculating the correlation index, represented by Pearson's correlation index, r. The data used in this correlation was the pre-service teachers' MKT scores on geometric pattern generalisation and the scores for each aspect of the quality of lesson measure. All the scores were converted to percentages (out of 100).

The pre-service teachers' scores are shown in Figure 13.

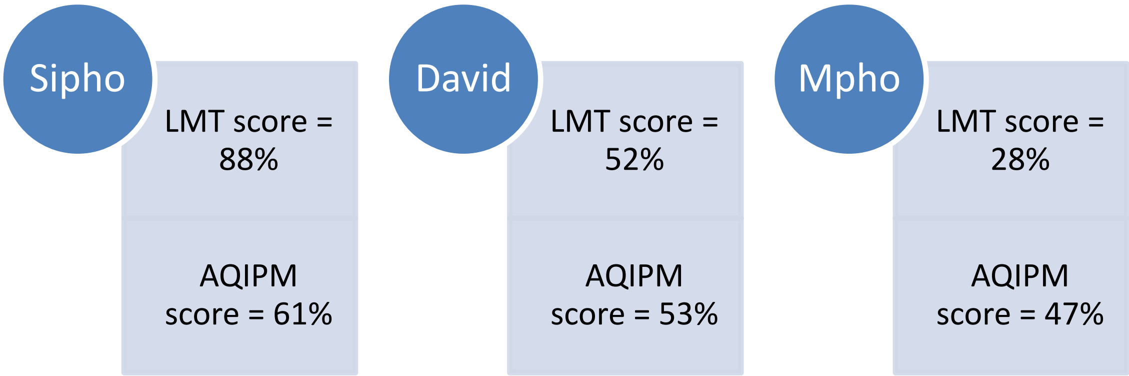


Figure 12. LMT and AQIPM scores for the pre-service teachers

The analysis was done using the SPSS 20 software. The correlation matrix for all the variables is shown in Table 21. The Pearson's coefficient ranged from -.05 to .99.

Table 21. Correlation of MKT scores with pre-service teachers' lesson quality using Pearson's, r, coefficient.

	1	2	3	4	5	6
1. MKT score	1					
2. Quality of Student-Content Interaction	.98*	1				
3. Quality of Instructor-Content Interaction	.92	.98	1			
4. Quality of Instructor-Student Interaction	.35	.17	-.05	1		
5. General Quality of Instruction	.97	.99	.99	.10	1	
6. Overall lesson score	.99*	.98	.90	.38	.96	1
Note. * Correlation is significant at the 0.05 level (2-tailed).						

The correlation analysis results show a high positive relationship (r = .99) between the pre-service teachers' MKT in geometric pattern generalisation and their overall lesson quality in pattern generalisation. This correlation was statistically significant ($p < 0.05$).

This means that pre-service teachers with high levels of MKT in geometric pattern generalisations are likely to present lessons of high quality in geometric pattern generalisations. Likewise, teachers with low levels of MKT in geometric pattern generalisations are likely to present lessons of low quality. By a high-quality lesson, we mean that the lesson has many affordances to facilitate student understanding, while low-quality lessons lack affordances for learning.

CHAPTER 6

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

6.1 INTRODUCTION

The results of this study were presented in the previous chapter. In this chapter, the researcher discusses the study's findings in line with the literature review and the theoretical framework. The discussion of the findings shall be done according to the research questions articulated in Chapter one. Section 6.1 is a discussion of the findings for research question one. These are the findings from the pilot study, which sought to investigate the adaptability of the MKT measures in Eswatini. In Section 6.2, the researcher discusses the findings for research question two.

This section discusses the pre-service teachers' cognition of MKT for geometric patterns. Section 6.3 focuses on the discussion of research question three. This section entails a discussion of the findings for the effect of the pre-service teachers' cognition of MKT for geometric patterns on classroom practice. The discussion in this chapter shall be guided by the theoretical framework that underpins the study. Section 6.4 presents the conclusion and recommendations from the study.

6.2 RESEARCH QUESTION 1: ADAPTABILITY OF THE LMT MEASURES OF MKT IN THE ESWATINI CONTEXT

The first research objective of this study was to determine the adaptability of the LMT measures to the Eswatini context. This question of adaptability arises from the fact that the MKT measures were developed in the USA teaching practice context, and teaching is dependent on culture (Hu, Leung, & Teng, 2018). This research contributes to the MKT theory by revealing whether the MKT measures are applicable in a developing African country like Eswatini.

An assessment of the adaptability of the LMT measures in Eswatini involved finding out if the tasks of teaching on which MKT is based are the same in Eswatini as in the USA. It also involved a psychometric analysis of data collected in a pilot study using the LMT measures. An assessment of the tasks on which MKT is based revealed that teaching tasks in the USA are the same as in Eswatini.

This equivalence of tasks was established through an analysis of the work of teaching in Eswatini as written in the curriculum framework and through an interview with practising teachers. However, it must be noted that even though the tasks of teaching are the same in Eswatini as in the USA, there are economic differences between these two countries. These economic differences have implications for the availability of resources for teaching. Non-availability of resources for teaching may hinder the successful carrying out of some of the teaching tasks.

For example, one of the tasks of teaching is presenting mathematical ideas. This presentation may require the use of teaching/learning aids. The appropriate teaching/learning aids may not be available in a country with a low economy. The researcher believes that these economic differences did not affect the validity of the results since Eswatini teachers are empowered with skills for improvisation. The teachers have been trained to use local materials to improvise for commercial teaching/learning materials.

The pilot study that was undertaken involved an administration of 21 LMT items, taken from the LMT measures developed by the Learning Mathematics project in the USA. These items were administered to 34 pre-service primary school mathematics teachers. The items were selected from the Patterns, Functions and Algebra (PFA) category. The items selected were developed to measure subject matter knowledge on geometric patterns. The items were developed based on teaching in the USA within the MKT framework.

To adapt the measures to Eswatini, the researcher first made a content analysis of the items. The items' content was compared to the Eswatini curriculum framework. The primary school mathematics syllabus, according to this framework, has the competencies listed in Figure 9. An analysis of these competencies and the content of the LMT measures revealed that the measures' assessment was applicable in Eswatini. The knowledge assessed by the LMT measures is the knowledge that the Eswatini teachers are expected to have.

The second stage in the adaptation process involved contextualising the LMT items to the local context.

This adaptation involved making superficial changes, e.g. changing names of people and currencies to replace them with local names and currencies, which did not change the difficulty level of the items. These changes were made to make the items relevant in the Eswatini context. To determine the applicability of the MKT measures, the researcher administered them to a sample of 34 pre-service mathematics teachers. The participants were pre-service primary school teachers in their final year of study. The data collected from this pilot project were statistically analysed using the SPSS version 20 software to find its psychometric properties.

The psychometric analysis was to check the performance of the adapted items in Eswatini. The psychometric analysis was based on the Item Response Theory (IRT) model with the assumption of unidimensionality of the items. Using this assumption, the Cronbach's alpha of .76 represented acceptable reliability of the measures (Bichi & Talib, 2018). The two parameters calculated for each item were the difficulty and the discrimination index. These parameters were calculated to identify items that were not performing well in Eswatini.

Items not performing well were those with a high difficulty index and a poor discrimination index. Therefore, the results of the two parameter calculation led to the elimination of two items (Item 10 and Item 11). Even though the instrument was of acceptable reliability, it had a high difficulty index ($p = .23$). This result means that the instrument is more suitable for identifying pre-service teachers with very high MKT levels. These findings are consistent with the results of studies in other countries, e.g. in Ghana (Cole, 2012) and Malawi (Kazima et al., 2016), who also found that the instrument measuring the MKT level was more suitable for identifying teachers with high MKT levels.

A closer analysis of the performance by the pre-service teachers revealed that one of the contributing factors that caused difficulty of the items was the use of the table items format in the test. Most of the items in the LMT measures used were in the table item format. Many pre-service teachers in the sample ($n = 94$) answered this type of question inappropriately ($n = 40$). This is about 43% of the total sample. These pre-service teachers treated the table items as multiple-choice items with (a), (b), (c) and (d) as the item options.

Hence, the pre-service teachers who answered this item format inappropriately would circle only one option, either (a), (b), (c) or (d) as their response to the item. In such cases, if the pre-service teacher circled (c) as their response (instead of circling 1, 2, or 3), the researcher would give a benefit of the doubt and score the pre-service teacher as correct.

However, the pre-service teacher would lose three marks for not stating whether the other expressions are correct or not correct. Some pre-service teachers had difficulty with this item format because this format is not commonly used in Eswatini. Therefore, the findings of this research suggest that the table items format should be removed from measures to be used in Eswatini. When the items are used, they should be used in conjunction with interviews to validate the results obtained. This finding is the same as that found in the use of the LMT items in Ghana (Cole, 2012).

Another reason contributing to the difficulty of the measures in Eswatini is cultural differences between the USA and Eswatini. Culture affects teaching, as posited by Hu et al. (2018). The USA culture encourages learners to take charge of their learning process, e.g. making postulates during problem-solving. The learner-centred approach is not common in Eswatini, where teaching is mainly teacher-centred (Dlamini, 2017; Ndlandla, 2017). Hence, questions framed around learners making propositions are not realistic to the Eswatini teacher.

Some of the questions were culture-based as they included games that are common in the USA but not common in Eswatini. An example is the baseball game. Baseball is common in the USA, but very few teachers in Eswatini know this game. Hence, the item framed around the baseball game was poorly done in Eswatini. The high difficulty index of the measures in Eswatini means that they are only valid in identifying teachers with high levels of MKT but not valid in identifying teachers with low levels of MKT.

6.3 RESEARCH QUESTION 2: THE PRE-SERVICE TEACHERS COGNITION OF MKT FOR GEOMETRIC PATTERNS

The second research objective of this study was to determine the pre-service teachers' cognition of MKT for teaching geometric patterns. The researcher alluded that learners' performance in geometry and geometric patterns is deficient in primary schools in Eswatini. This low performance of learners could be due to several factors. One of these factors could be a lack of teacher knowledge on this topic. However, there are very few studies on teacher knowledge in Eswatini. Many researchers believe that to improve learners' mathematics performance, and there is a need to improve teacher knowledge for teaching mathematics (Bardach & Klassen, 2020; Charalambous et al., 2020).

There are many frameworks of this teacher knowledge that have been proposed. The MKT framework is appropriate for this study since it is grounded in teaching mathematics. It also provides measures for the teacher knowledge required in mathematics teaching. Therefore, this framework is appropriate in this study, which focused on pre-service teacher knowledge in teaching geometric patterns. The pre-service teachers' cognition was investigated using a pen-and-paper test consisting of 18 items that were constructed to test subject matter knowledge on geometric patterns. These items were taken from the LMT measures developed based on the MKT framework. These measures were developed based on the USA context of teaching mathematics. Therefore, there was a need to adapt them to the Eswatini context and check that they performed well after being adapted to the local context. The adaptation process is explained in Section 4.5.1.

The adapted instrument was used to assess the pre-service teachers' cognition of MKT on geometric patterns. The test was administered and marked by the researcher. The pre-service teachers' scores on the test were quantitatively analysed to help understand their cognition of MKT for geometric patterns. The analysis of the scores from the cognition test revealed that the pre-service teachers' cognition of MKT for teaching geometric patterns was varied, with a low score of 11% and the highest score being 89%. However, most of the scores were low, as evidenced by the mean score of 45.8% and a median score of 44.0%.

The median was less than the mean, showing that the scores were positively skewed. A positively skewed distribution is characterized by most pre-service teachers falling below the mean score. Hence, a majority of the pre-service teachers had scored less than 45.8%. This finding reveals that the participating pre-service teachers' cognition of MKT for teaching geometric patterns was low. These findings are similar to Sahidin et al. (2019) assessments, which allude that teachers lack adequate MKT in many topics in the mathematics syllabus. The authors assert that mathematics teachers lack conceptual knowledge of mathematics.

Conceptual knowledge is only attained when learning follows the stage model of Information Processing (Lutz & Huitt, 2003). The pre-service teachers' low MKT for teaching geometric patterns is a cause for concern as research postulates that such teachers deliver lessons of low mathematical quality, leading to low performance of learners in mathematics (Ball et al., 2005). The results indicate a need to re-visit the pre-service primary school mathematics teachers' course on geometric patterns. The course on geometry and number patterns is done in the second year of the PTD programme in Eswatini.

The LMT test was given to final year (third year) pre-service teachers who had already done the course in their second year of study. This means that the teaching of the course on geometry and number patterns had not helped most of these pre-service teachers in increasing their level of MKT for geometric patterns.

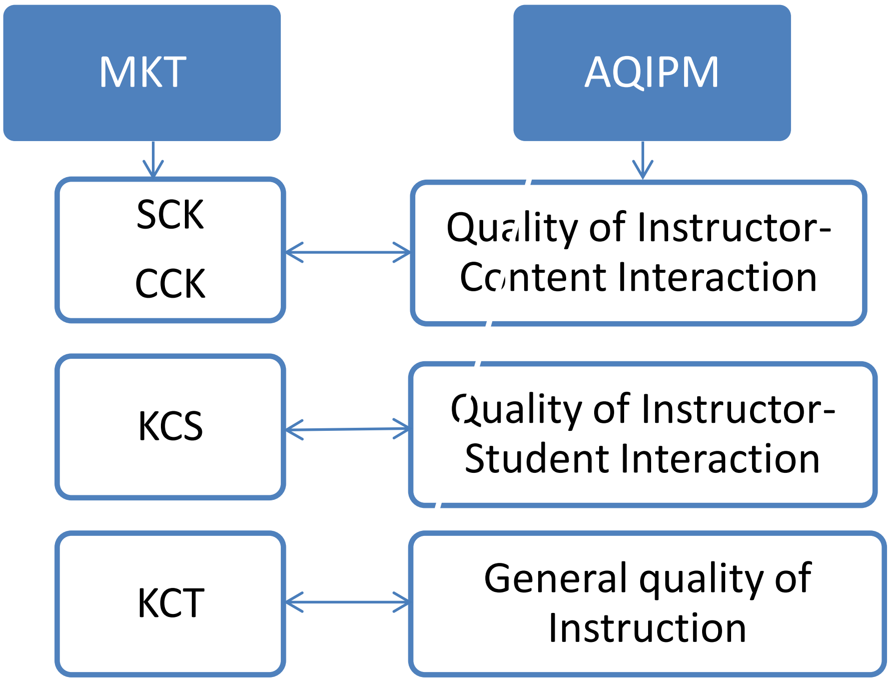
One unanticipated finding was a negligible and non-significant relation ($r = .11$) between the pre-service teachers' high school grades and their MKT score. This was surprising because, according to Sahidin et al. (2019), conceptual knowledge is necessary for the development of MKT. It was therefore, expected that pre-service teachers with good conceptual knowledge would develop high MKT in geometric patterns. However, the negligible and non-significant relation between the pre-service teachers' high school grades and their MKT score found in this study does not necessarily contradict what is reported in the literature. The researcher's interpretation of this finding is that the correlation in this study was between the pre-service teachers' MKT in geometric patterns and their high school overall grade in mathematics. The pre-service teachers' overall high school grade does not show their performance on geometric patterns, which was the focus of this study.

These findings suggest a need for the pre-service course on geometric number patterns to give more time and focus on the development of MKT through providing practical school-based content. School-based content, according to Kopanen (2017), involves activities of the application of mathematics in real-life situations as opposed to university mathematics which is mostly "conceptual and abstract" (p.92).

However, this is not to say that university mathematics is not necessary. Researchers' assessments are to the effect that mathematical content knowledge is a prerequisite for the development of PCK (Ma, 1999; Kopanen, 2017).

6.4 RESEARCH QUESTION 3 – THE EFFECT OF THE PRE-SERVICE TEACHERS COGNITION OF GEOMETRIC GENERALISATIONS ON CLASSROOM PRACTICE

Three pre-service teachers were selected using stratified sampling. The selected pre-service teachers were recorded while teaching mathematics lessons on geometric patterns. The recorded lesson videos were then analysed for quality of teaching using the Assessing Quality of Instruction in Primary Mathematics (AQIPM) instrument. The AQIPM instrument was developed based on the MKT framework. It has four categories, as shown in Table 8. Each category of the instrument reflects on a specific component of the MKT framework. The relation of the AQIPM instrument categories and the MKT framework is represented by Figure 7, herein reproduced;



The Quality of Instructor-Content Interaction codes were an assessment of the pre-service teachers' subject content knowledge and common content knowledge.

Likewise, the Quality of Instructor-Student Interaction codes assessed the pre-service teachers' knowledge of content and students. The knowledge of content and teaching was assessed in the General quality of Instruction category. There were 20 codes in total, as discussed in Section 4.6.2.

The pre-service teachers were assessed on each category of the AQIPM instrument. Each code was scored on a range of 0 – 5. The scores for the quality of instruction of each lesson are shown in Table 22.

Table 22. The pre-service teachers' quality of instruction scores

	Sipho	David	Mpho
Quality of Student-Content Interaction	8	5	4
Quality of Instructor-Content Interaction	20	18	18
Quality of Instructor-Student Interaction	18	20	16
General/Underlying Quality of Instruction	15	10	9
OVERALL SCORE	61	53	47

Table 22 shows that Sipho's lessons had the highest quality of instruction, followed by David's lesson, and Mpho's lesson had the lowest quality of instruction. It can be noted that, though Mpho had a very low score on the pen-and-paper cognition test, the quality of his lessons was close to that of David. Surprisingly, in the quality of Instructor-Student interaction, David's lesson had the highest score. In the category of quality of Instructor-Content interaction, all three pre-service teachers had almost the same performance.

An analysis of the scores on codes for each category revealed that the pre-service teachers' lessons were not only influenced by their cognition of MKT for Euclidean pattern generalisation. A pre-service teacher with a very low score on the LMT test had a fair lesson on pattern generalisation. Since the pre-service teachers did not get assistance from the researcher or the practising teachers in lesson preparation, the researcher believes that the curriculum materials that the pre-service teachers were using helped them a lot in lesson preparation and presentation.

These curriculum materials are the Teachers' Guides and Pupils' books prepared by the National Curriculum Centre (NCC) of Eswatini. The study's findings reveal that a teacher with low MKT can still deliver a lesson of acceptable quality of instruction if they are given enough guidance in lesson preparation. This result shows the importance of providing teachers' guides and learners' workbooks to schools to aid the teachers in lesson preparation. These curriculum materials enable teachers with low MKT to present lessons that result in learners developing conceptual understanding of mathematical concepts nearly the same way as teachers with high MKT.

This finding reveals that the mathematics curriculum materials produced by the government, through the NCC, are very useful in supporting teachers with varying mathematical knowledge for teaching to present quality mathematical instruction to learners. The results of the calculation of the correlation between MKT levels and the quality of instruction revealed a more profound linear relation between the pre-service teachers' MKT level and their general/underlying quality of instruction aspect of the AQIPM instrument. Sipho, who had a high level of MKT, had a high score in general quality of instruction. Mpho, who had a low level of MKT, also had a low score on the general quality of instruction. Likewise, David's (with an average MKT level) quality of instruction was average.

The pre-service teachers' MKT scores were correlated with the score on the quality of instruction. The results showed a positive and strong correlation ($r = .99$) between the teachers' MKT scores and their overall lesson quality. These results agree with those found in the literature (Hill et al., 2008; Cengiz et al., 2011; Charalambous & Litke, 2018). This means that in order to have an improvement in the teaching of geometric patterns, there must be an improvement in the MKT levels of the teachers. According to the assessments of Ball et al. (2008), an improvement in MKT levels of the teachers also results in the improved achievement of learners in mathematics.

The relation between MKT and the quality of Instructor-Student Interaction was very low ($r = .35$). David, the teacher with an average level of MKT, had the highest score on the quality of Instructor-Student Interaction. These findings also show that there was not much difference in the quality of Instructor-Content Interaction of the three pre-service teachers despite their very varied MKT levels.

The findings of this study suggest that it is the knowledge of content and teaching (KCT) that has a linear relationship with a teachers' classroom practice.

This means that KCT is the one category of the mathematical knowledge of teaching that has a greater effect on the quality of lessons delivered by a teacher. However, with a small sample size of three pre-service teachers, caution must be applied, as these findings might not be generalised to all pre-service teachers in Eswatini.

There is, therefore, a need for further research to establish the relationship between each category of MKT and the quality of the lesson.

6.5 CONCLUSION AND RECOMMENDATIONS

This study posed three questions to determine the pre-service teachers' cognition of MKT for geometric patterns and understand how this knowledge is used in classroom practice. The study questions were:

1. What is the adaptability of the LMT measures of MKT to the Eswatini context?
2. What is the pre-service primary teachers' cognition of mathematical knowledge for teaching geometric patterns in Eswatini?
3. How much does the pre-service primary teachers' cognition of mathematical knowledge for teaching geometric patterns to influence their lesson planning and presentation?

The first chapter introduced the study and outlined the problem statement and the research objectives. Chapter two presented a review of literature that situated the study in the context of mathematical knowledge for teaching. The third chapter presented the theoretical framework that guided the study. Chapter 4 outlined the methodology used to collect and analyse data to achieve the research objectives. In chapter five, the researcher presented the findings for the research. Chapter 6 discusses the findings, conclusion and recommendations for future research.

The adaptation of the LMT measures for the Eswatini context was carried out in a pilot study involving 34 pre-service teachers. The data collected from the pilot study were analysed using the SPSS 20 software. Using psychometric analysis, the researcher found out that the instrument was applicable in Eswatini. However, due to differences in the culture of the USA, where the measures were developed, and that of Eswatini, there was a need to adapt the measures for use in Eswatini.

Even though the measures have been adapted for use in other countries outside the USA, the researcher concludes that there is a need to adapt the measures for each country where they are used.

This is because teaching is influenced by culture (Hu et al., 2018). There is, therefore, a need to adapt the measures before they are used in each country because cultures vary with countries.

They were given a pen-and-paper test to determine the pre-service teachers' cognition of MKT for geometric patterns. It can be concluded that their MKT level is low. Most of the pre-service teachers' scores were below 45%. This finding has important implications for teacher training institutions to develop courses that will increase the MKT level of the pre-service teachers. The data from the pen-and-paper test was correlated with the data generated from the lesson videos. The results indicate a strong correlation between the teachers' MKT level and their classroom practice. This finding supports previous research that mathematical teacher knowledge influences the quality of instruction. The low level of the pre-service teachers' MKT in geometric patterns suggests a need to review the courses offered on geometry and number patterns to primary teachers' diploma pre-service teachers. This finding agrees with Bowie et al. (2019) assessments, who found out that there is a need to revisit primary teacher educator courses in South Africa better to prepare the teachers for their future teaching careers.

The results of this research have come out with an adapted instrument for measuring teachers' MKT level in the Eswatini context. The findings of this study have also confirmed the positive correlation between a teachers' MKT level and the quality of instruction in mathematics teaching in Eswatini. This finding agrees with findings in other parts of the world (Ball et al., 2008; Ribeiro, 2012; Chua, 2020). The positive correlation between MKT level and quality of instruction speaks to the need to increase the MKT level of teachers at pre-service education to improve student achievement in primary mathematics. The findings that the MKT level of the primary mathematics pre-service teachers is low could cause the low performance of learners in mathematics in our schools. Therefore, there is a need for teacher training institutions to structure their programme to develop MKT in pre-service teachers.

Due to limitations of resources and the outbreak of the COVID-19 pandemic, the study could not develop a teaching model to promote the development of MKT at pre-service as was envisaged. Therefore, it is recommended that future studies investigate teaching practices that would promote the development of MKT in pre-service education. Since this study was carried out on only one topic in mathematics teaching and one institution, there is a need for a more extensive study with a broader scope to inform pre-service education on the review required in pre-service mathematics education to improve the quality of teaching in the schools.

REFERENCES

Alam, M. J., Das, R. L., & Das, D. K. (2018). Complexity level of mathematics: An index based measurement.

Alex, J., & Mammen, K. J. (2018). Students' understanding of geometry terminology through the lens of Van Hiele theory. *Pythagoras, 39*(1), 1-8.

Andila, Y., & Musdi, E. (2020). *Practicality of geometry learning set based on van hiele theory to increase students' mathematical communication ability.* Paper presented at the Journal of Physics: Conference Series.

Aslan-Tutak, F. (2009). A study of geometry content knowledge of elementary preservice teachers: the case of quadrilaterals. *Unpublished doctoral dissertation, University of Florida*.

Aydoğdu, M. Z., & Keşan, C. (2014). A RESEARCH ON GEOMETRY PROBLEM SOLVING STRATEGIES USED BY ELEMENTARY MATHEMATICS TEACHER CANDIDATES. *Engineering Sciences & Technologies/Nauki Inzynierskie i Technologie, 4*(1).

Ball, D. L., Bass, H., Delaney, S., Hill, H., Phelps, G., Lewis, J., . . . Zopf, D. (2005). *Conceptualizing mathematical knowledge for teaching.* Paper presented at the Annual Meeting of the American Educational Research Association, Montreal, Canada.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching. *Journal of teacher education, 59*(5), 389-407.

Bardach, L., & Klassen, R. M. (2020). Smart teachers, successful students? A systematic review of the literature on teachers' cognitive abilities and teacher effectiveness. *Educational Research Review, 30*, 100312.

Barkman, R. C. (2018). See the World through patterns. *Psychology Today*.

Bichi, A. A., & Talib, R. (2018). Item Response Theory: An Introduction to Latent Trait Models to Test and Item Development. *International Journal of Evaluation and Research in Education, 7*(2), 142-151.

Bishop, J. (2000). Linear geometric number patterns: Middle school students' strategies. *Mathematics Education Research Journal, 12*(2), 107-126.

Borsuk, K. (2018). *Foundations of geometry*: Courier Dover Publications.

Bowie, L., Venkat, H., & Askew, M. (2019). Pre-service Primary Teachers' Mathematical Content Knowledge: An Exploratory Study. *African Journal of Research in Mathematics, Science and Technology Education, 23*(3), 286-297.

Bremner, N. (2021). The multiple meanings of 'student-centred'or 'learner-centred'education, and the case for a more flexible approach to defining it. *Comparative Education, 57*(2), 159-186.

Browning, C., Edson, A. J., Kimani, P., & Aslan-Tutak, F. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on geometry and measurement. *The Mathematics Enthusiast, 11*(2), 333-383.

Carpenter, S. (2018). Ten steps in scale development and reporting: A guide for researchers. *Communication Methods and Measures, 12*(1), 25-44.

Catalano, A., Asselta, L., & Durkin, A. (2019). Exploring the Relationship between Science Content Knowledge and Science Teaching Self-Efficacy among Elementary Teachers. *IAFOR Journal of Education, 7*(1), 57-70.

Charalambous, C. Y., Hill, H. C., Chin, M. J., & McGinn, D. (2020). Mathematical content knowledge and knowledge for teaching: exploring their distinguishability and contribution to student learning. *Journal of Mathematics Teacher Education, 23*(6), 579-613.

Charalambous, C. Y., & Litke, E. (2018). Studying instructional quality by using a content-specific lens: the case of the Mathematical Quality of Instruction framework. *ZDM, 50*(3), 445-460.

Charalambous, C. Y., & Pitta-Pantazi, D. (2015). Perspectives on priority mathematics education: Unpacking and understanding a complex relationship linking teacher knowledge, teaching, and learning. In E. L. & K. D. (Eds.), *Handbook of International Research in Mathematics Education* (Third ed., pp. 19-59). New York: Routledge.

Chinnappan, M., & Lawson, M. J. (2005). A framework for analysis of teachers' geometric content knowledge and geometric knowledge for teaching. *Journal of Mathematics Teacher Education, 8*(3), 197-221.

Chua, V. (2019). Mathematical knowledge for teaching: A literature review on ideology, instrumentation, and investigations. In.

Cifarelli, V. V., Pugalee, D. K., & Higbie, P. (2019). *EXAMINING CONTENT AND PEDAGOGICAL KNOWLEDGE OF PRE-SERVICE ELEMENTARY GRADES MATHEMATICS TEACHERS.* Paper presented at the International Symposium Elementary Mathematics Teaching.

Clark, V. L. P. (2019). Meaningful integration within mixed methods studies: Identifying why, what, when, and how. *Contemporary Educational Psychology, 57*, 106-111.

Cohen, L., Manion, L., & Morrison, K. (2013). *Research methods in education*: routledge.

Cole, Y. (2012). Assessing elemental validity: the transfer and use of mathematical knowledge for teaching measures in Ghana. *ZDM, 44*(3), 415-426.

Cresswell, W. (2013). Research Design; Quantitative and Mixed Methods. In: SAGE Journals.

Creswell, J. W. (2014). The selection of a research approach. *Research design: Qualitative, quantitative, and mixed methods approaches*, 3-24.

Dawadi, S., Shrestha, S., & Giri, R. A. (2021). Mixed-methods research: A discussion on its types, challenges, and criticisms. *Online Submission, 2*(2), 25-36.

Delaney, S., Ball, D. L., Hill, H. C., Schilling, S. G., & Zopf, D. (2008). "Mathematical knowledge for teaching": Adapting US measures for use in Ireland. *Journal of mathematics teacher education, 11*(3), 171-197.

Depaepe, F., & König, J. (2018). General pedagogical knowledge, self-efficacy and instructional practice: Disentangling their relationship in pre-service teacher education. *Teaching and Teacher Education, 69*, 177-190.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and teacher education, 34*, 12-25.

Douaire, J., & Emprin, F. (2017). *Teaching geometry to students (from five to eight years old)" All that is curved and smooth is not a circle".* Paper presented at the CERME 10.

Drageset, O. G. (2010). The Interplay between the Beliefs and the Knowledge of Mathematics Teachers. *Mathematics Teacher Education and Development, 12*(1), 30-49.

Du Plessis, E. (2020). Student teachers' perceptions, experiences, and challenges regarding learner-centred teaching. *South African Journal of Education, 40*(1).

ECESWA. (2019). *Examination Council of Eswatini, EPC examination report*. Retrieved from Mbabane:

Evens, M., Elen, J., & Depaepe, F. (2015). Developing pedagogical content knowledge: Lessons learned from intervention studies. *Education Research International, 2015*.

Fauskanger, J., Jakobsen, A., Mosvold, R., & Bjuland, R. (2012). Analysis of psychometric properties as part of an iterative adaptation process of MKT items for use in other countries. *ZDM, 44*(3), 387-399.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact.

Flores, D. (2018). *Standing in the middle: Insider/outsider positionality while conducting qualitative research with opposing military veteran political groups*: SAGE Publications Ltd.

Friel, S. N., & Markworth, K. A. (2009). A framework for analyzing geometric pattern tasks. *MatheMatics teaching in the Middle school, 15*(1), 24-33.

Herbst, P., & Kosko, K. (2014). Mathematical Knowledge for Teaching and its Specificity to High School Geometry Instruction. In (pp. 23-46).

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for research in mathematics education*, 372-400.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and instruction, 26*(4), 430-511.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American educational research journal, 42*(2), 371-406.

Hine, G. S. (2015). Strengthening pre-service teachers' mathematical content knowledge. *Journal of University Teaching & Learning Practice, 12*(4), 5.

Hoover, M., Mosvold, R., Ball, D. L., & Lai, Y. (2016). Making progress on mathematical knowledge for teaching. *The Mathematics Enthusiast, 13*(1), 3-34.

Hu, X., Leung, F. K., & Teng, Y. (2018). The influence of culture on students' mathematics achievement across 51 countries. *International Journal of Science and Mathematics Education, 16*(1), 7-24.

Hwang, W.-Y., Purba, S. W. D., Liu, Y.-f., Zhang, Y.-Y., & Chen, N.-S. (2018). An investigation of the effects of measuring authentic contexts on geometry learning achievement. *IEEE Transactions on Learning Technologies, 12*(3), 291-302.

Jankvist, U. T., Clark, K. M., & Mosvold, R. (2020). Developing mathematical knowledge for teaching teachers: potentials of history of mathematics in teacher educator training. *Journal of Mathematics Teacher Education, 23*(3), 311-332.

Kazima, M., Jakobsen, A., & Kasoka, D. N. (2016). Use of Mathematical Tasks of Teaching and the Corresponding LMT Meaures in the Malawi Context. *The Mathematics Enthusiast, 13*(1), 171-186.

Kellman, P. J., & Massey, C. M. (2013). Perceptual learning, cognition, and expertise. In *Psychology of learning and motivation* (Vol. 58, pp. 117-165): Elsevier.

Kellman, P. J., Massey, C. M., & Son, J. Y. (2010). Perceptual learning modules in mathematics: enhancing students' pattern recognition, structure extraction, and fluency. *Topics in cognitive science, 2*(2), 285-305. doi:10.1111/j.1756-8765.2009.01053.x

Khanna, T., Lakhani, K. R., Bhadada, S., Khan, N., Davé, S. K., Alam, R., & Hewett, M. (2021). Crowdsourcing Memories: Mixed Methods Research by Cultural Insiders-Epistemological Outsiders. *Academy of Management Perspectives, 35*(3), 384-399.

Klette, K., Blikstad-Balas, M., & Roe, A. (2017). Linking instruction and student achievement: Research design for a new generation of classroom studies. *Acta Didactica Norge-tidsskrift for fagdidaktisk forsknings-og utviklingsarbeid i Norge, 11*(3), 19.

Koeno, G., Figueiredo, N., Feijs, E., van Galen, F., Keijzer, R., & Munk, F. (2016). *Measurement and Geometry in Upper Primary School*: Springer.

Kohen, Z., & Kramarski, B. (2018). Promoting mathematics teachers' pedagogical metacognition: A theoretical-practical model and case study. In *Cognition, Metacognition, and Culture in STEM Education* (pp. 279-305): Springer.

Korn, J. (2014). Teaching conceptual understanding of mathematics via a hands-on approach.

Lachman, R., Lachman, J. L., & Butterfield, E. C. (2015). *Cognitive psychology and information processing: An introduction*: Psychology Press.

Larsen, K. R., & Eargle, D. (Eds.). (2015). *Theories Used in IS Research Wiki.*

linnette D'Sa, J., Alharbi, M. F., & liza Visbal-Dionaldo, M. (2018). The Relationship between Item Difficulty and Non-functioning Distractors of Multiple Choice Questions. *Amarjeet Kaur Sandhu, 10*(3), 48.

Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of teacher education, 59*(5), 389-407.

Lutz, S., & Huitt, W. (2003). Information processing and memory: Theory and Applications. *Educational Psychology Interactive.* Retrieved from http://www.edpsycinteractive.org/papers/infoproc.pdf

Maarouf, H. (2019). Pragmatism as a supportive paradigm for the mixed research approach: Conceptualizing the ontological, epistemological, and axiological stances of pragmatism. *International Business Research, 12*(9), 1-12.

Malmqvist, J., Hellberg, K., Möllås, G., Rose, R., & Shevlin, M. (2019). Conducting the pilot study: A neglected part of the research process? Methodological findings supporting the importance of piloting in qualitative research studies. *International Journal of Qualitative Methods, 18*, 1609406919878341.

Malterud, K., Siersma, V. D., & Guassora, A. D. (2016). Sample size in qualitative interview studies: guided by information power. *Qualitative health research, 26*(13), 1753-1760. Retrieved from https://journals.sagepub.com/doi/full/10.1177/1049732315617444?url_ver=Z39.88-2003&rfr_id=ori%3Arid%3Acrossref.org&rfr_dat=cr_pub%3Dpubmed

Mapolelo, D. C., & Akinsola, M. K. (2015). Preparation of Mathematics Teachers: Lessons from Review of Literature on Teachers¡¯ Knowledge, Beliefs, and Teacher Education. *American Journal of Educational Research, 3*(4), 505-513. Retrieved from http://pubs.sciepub.com/education/3/4/18

Martinovic, D., & Manizade, A. G. (2018). The challenges in the assessment of knowledge for teaching geometry. *ZDM, 50*(4), 613-629.

Matthews, M., Rech, J., & Grandgenett, N. (2010). The impact of content courses on pre-service elementary teachers' mathematical content knowledge.

MBABANE, S. (2011). THE SWAZILAND EDUCATION AND TRAINING SECTOR POLICY.

McCluskey, C., Mitchelmore, M., & Mulligan, J. (2013). Does an ability to pattern indicate that our thinking is mathematical?

Mello, P. A. (2021). *Qualitative comparative analysis: An introduction to research design and application*: Georgetown University Press.

Melo, H. S., & Martins, M. d. C. (2015). Behaviors and attitudes in the teaching and learning of geometry. *European Scientific Journal*(Special Edition), 98-104.

Mesa, V., Duranczyk, I., & Watkins, L. (2019). *The structure of EQIPM, a video coding protocol to assess the quality of community college algebra instruction.* Paper presented at the Eleventh Congress of the European Society for Research in Mathematics Education.

MoET. (2018). *Improving the quality of education for the sustainable and inclusive growth of Eswatini*. Mbabane: Government of Eswatini

Murimba, S. (2005). The southern and eastern Africa consortium for monitoring educational quality (SACMEQ): Mission, approach and projects. *Prospects*(1), 1-229. doi:http://www.sacmeq.org/?q=sacmeq-members/swaziland

Mutilifa, S. I., & Kapenda, H. M. (2017). Does Learner-Centred Approach Improve High School Learners' Understanding of Acids and Bases Topic? A Case of Two Selected Secondary Schools in Ohangwena Region, Namibia. *Creative Education, 8*(08), 1260.

Naidoo, J. (2013). Does social class influence learner reasoning in geometry? *Global Journal of Medical Research*.

Ndlandla, S. S. (2017). *Examining primary school teachers' understanding of teaching geometry through the problem solving approach in Swaziland.*

Ng, D. (2011). Indonesian primary teachers' mathematical knowledge for teaching geometry: implications for educational policy and teacher preparation programs. *AsiaPacific Journal of Teacher Education, 39*(2), 151-164.

Nhlengethwa, K. B. (2019). *Swaziland pre-service teachers' understanding and enactment of inquiry-based-science teaching: a case of a university in Swaziland.*

Pasani, C. F. (2019). Analyzing Elementary School Students Geometry Comprehension Based on Van Hiele's Theory. *Journal of Southwest Jiaotong University, 54*(5).

Patel-Junankar, D. (2017). Learner-Centered Pedagogy: Teaching and Learning in the 21st Century. *The Health Professions Educator: A Practical Guide for New and Established Faculty*.

Pi, Y., Liao, W., Liu, M., & Lu, J. (2008). Theory of cognitive pattern recognition. In *Pattern recognition techniques, technology and applications*: IntechOpen.

Poulou, M. S., Reddy, L. A., & Dudek, C. M. (2019). Relation of teacher self-efficacy and classroom practices: A preliminary investigation. *School Psychology International, 40*(1), 25-48.

Radford, L. (2008). Iconicity and contraction: A semiotic investigation of forms of algebraic generalizations of patterns in different contexts. *ZDM, 40*(1), 83-96.

Ramdhani, M., Usodo, B., & Subanti, S. (2017). *Discovery Learning with Scientific Approach on Geometry.* Paper presented at the Journal of Physics: Conference Series.

Rau, M. A., Sen, A., & Zhu, X. (2019). *Using Machine Learning to Overcome the Expert Blind Spot for Perceptual Fluency Trainings.* Paper presented at the International Conference on Artificial Intelligence in Education.

Ribeiro, C. M., & Carrillo, J. (2012). Discussing a teacher MKT and its role on teacher practice when exploring data analysis. *PNA. Revista de Investigación en Didáctica de la Matemática, 6*(3), 105-114.

Ro, S. (2019). The Influence of Euclidean Geometry on Current technologies and Various Applications of Euclidean Geometry. *Student International Journal of Research, 6*(1).

Sahidin, L., Budiarto, M. T., & Fuad, Y. (2019). *What Construct of Mathematical Knowledge for Teaching do Mathematics Teachers Need?(A Theoretical Framework and Conceptualization in Geometry).* Paper presented at the 1st International Conference on Advanced Multidisciplinary Research (ICAMR 2018).

Santagata, R., & Lee, J. (2019). Mathematical knowledge for teaching and the mathematical quality of instruction: a study of novice elementary school teachers. *Journal of Mathematics Teacher Education*, 1-28.

Saunders, M. N., Lewis, P., Thornhill, A., & Bristow, A. (2015). Understanding research philosophy and approaches to theory development.

Scheiner, T. (2018). Toward theory advancement in mathematical cognition and teacher cognition.

Schoonenboom, J., & Johnson, R. B. (2017). How to construct a mixed methods research design. *KZfSS Kölner Zeitschrift für Soziologie und Sozialpsychologie, 69*(2), 107-131.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard educational review, 57*(1), 1-23.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher, 15*(2), 4-14.

Simamora, R. E., & Saragih, S. (2019). Improving Students' Mathematical Problem Solving Ability and Self-Efficacy through Guided Discovery Learning in Local Culture Context. *International Electronic Journal of Mathematics Education, 14*(1), 61-72.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching, 77*(1), 20-26.

Skemp, R. R. (2006). Relational Understanding and Instrumental Understanding. *Mathematics Teaching in the Middle School, 12*(2), 88-95. Retrieved from http://www.jstor.org/stable/41182357

Steele, M. D. (2013). Exploring the mathematical knowledge for teaching geometry and measurement through the design and use of rich assessment tasks. *Journal of Mathematics Teacher Education, 16*(4), 245-268.

Stols, G. H. (2013). Does the use of technology make a difference in the geometric cognitive growth of pre-service mathematics teachers? Retrieved from Stols, G. (2012). Does the use of technology make a difference in the geometric cognitive growth of pre-service mathematics teachers? Australasian Journal of Educational Technology, 28(7), 1233-1247. http://hdl.handle.net/2263/20890

Stylianides, A. J., & Delaney, S. (2018). Pre-service Mathematics Teachers' Knowledge and Beliefs. In *Research Advances in the Mathematical Education of Pre-service Elementary Teachers* (pp. 219-228): Springer.

Subedi, D. (2016). Explanatory sequential mixed method design as the third research community of knowledge claim. *American Journal of Educational Research, 4*(7), 570-577.

Sugesti, I., Rukmini, D., Faridi, A., & Fitriati, S. W. (2020). *Teachers' Cognition and Their Teaching Practices in an EFL Classroom: A Correlational Study.* Paper presented at the International Conference on Science and Education and Technology (ISET 2019).

Sunzuma, G., & Maharaj, A. (2019). In-service Teachers' Geometry Content Knowledge: Implications for how Geometry is Taught in Teacher Training Institutions. *International Electronic Journal of Mathematics Education, 14*(3), 633-646.

Susilawti, W., Suryadi, D., & Dahlan, J. A. (2017). The improvement of mathematical spatial visualization ability of student through cognitive conflict. *International Electronic Journal of Mathematics Education, 12*(2), 155-166.

Tsao, Y.-L. (2018). The Effect of Constructivist Instructional-Based Mathematics Course on the Attitude Toward Geometry of Pre-Service Elementary School Teachers. *US-China Education Review, 8*(1), 1-10.

Tutak, F. A., & Adams, T. L. (2017). A study of geometry content knowledge of elementary preservice teachers. *International Electronic Journal of Elementary Education, 7*(3), 301-318.

Van de Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 6-8 (Volume III)(Teaching Student-Centered Mathematics Series).

Van den Kieboom, L. A. (2013). Examining the mathematical knowledge for teaching involved in pre-service teachers' reflections. *Teaching and Teacher Education, 35*, 146-156.

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development, 41*, 121-130.

Woolcott, G. (2020). Learning and Memory in Modern Cognitive Psychology and Integrative Biology. In *Reconceptualising Information Processing for Education* (pp. 3-7): Springer.

Yap, W.-L., Neo, M., & Neo, T.-K. (2016). Learner-Centred Teaching Contributes in Promising Results in Improving Learner Understanding and Motivation: A Case Study at Malaysia Tertiary Education. *Electronic Journal of e-Learning, 14*(4), 266-281.

Zakharov, A., Tsheko, G., & Carnoy, M. (2016). Do "better" teachers and classroom resources improve student achievement? A causal comparative approach in Kenya, South Africa, and Swaziland. *International Journal of Educational Development, 50*, 108-124.