

# **THE UNIVERSITY OF ZULULAND**



## **APPLICATION OF GEOGEBRA ON EUCLIDEAN GEOMETRY IN RURAL HIGH SCHOOLS - GRADE 11 LEARNERS**

**By**

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## DECLARATION

This study represents the original work by the author and has not been submitted in any form for any degree or diploma to any tertiary institution. Where the author has made use of work of others, it is duly acknowledged in the text.

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## **DEDICATION**

I dedicate this research to my late parents Ntombiyodwa Thelmah (MaKhuzwayo) and Thanduyise Wilson Mthethwa, my late son Makwande and my late brother Skhumbuzo.

## **ACKNOWLEDGEMENTS**

First and foremost I would like to thank God the Almighty for keeping me alive and helping me through this dissertation especially during the times I felt hopeless and even thought of giving up.

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## **ABSTRACT**

This research aims to establish the level of students' cognitive skills using GeoGebra, and investigates whether GeoGebra as a technological tool helps in improving poor performance in respect of Euclidean geometry or geometry of the circle. Students' interests, in learning about circle geometry in mathematics, are also being tested.

GeoGebra is an innovative, dynamic mathematics software which integrates algebra, geometry and calculus to aid students during the learning process. The specific sample in this research consists of 112 Grade 11 secondary school learners within the UMkhanyakude district, Hlabisa circuit, under the Empembeni and Ezibayeni wards. During this research, GeoGebra and the concept of circle geometry were introduced to students. Afterwards, students had to answer several geometry of the circle questions, entailing key theorems as prescribed by the National Mathematics pacesetter for Grade 11 and Grade 12.

As students answered the above questions, they solved problems and conducted discussions among themselves. At the end, students were individually required to answer questionnaires which consisted of 15 closed items relating to views on GeoGebra and its impact on Euclidean geometry and mathematics, as well as three open-ended questions which asked learners about their reflections on the application of GeoGebra.

The above methods provided a strong base to explore whether GeoGebra as a tool helps students in the learning process. The results showed that students endorsed the use of GeoGebra as a technological tool in the teaching of Euclidean geometry. Some students even suggested that GeoGebra be used in other mathematical topics. Students overall enjoyed the use of GeoGebra, finding it user-friendly and a highly significant learning motivator.

## TABLE OF CONTENTS

Description	Page
Declaration .....	ii
Dedication .....	iii
Acknowledgement.....	iv
Abstract.....	v
Table of content.....	vi
List of tables .....	xii
List of figures .....	xiii
List of abbreviations .....	xv
 Chapter One: Introduction and overview.....	 1
1.1 Introduction .....	1
1.2 Background.....	4
1.3 Problem statement and delimitation of the study .....	9
1.4 Research questions, aims and the significance of the study .....	10
1.5 Data collection method .....	13
1.5.1 Participants .....	13
1.5.2 Instruments .....	14
1.5.3 Procedures .....	15
1.5.3.1 Treatment of the experimental groups .....	15
1.5.3.2 Treatment of the control groups.....	15
1.5.4 Analysis of data.....	16
1.6 Ethical obligations.....	16
1.7 Conclusion.....	17
 Chapter Two: Literature Review and Theoretical framework.....	 18
2.1 Literature review.....	18
2.1.1 Introduction.....	18
2.1.2 Inclusion of Euclidean geometry.....	19
2.1.3 Umalusi on NCS and CAPS .....	23
2.1.4 Proof and verification .....	27
2.1.5 Dynamic geometry software.....	29

2.1.6 Students' performance in geometry .....	30
2.1.7 Nature and scope of geogebra software .....	30
2.1.8 Geogebra impacting on individual learning.....	32
2.1.9 Geogebra for model-centred learning .....	32
2.1.10 Geogebra on problem solving and attitude change .....	33
2.1.11 Why geogebra is different? .....	34
2.1.12 Hindrances of geogebra in mathematics classroom.....	38
2.2 Theoretical framework.....	40
2.2.1 Information Processing Theory.....	40
2.2.1.1 Sensory Memory.....	42
2.2.1.2 Short Term Memory .....	42
2.2.1.3 Long Term Memory.....	44
2.2.2 Cognitive Neuroscience and GeoGebra Application .....	46
2.2.2.1 Neuroscience on models.....	46
2.2.2.2 Neuroscience on students' school success.....	47
2.3 Conclusion .....	48
 Chapter Three: Research Methodology.....	 50
3.1 Introduction .....	50
3.2 Clarification and justification of the study .....	51
3.3 Research paradigms.....	51
3.3.1 Positive paradigm.....	51
3.3.2 Interpretive paradigm .....	52
3.4 Research design.....	53
3.4.1 Sampling of site and population.....	53
3.4.1.1 School setting.....	54
3.4.2 Achievement test.....	55
3.4.3 Sample size and selection.....	56
3.4.3.1 Sample size calculation.....	57

3.4.3.2 Procedure.....	59
3.5 Methods of data collection .....	60
3.5.1 Questionnaires.....	60
3.5.1.1 Questionnaire distributions and challenges.....	63
3.5.2 Open-ended questions.....	64
3.5.3 Document analysis.....	65
3.5.4 Triangulation of method, data and theory.....	66
3.5.4.1 Methodological Triangulation.....	66
3.5.4.2 Data Triangulation.....	66
3.5.4.3 Theory Triangulation.....	66
3.6 Data analysis.....	67
3.6.1 Multiple Regression Analysis (MRA) Application.....	67
a) The Multiplication Regression model.....	67
b) Interpreting Regression Coefficients .....	68
c) How good is the Regression? .....	68
3.6.2 Meeting the assumption of MRA usage.....	68
3.6.3 Variables and Models Evaluations.....	69
3.6.3.1 R Square Value.....	69
3.6.4 Qualitative Data Analysis.....	70
3.6.4.1 Data Capturing, Immersion, Coding, Reduction and Interpretation.....	70
3.6.5 Assumptions of Qualitative Data Analysis.....	70
3.6.6 The Need for Qualitative and Quantitative Data Analysis.....	72
3.6.7 Analysis of Data from Documents.....	72
3.7 Conducting the pre-test .....	74
3.8 Intervention using computer software (geogebra).....	74
3.8.1 Activities.....	81
3.8.1.1 Activity 1.....	81
3.8.1.2 Activity 2.....	83
3.8.1.3 Activity 3.....	84
3.8.1.4 Activity 4.....	87



3.8.1.5 Activity 5.....	87
3.8.1.6 Activity 6.....	89
3.8.2 Tasks.....	91
3.9 Conducting the post-test .....	93
3.10 Research quality.....	94
3.10.1 Trustworthiness.....	95
3.10.2 Validity.....	96
3.10.3 Transferability.....	96
3.10.4 Reliability .....	96
3.10.5 Dependability.....	97
3.10.6 Confirmability .....	97
3.11 Ethical issues .....	97
3.12 Conclusion.....	99
 Chapter Four: Data analysis.....	 100
4.1 Introduction.....	100
4.2 Context of findings.....	101
4.3 Analysis and assessment.....	103
4.3.1 Analysis on Question 1 .....	103
4.3.2 Analysis on Question 2.....	106
4.3.3 Analysis on Question .....	112
4.4 Meeting the assumptions of the MRA usage.....	122
4.5 Conclusion.....	124
 Chapter Five: Discussion of findings .....	 126
5.1 Introduction .....	126
5.2 Discussion of results .....	126
5.2.1 Section A.....	126
5.2.1.1 T-Tests.....	126

5.2.1.2 Discussion of the t-test results.....	127
5.2.2 Section B.....	127
5.2.2.1 The Self-designed Questionnaire.....	128
5.2.2.2 Discussion of results on questionnaire.....	128
5.2.3 Section C .....	129
5.2.3.1 Sample result on Regression Analysis .....	129
5.2.3.2 Discussion in respect to Regression Analysis.....	130
5.2.4 Section D.....	130
5.2.4.1 Findings from Question 1 .....	130
5.2.4.2 Findings from Question 2.....	134
5.2.4.3 Findings from Question 3.....	138
5.3 Students' feedback.....	143
5.4 Conclusion.....	147

## Chapter Six: Summary, Conclusions, Limitations and Recommendations .....

6.1 Introduction.....	148
6.2 Summary of the research .....	148
6.3 Conclusions.....	154
6.3.1 Conclusions from literature and theoretical framework.....	154
6.4 Recommendations.....	155
6.4.1 Recommendations for further research.....	155
6.5 Limitations.....	156
6.6 Summary.....	156

## REFERENCES .....

Annexure 1: Request for permission letter .....	178
Annexure 2: Ethical declaration form for research .....	180
Annexure 3: Clearance certificate for research.....	182

Annexure 4: Participant Informed consent-research study.....	184
Annexure 5: Practice Exercise.....	186
Annexure 6: Achievement test (pre-test and post-test).....	188
Annexure 7: Questionnaire.....	193
Annexure 8: Turnitin .....	196
Annexure 9: Editing report .....	197
Annexure 10: Editing certificate.....	198

## LIST OF TABLES

<b>Table 2.1:</b> Mathematics achievement.....	11
<b>Table 2.2:</b> Statistics on Euclidean geometry performance, RSA 2001-2007.....	12
<b>Table 2.3:</b> Content/skills coverage over various subtopics in NCS and CAPS.....	15
<b>Table 2.4:</b> Weighting per topic by percentage of time.....	16
<b>Table 2.5:</b> Weighting per topic by percentage of marks.....	16
<b>Table 3.1:</b> An extract of t-test confidence interval table.....	54
<b>Table 3.2:</b> An extract of z-test standard normal distribution table.....	54
<b>Table 3.3:</b> Composition of sample.....	54
<b>Table 4.1:</b> Results of independent t-test on the pre-test of both groups.....	98
<b>Table 4.2:</b> Results of independent t-test on the post-test of both groups.....	98
<b>Table 4.3:</b> Results of the paired sample t-test.....	99
<b>Table 4.4:</b> The students' responses and their percentage desirous of learning Euclidean geometry with GeoGbra.....	100
<b>Table 5.1:</b> Levels of Mathematics Competency.....	131
<b>Table 5.2:</b> Action verbs and products involved in mathematics.....	132

## LIST OF FIGURES

<b>Figure 2.1:</b> RSA in TIMSS results ranging from (1995-2011).....	9
<b>Figure 2.2:</b> Number of sub-topics per grade and overall.....	14
<b>Figure 2.3:</b> Weighting percentage of marks and time for each topic in CAPS.....	17
<b>Figure 2.4:</b> Proof and verification of interior angles triangle proof with GeoGebra application.....	19
<b>Figure 2.5:</b> Screenshot showing both algebra and geometry window.....	22
<b>Figure 2.6:</b> Information Processing Theory.....	38
<b>Figure 2.7:</b> Significant correlation between activation in left angular gyrus and mathematical-numerical intelligence .....	43
<b>Figure 3.1:</b> Diagram showing quasi-experimental study design.....	56
<b>Figure 3.2:</b> Screenshot of GeoGebra menu toolbar.....	74
<b>Figure 3.3:</b> Student exploring the size of a circle using GeoGebra.....	81
<b>Figure 3.4:</b> Student practising measuring angles and lines using GeoGebra tools .....	82
<b>Figure 3.5:</b> Student evaluating the lengths of radii using GeoGebra software .....	83
<b>Figure 3.6:</b> Student verifying perpendicular chord theorem using GeoGebra tools .....	84
<b>Figure 3.7:</b> Student verifying centre theorem using GeoGebra software.....	85
<b>Figure 3.8:</b> Student practising same segment theorem using GeoGebra tools.....	85
<b>Figure 3.9:</b> Student verifying same segment theorem using GeoGebra tools.....	86
<b>Figure 3.10:</b> Student evaluating semi-circle theorem using GeoGebra tools.....	87
<b>Figure 3.11:</b> Student comparing properties of supplementary theorem using GeoGebra tools.....	88
<b>Figure 3.12:</b> Student verifying supplementary theorem using GeoGebra tools.....	89
<b>Figure 3.13:</b> GeoGebra software on tangent-chord theorem.....	90
<b>Figure 3.14:</b> Student verifying tangent-chord theorem using GeoGebra tools ....	90
<b>Figure 3.15:</b> A diagram for students to answer task 1.....	92
<b>Figure 3.16:</b> A diagram for students to answer task 2.....	92
<b>Figure 3.17:</b> A diagram for students to answer task 3.....	92

<b>Figure 3.18:</b> A diagram for students to answer task 4.....	92
<b>Figure 3.19:</b> A diagram for students to answer task 5.....	92
<b>Figure 4.1:</b> A diagram showing main terminology in circle geometry.....	107
<b>Figure 4.2:</b> Snapshots of GeoGebra screen on teaching terminology.....	108
<b>Figure 4.3:</b> A diagram defining secant line in a circle.....	108
<b>Figure 4.4:</b> A diagram defining tangent line in a circle .....	108
<b>Figure 4.5:</b> A diagram defining a chord in a circle.....	109
<b>Figure 4.6:</b> A diagram defining a central angle in a circle.....	109
<b>Figure 4.7:</b> A diagram defining an arc in a circle .....	109
<b>Figure 4.8:</b> GeoGebra showing two tangents meeting at one point.....	109
<b>Figure 4.9:</b> Breakdown of results improvements.....	110
<b>Figure 4.10:</b> Breakdown on beliefs on performance in Euclidean geometry.....	111
<b>Figure 4.11:</b> Snapshot showing exterior angle of a triangle theorem.....	112
<b>Figure 4.12:</b> Breakdown on visualisation through GeoGebra.....	114
<b>Figure 4.13:</b> Breakdown on justifying and logical assumptions of theorems.....	115
<b>Figure 4.14:</b> Snapshot for understanding circle geometry using GeoGebra.....	117
<b>Figure 4.15:</b> Breakdown on understanding circle geometry concepts.....	118
<b>Figure 4.16:</b> Results showing in histogram.....	122
<b>Figure 4.17:</b> Simple linear regression line.....	123
<b>Figure 5.1:</b> A diagram showing a rider of circle multiple theorems.....	131
<b>Figure 5.2:</b> Snapshot showing GeoGebra tools in proof justification.....	141
<b>Figure 5.3:</b> Students' ratings of GeoGebra in learning Euclidean geometry.....	144

## **LIST OF ACRONYMS AND ABBREVIATIONS**

<b>ANA</b>	Annual National Assessment
<b>CAPS</b>	Curriculum Assessment Policy Statements
<b>BDE</b>	Basic Department of Education
<b>DBE</b>	Department of Basic Education
<b>DGE</b>	Dynamic Geometry Environment
<b>DGS</b>	Dynamic Geometric Software
<b>DOE</b>	Department of Education
<b>HSRC</b>	Human Sciences Research Council
<b>IPT</b>	Information Processing Theory
<b>KZN</b>	KwaZulu-Natal
<b>MCL</b>	Model-Centred Learning
<b>MFL</b>	Model-Facilitated Learning
<b>MST</b>	Mathematics, Science and Technology
<b>NCS</b>	National Curriculum Statement
<b>NCTM</b>	National Council of Teachers of Mathematics
<b>NRC</b>	National Research Council
<b>RSA</b>	Republic of South Africa
<b>SACMEQ</b>	Southern and Eastern African Consortium for Monitoring Education Quality
<b>TIMSS</b>	Trends in International for Mathematics and Science Study
<b>WEF</b>	World Economic Forum

# **CHAPTER ONE**

## **INTRODUCTION AND OVERVIEW**

### **1.1 INTRODUCTION**

Euclidean geometry has been for centuries the field, through which learners have a chance to understand the nature of mathematics (Orlov, 2005). As Ronan (2008) said Euclidean geometry teaches about logical reasoning and abstraction, and helps in rationalism, making judgments from reality especially for high learners. Classical geometric intuition is significant as this gives exposure to clear and rigorous arguments.

Studying Euclidean geometry leads to big advantages of being visual and readily accessible to intuition. Euclidean geometry is an important topic in the mathematics curriculum. According to Curriculum 2005, Euclidean geometry yields to the three distinctive sources; (i) philosophy of learner-centred education, (ii) Outcomes-Based education and (iii) to an integrated approach to knowledge.

Hanna (2009) in her research paper stated that Euclidean geometry leads learners to spatial thinking, graphical reasoning and the ability to interpret mathematical arguments. Euclidean is a pre-requisite of Linear Algebra, Pre-Calculus and Calculus, thus it helps learners in pursuing careers in engineering, economics, architecture and other related studies which are very important careers for bridging between rural and urban lives. Curriculum and Assessment Policy Statement (CAPS, 2010) and grade eleven mathematics pace setter, the class of Grade 11 year 2014 are expected to grasp and grapple the geometry of the circle in three weeks (approximately 15 hours), and accept results established in earlier grades as axioms. At the exit point they are expected to have investigated, proven and be able to apply the following theorems of the geometry of circle:

- The line drawn from the centre of the circle perpendicular to a chord bisects the chord.
- The perpendicular bisector of a chord passes through the centre of the circle.
- The angle subtended by the arc at the centre of a circle is double the size the angle subtended by the same arc at the circle.



- Angles subtended by a chord of the circle, on the same side of the chord are equal.
- The angle subtended by the diameter at the circumference of the circle is right angle.
- If the angle by a chord at the circumference of the circle is right angle, then the chord is diameter.
- Angles subtended by the chord of the circle, on the same side of the chord are equal.
- If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.
- The opposite angles of a cyclic quadrilateral are supplementary.
- If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- If the exterior angle of a quadrilateral is equal to the interior angle opposite angle of the quadrilateral, then the quadrilateral is cyclic.
- Two tangents drawn to a circle from the same point outside the circle are equal in length.
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.
- If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.

These are examinable in grade 12 final examination but do not fall on detailed work schedule (DOE, 2010). The debate of the inclusion Euclidean geometry as part of the compulsory exit examination for learners in the South Africa context has risen to fever pitch over the last few years (Van Niekerk, 2010)

In her research Bowie (2009) stated that some South African universities argue that the removal of Euclidean geometry from the core curriculum has created a lack of coherence in the study of space and shape and that the opportunity to work with proof has been diminished. Furthermore it has been envisaged that South African teachers are not familiar with the content in Euclidean geometry (Bowie, 2009). It depends on the teachers' attitudes and knowledge on whether they will or can teach Euclidean geometry.

School experts in South Africa have warned that the 2014 matric results could sharply drop as many Grade 12 teachers in state schools struggle to prepare learners to write compulsory sections on Euclidean geometry and Probability for the first time (Jansen and Dardagan, 2014).

Bowie (2009) commented that in Euclidean geometry, providing proofs in the answer is a strong element of the content which required preparedness by teachers. The content was re-introduced in 2010 curriculum planning due to universities' warnings that students signing up for engineering and related courses, were not copying because they had no background knowledge of Euclidean geometry. Basic Department of Education (BDE) said that with the introduction of new CAPS in 2012 in grade 10, the teaching and testing of Euclidean geometry and Probability learning was compulsory.

But Bowie (2014), a mathematics education expert from Witwatersrand University, said that teachers were not teaching these re-introduced sections while it was offered as an optional paper (Paper 3), because they lacked the knowledge about them. Learners are prone to struggle. Bowie (2014) even suggested that there should be gentle introduction of these topics.

This study investigated the usage of geometry software called GeoGebra in five rural high schools. The essence was to assess the level of improvement in Euclidean geometry due to the application. The participants in this study were pupils who learn mathematics in Grade 11.

The teaching and learning with the use of technology has many advantages such as providing greater learning opportunities for students, enhancing student engagement and encouraging discovery learning (Roberts, 2012 and White, 2012). In the teaching and learning of mathematics, and Euclidean geometry in particular, it is imperative that students take part in drawing, imagining, construction conjecturing, verifying, justifying shapes and making connections with the related facts of proofs and theorems. An opinion noted by Dogan (2010) suggested that a computer will assist students in imagining and understanding relevant constructs. It has also been highlighted that in a balanced mathematics program, the strategic use of technology strengthens mathematics learning and teaching (Dick & Hollebrands, 2011).

There have been studies on a number of technology tools available, including calculators, Geometer Sketchpads and GeoGebra resources, such as those by Laborde, Kynigos, Hollebrands and Strasser who said:

*“Research on the use of technology in geometry not only offered a window on students’ mathematical conceptions of notions such as angle, quadrilaterals, transformations, but also showed that technology contributes to the construction of other views of these concepts.” (Laborde et al., 2006, pp. 275-304)*

Laborde et al., (2006) further argue that:

*“Research gave evidence of the research and the progress in student conceptualisation due to geometrical activities (such as construction activities or proof activities) making use of technology with the design of adequate tasks and pedagogical organisation. Technology revealed how much the tools shape the mathematical activity and led researchers to revisit the epistemology of geometry” (Laborde et al., 2006, pp. 275-304).*

In addressing the issues by Laborde et al. (2006), the current study explores the use of GeoGebra software to learn Euclidean geometry, especially circle geometry in mathematics. Among other objectives the research primarily endeavours to address application of GeoGebra in Euclidean geometry by learners. It also seeks to explore how the use of GeoGebra improves learners’ understanding of geometry theorems. And lastly the practical and theoretical implications of GeoGebra on: Teachers’ confidence in teaching geometry; learners’ performance improvements as well as justifying proofs and theorems of geometry.

## **1.2 BACKGROUND**

Geometry (originally from Greek word, geo = earth; metria = measure) arose as the field of knowledge dealing with spatial relationships (Luneta, 2014). Geometry was one of the two fields of pre-modern mathematics, the other being the study of numbers (arithmetic) (Dani, 2009 and Eves, 1990). Geometry was revolutionized by the Greek mathematics Euclid, who introduced mathematical rigor about the axiomatic methods still in use today.

Shuttleworth (2010) said Euclidean entered as one of the greatest of all mathematicians and he is often referred to as the father of geometry. The standard geometry mostly taught in school is Euclidean geometry. Euclidean geometry is sometimes termed to be “the Elements” from his famous book. Euclid based his approach upon 10 axioms (statements) that could be accepted as truths, as a results he termed postulates. He divided them in two groups of five. The first group common to all mathematics, the second group is specific to geometry. Though some of these postulates are self-explanatory, but Euclid operated upon principle that no axiom could be accepted without proof.

Hielbert (2013) sufficiently gave precise axiomatisation into;

#### A. The Common Notions

- Things which are equal to the same thing are also equal to each other.
- If equals are added to equals, then the results are equal.
- If equals are subtracted from equals, the remainders are equal.
- Things that coincide with each other are equal to each other.
- The whole is greater that the part.

#### B. The Specifically Related to Geometry

- A straight line can be drawn between any two points.
- Any finite straight line can be extended indefinitely in a straight line.
- For any line segment, a circle can be drawn using the line segment as
- The radius and one endpoint as the centre.
- All right angles are congruent (the same).
- If a straight line falling across two other straight lines in the sum of the angles on the same side less than two right angles, then the two straight lines, if extended indefinitely, meet on the same side as the side where the angles sums are less than two right angles.

Euclid included common words points and lines to cover up semantic errors. As a result he built the theory of plane geometry that has shaped mathematics, science and philosophy. Euclid was so influential to Newton and Descartes with his structure and format, moving from simple first principle to complicated concepts.

Pythagonacci.com (2010) pointed that Euclidean geometry is divided into two-dimensional otherwise called plane geometry which deals with figures in a plane like circle, line and polygons, and three-dimensional solid geometry that deals with solids concerned with polyhedrons, spheres; lines in three space.

According to Coetzee (2012) many teachers still do not know how to work with concrete (body-nobody), representative and abstract concepts to teach mathematical concepts. Orlov (2005) stated that every learner needs to develop formal thinking to enter formal operational stage (phase) of Piaget. Mainly the problems in teaching and learning Euclidean geometry are teaching Euclidean geometry theorems' proofs by heart. New methods or strategies of teaching and learning Euclidean geometry to the ability of individual learners as agreed by de Villiers (2009), the use of Sketchpad and GeoGebra, thus more visualization helps accommodate both average and less average learners. Ndlovu, Wessels and de Villiers (2013) in their research highlighted that the teacher competence has been a key issue in mathematics education reform as a quality of an education system is fundamentally defined by the quality of its teachers when using software in teaching geometry.

And furthermore Ndlovu (2014) highlighted the basic understanding of dynamic geometry definitions is a challenge to educators thus scaffolding is imminent. Technology is so fascinating even to teachers. As Chan, Kim and Tan (2010) founded out that more than 90% of teachers use software primarily for societal networking and expedient information retrieval. Jones (2009) clear stated that the embedding of technology into mathematics teaching is known to be a complex process, yet GeoGebra, the opened-source dynamic mathematics software that incorporates geometry and algebra into single package, is proving popular with teachers. The problem is solely having access to such technology can be insufficient for successful integration of technology into teaching and learning.

GeoGebra and Sketchpad illustrate how learners can move from empirical, visual description of spatial relations to a more theoretical abstract one (Sinclair and Jones, 2009). The arguments by learners during the lesson transcend empirical arguments, providing evidence of how young learners can be capable of engaging in aspects of deductive argumentation. Spatial visualization is an important skill that deserves further instructional attention. Christou, Sendova, Matos, Jones, Zacharides, Pitta-

Pantazi, Monsonlides, Pittalis, Boytchev, Mesquita, Chehlorova and Lozanov (2007) stated that one way to improve learners' spatial visualization and reasoning abilities is to provide learning activities that exploit the possibilities of exploring the properties of 3D objects in appropriately developed dynamic and interactive computer applications. That is where again GeoGebra comes on board.

Department of Basic Education (2011) clearly stated that a child's formative study of geometry in school is quite heavily centered towards Euclidean geometry which consists of understanding the patterns and properties of shape. It could be argued that learners have some knowledge of the connectionist properties of Euclidean geometry. In learning geometry learners seem to develop from pure and synthetic geometry (Euclidean), but need to have an understanding of algebra to attain more sophisticated levels of analytical (algebraic geometry) (Curran, 2013).

Fulton (2013) said many learners hit the geometry wall in high school and their mathematical journey ends. This wall often prevents them continuing in mathematics courses and having a successful transition to college. There are various reasons for this difficulty. In the first instance it may be due to many textbooks and multiple district pacing guides that emphasize numeracy arithmetic and algebraic reasoning. Geometry is often tucked into the third term and as last chapters of books. Geometric thinking needs development and understanding as stipulated by Van Hiele's levels (Mason, 2014). These are sequential to geometric thinking. It is not a matter of cognitive development dependent upon age, moving from one level to the next higher one, but rather hinges upon exposure to these geometric experiences.

Katzman (2014) stated that a computer can really help in geometry teaching, since it allows a dynamic interactive manipulation of figures. A learner can move, relate or stretch the figure, and observe what properties stay the same. It is important that mathematics teachers help learners to understand and construct proofs, and this will make easier for the learners to apply these proofs on their own.

Formal logic does not come naturally to most people. In high school geometry, learners are only given a crash course in logic. Learners still develop strong, logical basics of proofs. Another reason for learners to struggle in geometry is that in geometry there are multiple ways to prove something. And too in geometry there is also no algorithm for doing proofs. Each problem is unique and must be thought

through carefully. Unlike in algebra where one can more or less reverse order of operations to solve an equation, there is no one specific way to do a proof in geometry. Mthembu (2007) cited a need for a shift from the traditional way of teaching to an outcomes-based education system, as recommended by the National Curriculum Statement.

In AMESA (Association for Mathematics Education of South Africa) News 53 (2013) it was agreed that for Euclidean geometry to be well understood by learners there is a need to use programs such as Sketchpad and GeoGebra by both teachers and learners. But Van Putten, Howie and Stols (2010) in their recent research on the preparedness of teachers, did notice a slight improvement in teachers' attitudes but still there is no sufficient improvement in their level of understanding to teach Euclidean geometry.

As stated GeoGebra can be freely downloaded via internet and it is relatively easy to use even by novice teacher and learner. The biggest advantage of using GeoGebra it is more learner-centred as learners can grapple with the information and can assess his or her own work. And once you have it, it can be used for various mathematics topics like transformation in algebra and trigonometry. GeoGebra it is more advantageous to use in the geometry of the circle because the mouse could be used to drag the vertices and to create more special cases. GeoGebra will measure the segments immediately and also update any calculations (Stols, 2012). Guven (2012) suggests that GeoGebra provides environment in which learners can experiment freely and learners can easily test their intuitions and conjectures in the process of looking for patterns. Also concurred by Stols and Kriek (2011), the inductive nature of the GeoGebra gives learners an opportunity to learn Euclidean geometry via explorations that promote their conjecturing process.

The researcher chose UMkhanyakude district as it has been lagging behind for sometimes in terms of results in the whole KwaZulu-Natal province. Grade 11 was chosen because it is the backbone for the Grade 12 results and again that is where these theorems of the geometry of the circle are treated, the expectations are always neglected to take Grade 11 with high caution.

Venema (2013) studied learners' utilization of the dynamic geometry program GeoGebra to explore many of the interesting theorems. Webber (2013) noted that

proof is a notoriously difficult mathematical concept for learners. Okazaki (2008) and Rollick (2009) found out in their studies that learners have difficulties in understanding the Euclidean geometry concepts and their applications. Furthermore, some studies have indicated a positive effect on using dynamic geometry programs on learners' problem solving and posing (Christou et al, 2007), and discovering and conjecturing (Habre, 2014).

The demise of Euclidean geometry is that it has been subsumed in algebra. Way (2014) suggested that despite some natural development of spatial thinking, deliberate instructions needed to move learners through several levels of geometric understanding and reasoning skills. Way (2014) further suggested that it is inappropriate to teach learners Euclidean geometry following the same logical construction of axioms, definitions and proofs. Learners do not think on a formal deductive level, and therefore can duly memorize geometric facts and rules but not understand relationships between the ideas, if taught following this approach. Then it is imminent that professional development for teachers to juggle between technology and constructivism in their advancement of Euclidean geometry to learners.

This current study sought to drill more on proofs, integration of the other types of geometry and geometry of the circle. These are fundamentals to the teaching and learning of Euclidean geometry. The literature tried to find insight in learners' taking of Euclidean geometry.

### **1.3 PROBLEM STATEMENT AND DELIMITATION OF THE STUDY**

Geometry particular Euclidean section puts a lot of bad stains in mathematics results in South African rural public high school learners. Learners tend to struggle a lot in proofs (they mix statements), and too to the application of theorems in conjunction with the previous learned materials from lower grades. It has been proven that when learners try to correct and fail to recall concepts accordingly they become demoralized. Teachers' applications of modern or traditional teaching approach are not helping either and it has been envisaged that teachers' attitudes and their beliefs of how they were taught has a negative impact on learners grasping of the geometry of the circle. As a result teachers and learners fail to apply the theorems correctly. Again the lack of basic ideas into how Euclidean geometry will impact on learners' future careers makes learning Euclidean geometry meaningless to learners.



Also the fact that Analytical geometry has a mixture of Euclidean geometry makes learners fail to distinguish between the two and thus bring more misconceptions.

There is an influx of new teachers in grades 10, 11 and 12. Some were not trained to be teachers and those who were trained to be teachers, were never exposed to the geometry of the circle both at high school and at tertiary level as it was phased out in the main curriculum plan. This is an inadequate preparation leading to learners struggling in Euclidean geometry.

This is the situation that the current research sought (i) to determine the degree of impact of GeoGebra on learners' performance in Euclidean geometry, (ii) to explore the understanding of GeoGebra software amongst rural learners in Euclidean geometry class, (iii) to explore the understanding of GeoGebra software amongst rural teachers in Euclidean geometry class and (iv) to what extent do the challenges of GeoGebra application impede on the performance of learners.

The focus of the study will be on learners in grade 11 mathematics in the UMkhanyakude district of KwaZulu-Natal province. Mathematics Curriculum and Assessment Policy Statements (CAPS) (DOE, 2011) has reintroduced the geometry of the circle in grade 11 learners, so the researcher saw it fit to come up with such study to address this in an innovative learner-centred way.

#### **1.4 RESEARCH QUESTIONS, AIMS AND THE IMPORTANCE OF THE STUDY**

Regarding the background and problem statement, the following research questions came out;

1. How does application of GeoGebra in Euclidean geometry impact learners' performance?
2. How does the use of GeoGebra improve learners' understanding of geometry theorems?
3. What are practical and theoretical implications of GeoGebra on:
  - 3.1 Teachers' confidence in teaching geometry?
  - 3.2 Learners' performance improvements?
  - 3.3 Justifying proofs and theorems of circle geometry?

Aims of the study:

- Providing guidance, materials and resources that will harness the power of technology so that learners will be better able to understand and apply mathematics.
- Engaging mathematics teachers in mathematical problem solving.
- Establishing a community that promotes an ongoing conversation about mathematics with learners, teachers, and professional mathematicians.
- Assisting educators in enhancing facilitation that is in line with trends of comprehensive circle geometry teaching

The study intended to drive learners' zest into their multiple careers with its advancement, adequacy and appropriation, thus South Africa, most especially rural areas will be catapulted at another level in science relatively stream. Again the study was mostly intended to assist Basic Department of Education to align teaching and learning processes to help both learners in Grade 11 and their teachers in addressing Euclidean geometry that has been reintroduced into curriculum. And too the study sought to help Higher Education Department to include courses that will enhance novice teachers to come prepared in the working environment. Curriculum planners will be alleviated into inclusion of technology aspects. The study sought to help Grade 11 learners most especially from rural areas the importance of studying Euclidean geometry for their future.

There have been numerous reasons for the aforementioned assertion. They include but are not limited to: poor preparation of primary classes in geometry and weak performance of learners who often struggle here, even highlighting generational stories regarding the perceived difficulty of geometry, let alone circle geometry. Thus, it is likely that both teachers and students might develop negative attitudes and stigmas toward Euclidean geometry even before it is taught.

In responding to this crisis, the South African government has organised workshops to improve mathematics teachers' skills cited Department of Basic Education (DBE) where she states:

*"We have already established the Mathematics, Science, and Technology (MST) office, with its main job being to drive the MST curriculum and assessment*

*targets in the sector. The national MST institute and nine provincial institutes will in the next five years increase outputs in order to turnaround MST by 2019.” (DBE, 2015, p. 16-56)*

DBE further argued that they have equipped and connected new teacher centres throughout the country for improvement in digital learning. As the head of department of mathematics and science in three schools, the researcher has witnessed situations where there are insufficient suitably qualified mathematics educators. In the workshops, dire situations arose where teachers confessed that they spend little of their time on Euclidean geometry because they said they grossly lack knowledge on this topic. Novice teachers even said it was highly complex, while experienced teachers stated that they started teaching mathematics after circle geometry was phased out of the curriculum. These confirm the shortage of suitably qualified and knowledgeable teachers, which can reflect on other areas too, and ultimately prejudice learners, including potentially some of tomorrow's stop mathematicians.

The current study explored how best GeoGebra can be used to improve learning of Euclidean geometry. The researcher chose the use of technology as it is appealing to young, curious minds, especially in today's tech-savvy times. It would be fascinating to further ascertain the level of satisfaction and learning benefits students enjoy during the application of GeoGebra software.

Underachievement by learners in mathematics in South Africa is a great concern and various authors have emphasised this issue. South Africa's score according to Global Information Report 2014, on Quality of Mathematics and Science is at 1.9 out of 10. This means the country needs drastic changes when it comes to the learning and teaching of these subjects. Only 19% of students are reportedly capable of mastering mathematics and science in South Africa. "South Africa low performance in mathematics, no matter the report" revealed by Bates (2014). According to the TIMSS report, almost two-thirds of mathematics learners and half of science learners in 2011 were taught by teachers who had not completed a university degree. But the report by Human Science Research Council (HSRC) revealed though South Africa ranked at the bottom, there were notable and progressive improvements in mathematics. And more learners are currently being taught by teachers with

university degrees. This observation also prompted the researcher to conduct this investigation.

Different studies revealed that high school students struggle in mathematics in that students may be mentally distracted focusing on multistep problems and procedures (Sherman, Richardson, & Yard, 2010), and effective teachers should employ the use of attention-grabbers such as drawings and learning aids. This again pointed to the relevance of this study.

The current research used quasi-experimental design, where Grade 11 only science learners were subdivided into control and treatment. The treatment group was the one administered using GeoGebra software after the first session, whereas all others continued to be taught using the traditional method. Control group was taught by the use of traditional method for the entire study duration. Achievement tests and questionnaires were data collection tools.

Further this chapter sought to reveal the motivation yielding the commencement of the current study, while also highlighting the research methodology used.

## **1.5 DATA COLLECTION METHOD**

### **1.5.1 Participants**

The participants in the study with were 112 Grade 11 learners ranging in age from 16 to 20 years or older, from five different classes of an Euclidean geometry (The geometry of the circle) in five rural high schools in UMkhanyakude District, KwaZulu-Natal province, RSA. There were 56 learners in the experimental group, classes which were taught using GeoGeBra and the remaining 56 learners in the control group, which were taught by the use of dotted worksheets. A quasi-experimental was used to situate the study in five schools that are well equipped in terms of computer laboratories and technological devices.

The research was carried out during the spring semester of 2014 academic year. The classes were randomly assigned as either experimental or control groups. Even though these was Grade 11 work some of the work learners have dealt with in grade 8, 9 and 10 (triangles and quadrilaterals).

### **1.5.2 Instruments**

The Euclidean Geometry Achievement Test (EGAT) and questionnaire were used as data collection instruments in this research.

#### **Euclidean Geometry Achievement Test (EGAT)**

This was a multiple choice Geometry achievement test, initially consisting of 20 questions covering, calculation and application of information. It was designed by the researcher, to measure learners' learning the geometry of the circle. The test was based on the prescribed learners textbooks and teacher guides for grade 8, 9, 10 and grade 11 distributed by National Department of Basic Education in line with the CAPS document. The researcher designed the test taking into consideration learners' levels, learners' achievement in the pre-Euclidean learning domain, and the goals of the study. The completed questions were then moderated by two to three teachers with more than 15 years of teaching Mathematics geometry and revised in accordance with their feedback. In its final version, the test was administered as a pre- and post- test before and after the study.

A pilot study using EGAT was conducted from 21 July 2014 to the 8th August with 43 Grade 11 learners in the same schools. In consideration of the results of the pilot study checked on study's reliability, items were modified according to the strength of the reliability from the EGAT. Again the researcher will be very cautious of content validity. The best reliability coefficient will be chosen accordingly in this case Spearman- Brown will be used. The test questions will be stratified to cover the desired measurements of the geometry of the circle.

After the study there will be new pilot study with open-ended questions to determine learners' learning of the circle of geometry with 20 learners chosen from learners who were participant on the study.

The actual study then commenced on the 18th August 2014 to 5th September 2014. One week after the study, clinical interviews with the same open-ended questions were administered on the same learners. The researcher's questions will stick to "why" and "how" questions. These two sets of results were compared to check on the consistence.

### **1.5.3 PROCEDURES**

#### **1.5.3.1 Treatment of the experimental group**

Before the treatment, learners in the GeoGebra group (experimental), were trained on how to draw line segment, and measure angles etc. The teacher spent five hours drilling learners into using a GeoGebra with no application on the geometry of the circle with learners. It was mainly on technical characteristics and basic uses of the software that will be summarized. The researcher then administered pretest as precautions to guide against shambles. The learners received lessons with the features of GeoGebra. Learners then studied the geometry of the circle on the computer independently from given worksheets. In worksheets the theorems were presented but learners had to study on their own and drew conclusions in groups and individually.

Observation results were written by learners on the worksheets. Group were allowed to cater for technical problems and this could enhance collaborative and cooperative group work. While learners were applying their news concepts, learners learned

- (a) Step by step constructing and measuring of angles
- (b) Dynamic observation of similarity of angles
- (c) Exploring the characteristics in line with theorems
- (d) Discussion of the observed results in the classroom
- (e) Making judgments and aligning the theorems in worksheets

GeoGebra was the main instrument to explore characteristics of the geometry of the circle, test the discovered characteristics and observe the applications of geometry of the circle. Learners were given more materials to revise and check on the computer screen. The researcher acted as a facilitator by advancing healthy classroom discussion.

#### **1.5.3.2 Treatment of the control group**

Learners mainly used dotted worksheets depending on the theorems to be addressed. The material taught in this group was the same to that in the

experimental group. All activities were carried out interactively in the computer environment by the experimental group will be completed by the control group using pencil and paper.

The researcher was in charge of both experimental and control groups to prevent the researcher as a main source of possible differences between academic achievements and comprehension levels of groups. The researcher has an 18 year experience in mathematics teaching and has degree in mathematics education. The researcher has been trained in the use of GeoGebra and other technological education material

#### **1.5.4 Data Analysis**

After the data was collected with the instruments of academic achievement test on Euclidean geometry and questionnaires, MathPortal.org software was used for statistical analysis. Furthermore, the researcher will use frequency distribution (histogram) as a data analysis technique. Frequency distribution showed how frequently the specific values (practical and theoretical assistances of application GeoGebra in improvement of teaching and learning geometry of the circle) and what their percentages are for the same variable.

#### **1.6 ETHICAL OBLIGATIONS**

The researcher followed among others, the code of ethics and principles as suggested by Hamersley and Traianou (2012). Those are the following Harm: The researcher requested the permission from KwaZulu-Natal Department of Education for using the five schools in the UMkhanyakude district in the form of informed consent letters. The school principals and various participants were drilled first on the purpose of the research in which to outline voluntary participation **Autonomy**: Through workshop participants were informed that they have a right to deny or participate, and that no participant(s) will be forced. The researcher stated that no reward must be expected from their involvement in the study.

**Privacy**: A healthy environment was chosen to conduct all elements of the study that suited comfort, privacy and confidentiality of all participants. The researcher assured the participants that pseudo names will be used to conform to privacy and anonymity obeying international research trends.

**Reciprocity:** The researcher ensured that the data was accessible and available for interviewees, even filled up questionnaire but under the jurisdiction of the supervisor.

**Equity:** The researcher ensured gender related matters are observed at all cost. And no participants or stakeholders to feel his or her right(s) jeopardized by the study in progress.

## **1.17 CONCLUSION**

The study was set out to explore the concept of circle geometry using dynamic geometry software GeoGebra, and has identified the nature and form exploration in mathematics in UMkhanyakude district, KwaZulu-Natal, South Africa, the reasons and motivation for utilising technology, and the role and impact of intervention on rural students and hence their performances. The study has also sought to find out the effectiveness of GeoGebra software in the mathematics results improvement particularly in Euclidean geometry. The study sought to answer the following questions:

- a) How does application of GeoGebra in Euclidean geometry impact learners' performance?
- b) How does the use of GeoGebra improve learners' understanding of geometry theorems?
- c) What are practical and theoretical implications of GeoGebra on:
  - i. Teachers' confidence in teaching geometry?
  - ii. Learners' performance improvements?
  - iii. Justifying proofs and theorems of circle geometry?



## **CHAPTER TWO**

### **LITERATURE REVIEW AND THEORETICAL FRAMEWORK**

#### **2.1 LITERATURE REVIEW**

##### **2.1.1 Introduction**

Technology integration in the teaching and learning process mostly in the postmodern era has attracted a lot of attention. There is an urgency to incorporate technology in the Euclidean geometry classroom as it provides a rich learning environment that promotes social interaction, critical thinking skills and comprehensive understanding of students' learning experiences (Nikoloudakis & Dimakos, 2009; Shadaan & Leong, 2010).

The literature revealed that a considerable number of studies has been conducted on GeoGebra on geometry, but not specifically on Grade 11 rural high school learners of UMkhanyakude District, KwaZulu-Natal province, South Africa. In addressing questions 1, 2 and 3.3, the researcher sought to reveal the impact of GeoGebra tools in Euclidean geometry to enhance students' performance and conceptual understanding of theorems.

For questions 3.1 and 3.2, the study dwelt on different modes in which teachers can use GeoGebra as a tool either for teaching and researching, as a checking or testing tool as well as to verify thinking or as a demonstration tool. The chapter will also address theories that can improve cognitive and learning attitudes toward geometry.

The researcher intended to describe in detail challenges in: teaching and learning Euclidean geometry; inclusion of Euclidean geometry as compulsory exit examination section for high school students; GeoGebra on proof and verification in mathematics geometry section; dynamic geometry software; students' performance in geometry; on meaning of Euclidean geometry; nature and scope of GeoGebra software; GeoGebra impacting on individual learning; GeoGebra for model-centred learning; GeoGebra use on problem solving and attitude change; uniqueness of

GeoGebra; hindrances of GeoGebra in mathematics classroom; information processing theory; and lastly on cognitive neuroscience and GeoGebra application.

Furthermore, the current research intended to integrate students' spatial visualisation that might be evidenced, through understanding of analytical knowledge to synthetic knowledge, which is highly viable in Euclidean geometry through the use of GeoGebra. This will thereafter improve core understanding of proofs and theorems and ultimately improve the performance.

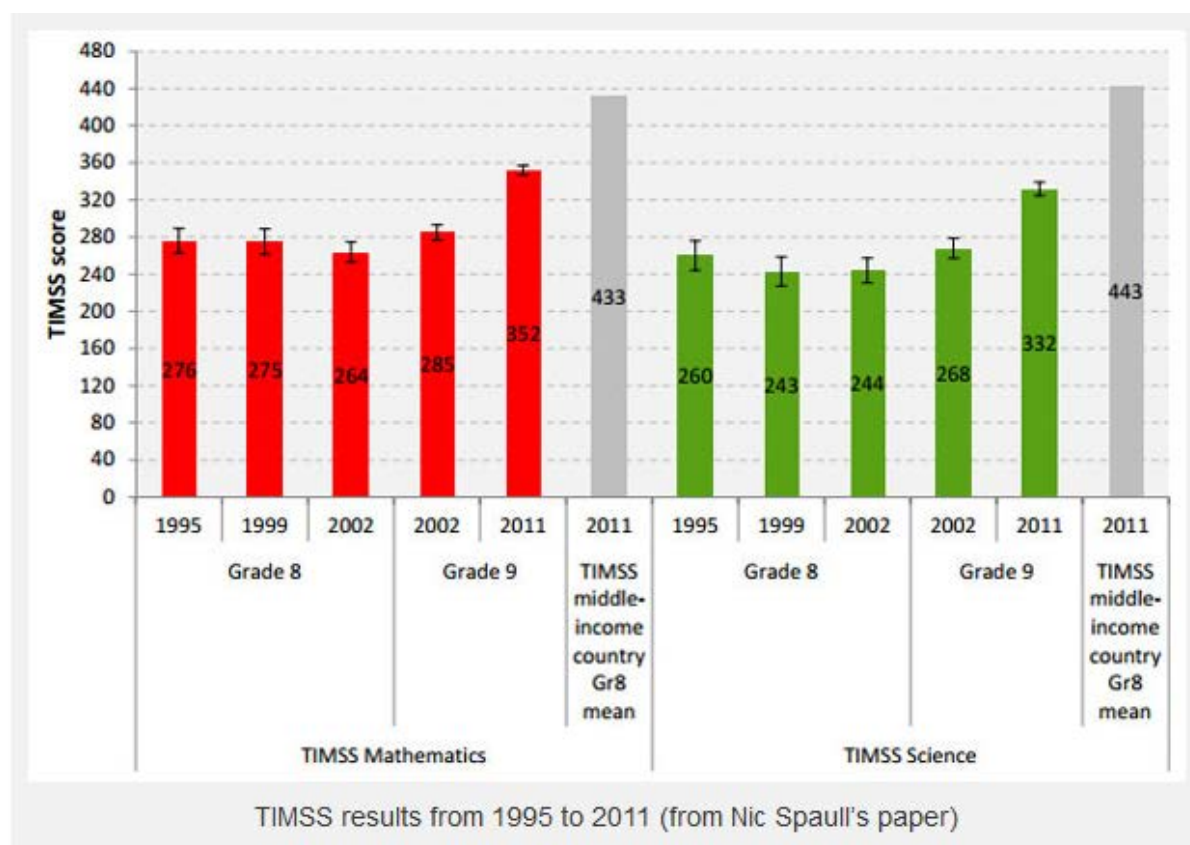
## **2.1.2 Inclusion of Euclidean Geometry**

Curriculum and Assessment Policy Statement (CAPS, 2010) and Grade 11 mathematics pace setter state that the class of 2014, Grade 11, were expected to grasp and grapple at the geometry of the circle within the first three weeks of the 3<sup>rd</sup> term (approximately 15 hours), and accept results established in earlier grades as axioms. There are four examinable theorems as proofs and students have to master the remaining ones especially their applications in both their year and during exit year, being Grade 12 (Department of Education, 2010 & 2014).

The Department of Basic Education is concerned because of uncertainty results on teachers teaching Euclidean geometry, do not mean that there was improvement. Rural mathematics teachers should be supported by means of other strategies, like bringing technology into the fold. Furthermore, teachers are reluctant to teach in rural areas. So why not develop the existing teachers in terms of using technology in their daily teaching practices?

Although geometry provides numerous benefits, students in South Africa particularly in rural schools do not like geometry related topics and eventually most fail. According to the Trends in International Mathematics and Science Study (TIMSS) results of 2011 South Africa after long participation is ranked bottom, 146<sup>th</sup> out of 184<sup>th</sup> countries. Using findings of the TIMSS report, the World Economic Forum (WEF) declared that South Africa had the worst mathematics and science education in the world. Although these may be disputed because of better performance of students in urban schools, it should be noted that these apply to former model C schools, whose average resembles top performing nations and places such as Singapore, Taipei, Hong Kong, Japan and Korea Republic according to the

arguments by DBE (2012) report. However the majority of students are in public schools and mostly in rural areas (72%) composed to those in urban areas (28%), which tells that much has to be done to improve mathematics and science education nationally (DBE, 2011). Even though Grade 9 students wrote a Grade 8 paper at international level, there was also only an unconvincing, slight improvement.



**Figure 2.1 RSA TIMSS results ranging from 1995 – 2011**

This might enhance the outlook in terms of how South Africa has been doing since their entrance in TIMSS in 1995 as illustrated in Figure 2.1, and Tables 2.1 and 2.2. Further reports for Southern and East Africa have nevertheless pitted South Africa as the lowest country in mathematics performance. To date, South Africa has participated in four TIMSS assessments: 1995, 1999, 2002 and 2011. Table 2.1 indicates that there is less improvement or none at all for mathematics, even if Grade 9's were administered with Grade 8 tests. The reality is that all stakeholders need to take a giant step towards uplifting the pupils who can compete with the first world students in mathematics in future. Table 2.1 clearly illustrates where South Africa stands in terms of mathematics performance out of 146 countries internationally. Furthermore, according to the TIMSS report almost two-thirds of mathematics

students, and half of science in 2011, were taught by teachers who had not finished a university degree (ENA / Erin Bates, 2011).

School experts in South Africa have warned that the matric results could sharply drop as many Grade 12 teachers, more especially in state schools, struggle to both introduce and prepare students for compulsory sections like Euclidean geometry and Probability (Jensen & Dardagan, 2014).

One of the other reasons for Euclidean geometry failure, stems from teachers directing students towards memorising during the period designed for acquiring geometric knowledge and skills through understanding (Kondratieva, 2011). Kutluca (2013) further highlighted condensed existence of geometry topics in the curriculum as another reason for the failure in Euclidean geometry.

**Table 2.1 TIMSS Mathematics Achievement**

<b>Country</b>	<b>Average Scale score</b>	<b>SE</b>
<b>Botswana</b>	<b>397</b>	<b>2.5</b>
<b>South Africa</b>	<b>352</b>	<b>2.5</b>
<b>Honduras</b>	<b>338</b>	<b>3.7</b>

In South Africa, most of geometry topics are in the third or even fourth term of the academic year. The problems faced in South Africa are that various instructional materials should be developed and applied in geometry instruction.

Darling-Hammond, Boron, Pearson, Schoenfeld, Stage, Zimmerman, Cervetti, Tilson and Chen (2008) indicate there is growing evidence that teacher preparation is a powerful predictor of students' achievement, perhaps even overcoming socioeconomic and language background factors.

In South Africa, Euclidean geometry has had some impact on students' poor performance (KwaZulu-Natal Basic Department of Education, 2012) as illustrated by table 2.2. The table clearly shows that students were performing in Euclidean geometry way below 30%, prior to its scrapping out of the main curriculum.

Visualisation is core to the discovery process in geometry and manipulating spatial images. Multiple representations increase rich conceptual understanding. Using

GeoGebra, however, cannot offer substitution for ineffective mathematics teachers. It is primarily intended to supplement good and effective teaching as depicted in Japan, where the blackboard is still used positively.

Table 2.2 Statistics on Euclidean Geometry Performance, RSA 2001 – 2007

YEAR	% Higher Grade (HG)	% Standard Grade (SG)	% HG & % SG
2001	4.3%	16.1%	20.4%
2002	4.6%	22.8%	27.4%
2003	5.3%	23.8%	29.1%
2004	5.2%	23.4%	28.6%
2005	5.2%	22.1%	27.3%
2006	4.8%	20.9%	25.7%
2007	4.5%	21.9%	26.4%

However young minds, most especially in rural areas, are used to the opposite, despite GeoGebra being such a powerful tool that could supplement knowledge and content in classrooms. The researcher agrees with Stols' view that in underperforming schools, though GeoGebra can be used, contact lessons shouldn't be neglected, for best results (Stols, 2012). However, to maximise use of GeoGebra, applications must be well aligned with NCS and CAPS ideologies. The teacher's role as a facilitator, should thus always guide students and serve to help clarify misconceptions.

Even in rural areas students cannot imagine a world without a remote control or a mouse. Students are highly connected and technologically eager and see innovation as an essential part of their lives, and thrive on an appropriate digital diet. Students' expectations include flexibility, self-discovery, instant feedback, collaborative learning and a digital approach that, incidentally, is highly embedded within GeoGebra software. It prepares students for their higher education and in turn for their careers in future (Olivier, 2014). Technology is the driving pedagogy because of its increasing portability and convenience. Burger (2014) indicated that students who

have been exposed to the use of mobile technology still do not want to do away with teachers. They want to have a relationship in which their teachers awaken their curiosity and embrace our technological era.

Spector (2004) stated that new tools and technology software should be used in ways that support what is known, to boost understanding of key concepts. In any event, students do actually create internal representations to make sense of new experiences and puzzling phenomena. The importance of internal representations is for the development of critical reasoning skills required in mathematics, particularly in Euclidean geometry.

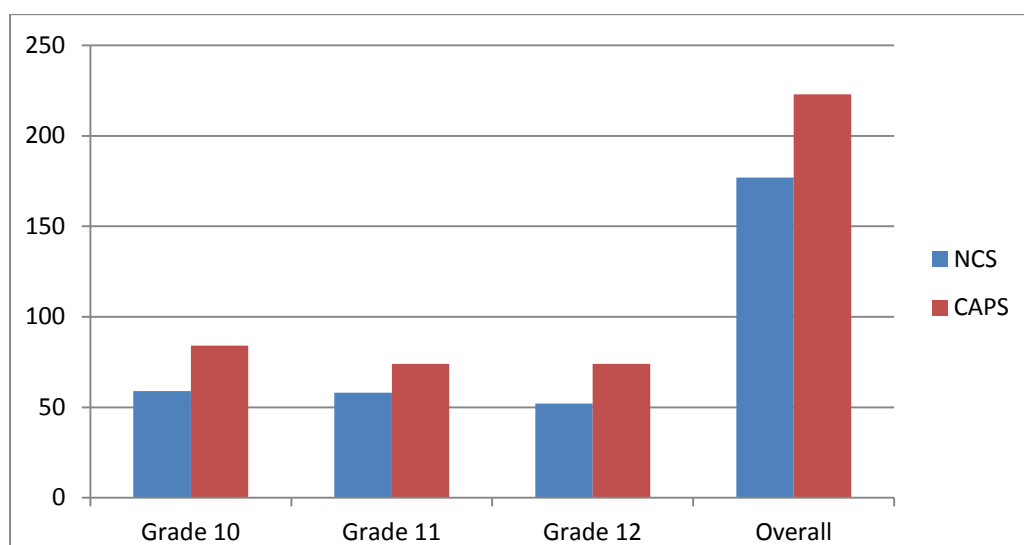
The availability of projection technology has increased the usage of dynamic representations in teaching and learning mathematics. Sinclair and Yurita (2008) cited that students were not previously exposed to exploring geometric constructions in a Dynamic Geometry Environment (DGE). Multiple reports noted that the integration of dynamic geometry software, such as GeoGebra into the teaching and learning of Euclidean systems, is more effective than the traditional approach in stimulating students' mathematical thinking skills (Idris, 2009; Dimakos & Zaranis, 2010; Chew & Lim, 2013). According to Hacıomeroglu and Andreasen (2013) dynamic software improves students' understanding of mathematics concepts as they explore more and form conjectures. Technology with its structural dynamism allows students to engage with visual representations of geometric structures and gives students opportunities to discover constraints, abstracts as well as construct their own structures. Goldenberg (1999) stated that visual media contributes to students geometry achievement and facilitates their active involvement.

The inclusion of Euclidean geometry as a compulsory section requires rigorous attention in respect of the performance of learners. This study seeks to find an alternative way of curbing poor understanding of geometry proofs and theorems which answers questions 1 and 2 of the study. GeoGebra with its attributes in terms of visualisation can assist in improving students' understanding of these mathematics concepts.

### **2.1.3 Umalusi on the NCS and CAPS**

The mathematics in CAPS is deemed to be significantly more demanding than the NCS, since the CAPS content exceeds that of the NCS in both breadth and depth.

Euclidean geometry tends to demand insight and involves an understanding of proof in theorem and riders. On the content base the subtopics of Euclidean geometry have increased from a total of 7 in NCS to 32 in CAPS. There was an increase of 8 subtopics for both Grades 10 and 11, from 6 subtopics to 15 subtopics and 1 subtopic to 11 subtopics respectively. This is too extensive to accommodate; see Table 2.3 and Figure 2.2.



**Figure 2.2: Number of subtopics per grade and overall**

Euclidean geometry and Probability are cognitively demanding sections which take time to teach and learn (NCS Technical report, 2014). This has increased the amount of work to be covered and as a result many teachers may omit certain subtopics or compromise the depth at which the subtopics are dealt with.

**Table 2.3: Content / skills coverage: No. of subtopics in NCS and CAPS**

Topic Content / Skills	National Curriculum Statement				Curriculum and Assessment Policy Statement			
	GR. 10	GR. 11	GR. 12	TOTS	GR. 10	GR. 11	GR. 12	TOTS
Euclidean geometry	6	1	0	7	15	11	6	32

The Figure 2.2 and Table 2.3 clearly show that the inclusion of Euclidean geometry in the mathematics curriculum could stunt the performance of relevant students, thus it is imperative for the study to unveil an alternative in alleviating the poor performance per research question 1. There were some aspects of Euclidean geometry in NCS, but that introduced in CAPS seems to be revised and with a higher level of demand, in common with Probability. This means that CAPS is significantly more demanding.

The overall mathematics time allocation has not been affected and stands at 4.5 hours per week. Amazingly the weighting of Algebra is lower than Euclidean geometry by percentage of marks that is 12.5%:16.3%. Meanwhile Algebra has more weighting by percentage of time, even though Algebra is deemed slightly demanding compared to highly demanding Euclidean geometry, which is 14.8%:12.5% (see Tables 2.4 and 2.5, and Figure 2.3). Trigonometry is deemed less demanding than Euclidean geometry and according to percentage allocation of marks, it is the same as Euclidean geometry, yet Trigonometry is given more teaching and learning time percentage than Euclidean geometry.

**Table 2.4: Weighting per topic by percentage of time**

Topic	CAPS (Percentage of marks)			
	Grade 10	Grade 11	Grade 12	Total time %
Algebra	22.6	18.8	0	14.8
Euclidean geometry	16.1	12.5	8.0	12.5

**Table 2.5: Weighting per topic by percentage of marks**

Topic	NCS (Percentage of marks)				CAPS (Percentage of marks)			
	Gr. 10	Gr. 11	Gr. 12	Totals	Gr.10	Gr. 11	Gr. 12	Totals
Algebra	12.5	8.3	6.7	8.8	15.0	15.0	8.3	12.5
Euclidean geometry	5.0	3.3	0	2.5	15.0	16.7	16.7	16.3
Trigonometry	12.5	16.7	20.0	16.9	20.0	16.7	13.3	16.3

This shows some flaws in the curriculum planning and the researcher believes that some reshuffling needs to be done in order for geometry, particularly Euclidean geometry, to yield proper improvement, per research questions 1 and 2. In the light

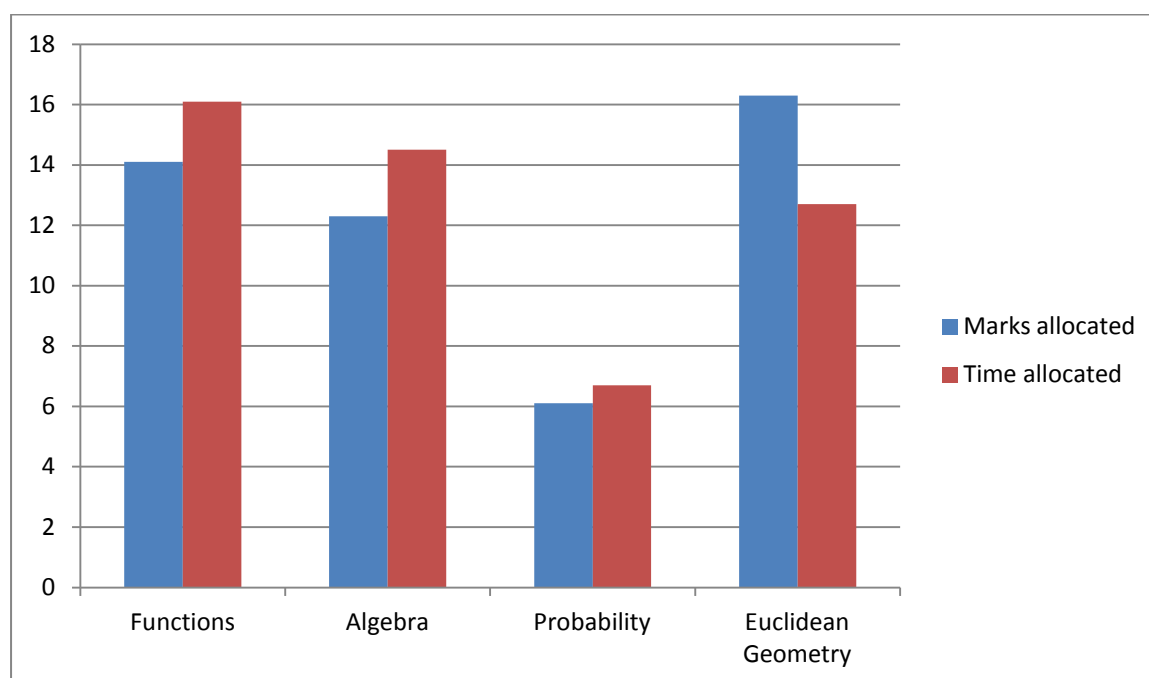


of these, the researcher proposes the application of GeoGebra to highly demanding topics (sections), starting with Euclidean geometry.

Table 2.5 shows that there are three topics weighted more heavily in the CAPS than in the NCS and these are highly demanding topics. In pacing, CAPS requires high and fast pacing as compared to NCS, and pedagogical approach to CAPS is more leaning to high cognitive demand.

At an Exit – Level Outcomes for Mathematics for FET on content / skills / competences in Euclidean geometry and Measurement (CAPS, 2012), students are expected to:

- Work with geometric definitions and deductive reasons to prove theorems and riders;
- Work with the geometry of triangles, similarity and proportionality in the study of the triangle deductively;
- Work with the geometry of quadrilaterals deductively;
- Work with the geometry of circles deductively;
- Understand congruency and similarity;
- Calculate perimeters, areas and volumes.



**Figure 2.3 Weighting: percentage of marks and time for each topic in CAPS**

This firmly concurs with the study questions outlined. The current study seeks to address the issues of exploring ways for learners to perform better in highly demanding topics, especially in Euclidean geometry which has relatively more marks allocated, but has less time dedicated to its teaching as compared to other topics (see Figure 2.3).

#### **2.1.4 Proof and Verification**

In the previous sections the focal point was on the challenges of Euclidean geometry to learners and its inclusion in the mainstream mathematics curriculum. In this section the researcher elucidates GeoGebra application to boost both teachers' and students' confidence, thus improving students' performance. The study examines what impact GeoGebra has on justification of proofs and theorems in Euclidean geometry, questions 2 and 3. The use of technological tools brings the possibility for different types of conceptualisations of mathematical objects that may help or hinder the processes involved in the development of proofs. Sanchez, Isabel and Miguel (2010) indicate that technology promotes and develops the 'functional language' that is very necessary for the construction of 'intellectual proofs'. This concurs with the current study where the main focus is on GeoGebra proving Euclidean geometry proofs and theorems, and circle geometry proofs and theorems in particular. It shows that technology for some decades has played a pivotal role in geometry proofs, making it easy to better understand theorems.

By GeoGebra application for example a simple triangle theorem can be seamlessly proven, see Figure 2.4. A proof is a theoretical confirmation that a statement (for example, "the bisector lines of a triangle are concurrent") is always true. In mathematics, a proof is a demonstration that, given certain axioms, some statement or interest is necessarily true. Formal verification is the act of proving or disproving the correctness of intended algorithms underlying a system, with respect to a certain formal specification or property, using formal methods of mathematics (Pavlovic & Stojanovic, 2008, pp. 9).

1. **Model checking** – a systematically exhaustive exploration of the mathematical model.

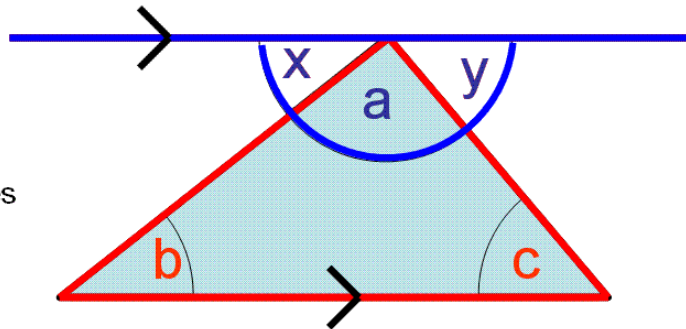
2. **Logical inference** – using a formal version of mathematical reasoning about the system (Pavlovic & Stojanovic, 2008, pp. 9).

## Geometric Proof

Prove that the sum of the angles in a triangle is  $180^\circ$ .

Draw a line parallel to one side.

Let  $x$  and  $y$  be the other two angles formed with the line.



Then  $x = b$  (alternate angles)  
and  $y = c$  (alternate angles)

We can also see that  $x + y + a = 180^\circ$ . (angles on a line)

Therefore,  $a + b + c = 180^\circ$ .

The question is can GeoGebra justify the proofs and theorems on Euclidean geometry?

Interior angles of a triangle add up to  $180^\circ$

$$58,69^\circ + 53,01^\circ + 68,3^\circ = 180^\circ$$

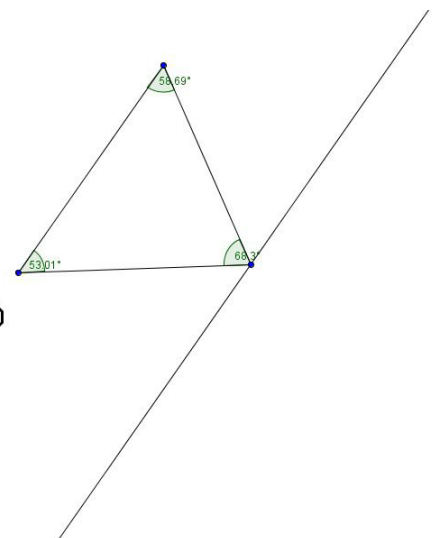


Figure 2.4: Proof and verification of interior angles of a triangle with GeoGebra application.

Jones (2012) indicated that students through GeoGebra can be highly engaged in activities where they create mathematical definitions and discover mathematical properties (Figure 2.4). An essential part of the process is that students must defend their ideas in a way that will convince their classmates that their definitions or properties are correct. As a result, the students realise the benefits of clear, logical proofs to meet their own needs and not merely fulfil a homework assignment. Shellhorn (2011) completely agreed by stating that free GeoGebra helps students understand constructive proofs in Euclidean geometry. GeoGebra helps in conjecturing, justification and thus verification about properties of geometric objects.

In the following section the researcher will interrogate Dynamic Geometry Software (DGS) on triangles, quadrilaterals and on circles.

### **2.1.5 Dynamic Geometry Software**

The first software to be developed was Geometer Supposer by Judah Schwartz and Michal Yerushalmy (Naftaliev & Yerushalmy, 2013). It contained three different programs; triangles, quadrilaterals and circles. Ruthven, Hennessy and Deaney (2008) in their report on Supposer found out that its key features was to create geometrical figures by repeating construction using different starting conditions. Supposer was not a dynamic environment but particular examples of a general case could be generated and geometric objects could be measured.

Interactive geometry software like CABRI Geometry and Geometers' Sketchpad (GSP) were developed independently around the same time (Bu & Schoen, 2011). The first free Dynamic Geometry Software (DGS) environment is WinGeon developed by Rick Parris with two versions: one for two dimensions (2D) and the other for three dimensions (3D) (Math Forum, 1994). The mentioned interactive software options, although useful in improving the understanding of geometry concepts, are not easily accessible to the learning community of UMkhanyakude District as they are not free. Thus, these can address the current study questions where learners outside school premises cannot afford such programs as they aren't free, as in the case of GeoGebra software.

The next section will look at how students fare in geometry, particularly in problems involving two-dimensional and three-dimensional figures.

### **2.1.6 Students' Performance in Geometry**

Geometry involves 2D and 3D problems and also forms part of coordinate geometry and trigonometry. National Basic Education report of 2011 cited students struggling in 2D and 3D, and that insufficient development in spatial perception, further showed that students lack deeper conceptual understanding. This is due to the traditional approach of mathematics teaching that has engulfed “stimulus-response” methods. Again when the traditional approach is used it leads to compartmentalisation and subsequently students not being able to integrate concepts into other topics. The reason for poor performance of students is linear justification, where students cannot reverse their thinking. The study through application GeoGebra, will drill students in working backwards, thus practising the process of reversing their thinking.

This is mostly required in learning by understanding of geometry theorems, duly addressing the study question 2: How does the use of GeoGebra improve learners' understanding of geometry theorems?; also question 1: How does application of GeoGebra in Euclidean geometry impact learners' performance?; and some parts of question 3: What are the practical and theoretical implications of GeoGebra on learners' performance improvements and justifying proofs and theorems of geometry?

### **2.1.7 Nature and Scope of GeoGebra Software**

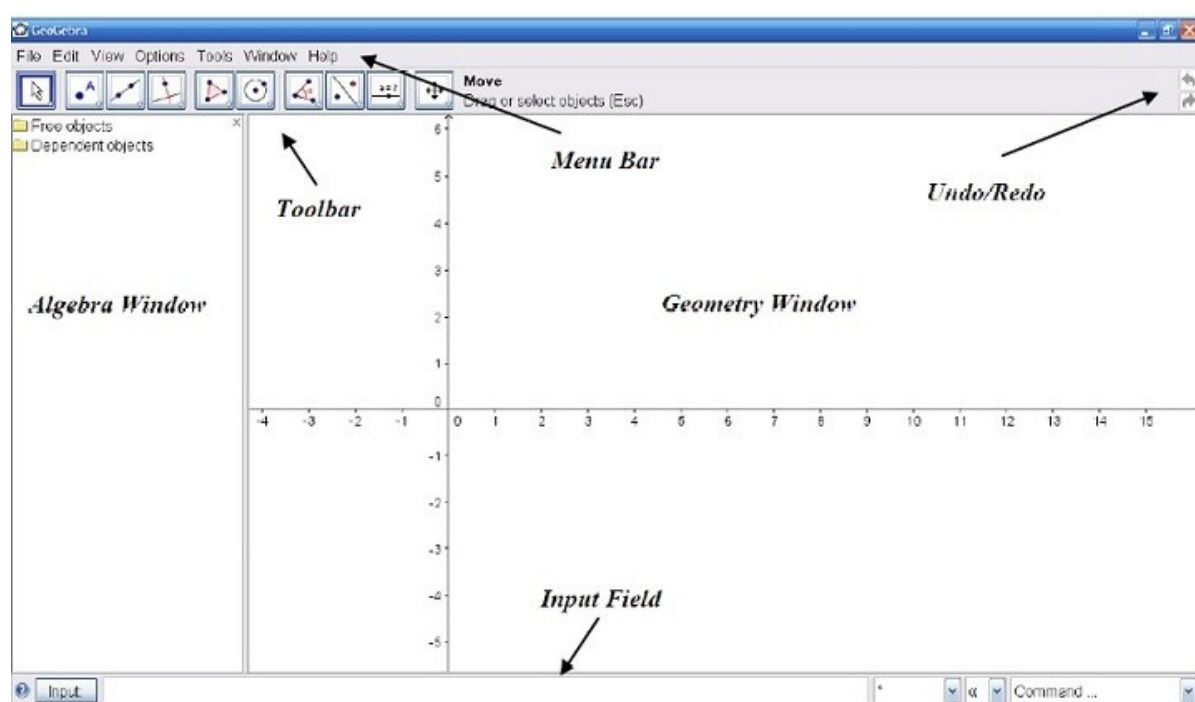
GeoGebra is an interactive geometry, algebra, statistics and calculus application, intended for learning and teaching mathematics and science from elementary school to university level. GeoGebra was designed by Marcus Hohenwarter as open source dynamic mathematics software that incorporates multiple mathematics trends into a single, open-source, user-friendly package (Hohenwarter, Jarvis & Lavicza, 2011). It combines features of older software programs such as Maple, Derive, Cabri and Geometer Sketchpad (Sahaa, Ayub & Tarmizi, 2010). GeoGebra is a free and easy-to-apply software that connects geometry and algebra (White, 2012).

As compared to traditional face-to-face classrooms, GeoGebra facilitates a learning environment and scenario that offers increased potential for students to make effective choices about their learning pace and sequence. In extreme cases students

can jump freely from one topic to another, while in other cases students may be required to follow a predetermined order and / or pace for best results.

The software includes geometry, menu toolbar (Figure 2.5) as well as an algebra and input field, a menu-bar, construction protocol and a navigation bar.

The construction protocol offers the researcher a step-by-step record of the students' computer interaction, which represents an important part of the pupils' choices and actions. Thus it enables the researcher to obtain a relatively precise image of the strategies used by students to solve a given problem.



**Figure 2.5: Screenshot showing both algebra and geometry windows**

GeoGebra can be used in many ways in the teaching and learning of mathematics: For demonstration and visualisation, since it can provide different representations; as a construction tool since it has the abilities for constructing shapes; for investigation to discover mathematics since it can help to create a suitable environment and situation for learning; and for preparing teaching materials using it as a cooperation, communication and representation tool.

The features of GeoGebra stimulate students to learn by understanding, thus helping them to master Euclidean geometry theorems' proofs. This mainly addresses the study question 2: How does the use of GeoGebra improve learners' understanding of

geometry proofs? And study questions 3.2 and 3.3: What are the practical and theoretical implications of GeoGebra on learners' performance improvements and justifying proofs and theorems of geometry? This is necessary in the study as GeoGebra, through its multiple helpful and seamless features, also addresses the core aim of the study: To provide guidance, materials and resources, that will harness the power of technology so that learners will be better able to understand and apply mathematics.

In the next subtopic the study will scrutinise GeoGebra software on Euclidean geometry proofs and theorems, as well as its impact on making these proofs and theorems simple for all students.

#### **2.1.8 Geogebra Impacting on Individual Learning**

Mathematics learning requires students to apply the principle of individualisation so that they can reflect better on concept understanding.

Language comprehension: individual learning is more important when students are home practising what they have learnt at school.

Interactive feedback: GeoGebra has a positive effect, especially for developmental mathematics on students who lack self-confidence or are intimidated by mathematics. Students can work individually, make mistakes along the way and still get corrected or helped without undue pressure. A student can therefore remain highly motivated, thus facilitating effective knowledge acquisition.

The study sought to expose rural Grade 11 learners to Euclidean geometry based on GeoGebra applications. The idea was to show them that irrespective of poor backgrounds in terms of socio-economic status and illiterate parents, they can nonetheless monitor their own work individually and improve their understanding of geometry' theorems as well as improve their mathematics achievements as per study questions 1 and 2.

#### **2.1.9 GeoGebra for Model-Centred Learning**

GeoGebra gives multiple ways of presenting a phenomena in various domains of mathematics, and a rich variety of computational utilities for modelling and simulations (Bu and Schoen, 2011). Models in this context are utilised to enact

realities to students so as to better grasp mathematical concepts. GeoGebra seeks to facilitate optimum mathematical understanding and proficiency for mathematics teaching and learning. A mathematically competent student can coordinate various representations of a mathematical idea in a dynamic way and further gain valuable insight into mathematical structures.

A model-centred framework on learning and instruction helps to understand the cognitive processes of mathematical sense-making and learning difficulties. GeoGebra is essentially a kind of synergy or concerted effort between technology and theory.

Bu and Schoen (2011) indicated that GeoGebra has created a positive attitude, centred on technology integration in mathematics teaching and learning. GeoGebra in model-centred mathematics teaching and learning goes beyond traditional mathematics instruction in content and coverage of concepts. GeoGebra is a conceptual tool, a pedagogical tool, a cognitive tool, and a transformative tool in mathematics teaching and learning. Dynamic GeoGebra models and simulations build a bridge between students' empirical investigations and mathematical formalisations (Burke and Kennedy, 2011). This approach to abstract mathematics illustrates the didactical conception of vertical mathematisation, the process that mathematical ideas are reconnected, refined and validated to higher order formal mathematical structures (Gravenmeijer and van Galen, 2003). Model-based conceptual interventions support students' development of valid mental models for formal mathematics.

Novak, Fahlberg-Stojanova and Renzo (2010) reported that GeoGebra seeks students' deep conceptual understanding of Euclidean geometry and underlying mathematics. The study seeks to reveal GeoGebra as flexible in that, when correctly utilised, improves mathematics abstraction and transforms the minds of students to conceptualise the key elements of Euclidean geometry into productive re-enactment.

#### **2.1.10 Geogebra on Problem-Solving and Attitude Change**

GeoGebra based modelling (a cognitive activity) helps students diagnose their mathematical conceptions, visualise the problem situations, and overcome algebraic barriers and thus focus on the geometric reasoning behind learning tasks. Iranzo and



Fortuny (2011) showcased that GeoGebra tool use, enhances students' prior mathematical and cognitive background. GeoGebra as a conceptual tool helps students make connections between real world situations and mathematical ideas. The study seeks to make connections between students' existing knowledge of geometry; points, lines and angles, as these are foundational to the learning of circle geometry.

GeoGebra models real problems and supports problem-solving, in providing visualisation and interactive illustrations, helping to increase students' motivation as well as cognitive development. GeoGebra thereby broadens students' mathematical exploration and visualisation skills. GeoGebra further has educational implications to real-world modelling problems in terms of mathematics' connections and the ever expanding learning opportunities that arise, sometimes unexpectedly in the modelling process (Bu & Alghazo, 2011).

GeoGebra software facilitates the engagement of student terms in collaborative knowledge-building and group cognition in problem-solving tasks of dynamic geometry; it increases the quality and quantity of productive mathematical discourse; and develops effective team practices in exploration, construction and explanation of the design of dependencies in dynamic geometry (Stahl, 2014). GeoGebra highly enhances students' positive attitudes on collaborative group work. GeoGebra fosters students' perseverance, curiosity, inductive attitudes and inclination to seek accuracy and rigor in geometric learning tasks. Ronchi (2010) views GeoGebra as a methodological resource that supports the teaching and learning of mathematics by helping teachers and students visualise formal mathematics knowledge and promote their sense of ownership through dynamic instruction. Conceptual models, in particular, mediate human understanding, where mental models play a control role, pervading, enabling or even disabling the cognitive processes.

#### **2.1.11 Why is GeoGebra Different?**

Unlike other software, GeoGebra with its versatility increases the possibility of exploring in a mathematics classroom or online resource, dynamic or step-by-step constructions with the purpose of simplifying the learning processes from students' perspectives Fry (2013). Tran, Nguyen, Nong, Maher and Nguyen (2014) argue how effective GeoGebra could be used for discovery learning, putting a student-centred

approach in place. In their analysis, they have found that students fondly enjoy using GeoGebra when learning the geometry of the circle.

GeoGebra is a very innovative technology that can easily support the progressive development of mental models appropriate for solving complex problems in mathematical relationships. GeoGebra duly can personalise students learning in Euclidean geometry. GeoGebra can be freely downloaded, no licence is required. GeoGebra can also help in lessons and activities aligned with standards, goals and objectives of CAPS. Furthermore, GeoGebra with its multiple features of dynamic modelling contributes immensely to improving students' general attitudes toward mathematics learning.

In the postmodern era where a student has to direct his or her learning, GeoGebra software is so vital in affording confidence to students, and because it is free, it enables students to work in any environment solving moderate to complex problems. This could lead to more motivated, committed students, thus learning with deep understanding will be the result, leading to improved performance and achievement in Euclidean geometry. Students may still take a long time in justifying multiple proofs and theorems. All these will address the study questions 1, 2, 3.2 and 3.3.

Herceg and Herceg (2010) in their study suggested that GeoGebra is helpful to students who face difficulties in mathematics problems. In Malaysian secondary schools Bakar, Ayub, Luan and Tarzimi (2002) found out that students using GeoGebra software, in transformation geometry topics, achieved better results than those exposed to the traditional approach. GeoGebra helps students in moving shapes and, or even creating their own geometric shapes. GeoGebra with its structural dynamism allows students to experience visual representations of the geometric structures and gives students opportunities to discover constraints and abstract mathematical relations simultaneously.

Every expression in the algebra window corresponds to an object in the geometry window and vice versa providing a deeper insight in the relations between geometry and algebra. GeoGebra provides the facility to move between the algebra window and geometry window. On one hand, the geometric representation can be modified by dragging it with the mouse like any other dynamic geometry software system, whereby the algebraic representation is changed dynamically. On the other hand,

algebraic representation can be changed using the keyboard causing GeoGebra to automatically adjust the related geometric representation. The study questions are for Euclidean geometry, but double partial measurement between algebra and geometry helped the students as these showed that algebra and geometry are inseparable. That brought improved attitudes and willingness to learn within students by familiarising them with geometry theorems, while using the knowledge of algebra at the same time.

Antohe (2009) agreed that GeoGebra could be an efficient platform for e-learning. Important theorems can be solved by using GeoGebra and the investigation is a constructive critical way, not so often restrictive. Using GeoGebra students can see abstract concepts, and can make connections and discover mathematics in an easy to learn fashion. This concurs with Blossier (2014) who stated that:

Students love GeoGebra because:

- ❖ It makes mathematics tangible by making a link between geometry and algebra in an entirely new, visual way where students can finally see, touch and experience mathematics.
- ❖ It makes mathematics dynamic, interactive and fun by teaching students mathematics in a new and exciting way that goes beyond the blackboard and leverage media.
- ❖ It makes mathematics accessible and available by allowing students to connect with mathematics anywhere and at any time – in school, at home, on the go.
- ❖ It makes mathematics easier to learn by creating the interactions that students need in order to absorb mathematical concepts.

Teachers love GeoGebra because:

- It allows them to continue teaching. GeoGebra does not replace teachers but it helps them to do what they do best – teach.
- It allows teachers to plan and deliver better lessons as it gives them the freedom to be themselves, creating lessons they know their students will find interesting.

- It allows teachers to connect with other teachers: GeoGebra teachers are part of a global mathematics community.

Students who use GeoGebra are more motivated and are likely to get better results. Emeny (2010) indicates that GeoGebra removes complexity of mathematics understanding and very quickly creates an intuitive-to-use platform in a form of a webpage. It is also less time consuming, plus it is natural that students gravitate towards simple, easy-to-use solutions. GeoGebra elucidates new things once and that is more appealing to students. Some other resources, as Emeny (2010) further warns, focus on engaging students rather than on being productive learning tools. Edutainment for enjoyment is totally different to learning for knowledge.

There are similarities to Weinberg (2012) who states that at the end of the day investigation should lead to genuine learning. Having interesting investigation or exploration is indeed great, but it needs to translate to actual student learning, to really be worth classroom time. In tests and examinations, students use pencil and paper, possibly with a graphing calculator. The question is: 'How can students get feedback on their pencil and paper in terms of mathematics?' Giving students' feedback on their work is the most important element of the learning process.

GSP, Cabri, Mathematica are also fantastic pieces of software. GeoGebra students know right away whether their answers are right or wrong. As agreed by Antohe (2009) that the ability to access students' solutions electronically may promote students' interests towards mathematics and advance students cognitive abilities. GeoGebra offers an excellent opportunity to explore mathematical ideas, and can be utilised to improve the teaching and learning of the subject. Briscoe (2012), found out that use of GeoGebra can aid in building dynamic demonstrations, creating dynamic relationships between objects on the screen live in front of a class. This is naturally preferable to using an off the shelf java applet or a static whiteboard. Students are able to build their own dynamic GeoGebra files, thereby being able to efficiently explore key mathematical ideas. GeoGebra is conducive to experimental learning where students can take ownership in and personalise their work.

GeoGebra further supports multiple representations that combine many of the features of a computer algebra system and dynamic geometry program. It also has a built-in spreadsheet and students can solve problems by exploring mathematics

dynamically. GeoGebra encourages students to think like a mathematician, especially in defining relationships between objects. GeoGebra, as mentioned, is an open-source resource making it widely and easily available to all learners.

These are further supported by Little (2008) who states that GeoGebra is built on commercial packages such as Geometer Sketchpad and Cabri that offer the user interplay between geometry and algebra. Due to its web-based platform and the universal nature of GeoGebra used anywhere, the accessibility of GeoGebra, with its worksheets and investigations, allows students' self-assessment outside the school premises. GeoGebra takes a role almost as extension of human minds, with its effective speed and massive memory. GeoGebra also brings flexibility to shy students by evaluating their process, providing remedial exercises discreetly with no embarrassment from peers and no fear of rejection from less-patient teachers. It further determines the pacing thus making mathematics lessons more stimulating and interesting, entrancing learning and retention even to a usually disinterested student. GeoGebra encourages all students to participate and have a greater eagerness to confirm their answers. The noise that is usually expected from students is when they are discussing among themselves, thus bringing more equilibrium to their knowledge levels, and helping the group collectively.

GeoGebra develops students' reasoning abilities, thanks to its flexibility and it can accommodate students with divergent cognitive aptitudes. This impressive learning tool consequently helps in students' development of critical thinking. GeoGebra goes on to lessen the adverse limitations of past prejudice that has affected mathematics teaching and learning, in both modern and postmodern eras. GeoGebra diminishes the likelihood of unhealthy competing and comparing among students during vital lessons. The principle of individualisation is enhanced and advanced.

#### **2.1.12 Hindrances of GeoGebra in Mathematics Classroom**

It is noted that using GeoGebra has its problems like the lack of experience in teaching as well as in staff qualified for teaching using computers:

- Language barriers pose a national problem, slowing down cooperation on the introduction of computer systems in teaching. Computer literates may

have knowledge of English, but it becomes a problem when teaching staff and students.

- An absence of pedagogical science of computer-assisted learning and teaching.
- Lack of didactically usable programs for teaching.
- Lack of communication between teachers.
- Schools rely heavily on external technicians or informatics experts.

Technology must supplement mathematics classroom experiences and enhance learning (Stols, 2014) and Gyongyosi Wiersum (2012).

Bennet (2014) believes computers can teach students without an intermediate human instructor. This is too idealistic and at times impossible to accomplish in the postmodern era. The researcher agrees with Gyongyosi Wiersum (2012) who firmly believes on the availability of effective teachers as facilitators. It is very important for teachers to guide students and see to it that the standard of learning is maintained at supreme level and to set appropriate targets. Computers and installed software can require technical attention at any time, thus an experience technician must be at hand to cater for such unforeseen circumstances. For every complete session a teacher must assess the level of progress in terms of knowledge advancement.

GeoGebra.exe error is one which might be caused by:

Related registry files being damage or corrupted, windows or drivers being outdated, malicious spyware or virus invasion, GeoGebra.exe file being corrupted or deleted mistakenly and improper program installation or removal. These may lead to serious problems such as these listed next:

- It takes a long time to start up / shut down the computer, open a website or launch a program;
- Some programs cannot be activated and used as normal;
- Instead malicious programs are downloaded or install unawares;
- Annoying error messages constantly pop up on the computer;
- Blue 'screen of death' happens occasionally;
- System sometimes crashes;
- Windows settings can be changed adversely.

The GeoGebra.exe file plays a core role in terms of the windows system. It must not be deleted at any cost as erroneous removal may cause the computer to crash. It is very important to modify registry settings and change file associations in order to run any file on the computer for ensuring that GeoGebra.exe error does affect the computer. An important question is: 'How does one detect a virus?' There are various 'symptoms' or red flags to look out for:

- Symptom 1: Slow performance, replication of viruses themselves to imitate system files thus consuming extra memory, limiting hard drive defragmentation and causing accumulation of junk files that require cleaning;
- Symptom 2: Computer receiving strange error messages or programs starting automatically or shutting down without notice;
- Symptom 3: Modem or hard drive works overtime and even produces sounds while running.

Only trusted antivirus software options are recommended and these must be timeously updated. It is also vital that one stops the GeoGebra.exe virus from infecting other programs.

## **2.2 THEORETICAL FRAMEWORKS**

The previous sections of this chapter presented literature review. The aim of the following sections is to discuss the theoretical frameworks on geometry, particularly in Euclidean geometry in mathematics education. The focus will mainly be on Information Processing and Cognitive Neuroscience theories. This section continues to describe how these theories are used as important components relevant to the implementation of Euclidean geometry through the application of GeoGebra, as per the current study. Outlaying an understanding of mathematics will also lay a foundation for the theories in the current study.

### **2.2.1 Information Processing Theory**

The Information Processing Theory (IPT) is a cognitive approach to understanding how the human mind transforms sensory information. In developmental stages, information is being processed with much greater efficiency. Environmental information must be captured, analyses and interpreted in order to make sense of stimuli and retain appropriate information for sustained periods. The development of

word usage to measure mental representation is quite effective in studying changes in memory (Shaki & Gevers, 2011).

In line with goal 5 of the study and hypothesis 2, the Information Processing Theory, formerly known as Stage Theory model, proposes that information is processed and stored in 3 stages. According to Huitt (2003) information is thought to be processed in a serial, discontinuous manner as it moves from one stage to another stage. Firstly, there are levels of processes to this proposition and students utilise different levels of elaboration. It is mainly done on a continuum from perception, through attention, to labelling and finally meaning all stimuli are constant. The most significant principles guiding the Information Processing Theory are:

- Assumption of a limited capacity of the mental system; these are some constraints or restrictions of information at specific points.
- Control mechanism; that is where encoding, transforming, processing, storage retrieval and utilisation of information is assessed or benchmarked against. Not all the processing capacity of the system is available and the advantage is when a student is confronted with new tasks, the schemata is properly or fully utilised, unlike in the case of routine exercises.
- Two-way information; this happens in a dynamic process as students develop or attach meaning to an environment and relations to it. Bottom up and top down processing is advanced. This is similar to inductive and deductive reasoning that's generated through imagination.
- The general preparedness to process and organise information in specific ways.

The Information Processing Theory is often referred to simply as Cognitive Information Processing (CIP) theory, and is applied to various theoretical perspectives like neuroscience, dealing with sequence and execution of cognitive events. Information goes through the cognitive systems, being subjected to several mental processes as cited by Shunk (1996) who states that information processing theories focus on how people:

- Respond to environmental events.



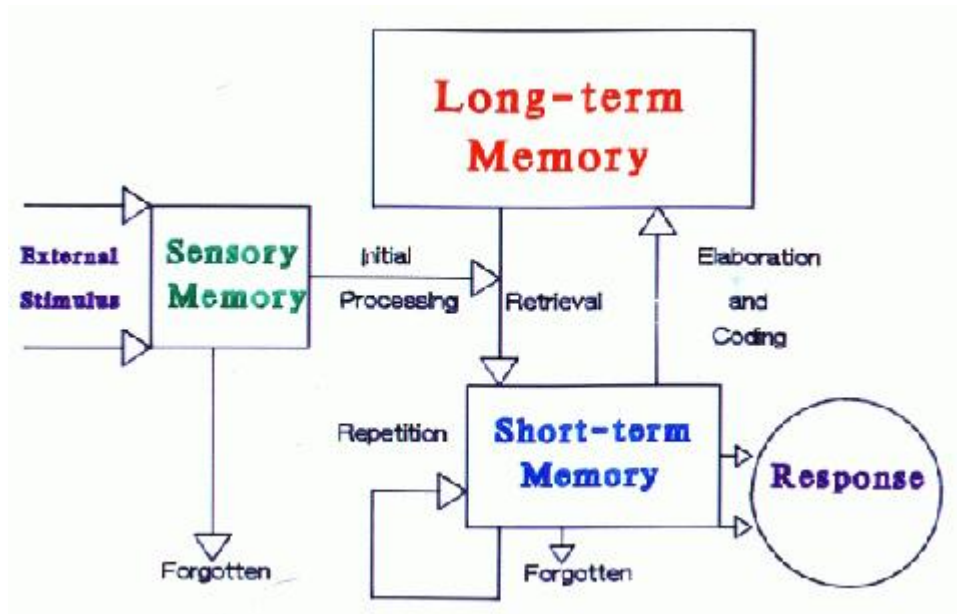
- Encode information to be learned and relate it to knowledge already in memory.
- Store new knowledge in memory.
- Retrieve information when it is needed.

Students are viewed as active seekers and processors of information through the basic components (Figure 2.6), which are:

- ❖ Sensory Memory (for visual, auditory, etc.)
- ❖ Short-Term Memory (STM), and
- ❖ Long-Term Memory (LTM).

#### **2.2.1.1 Sensory memory**

This is affiliated with the transduction of energy that is where memory is created, but for a very short time. It is pivotal that a student goes through this initial phase for smooth transition to the next phase. It holds unplanned information mainly associated with the senses (visual, auditory etc.), to be processed at latter phase(s). Mathew and McCrudden (2013) cited that the sensory memory phase screens incoming stimuli and processes only those stimuli that are relevant at the present point in time, allowing them to be transformed to the next phase. Huitt (2003) indicated two major concepts for getting information to short-term memory (STM), these are when students tend to pay attention to stimuli if there are interesting features to them. This is in line with study question 1 and hypothesis 2. Secondly students pay attention if the stimulus is always a known pattern.



**Figure 2.6: Information Processing Theory**

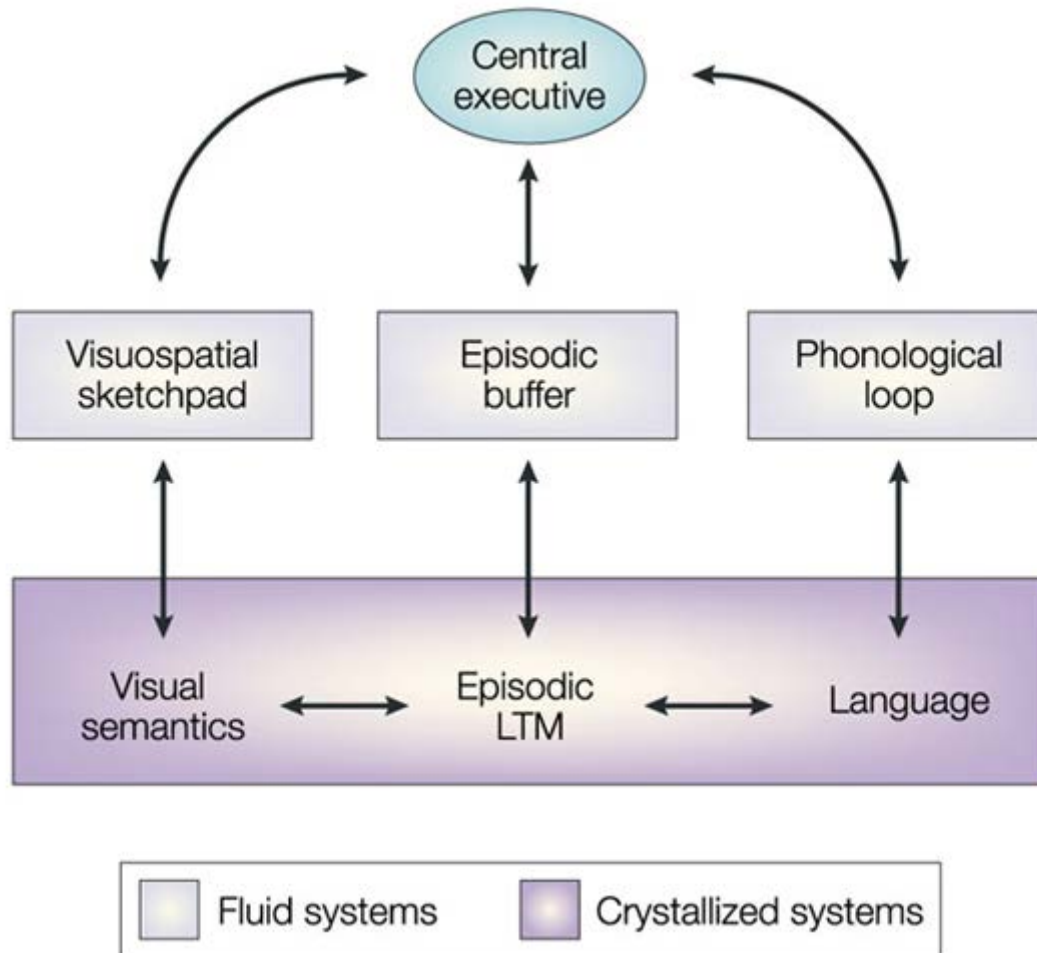
### 2.2.1.2 Short-term memory (STM)

This is sometimes termed “working memory”. It is indeed the centre of conscious thought, analogous to the central processing unit of a computer, where information from long-term memory and the environment is combined to assist in problem solving. In the working memory, abilities of humans to solve problems are limited due to its short span to process necessary information.

Working memory is created by attention to external stimulus, internal, or both. It is highly retained by organisation and repetition. Organisation is best described by these elements or attributes:

- i. Component (classification according to category)
- ii. Sequential (cause / effect)
- iii. Relevance central idea (unifying), and
- iv. Transitional (connective).

Chunking or grouping helps keep information in short term memory and to get the information transferred into the long term memory bank.



## BADDELEY'S MODEL OF WORKING MEMORY

- Central executive: Part of working memory where information is controlled.
- Visual / spatial: Visual and spatial information.
- Phonological loop: Sounds
- Episodic buffer: Where information is brought to the forefront, used and constructed.

### 2.2.1.3 Long-term memory (LTM)

Most of the information is retained indefinitely here. Long-term memory has both explicit and implicit systems. This phase is referred to as preconscious (where information is easily recalled) and unconscious memories (only available during normal consciousness). Processes of elaboration and distributed practice play key roles in long term memory. The following are organisational structures of long term memory:

- 🌈 Declarative memory: With their semantic memory, goals of the study will be addressed, as students will be using schemata networks of connected ideas or relationships, propositions (interconnected sets of concepts and relationships).
- 🌈 Procedural memory: In terms of Information Processing Theory, concepts formation is withheld.

There are guiding principles that lead to concept development, in naming and defining the concept to be learned, and involve both inductive and deductive reasoning. These principles are, as suggested by Huitt (2003):

- Gain the students' attention.
- Bring to mind relevant prior knowledge.
- Point out important information.
- Present information in an organised matter.
- Show students how to categorise (chunk) related information.
- Provide opportunities for students to elaborate on new information.
- Show students how to use coding when memorising lists
- Provide for repetition of learning, and provide opportunities for over-learning of fundamental concepts.

To Shukla (2010) the term information processing pertains to the process employed by the intelligent system to alter a given set of data, to help in the full understanding and perception of such data, by the system. Information processing is a cognitive developmental theory inclusive of linguistic skills, cognitions, conceptions, reading and thinking processes, problem solving, etc.

According to McLeod (2013) the human brain follows certain fundamental steps in understanding and interpreting the world around it. The perceptions and understandings are not automatic processes, but are a consequence of complex mechanisms through which the brain takes in external data (Shukla, 2010). The data comes in the form of sensory perceptions, that the brain processes using logic, reasoning and responses to produce the output. Shukla (2010) describes four underlying beliefs that uphold the structure of information processing:

- Thinking: This includes activities of perception of external stimuli, encoding the same, and storing the data so perceived and encoded in one's mental recesses.
- Analysis of stimuli: Encoded stimuli are engaged to suit the brain's cognition and interpretation processes, enabling decision-making. This is based on encoding, stratification, generalisation and automatisisation.
- Situational modification: Entails use of experience to handle similar situations in future.
- Obstacle evaluation: The nature of problem, when evaluating the subject's intellectual, problem solving and cognitive acumen.

#### Learning and Memory, Strategies and Knowledge:

Memory formation follows a three step mode of:

- a) Encoding.....information put into codes.
- b) Storage.....keeping information until it is needed.
- c) Retrieval.....bringing back stored information for usage.

High school students when faced with information that is unfamiliar to them, they tend to develop strategies to encode the information so as to store it and accurately and easily access it at a later stage (Miller, 2011). As children grow, they experience increased cognitive abilities, increased memory capacity, and other social / cultural factors which all serve as major contributors to development. Older children are likely to develop memory strategies independently, better note appropriate memory strategies for particular scenarios as well as have the ability to select important information and sift out irrelevant information.

The children are able to often change their strategies used (Miller, 2011), and their level of comprehension is integrally connected with their memory. Older children take more information faster, allowing for better efficiency of information processing. According to Information Processing Theory, memory and knowledge enables the child to readily access information from their long term storage and utilise it in appropriate situations. It has been envisaged that more associations increase the complexity of network association, thus making information recall better.

Children as they grow can better their information gathering skills, thus making inferences, judgements and going beyond pure recall (Miller, 2011). Meta-memory and meta-cognition assist children in that they can better understand how their memory works as well as human cognition function. The principle of constructivism is strongly applied where reviving of schemata is critical for learning. Expert learners (2011) suggested that for learning and instruction to be meaningful and relevant, it must be built upon students' prior knowledge and help learners to make connections between what they already know and what they are about to learn.

## **2.2.2 Cognitive neuroscience and GeoGebra application**

This theory will help the researcher to understand students' cognition as this is the primary function of the human brain (Howard, 2011). Howard (2011) elucidated that cognition can be concisely described with mathematically specific descriptions, and that these equations provide large-scale constraints on the collective activity of neuron volumes. The observed ensembles of neurons place constraints of mathematical models on cognition.

### **2.2.2.1 Neuroscience on models**

Hoskin (2011) stated that the goal of theoretical cognitive neuroscience is to develop physically-constrained models of cognition, with special attention to learning memory. As per study question 1 and hypothesis 1, as cited by Howard (2011) "our theoretical work uses mathematical analysis and computational tools while our empirical work requires behavioural experimentation and collaborative work with a cognitive and systems neuroscientist to constrain theories". Students need a variety of skills developed at different levels, hence there is need for students to place emphasis on cognitive science. Cognitive neuroscience seeks to understand the neural bases of mental abilities such as perception, memory, attention, categorisation, self-awareness, reasoning, motor control and language. Cognitive neuroscience basically describes how the brain creates the mind.

Dannay (2013) states that encouraging the development of social and creative thinking in students is now recognised as central to education. In order to elevate students to fall in line with the postmodern era trend, it is vital to understand the mental processes of mathematics students before even posing a problem for them to

solve. Intelligent Quotient (IQ) has been the leading measure of mathematics students to date.

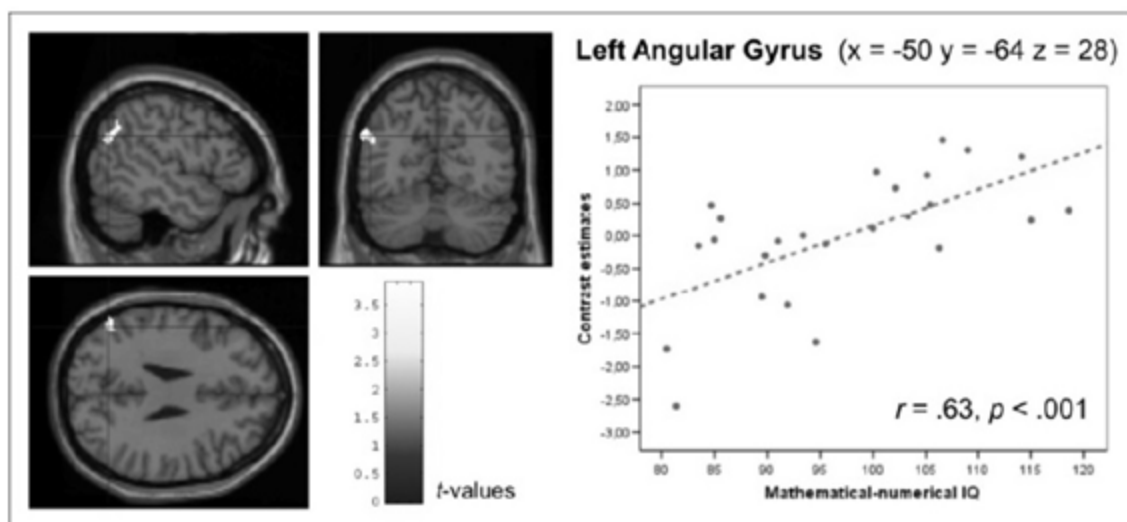
#### **2.2.2.2 Neuroscience on students' school success**

Ansari (2013) indicates in his findings that young minds when active, and online self-directed activities, engage their brains in deeper, faster and better learning as opposed to sitting passively in the classroom. Neuroscience asserts that the frontal lobes of the brain, which regulate functions such as attention, self-control, focus, and decision-making, are so critical for school and career success.

Executive brain structures can actually be trained through methods such as brain strategies, exercises, mediation, and software, to improve executive and academic skills in students. Andreasen (2013) discovers that the creative process influences the brain and highlights the importance of arts and creativity for child development. It should be noted that aspects like positive psychological and environmental circumstances may stimulate students' creative insights. Ultimately, there is a meaningful connection between the understanding and functioning of the brain and learning mathematics.

There is evidence that numerical cognition is intimately related to other aspects of thought, spatial cognition in particular. Numbers are mentally represented with a spatial layout. Behavioural studies also reinforce the connection between numerical and spatial cognition.

Ansari (2012) indicated in his neuroimaging studies that the association between number and space also shows up in brain activity. Regions of parietal cortex show shared activation for both spatial and numerical processes. The parietal system is active in children and adults during numerical tasks, and over a period of time it develops individually.



**Figure 2.7 Significant correlation between activation in left angular gyrus and mathematical-numerical intelligence. (Grabner et al., 2009)**

Basic number sense or numerical information can be stored verbally in the language system, a system that neuroscience reveals as qualitatively different at the brain level. The number sense system stores information about verbal sequences like days of the week and with numerical processing it supports counting and learning multiplication tables. Arithmetic skills are supported by different brain mechanisms offering deeper understanding of learning processes that underlie arithmetic proficiency.

## 2.3 CONCLUSION

This chapter reviewed literature and theoretical frameworks on the application of GeoGebra on Euclidean geometry and the impact GeoGebra has on teachers' confidence as well as improving students' mathematics performance in various countries around the globe. Macekova (2013) indicate that the use of GeoGebra evoked new platforms of Mathematics for students. Further development need to fast-track the implementation of new technology such as GeoGebra, as a teaching aid into educational process. However technology can make mathematics lessons vague to students, teachers should also prepare thorough to select suitable materials.

The topics covered are GeoGebra on proofs and theorems, benefits of GeoGebra to learners, South African studies on Euclidean geometry and International studies.



Furthermore, TIMSS results are presented in table and diagram formats. The next chapter will focus on the research methodology.

## **CHAPTER THREE**

### **RESEARCH METHODOLOGY**

#### **3.1 INTRODUCTION**

In the previous chapter, literature review and theoretical frameworks related to the current study were discussed, which among others cite that GeoGebra helps both teachers and learners in planning, researching, testing and demonstrating results when doing Euclidean geometry, and further fosters more independent student activity and increases the probability of success which then leads to improved educational outcomes. The current study addresses the following research questions:

- a)** How does application of GeoGebra in Euclidean geometry impact on learners' performance?
- b)** How does the use of GeoGebra improve learners' understanding of geometry theorems?
- c)** What are the practical and theoretical implications of GeoGebra on:
  - i.** Teachers' confidence in teaching geometry?
  - ii.** Learners' performance improvements?
  - iii.** Justifying proofs and theorems of the circle geometry?

This chapter focuses on the research methodology. The chapter includes research paradigms, research design, data collection methods, data analysis, research quality and ethical issues. According to Biyane (2007), the research contains two main stages: One is the stage of planning and other is the stage of implementation. During the planning phase, the researcher constructs a design, a proper plan of the research, and during the second phase data is collected and analysed. The design is the key to the study as it explains in some details how the researcher intends to conduct the study, namely how the research questions will be addressed. It indicates the information gathered as well as the methods, procedures and instruments used in the research.

## **3.2 CLARIFICATIONS AND JUSTIFICATIONS OF THE STUDY**

A significant number of studies have been conducted regarding the use of GeoGebra application in mathematics. Most of these studies globally have focused on geometry, and some of them on algebra.

## **3.3 RESEARCH PARADIGM**

A scientific paradigm is a framework containing all of the commonly acceptable views about a subject, a structure of what direction research should take and how it should be formed (Shuttleworth, 2008). Kuhn as cited by Shuttleworth suggested that a paradigm defines “the practices that define a scientific discipline at certain point in time”. Paradigms are also deemed unique and culturally-based. The two paradigms were adopted namely positivism and interpretivism.

### **3.3.1 Positive Paradigm**

Positivism adheres to the view that only factual knowledge gained through observations (the senses), including measurement, is trustworthy. Interpretation of data is via objective approach and the research findings are usually observable and measurable. Principles of positivism are subsequently dependent on quantifiable observations that lead to statistical analysis. Positivism is in accordance with the empiricist view that knowledge stems from human experience. It has an atomistic, ontological view of the world as comprising discrete, observable elements and events that interact in an observable, determined and regular manner (Collins, 2011).

The researcher remains independent and unbiased within the study and there are no provisions for personal interests. The researcher however assumed some elements of the positivist approach to the study in order to ensure pure objectivity throughout. By independent it is implied that the researcher maintained minimal interaction with research participants when carrying out this study, as agreed by Wilson (2010).

To some extent the key features of positivism as presented by Ramanathan (2008), include the following:

- The observer must be independent.
- Human interests should be irrelevant.

- Explanations must demonstrate causality.
- Research progresses through hypotheses and deductions.
- Concepts need to be operationalized so that they can be measured.
- Units of analysis should be reduced to simplest terms.
- Generalisations through statistical probability.
- Sampling requires large numbers selected randomly.

### 3.3.2 Interpretive Paradigm

The interpretivism paradigm is based on the observation that there are fundamental differences between the natural and social worlds. The aim of interpretivism is to understand the subjective experiences of those being studied, how they think and feel and how they act or react in their habitual contexts. Its core is an assumption that social actors generate meaningful constructs of the social world in which they operate (Crofts, Madden, Franks, & James, 2011).

The researcher's position is founded on theoretical belief that *"reality is socially constructed"* (Mertens, 2005) and integrating *"this social / settings and relations with participants are important"* (Creswell, 2003). In lieu of this perspective, validity or truth cannot be grounded solely in an objective reality. Valid claims to knowledge dominant of qualitative method but there are some qualitative elements in it:

- Assumes that we cannot separate ourselves from what we know (subjectivist).
- Truth negotiated through dialogue.

Positivism paradigms provide a high level of measurement precision and statistical power, thus high levels of reliability with gathered data. Madrigal and McClain (2012) state that statistical analysis lets us derive important facts from research data, including preference trends and differences between groups.

Interpretivist paradigms describe the qualities or characteristics of a phenomenon. It includes information about participants' needs, desires and variety of other information that is essential in producing what is beneficial in participants lives (Madrigal & McClain, 2012). These further require flexibility, allowing participant to respond to data as it emerges during a session, and may be in the form of naturalistic or structured interviews. Identifying patterns and needs when analysing,

are important. As also McKenzie and Knipe (2006) stated, both of these paradigms in one study can be extremely effective in that interpretivism identifies factors that affect the areas under investigation, then make use of that information to quantify, by assessing how these factors would affect participant preferences.

### **3.4 RESEARCH DESIGN**

Research design refers to the overall strategy that the researchers choose to integrate the different components of the study in a coherent and logical manner, thereby ensuring the study will effectively address the research problem. It constitutes the blue print for the collection, measurement and analysis of data (Labaree, 2013).

Research design essentially sticks the whole research project together. A design was utilised to structure the research, thus to elucidate how all major parts of the research project would flow i.e. the samples used (groups), measures applied, treatments or intervention programs and assigned methods – all working together, trying to address the main research questions. The research design employed in the current study was a quasi-experimental research design.

#### **3.4.1 Sampling of Site and Population**

Sampling is a statistical method of obtaining representative data or observations from a group (population) according to BusinessDictionary.com. Sampling is the process of selecting a group of subjects, which can be people, events, behaviours, or other elements with which to conduct a study. In a statistical context, the “population” is a complete set of elements (persons or objects) that possesses some common characteristics defined by the sampling criteria established by the researcher. The “target population” is a subject of individuals with specific clinical and demographic characteristics in whom one want to study one’s intervention, while a “sample” is a portion, piece, or segment that is representative of a whole (Kadam & Bhalerao, 2010). In this study the target population is the entire group of students in the rural areas of UMkhanyakude district in KwaZulu Natal province in the Republic of South Africa. The accessible population to which the researcher has reasonable access in this current study is the target population. The researcher chose five

schools under Hlabisa, which are at a radius of thirteen kilometres around Hlabisa town.

In many cases the population is too large and cannot be used due to inability of a researcher to handle it based on limited resources. In this case the portion of subjects of the population used for the study is called a sample (Samkange, 2009).

UMkhanyakude district is one of the rural districts in KwaZulu Natal that has been performing poorly in mathematics for the last several years. Again, it has been underprivileged in terms of getting quality mathematics teachers. There are three circuits namely Ingwavuma, Ubombo and Hlabisa. The research sample was selected from the sample frame. A sample frame is a set of information used to identify a sample population for statistical treatment. A sampling frame includes a numerical identifier for each individual, plus other identifying information about characteristics of the individuals to aid in analysis and allow for division into further frames for more in-depth assessment (BusinessDictionary.com).

The researcher selected five schools in two wards under Hlabisa circuit, three from Empembeni ward and two from Ezibayeni ward. All these schools fall into one mathematics cluster for the past ten years due to their demarcation.

#### **3.4.1.1 School setting**

##### **Ezibayeni ward**

The first school has an enrolment of 613 students, with 20 educators with a mixture of temporary and permanent educators. There are three mathematics educators teaching Grade 8 to Grade 12 classes. There are 21 Grade 11 mathematics students in science stream. The second school has 879 students, 29 educators and 33 Grade 11 mathematics students in science stream.

##### **Empembeni ward**

The first school has 16 Grade 11 mathematics students. The second school has 19 Grade 11 mathematics students, and the third school has 66 Grade 11 mathematics.

The sample consisted of 155 Grade 11 mathematics students. All schools observe a five days cycle curriculum timetable with one break. All schools are also in quintile 1,

except one which is in quintile 2. Quintile statistically refers to each of any set values of a variate which divide a frequency distribution into equal groups, each containing the same fraction of the total population according to Oxford dictionaries. In the South African perspective, schools are classified in quintile 1 to 5; quintile 1 involves schools which are deemed to be the poorest of the poor and where learners are not obliged to pay school fees. Quintile 2 schools are previously disadvantaged with some exemption from paying school services fees. The socioeconomic background is one resembling majority of rural students in UMkhanyakude district and the mathematics educators are Africans by classification. All five schools have computer laboratories that are being used for other computer related subjects. This made the researcher see the need to integrate mathematics teaching and learning in this fold, in using these technological centres for the advancement of mathematics results.

#### **3.4.2 Achievement Test**

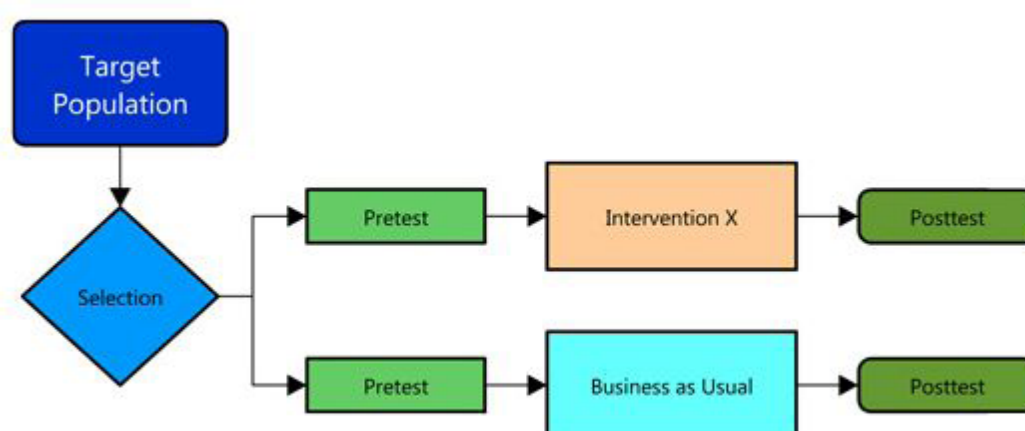
By examining the target behaviours determined by the Department of Basic Education and 2014 mathematics pace setter for Grade 11 and Grade 12, for the unit of Euclidean geometry theorems, the achievement test involved 14 multiple choice items (See Annexure 6). In line with targets of the given units, the achievement test consisted of 14 items which students solved by first calculating and then choosing the best option that was precisely related to the corresponding item. Different textbook questions and questions previously asked in high school exit examinations for Grade 11 and Grade 12 were integrated. The achievement test was designed to measure the following objectives that students in both groups were expected to achieve during the current study.

The test was prepared by the researcher and checked by six mathematics educators all who have more than 15 years of experience in mathematics teaching. The achievement test was first piloted on 45 students doing mathematics, specifically Euclidean geometry; 43 Grade 11 students. All these students were not part of the study. The main purpose was to determine students' difficulties in understanding the tasks in the test. The pilot study assisted in checking the ambiguity of items, then changes were effected in the main study.

#### **3.4.3 Sample Size and Selection**

A quasi-experimental study is a type of evaluation which aims to determine whether a program or intervention has the intended effect on a study's participants. The researcher chose quasi-experimental but non-equivalent group design (meaning assignment of subjects to groups was not random and the use of intact groups are similar), where there was control over assignment of the treatment. But some criteria were used, other than random assignment, to determine which participants receive treatment (Bradley, 2009).

An achievement test served as pre-test and post-test, and was administered to both groups as shown in study design (Figure 3.1).



**Figure 3.1 Diagram showing quasi-experimental study design**

The selection took only three days. The researcher taught for 10 days, using a one and half hour period per day, and employing the traditional approach. Two schools under Ezibayeni ward were clustered together for the initial employment of traditional approach before pre-test was administered, because these are neighbouring schools. And two schools in Empembeni were combined due to their number of students and the last one was taught separately.

The researcher taught all Euclidean geometry theorems of circle geometry using pencil and paper. Then pre-test in the form of achievement test was administered (See Annexure 6).

For a sample to be representative of a whole population it should have the following attributes as presented by Kadam and Bhalerao (2010, pp.55-57):



- Every individual in the chosen population should have an equal chance to be included in the sample.
- Ideally, choice of one participant should not affect the chance of another's selection (hence we try to select the sample-randomly, thus, it is important to note that random sampling does not describe the sample or its size as much as it describes how the sample is chosen).

### 3.4.3.1 Sample size calculation

The sample size is the number of participants in a sample. It is a basic statistical principle to define the sample size before the commencement of the study so as to avoid bias in results interpretation. The calculation of an adequate sample size is crucial and is the process by which the researcher calculates the optimum number of participants required to be able to arrive at ethically and scientifically valid results.

Kadam and Bhalerao (2010) describe the following principles and methods to calculate the sample size.

The sample size depends on the:

- Acceptable level of significance – The “ $p$ ” which is acceptable at  $p < 0.05$ . A confidence level tells how likely it is that the interval estimate actually captures the truth we are seeking (Utts and Heckard, 2007, pp. 405).
- Power of the study is its ability to detect a difference, the “Type II error”, if the “Type I error” exists. Statistical power is influenced by the magnitude of the true difference, the standard deviation of the population means, and the sample size. When the hypothesis is true, the probability of making the correct decision is called the power of a test (Utts and Heckard, 2007, pp. 509). Power is simply the flipside of the risk of a type II error. A power of 80% is often chosen, hence a true difference will be missed 20% of the time.
- Expected effect size is a quantitative measure of the strength of a phenomenon. Cohen, Mannarino and Dellbinger (2010, pp. 295-311) suggested that  $d = 0.2$  be considered a “small” effect size,  $d = 0.5$  considered a “medium” effect size and  $d = 0.8$ , a “large” effect size. This means that if two groups' means don't differ by standard deviation of 0.2 or more, the

difference is trivial, even if it is statistically significant. The relative reduction with the test intervention is 50%.

- Underlying event rate in the population is the proportion of the population.
- Standard deviation in the population is the measure of dispersion or variability in the data. Smaller standard deviation leads to smaller variance and smaller sample size: This result in a homogenous population, meaning a good representation.
- Expected drop-out rate.
- An unequal allocation ratio.
- The objective and design of the study.
- Cost considerations (e.g. maximum budget, desire to minimise cost).
- Administration concerns (e.g. complexity of the design, research deadlines).
- Minimum acceptable level of precision.
- Variability with the population or subpopulation (e.g. stratum, cluster) of interesting sampling method.

The researcher firstly calculated the effect size using Cohen's *d* calculator for *t* test. In statistical analysis, effect size is the measure of the strength of the relationship between the two variables, and Cohen's *d* is the difference between two means divided by standard deviation.

#### Formula:

$$r = \sqrt{[(t^2) / ((t^2) + (df \times 1))]}$$

$$d = 2t / \sqrt{df}$$

Where, *r* = effect size, *d* = Cohen's *d* value (standardised mean difference), *t* = *t* test value and *df* = degree of freedom. The effect size *r* is generally classified into small if 0.2, medium if 0.5 and large if 0.8. The study effect size was calculated as follows. The effect size is large. The sample size is calculated using the following formula:

$$n = (2(Z_{\alpha} + Z_{1-\beta})^2 \times \sigma^2) / \Delta^2 r = \sqrt{[(1.96)^2 / ((1.96)^2 + (2 \times 1))]} = 0.8109426903982$$

Where *n* is the required sample size. For *Z<sub>α</sub>*, *Z* is a constant (set by convention according to the accepted *α* error and whether it is a one-sided or two-sided effect) as shown in table 3.1.

**Table 3.1 an extract of t-test confidence interval table**

$\alpha$ - error	5%	1%	0.1%
2 - sided	1.96	2.5758	3.2905
1 - sided	1.65	2.33	

For  $Z_{1-\frac{\alpha}{2}}$ , Z is a constant (set by convention according to power of the study) as shown in Table 3.2.

In the formula  $\alpha$  is the standard deviation (estimated), and  $\Delta$  the difference in effect of two interventions which is required (estimated effect size). The sample size of the current study was:

$$n = [2(1.96 + 0.8416)^2 \times (0.532)^2] / (0.2)^2 = 111.072239 \approx 112$$

**Table 3.2 Extract of z-test standard normal distribution table**

Power	80%	85%	90%	95%
Value	0.8416	1.0364	1.2816	1.6449

The sample size of the study, as illustrated in Table 3.3, is 112 participants with 56 in the control group and 56 in the treatment group. The other 43 learners did receive treatment during the pilot study prior to the main research but assisted in adjustment of items used for the research.

**Table 3.3: Composition of sample**

No. of students	Group of students	No. of breakdown	Percentage (%)
112	Experimental	56	50
	Control	56	50
Total	112		

### 3.4.3.2 Procedure

The experimental group underwent an intervention where they learnt circle geometry using GeoGebra software for two weeks while the control group, on the other hand, continued learning circle geometry using traditional approach, not involving

GeoGebra at all. Ten intact classes consisting of 112 students in Grade 11 in total were used for this study from a population of 155 students.

There were three guiding reasons accounted for site selection. Firstly, the schools were readily accessible to the researcher since these schools are a mere 12 kilometres from where the researcher stays. Secondly, the researcher frequently visits all these schools when in cluster team teaching, and thirdly all schools offer computer classes which make it possible to conduct the study. The participants are in the ages between 16 and 19 years. All of them were pursuing pure mathematics Grade 11 in the year 2014. In the sample of 112 participants, 49 were male students and 63 were female students, meaning about 44% participants were male and 56% were female students. The following procedure was used in selecting the participants for treatment:

- ✚ From each school a list of students' names according to their performance in their March and June common test was drawn. The list was further subdivided into three sections; the top, the average, and the bottom, and each student in the list had a number assigned to his / her name.

All participants are being taught using the same CAPS, same District Intervention Programme for Mathematics, same work schedule, writing the same September common test that is on Euclidean geometry and in the same cluster (Empembeni Mathematics cluster).

### **3.5 METHODS OF DATA COLLECTION**

Data collection is the process of gathering and measuring information on variables of interest, in an established systematic fashion that enables one to answer stated research questions, test hypotheses, and evaluate outcomes according to free encyclopaedia. The technique of collection depends largely on the type of tools used, and these may include questionnaires and open-ended questions.

#### **3.5.1 Questionnaires**

Quantitative research is concerned with testing hypothesis derived from theory and / or being able to estimate the size of a phenomenon of interest. Questionnaires are one of typical quantitative data gathering strategies. Questionnaire is a form

containing a set of questions (items), especially one addressed to a statistically significant number of subjects as a way of gathering information for a survey. Questionnaires help gather information on knowledge, attitudes, opinions, facts, and other information (Radhakarishna, Nagaran & Vijayanandhan, 2014).

The researcher followed a sequence while developing the study questionnaire (Radhakarishna et al., 2014)

- **Target audience background:** The researcher examined the purpose, research questions, and hypotheses of the study. This was done to assess subjects' educational / readability levels, access, and the process to select the respondents (sample vs. population).
- **Conceptualisation:** The researcher then embarked on the content (from both literature and theoretical framework), by transforming it into statements / questions. The researcher indicated what exactly the questionnaire was to measure. Independent and dependent variables were clearly indicated.
- **Item Formulation and Data Analysis:** The researcher focused on writing statements / questions, selections, selection of appropriate scales of measuring, questionnaire layout, format, item ordering, font size, front and back cover, and data analysis. Scales selected are used in quantification and data analysis, including measuring a subject's response on a particular variable.
- **Validity:** After the first steps, a draft questionnaire was prepared. Validity refers to the amount of systematic error in measurement and a field test was used to check content validity.
- **Reliability:** The researcher, as the last step of questionnaire development, piloted the instrument. Test-retest was used to assess reliability of knowledge. The researcher used 45 subjects for pilot testing, who are in Grade 11 and Grade 12, doing pure mathematics, but all were not part of the main study. SPSS (Statistical Package for Social Sciences) was used to analyse the data collected from the pilot test.

After the researcher ensured that efficiency can be attained using analysis from the pilot test, the main research questionnaire was developed (see Annexure 4). In formatting the questionnaire, the researcher focused on four sections targeting the

main aims of the study. Section 1 dealt with biographic and general information. Section 2 contained 15 items which are closed-ended for testing students' content, knowledge, attitudes, opinions and facts. These were in the form of a four-point Likert scale. In a Likert scale, statements that express an opinion or feeling about an object are written, and it also serves as a self-coding for any explanation given. The statements are listed and to the right of each statement is a space for the respondent to indicate the degree of agreement or disagreement. The Likert scale was therefore used in providing an attitude continuum for each statement ranging from strongly disagree (SD) = 1 key, disagree (D) = 2 key, agree (A) = 3 key, and strongly agree (SA) = 4 key. The respondents had to indicate their responses to the particular item by means of a cross. The Likert scale gives a wider range of responses than the mere yes / no or agrees / disagree types of responses. It is advantageous to use Likert scale in that it provides precise information about the respondent's degree of agreement or disagreement to the detail that the researcher requires.

Questionnaires were used because students' responses would remain anonymous. Students in this context may be more truthful than they would be in a personal interview or when discussing a topic with the researcher. The researcher using data analysis from pilot test, followed the qualities underlying a good questionnaire as stated by Husain and Farooq (2013, pp. 43-57):

- All answers were supposed to be direct and accurate.
- Items were described precisely and correctly.
- The language used was easy and simple.
- The length was a proper one.
- The answers were supposed to be relevant to the problem.
- Items were moving around the theme of the investigation.

Section 3 consisted of three open-ended questions, wherein students had to mention problems and impressions they encountered while learning circle geometry through GeoGebra, and had to make suggestions on what could be done to improve the use of GeoGebra in their learning.

### **3.5.1.1 Questionnaire distributions and challenges**

The researcher had planned the dates and timeframe for completion of the questionnaire. The allocated time in each school for completion was one hour. This took the researcher two days to distribute and collect all questionnaires. After explaining this to respondents, questionnaires were hand delivered by the researcher and picked up immediately after completion in each school.

The main disadvantage of using a questionnaire is probably a low response rate. The researcher will mostly depend on the willingness of the respondents. Another disadvantage is that questions had to be simple and straightforward enough to be understood with the help of printed instructions and definitions. Questionnaires lack probing and as a result some answers tend to be superficial. The rate of responses was addressed by the researcher as respondents were requested to respond, moreover all respondents were present in their respective schools due to strict school management teams of the schools who are eager to find a viable solution to curb out the scourge of poor performance in mathematics. The respondents were continuously informed of the aims of the study and the significance of their participation. The reason for the pilot sample was intended to increase the response rate. This assisted in making the questionnaire have less ambiguous, simple and straightforward items. The other challenges were respondents who continually pleaded with the researcher for more clarification of items, mostly translation queries.

It was very helpful that the same questionnaire was distributed to a number of respondents. And the fact that the researcher was well acquainted with most respondents, having taught them before and was conducting the study himself, that placed respondents at ease in taking initiative to respond.

At the beginning of the study both experimental group and control group took a pre-test to gauge their abilities on the concepts of circle geometry involving all theorems and riders. The pre-test and post-test contain similar items.

### **3.5.2 Open-Ended Questions**

Open-ended questions, also called open, unstructured, qualitative questions, refer to those questions for which the response patterns or answer categories are provided by the respondent, not the interviewer (Frey, Daalem & Peyton, 2004). This is in

contrast to close-ended questions or structured questions, for which the interviewer provides a limited number of response categories from which the respondent makes a selection.

Open-ended questions are mostly appropriate for situations in which one wants to ask questions that elicit depth of information from relatively few respondents. As McIntyre (2013) stated, open-ended questions certainly prove valuable for learning, but they also have their own challenges. Open-ended questions allow researchers to hear from the respondents, in their own words, why they have answered closed-ended questions in certain ways, how we can satisfy them better, what their suggestions might be for future initiatives and so on. The researcher, in line with interpretive paradigm, designed three open-ended questions (McIntyre, 2013) following its yielding opportunities:

- Open-ended questions allow respondents to provide their opinions in their own words, with all the subtleties and nuances this implies. Respondents can make distinctions and add conditions that put meat on the bones of closed-ended responses as opposed to rating scales, interval scales, ranking or lists.
- Respondents can add examples and context to expand on and illuminate their answers in a way closed-ended responses simply cannot, adding richness and depth to the research findings.
- Open-ended questions give the researcher insights into how respondents talk about the software, and this is helpful in understanding their behaviours or motivations, as well as in crafting future quantitative research questions.

The researcher again resorted to open-ended questions to supplement the information from closed-ended items in line with ideas of Downey (2010), in those open-ended questions:

- Facilitate enhanced levels of cooperation and understanding.
- Provide opportunities for others to express themselves more openly and honestly.
- Encourage others to provide information including their ideas, concerns and feelings.
- Assist in creating a positive learning and sharing experience.
- Allow others to share what is presently relevant to them.



- Show respect and interest in others.
- Encourage others to flow with their thoughts and feelings and allows you to support this flow.
- Depicts your willingness to constructively invest time in others.

The challenges are:

- Coding open-ended responses is time consuming and costly.
- These are often daunting to implement and successfully coordinate in strict alignment with aims.

The researcher consulted three knowledgeable educators, with more than 10 years' experience each, to help in wording questions clearly and carefully, to make it as easy as possible for those taking the survey to frame their responses.

### **3.5.3 Document Analysis**

Document analysis is the systematic examination of programme documents. In planning the document analysis the researcher considered students educational background, motivational levels, and their skill levels. Document analysis helped the researcher in gaining insight in the programme activity or service, examining trends, patterns, consistency, and provided the best preliminary study for survey questions, goals of the research and the ability to implement changes as to how the organisational contexts would be impacting the research programme. This highlights critical reasons for respondents' participation. Furthermore, a look at the essential needs of the entire population helped in narrowing the focus, determining how the results will be used and finally helped develop document analysis criteria (IAR, 2011).

### **3.5.4 Triangulation of Method, Data and Theory**

Validity, in qualitative research, refers to whether the findings of a study hold and are "true" in the sense that study findings accurately reflect the situation, and "certain" in the sense that findings are supported by the evidence. Triangulation is a method used by qualitative researchers to check and establish validity in their studies by analysing a research question from multiple perspectives. Patton (2002) as cited by Guion, Diehl and McDonald (2011), cautions that it is a common misconception that

the goal of triangulation is to arrive at consistency across data sources or approaches; in fact, such inconsistencies may be likely given the relative strengths of different approaches. In Patton's view, these inconsistencies should not be seen as weakening the evidence, but be viewed as an opportunity to uncover deeper meaning in the data.

#### **3.5.4.1 Methodological triangulation**

Methodological triangulation involves the use of multiple qualitative and / or quantitative methods to study the program. The researcher compared results from the questionnaire and open-ended questions, and established similarities. The validity of results was thereby confirmed.

#### **3.5.4.2 Data triangulation**

Data collection involves using different sources of information in order to increase the validity of a study. Sources were participants and staff who helped in the organisation of the programme. While analysing, feedback further showed that there were areas of divergence and areas of agreement.

#### **3.5.4.3 Theory triangulation**

Theory triangulation entails the use of multiple perspectives to interpret a single set of data. The researcher while looking for information on technology, consulted with Information Technology staff to gain insight into how computers can be upgraded in rural areas, to assist in achieving the required results.

The benefits of triangulation include "increasing confidence in research data, creating innovative ways of understanding a phenomenon, revealing unique findings, challenging or integrating theories and providing a clearer understanding of the problem. These benefits largely result from the diversity and quantity of data that can be used for analysis. Using open-ended questions and questionnaires added a depth to the results that would not have been possible using a single-strategy study, thereby increasing the validity and utility of the findings. On the other hand triangulation is time-consuming, requiring planning and organisation of resources. Triangulation can however be effectively used to deepen the researchers'

understanding of the underlying issues and maximise their confidence in the findings of qualitative studies.

### **3.6 DATA ANALYSIS**

Analysis of data is a process of inspecting, cleaning, transporting, and modelling data with the goals of discovering useful information, suggesting conclusions and supporting decision-making.

#### **3.6.1 Multiple Regression Analysis (MRA) Application**

Multiple regression is a statistical tool used to drive the value of a criterion from several other independent, or predictor, variables. It is the simultaneous combination of multiple factors to access how and to what extent they affect a certain outcome. Multiple regression analysis is a powerful technique used for predicting the unknown value of two or more variables. Multiple regression examines the relationship between a single outcome measure and several predictor or independent variables (Jaccard, Guilamo-Ramos, Johansson, & Bouris, 2006).

By multiple regression, we mean models with just one dependent and two or more independent (exploratory) variables. The variable whose value is being predicted is known as the dependent variable and the ones whose known values are used for prediction are known independent (exploratory) variables. Multiple regression with its flexibility helped the researcher test hypotheses of linear associations among variables, examining associations among pairs of variables while controlling for potential confounds as defined by Hoyt, Leierer and Millington (2006), and to test complex associations among multiple variables. This helped the researcher make inferences and generalisations about the theory to be valid and reliable.

##### **a) The Multiple Regression Model**

In general, the multiple regression equation of Y on  $X_1, X_2 \dots X_k$  is given by:

$$Y = b_0 + b_1X_1 + b_2X_2 + \dots + b_kX_k$$

##### **b) Interpreting Regression Coefficients**

Here  $b_0$  is the intercept and  $b_1, b_2, b_3, \dots, b_k$  are analogous to the slope in linear regression equation and are also called regression coefficients. These can be interpreted the same way as slope. Thus if  $b_1 = 2.5$ , it would

indicate that  $Y$  will increase by 2.5 units if  $X_i$  increased by 1 unit. The appropriateness of the multiple regression model as a whole can be tested by the  $F$ -test in the ANOVA table. A significant  $F$  indicates a linear relationship between  $Y$  and at least one of the  $X$ 's.

### **c) How Good Is the Regression**

Once a multiple regression equation has been constructed, one can check how effective it is (in terms of predictive ability) by examining the coefficient of determination ( $R^2$ ).  $R^2$  always lies between 0 and 1.

$R^2$  – coefficient of determination

The closer  $R^2$  is to 1, the better the model and its predictive capability is. A related question is whether the independent variables individually influence the dependent variable significantly. Statistically, it is equivalent to testing the null hypothesis that the relevant regression coefficient is zero.

This can be done using  $t$ -test. If the  $t$ -test of a regression coefficient is significant, it indicates that the variable in question influences  $Y$  significantly, while controlling for other independent explanatory variables

### **3.6.2 Meeting the Assumption of MRA Usage**

The multiple regression technique, however, does not test whether data is linear. It proceeds by assuming that the relationship between the  $Y$  and each of  $X_i$ 's is linear. It is prudent to always look at the scatter plots of  $(Y, X_i)$ ,  $i = 1, 2, \dots, k$ . If any plot suggests non linearity, one may use a suitable transformation to attain linearity. The researcher assumed that the relationship between variables is linear. The researcher relied on bivariate scatterplot of the variables of interest, to check on the linearity.

Another important assumption is non-existence of multicollinearity, where the independent variables are not related among themselves. Collinearity is a statistical phenomenon in which two or more predictor variables in a multiple regression model are highly correlated, meaning that one can be linearly predicted from the others with a non-trivial degree of accuracy. At a very basic level, this can be tested by

computing the correlation coefficient between each pair of independent variables. On assumption of normality, the researcher assumed that residuals are distributed normally. The researcher, to check for normality, reviewed the distributions of the major variables of interest. Histograms for residuals and normality probability plots were produced for inspection of the distribution of residual values.

The researcher included only 15 closed-ended items for the questionnaire, to bring about stability in the regression line (see Annexure 7).

The researcher used multiple regression analysis in predicting a continuous dependent variable from a number of independent variables. If the dependent variable is dichotomous, then logistic regression should be used.

### **3.6.3 Variables and Models Evaluations**

#### **3.6.3.1 R Square value**

R-squared in statistics, the coefficient of determination  $R^2$ , is the proportion of variability in a data set that is accounted for by a statistical model. In this definition, the term “variability” is defined as the sum of squares. R-squared measures how close the data are to the fitted regression line. It is also known as the coefficient of determination, or the coefficient of multiple determination for multiple regression. 0% indicates that the model explains none of the variability of the response data around its mean.

Adjusted R-square is a modification of R-square that adjusts for the number of terms in a model. R-square always increases when a new term is added to a model, but adjusted R-square increases only if the new term improves the model more than would be expected by chance.

#### **3.6.4 Qualitative Data Analysis**

Qualitative data analysis refers to assessing data that approximates or characterises but does not measure the attributes, characteristics, properties and so on of the subject in question.

The researcher employed qualitative data analysis in order to delve more on the depth of the study. The purpose was to describe a situation and gain insight on the

use of GeoGebra in circle geometry, theoretically. The researcher wanted an in-depth explanation from a small sample, intended to then draw out patterns from assessment of illustrative explanations and individual responses, to then gauge concepts and insights. . The researcher designed open-ended questions with no pre-determined response categories and questions were kept broader, contextual and flexible.

#### **3.6.4.1 Data Capturing, Immersion, Coding, Reduction and Interpretation**

Data capturing involves inputting of data, not as a direct result of data entry but instead as a result of performing a different, but related, activity. Data immersion on the other hand is the process of reading and rereading each set of notes or transcripts until you are intimately familiar with the content ([www.path.org](http://www.path.org)). The researcher read data timeously to detect relationships and contradictory factors. The researcher further did data coding to categorise themes for easy sorting, comparing and for later retrieval. The researcher did data reduction by selecting, focusing, simplifying, abstracting, and transforming the raw data and summarised patterns based on the original objectives of the study. The data reduction continued until the final report. The researcher continuously, during data collection, considered the meaning of each set of information gathered, carefully noting patterns and explanations.

#### **3.6.5 Assumptions of Qualitative Data Analysis**

The underlying assumptions of applying a qualitative data analysis are:

- Data analysis is determined by both the research objectives (deductive) and multiple readings and interpretations of the raw data (inductive). Therefore the findings are derived from both the research objectives outlined by the researcher and findings arising directly from the analysis of the raw data.
- The primary mode of analysis is the development of categories from the raw data into a model or framework that captures key themes and processes judged to be important by the researcher.
- The researcher's findings result from multiple interpretations made from the raw data, by researchers who code the data. Inevitably, the findings are shaped by the assumptions and experiences of the researchers conducting

the research and carrying out the data analyses. In order for the findings to be usable, the researcher (data analyst) must make decisions about what is more important and less important within the data.

- The trustworthiness of findings can be assessed by a range of techniques such as (a) independent replication of the research, (b) comparison with findings from previous research, (c) triangulation within a project, (d) feedback from participants in the research, and (e) feedback from users of the research findings.

In the current study the researcher was concerned primarily with process, meaning how participants make sense of their experiences in learning circle geometry using traditional approach and technological approach (involving GeoGebra). The researcher used descriptive measures as he was interested in process, meaning, and understanding gained through words and pictures. Criteria for inductive reasoning were met as the researcher built abstractions, concepts, hypotheses, and theories from details. Reality is socially constructed, primacy of the subject matter, variables are complex, interwoven, and difficult to measure and mimic (participant's point of view), were the underlying assumptions.

Basing on interpretive frameworks, the researcher used the following philosophical assumptions as suggested by Creswell (2012) and Carnaghan (2013), in shaping the direction of the current study:

- **Ontological assumptions (The nature of reality):** This relates to the nature of reality and its characteristics. The researcher embraced the notion of multiple realities and reported on these multiple realities by exploring multiple forms of evidence from different individuals' perspectives and experiences.
- **Epistemological assumptions (How researchers know what they know):** The researcher was very meticulous with participants during the study. Thus the researcher did endeavour to obtain subjective evidence based on individual points of view from participants.
- **Axiological assumptions (The role values in research):** The researcher elucidated on the values in the study and actively reported their values and biases as well as the value-laden nature of information gathered from the field.

- **Methodology (The methods applied in the research):** Inductive, emerging approaches adopted, while being shaped by the researcher's experience during data analysis and collection.

### **3.6.6 The Need for Quantitative and Qualitative Data Analysis**

Quantitative and qualitative data are, at some level, virtually inseparable. In the above views the researcher looked for explanatory, confirmatory, deductive and inductive classification relating to the current study, thus the researcher applied both, in that each:

- Involve the use of observation to answer research questions.
- Use safeguards to minimise bias and invalidity.
- Attempt to triangulate data.
- Attempt to provide explanations of findings.
- Interpret and create narrative conclusions about findings.
- Select and use analytical techniques to gain maximum meaning.
- Attempt to explain complex relationships.
- Utilise techniques to verify the data.
- Tend to use data reduction techniques.

### **3.6.7 Analysis of Data from Documents**

Document analysis is the detailed examination of documents produced across a wide range of social practices, taking a variety of forms from written word to the visual image (Jupp, 2008). Document analysis requires that data be examined and interpreted in order to elicit meaning, gain understanding, and develop empirical knowledge (Corbin & Strauss, 2008: Rapley, 2007, pp. 273-290).

Documents collected may be used in future for systematic evaluation as part of the study. Documents included are textbooks, diaries, agendas, attendance registers, minutes of meetings, and letters. The analytical procedure entailed finding, selecting, appraising, and synthesising data contained in documents. Documents, as suggested by Bowen (2009, pp. 27-40) can be used to:

- Provide data on the context within which research participants operate. Documents provide background information.



- Suggest some questions that need to be asked and situations to be observed as part of the research.
- Provide supplementary research data.
- Provide a means of tracking change and development.
- And can be used as a way to verify findings or corroborate evidence from other sources.

The researcher skimmed (superficial examination), read and interpreted data in this study's document analysis. In so doing, content analysis and thematic analysis were combined.

Bowen (2009) described content analysis as the process of organising information into categories related to the central questions of the research. But as Franzosi, Doyle, McCelland, Rankin and Vicari (2013) contend content analysis is an attempt to characterise the meanings in a given body of discourse in a systematic and quantitative fashion, it is the statistical semantics formulations, directed toward empirical problems and its statistical character is one of its most distinctive attributes.

But the researcher strongly aligned with Bowen (2009), who contends content analysis excludes the quantification section, and entails a first-pass document review, where meaningful and relevant passages of text or other data are identified. The researcher demonstrated this by identifying pertinent information and separating it from that which is not pertinent as suggested by Corbin and Strauss (2008).

Thematic analysis is a form of pattern recognition within the data, with emerging themes becoming the categories for analysis (Fereday & Muir-Cochrane, 2006). The researcher was careful and detailed when handling data, always re-reading and reviewing the data at every stage. The researcher also selected, coded and categorised constructions based on the data's characteristics in addressing themes that are pertinent to application of GeoGebra on circle geometry.

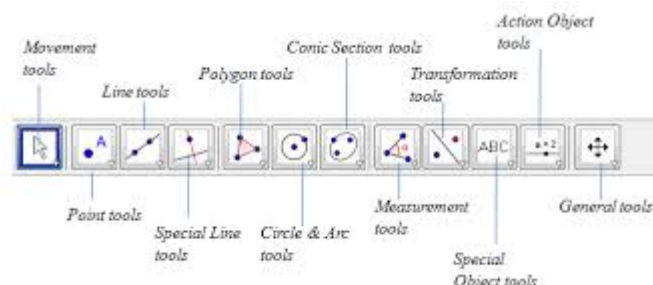
### **3.7 CONDUCTING THE PRE TEST**

The researcher taught in both experimental and control groups in all five schools using paper and pencil, for two weeks. Thereafter an achievement test in the form of pre-test of 14 multiple choice questions was administered and results recorded (Annexure 5). The students calculated first before choosing the correct option from


those given in each item. The achievement test covers all four basic circle geometry theorems and most riders. The main themes were on the basic understanding of the inscribed angles in a circle, lines inside (chords) and outside (tangents). The terms like bisect and perpendicular formed part of the themes, so was the Theorem of Pythagoras and congruency. The scores were put aside and further teaching continued. In the next phase of teaching, groups were separated as assigned into experimental group who were next taught with the use of GeoGebra software, while the control were further taught best by the researcher who has 19 year experience in mathematics teaching.




### 3.8 INTERVENTION USING COMPUTER SOFTWARE (GEOGEBRA)

The experimental group of students were introduced to computer set of tasks within GeoGebra. This was done in all five school sessions. The first session took one and a half hours in each school. The focus was to orientate students to GeoGebra software; exploring and introducing the different menu options as well as observing tutorials and presentations built into the GeoGebra program.









**Figure 3.2 Screenshot of a GeoGebra menu toolbar**








Movement tools are by default grouped  icons in the toolbar:


- Move: 
- Rotate around the point: 
- Record to spreadsheet tool: 





Point tools are by default grouped  icons in the toolbar:





- Point: 
- Point on object: 
- Attach/ detach tool: 
- Intersect tool: 
- Midpoint or centre tool: 
- Complex number tool: 

Line tools are by default grouped  icons in the toolbar:





- Line tool: 
- Line segment tool: 
- Segment with given length tool: 
- Ray tool: 
- Vector tool: 
- Polygon line tool: 
- Vector from point tool: 


Special line tools are by default grouped  icons in the toolbar:










- Perpendicular tool: 
- Parallel tool: 
- Perpendicular bisector: 
- Angle bisector: 


- Tangents tool: 
- Polar diameter tool: 
- Best fit line tool: 
- Locus tool: 





Polygon tools are by default  grouped icons with:

- Polygon tool: 
- Regular polygon tool: 
- Rigid polygon tool: 
- Vector polygon tool: 







Circle and arc tools which are by default grouped  icons have the following tools:


- Circle with centre through point tool: 
- Circle with centre and radius tool: 
- Compass tool: 
- Circle through 3 points tool: 
- semicircle through 2 points tool: 
- circular arc: 
- Circumcircular arc tool: 
- Circular sector tool: 
- Circumcircular sector: 







Conic section tools are by default grouped  icons, and include:

- Ellipse tool: 
- Hyperbola tool: 
- Parabola tool: 
- Conic through five points: 



Measurement tools are by default grouped  icons, and comprise of:

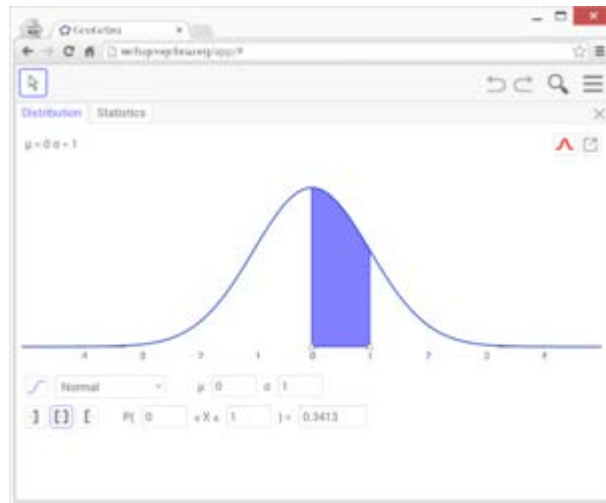
- Angle tool: 
- Angle with given size tool: 
- Distance or length tool: 
- Distance with given size tool: 
- Area tool: 
- Slope tool: 

Transformation tools are by default grouped  icons, and include:

- Reflect about line: 
- Reflect about point: 
- Reflect about circle: 
- Translate by vector: 
- Dilate from point: 
- Rotate around point: 

Special object tools are by default grouped <sup>ABC</sup> icons which have:

- Text tool: <sup>ABC</sup>
- Image tool: 
- Pen tool: 
- Relation tool: <sup>?</sup>  $a = b$



- Probability tool:



Z Test of a Mean

Hull Hypothesis  $\mu = 7$

Alternative Hypothesis  $\mu < 7$

Sample

Mean 7

$\sigma$  7

N 7

Result

Z Test of a Mean

Mean 7


$\sigma$  7

SE 7





N 7


Z 7










P 7

- Statistics tool:
- Function inspector tool: 

Action object tools are by default grouped  $\xrightarrow{a=2}$  icons with the following tools included:

- Slide tool: 
- Check box tool: 
- Button tool: 
- Insert/input box tool: 

General tools are by default grouped  icons with:

- Move graphic view tool: 
- Zoom In tool: 
- Zoom Out tool: 
- Show/hide object tool: 
- Show/hide label tool: 
- Copy visual style tool: 
- Mode delete:  or  or 

The researcher introduced GeoGebra as a computer tool for learning mathematics, beginning with selecting points, parallel lines, measuring lines and angles, deleting and going clockwise and anticlockwise when measuring angles, circle etc. Specifically, orientation was on basics like which icons are used for algebra or geometry.

There were no planned activities and no explicit instructional goals in this session. The main idea was to let students experience this type of software and develop an interest so that the researcher could identify factors for more structured analysis.

In the following sessions, the researcher worked with students individually as well as with small groups. The aim was for students to explore the meaning of the concepts of perpendicular bisector, inscribed angles and tangents in a circle by interacting with computer based activities. These sessions were observed and notes were taken by

the researcher. Following the completion of this session, students were asked to carry out tasks based on new conceptualisation of each facet of circle geometry. The tasks were mainly designed by the students themselves to drill and assess their own understanding. The main goal of the researcher was to observe how students used GeoGebra software and what they learned from activities. Timeously the researcher participated as a facilitator and interviewer when required.

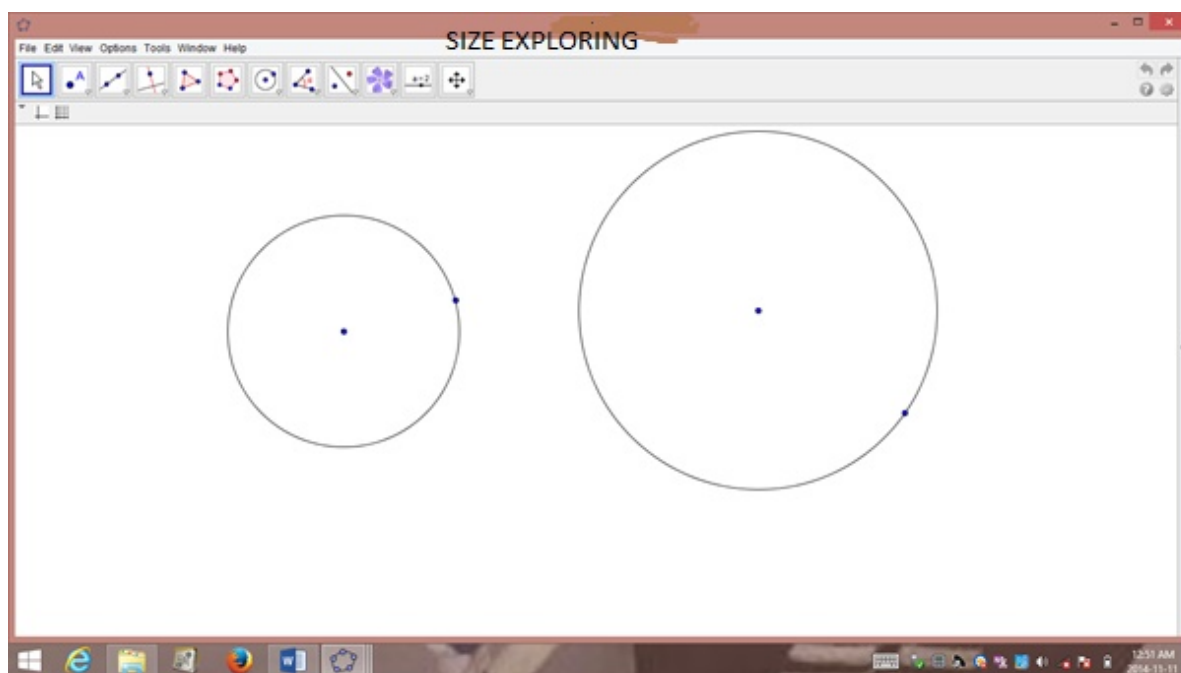
As a facilitator, the researcher ran the software and guided students in reaching intended tasks as suggested by Thimbu (2007). Acting as an interviewer, the researcher asked for more explanations to evaluate nature, breadth and depth of their underlying understanding of each sub-topic in line with the whole concept of circle geometry. When students had technical difficulties with the software, the researcher limited interventions by providing little help to avoid interfering with results and relevant activities. Only when serious difficulties arose was the researcher compelled to providing direct tutorials for graphing.

### **3.8.1 Activities**

#### **3.8.1.1 Activity 1**






Individually and in a small group, students were introduced to the icons and their usage on the spreadsheet of the GeoGebra toolbar. They then practised moving the cursor of each icon and further learned how to expand the circle by moving the cursor on the circumference of the circle away from the centre of the circle as done by a student (Figure 3.3). This acquainted each student in using a drag mode in the toolbar.

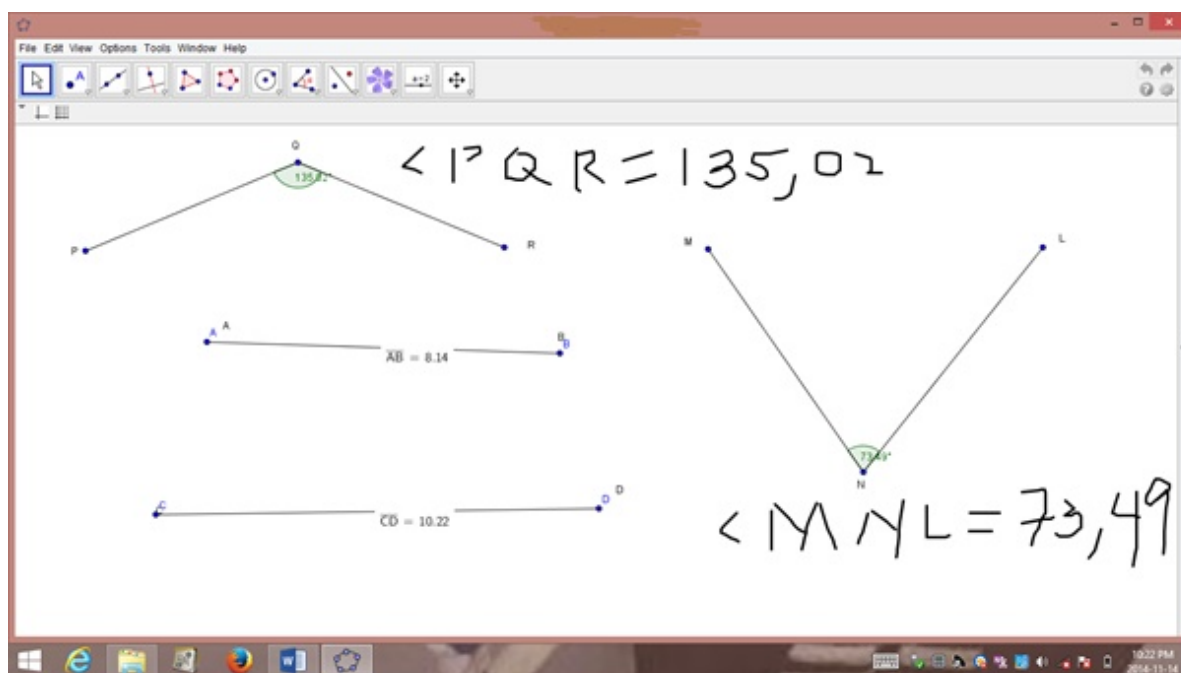




**Figure 3.3 Student exploring the size of a circle using GeoGebra software**







In the next figure 3.4, diagrams show one student practising measuring angles and lines. Angles: ( $\angle PQR = 135.02^\circ$ ) and ( $\angle MNL = 73.49^\circ$ ). And lines were measured:  $AB = 8.14\text{cm}$  and  $CD = 10.22\text{ cm}$ . Students were utilising multiple tools from GeoGebra software.

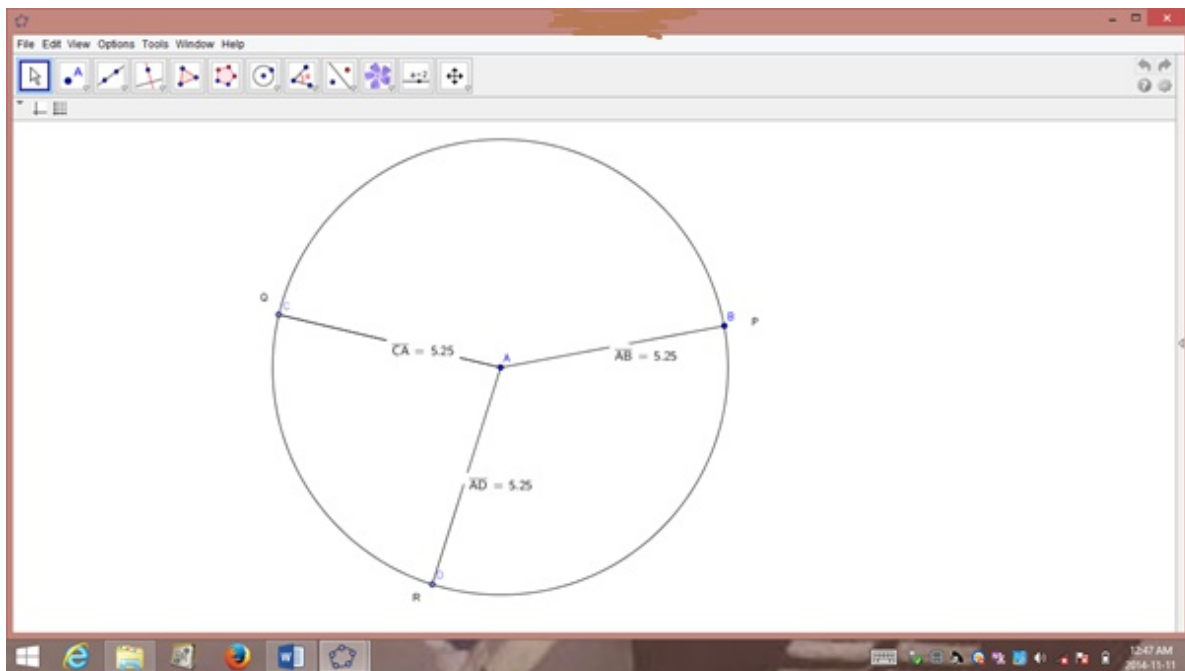
Specifically they practised through dragging mode (move tool ) , point tool  for plotting points, line segment tool  for joining points, distance tool  for measuring line segments between points, angle tool  for measuring angles between line segments, and text tool  $ABC$  for labelling points.



**Figure 3.4 Student practising measuring of angles and lines using GeoGebra tools**

The students verified, using GeoGebra software, the key in all circle geometry: All radii have the same size (equal length). As shown in the Figure 3.5 below if A is the centre and points P, Q, and R, are found on the circumference of the circle, the AP, AQ, and AR, are radii. The student, as other students did on their explorations, measured the radii to be all equal to 5.25 cm. The above introductory activity addressed research question 3: “practical and theoretical assistance of GeoGebra on justification and verification in geometry.”

The following tools were used to further verify that radii of the same circle are equal in size (length). Circle with centre tool  for constructing circle with A given centre, point tool  for plotting points on the circumference of the circle, line segment tool  for joining points and distance tool  for measuring the length of line segments, move tool  for dragging and text tool  for labelling.










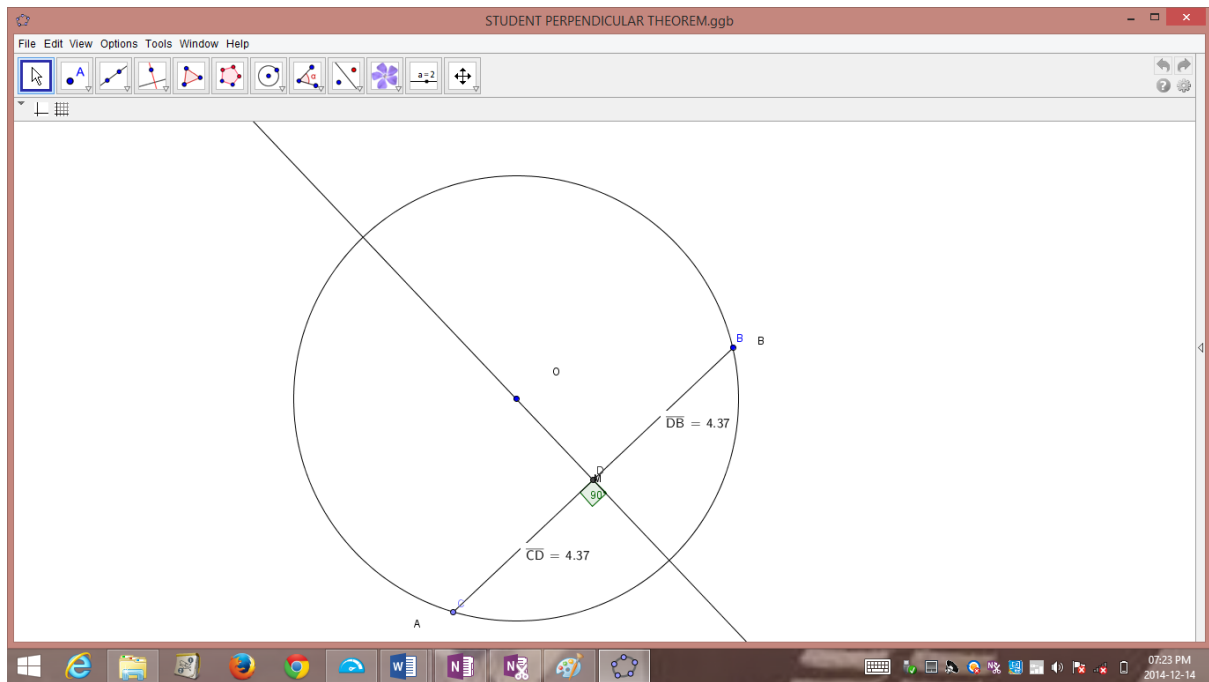
**Figure 3.5 Student evaluating the lengths of radii using GeoGebra software**

### 3.8.1.2 Activity 2

Students proceeded with the theorem that the line from the centre of the circle to the midpoint is perpendicular to the chord (Figure 3.6). The student drew the circle with centre O, the chord BC. Then the student plotted point D on chord BC, thereafter joined the line from centre perpendicular to BC at D. The student measured  $\angle ODB$  and found out it is  $90^\circ$ , then measured CD and DC of which both were equal in length. This was addressing the research question 2 “How does the use of GeoGebra improve learners’ understanding of circle geometry theorems?” and research question 3 “The practical assistance of GeoGebra on teachers’ confidence, learners’ performance improvements and justifying proofs and theorems.”

The following GeoGebra tools were used by students to prove the circle geometry theorem: The line from the centre of the circle to the midpoint of the chord is perpendicular to the chord, and its corollary ‘the line from the centre of a circle which is perpendicular to the chord bisect the chord.’

Move tool , angle tool , point tool , line segment tool , distance tool  cm, circle centre tool , and text tool .








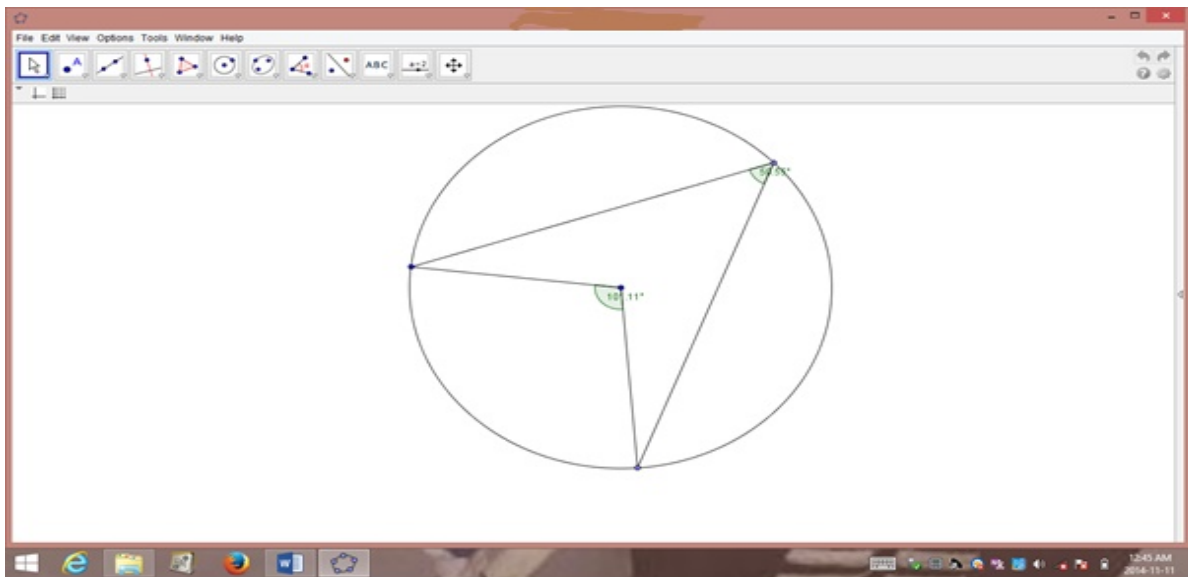
**Figure 3.6 Student verifying perpendicular chord theorem using GeoGebra tools**

### 3.8.1.3 Activity 3

We note that the angle in the centre is doubled that subtended in the circle (Figure 3.7). A student was trying to verify that “an angle at the centre of the circle is twice an angle it subtends in the circle.” This addressed research question 2: “The use of GeoGebra in improving learners’ understanding of geometry.”, and “Practical assistance of GeoGebra on justifying proofs and theorems of circle geometry.” which is research question 3.3.

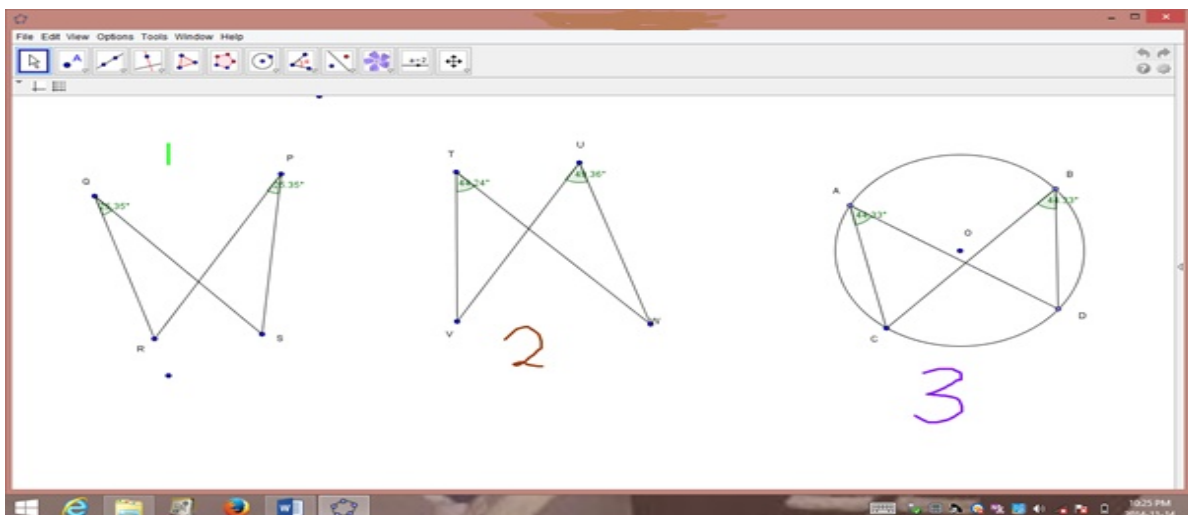
Students used the following GeoGebra tools to justify and verify the circle geometry

theorem. Move tool  for dragging, circle with given centre tool  for constructing circle, point tool  for plotting points, line segment tool  for joining points, angle tool  for measuring angles between line segments.

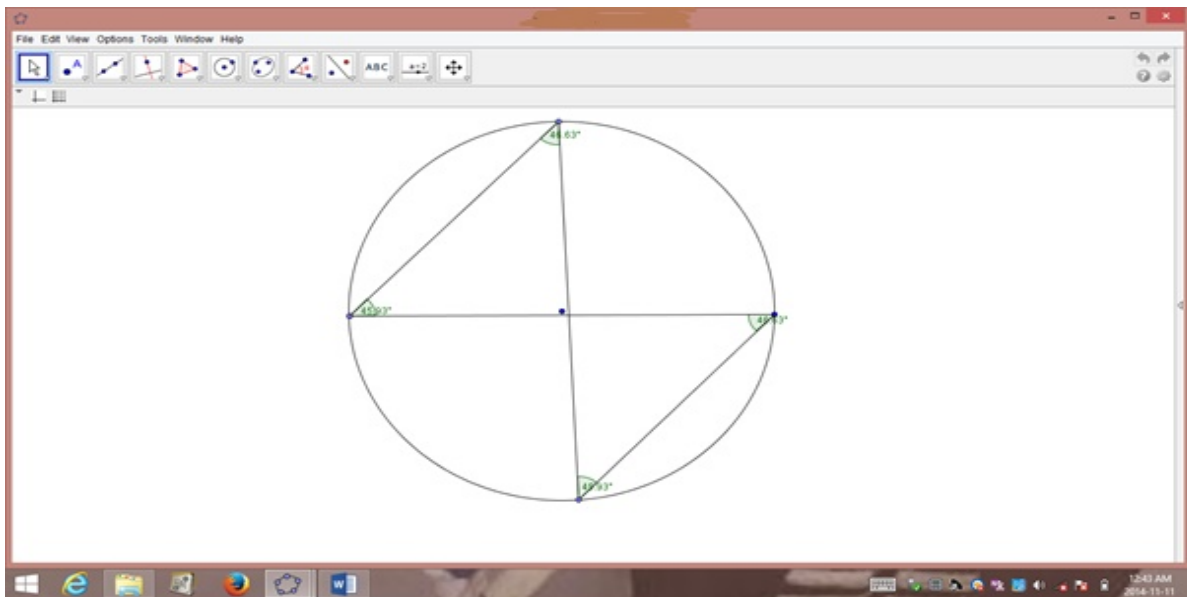


**Figure 3.7 Student verifying centre theorem using GeoGebra software**

A student was not accurate enough but the angle at the circle is  $50.55^\circ$  and the angle at the centre is  $101.11^\circ$ . The angle at the centre should be  $101.10^\circ$ . That is where the researcher as a facilitator came on board to observe whether students realised where they may have made mistakes, and allow them to address the issue on their own.



**Figure 3.8 Student verifying circle geometry theorem using GeoGebra tools**



**Figure 3.9 Student verifying same segment theorem using GeoGebra tools**

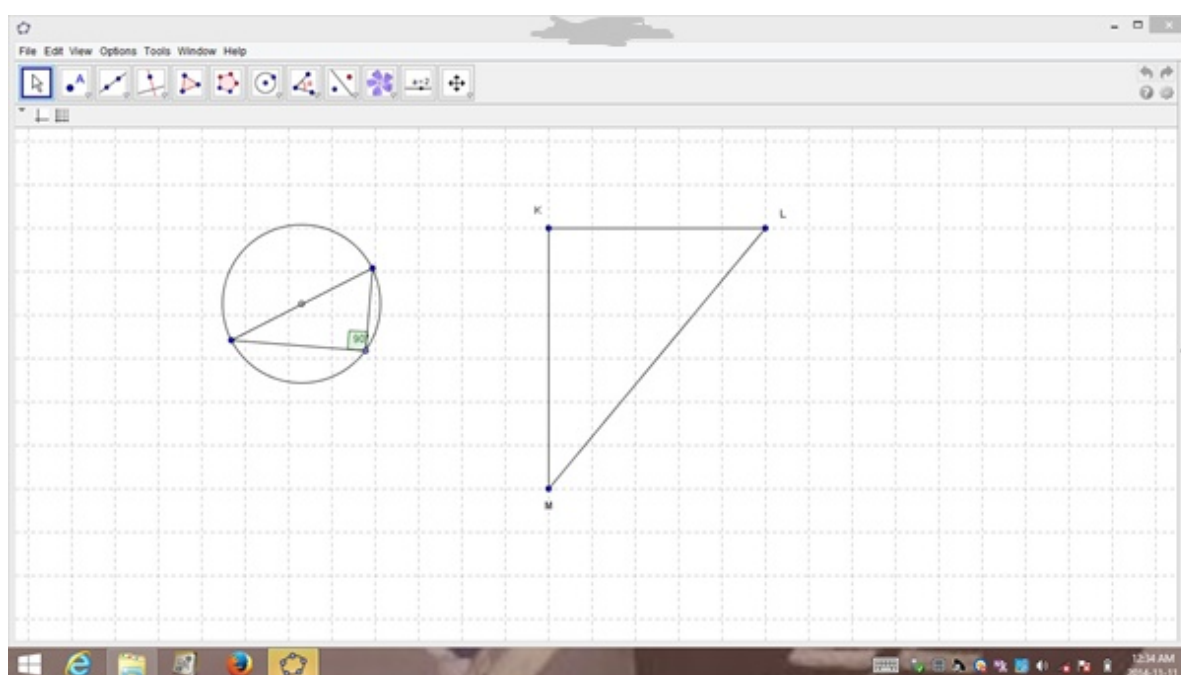
Angles subtended by same arc are equal (Figure 3.8). Figure 3.8, diagram 1 have  $\angle RQS = 25.35^\circ$  and  $\angle RPS = 25.35^\circ$ ; conclusion can be made that points P, Q, R, and S lie on the circle because two angles  $\angle RQS = \angle RPS$  and these are subtended by the same arc RS. But in the same Figure 3.8, diagram 2 has  $\angle VTW = 44.24^\circ$  and  $\angle VUW = 49.36^\circ$  which are not equal, yet subtended by the same arc VW, then points T, U, V, and W, do not all lie on the same circle. In the same Figure 3.8 diagram 3 verifies that when four points lie on the circle, the two angles subtended by the same arc will be equal, as can be seen that  $\angle CAD = \angle CBD = 44.33^\circ$  both are subtended by same arc CD. Accuracy is what the researcher stressed when facilitating, as it is imperative for all problems in mathematics. Multiple corollaries were drilled using GeoGebra software. This activity addressed research question 2: “The use of GeoGebra improves learners’ understanding of circle theorems and riders” and research question 3.3 “Practical assistance through GeoGebra on justifying proofs and theorems of geometry.”

The rider “angles in the circle which are subtended by the same arc are equal in size.” This is confirmed in either way by Figure 3.9. A student verified that where corresponding angles are subtended by the same arcs, they are equal in size.

#### 3.8.1.4 Activity 4

The aim of the activity was showing “How the use of GeoGebra in improving learners’ understanding of circle”: Research question 2 and research question 3.1: “Practical assistance through GeoGebra on teachers’ confidence in teaching geometry” and research question 3.3: “Practical assistances through GeoGebra on justifying proofs and theorems of circle geometry.”

In Figure 3.10, we observe verification of the rider, being “an angle subtended by the diameter is a right angle”.



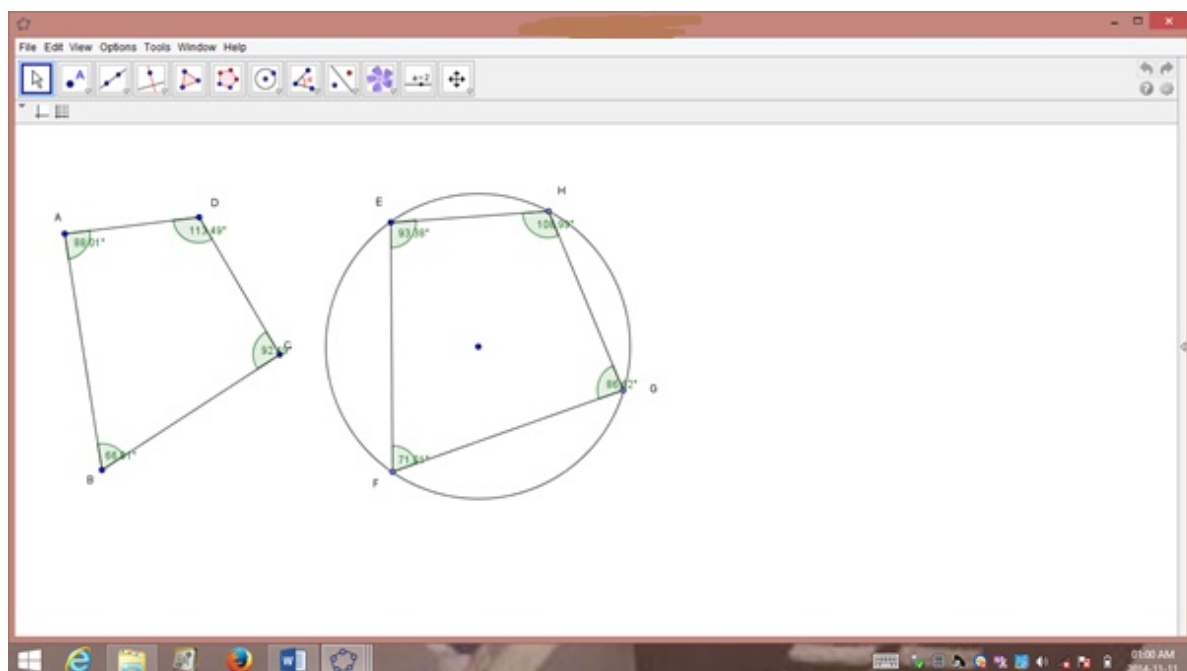
**Figure 3.10 Student evaluating semi-circle theorem using GeoGebra tools**

#### 3.8.1.5 Activity 5

This activity was designed to practise, verify and justify the theorem, “opposite angles of a circle added, are equal to  $180^\circ$ , meaning they are supplementary”, as well as related riders. Again the activity covers all research questions that number 1: “The impact of application of GeoGebra in Euclidean geometry in improving learners’ performance”, number 2: “The use of GeoGebra in improving learners’ understanding of GeoGebra theorems” and number 3: “Practical assistances of GeoGebra on teachers’ confidence in teaching geometry, learners’ performance

improvements, and justifying proofs and theorems of circle geometry". Students explored that for any quadrilaterals, if the sum of their interior opposite angles do not add up to  $180^\circ$ , the quadrilateral automatically cannot be cyclic (Figure 3.11) as explored by a student. In quadrilateral ABCD,  $\angle A = 88.01^\circ$  is opposite to  $\angle C = 92.49^\circ$ . But  $\angle A + \angle C = 88.01^\circ + 92.49^\circ = 180.5^\circ$  which is  $0.5^\circ$  more. And  $\angle B = 66.01^\circ$  opposite to  $\angle D = 113.49^\circ$ . And  $\angle B + \angle D = 66.01^\circ + 113.49^\circ = 179.5^\circ$  this is  $0.5^\circ$  less than  $180^\circ$  thus not being supplementary once again. Therefore points A, B, C, and D do not lie on the circle. The quadrilateral in diagram 2 of Figure 3.11 verifies and justifies that opposite angles are supplementary. A student had measured  $\angle F = 71.01^\circ$  and  $\angle H = 108.99^\circ$  and when added gave  $180^\circ$ , therefore  $\angle F$  and  $\angle H$  are supplementary. And  $\angle E = 93.38^\circ$  and  $\angle G = 86.62^\circ$ , which when added together give  $180^\circ$  as well. So points E, F, G, and H are cyclic (meaning these points lie on the circle).

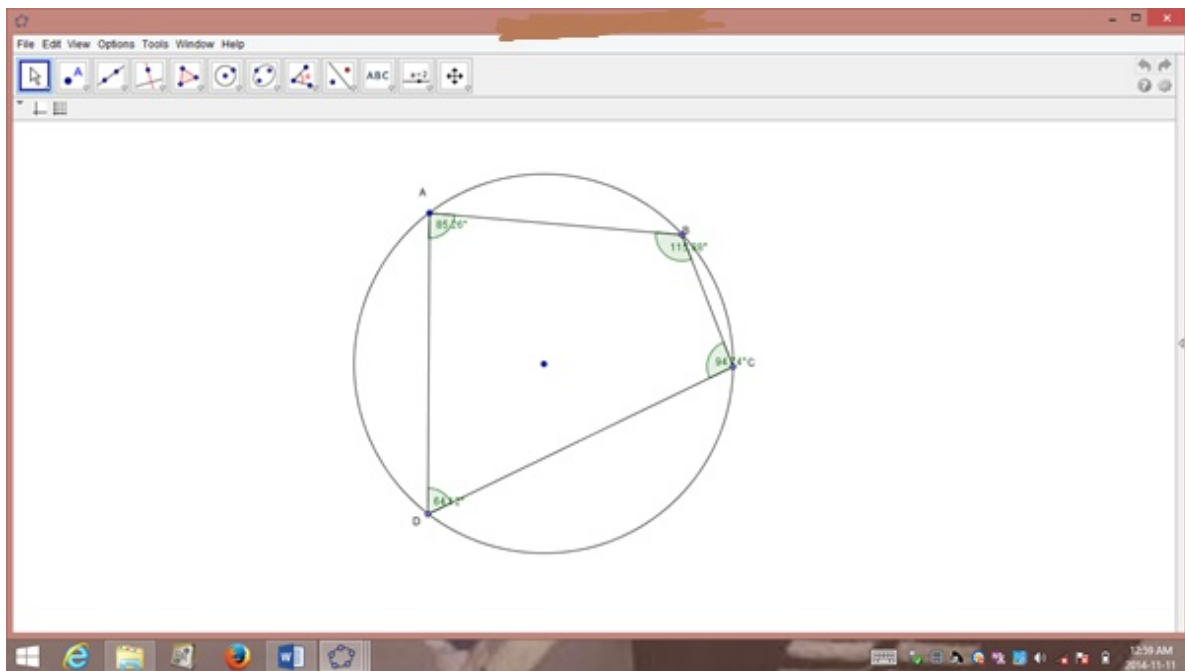
A student did her own verification of the theorem and the researcher as the facilitator (Thimbu, 2007), stressed the issue of accuracy.



**Figure 3.11 Student comparing properties of supplementary theorem**

Another student verified, by drawing the different figures, shown in Figure 3.12, to justify the proof and theorem of opposite angles of a quadrilateral are supplementary. In her figure  $\angle A = 85.26^\circ$  which when added to  $\angle C = 94.74^\circ$  sum up to  $180^\circ$





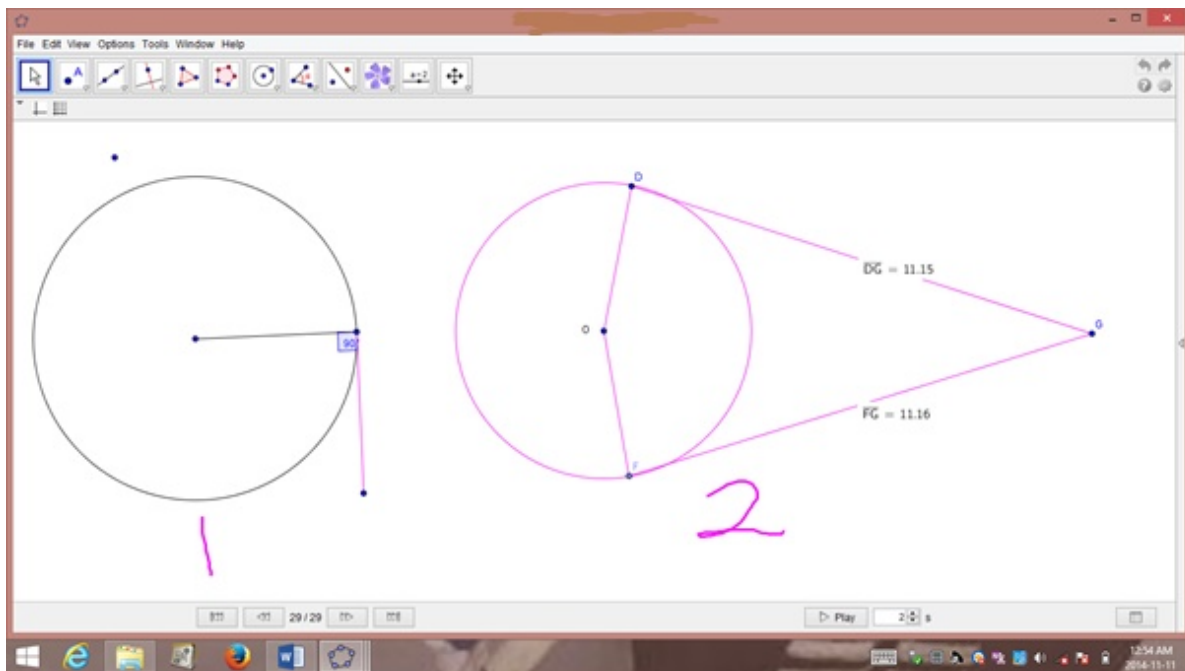
**Figure 3.12 Student verifying supplementary theorem using GeoGebra tools**

### 3.8.1.6 Activity 6

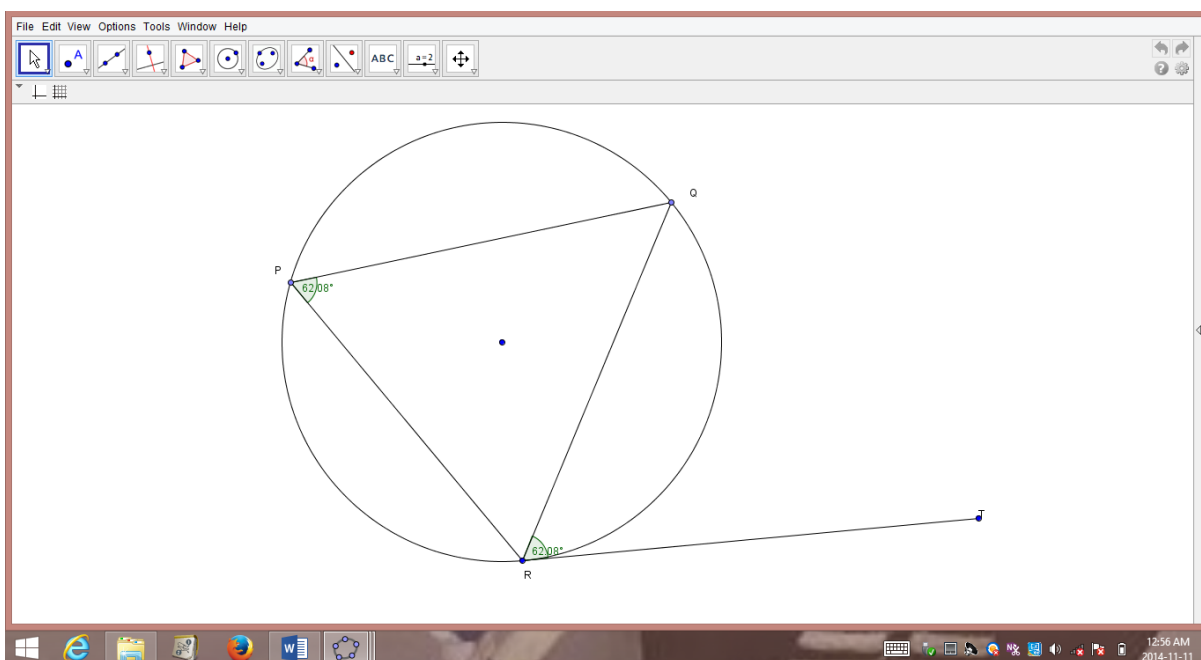
The researcher designed this activity in order to drill students on this challenging theorem and its related riders.

The first issue was based on binding axioms of tan chord theorem. A student as seen in Figure 3.13, diagram 1, verified and justified that “the radius is perpendicular to the tangent”. This again addresses research question 3.3: “Practical assistance of GeoGebra on justifying proofs and theorems of circle geometry.”

In the same Figure 3.13, but diagram 2, a student was trying to prove that “tangents touching one circle from different points of contact but which meet outside the circle at one point are equal in length.” This was the underlying postulate into understanding the preceding theorem. By letting students practise more on these the intention was to make it easier for them to follow the theorem with precise reasoning.



**Figure 3.13 Student practising the postulates of tangent chord theorem**



**Figure 3.14 Student verifying tan chord theorem using GeoGebra tools**

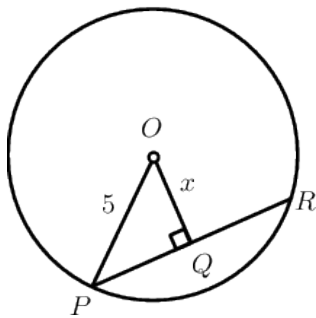
The next step was for students to verify the theorem; “the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.” In the Figure 3.14 the student explored this through GeoGebra software, by drawing and measuring the corresponding angles to justify and verify the theorem.  $\angle QPR = \angle QRJ$  each equal to  $62.08^\circ$  which confirm the theorem.

### 3.8.2 Tasks

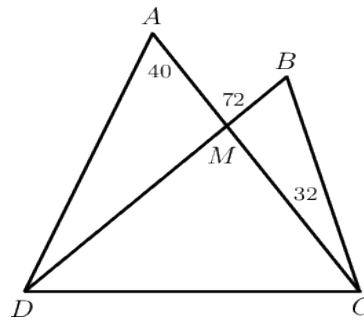
After carrying on with these activities and once students became confident in constructing circles, chords, triangles inside circle, quadrilaterals, inscribed angles and tangents on the circle, they were asked to do these five specific tasks. The main purpose to present students with tasks was “to provide opportunity for them to apply abilities and knowledge gained, in order to demonstrate their understanding of the concepts of tangent lines, inscribed and central angles in a circle and their related riders”.

Feedback is one of the most powerful influences on learning and achievement, but this impact can be either positive or negative (Hattie and Timperley, 2007). Timeously, in order to properly complete tasks, students felt the need to return and repeat activities carefully and practise more features of the dynamic software. 85.7% (48) of the students had to redo all activities and they tried to find new insight to help them better understand the underlying features of each theorem and its related rider(s).

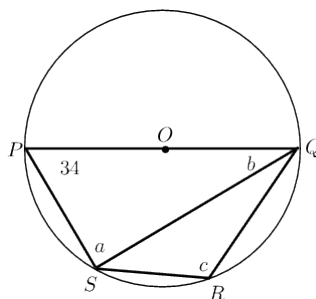
The researcher mainly used these tasks to assess students' learning. This was to help both the researcher and students in which academic peers (other students) critically appraised and provided valuable feedback, which could then be used to improve their work (Mulder, Pearce and Baik, 2014). Students' accomplishment in completing these tasks ensured the researcher that students had mastered the concepts of theorems of geometry of the circle and their applications. Some students found it appealing in helping one another and showed a positive learning experience (Moore and Teather, 2013; Vickerman, 2009), others were reluctant to evaluate another student's work because of concerns relating to bias and fairness. Results from some studies suggest that students are anxious about their own abilities, or the abilities of their peers, when it comes to providing critical feedback (Cartney, 2010). They feel that assessment should remain the responsibility of the instructor (Biggs & Tang, 2007).



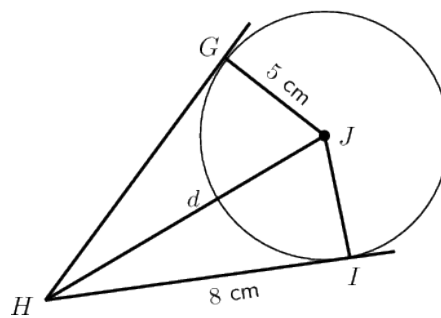
**Figure 3.15**



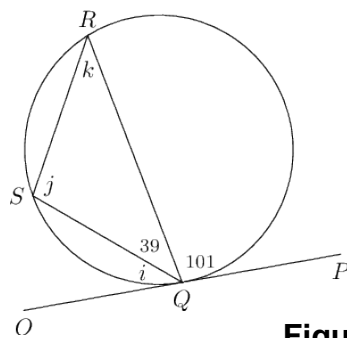
**Figure 3.16**



**Figure 3.17**



**Figure 3.18**



**Figure 3.19**

### Task 1

1. Complete the following statement: The line drawn from the centre of a circle to the midpoint of a chord is .....
2. Refer to the figure 3.15 above find the value of  $x$  given  $OP = 5$  units and  $PR = 8$  units.

### Task 2

1. Complete the following statement: Angles subtended by the same segment (arc) of a circle are .....
2. Refer to the figure 3.16 above to prove that ABCD form a cyclic quadrilateral.

### Task 3

1. The opposite angles of a cyclic quadrilateral are .....
2. Refer to the figure 3.17 to calculate the values of
  - i.  $a$
  - ii.  $b$
  - iii.  $c$

### Task 4

1. Complete the following statement: If two tangents are drawn from the same point outside of a circle, then .....
2. Refer to the figure 3.18 to calculate the value of  $d$ .

### Task 5

1. Complete the following statement: The angle between a tangent to a circle and a chord drawn at the point of contact is .....
2. Refer to figure 3.19 to find the values of
  - i.  $i$
  - ii.  $j$
  - iii.  $k$

## 3.9 CONDUCTING THE POST-TEST

Both the control and experimental groups took a post-test after lengthy teaching in experimental grouping with more learning through GeoGebra, and the control group who had continued being taught in the traditional approach. Firstly, all participants in two groups took the same achievement test that they took in the pre-test. The scoring is presented in the next chapter. Then experimental group participants were given a questionnaire with closed-questions and open-ended questions (Annexure

4). The administering of questionnaires was aimed at aiding in the confidence, credibility and applicability of results, and further help in making informed decisions (Jensen & Dardagan,2014), while open-ended questions were aimed at providing useful information regarding students' understanding of relevant concepts.

Data collected at this stage could reveal possible changes and improvement in understanding a topic in mathematics after interacting with dynamic software, and showed in what ways this software helped students change their understanding of Euclidean geometry, in particular circle geometry.

The qualitative approach was used to collect data by focusing on students' answering worksheets, questionnaires and open-ended questions before and after interacting with digital technologies in an attempt to make a comparison. The qualitative research was chosen since it assists in gaining more insight about the nature of a particular phenomenon, and again it facilitates the study of underlying issues in depth and detail.

Educators are directly responsible for most learning in the classroom. They create a learning atmosphere for students in mathematics classes. Open-ended questions were further complemented by observations of students' interactions with the GeoGebra software in solving and doing the tasks designed in a dynamic environment. The open-ended items effectively reflected the students' responses and their observation of the GeoGebra software. These in turn assisted the researcher in the empirical study to identify not only the students' answers but also their process of thinking and possible misunderstandings or misconceptions that could arise. Open-ended questions were used again to address any unfortunate, unplanned and unexpected actions. The open-ended responses were carefully collected and students were given two days in each school to finish, so as to facilitate a more in-depth analysis. These helped the researcher truly grasp the nature of the change in students' understanding of geometry of the circle, being influenced by the technology based activities in a dynamic environment.

### **3.10 RESEARCH QUALITY**

Qualitative research methods help clarify what readers need and expect before the drafting of formalised satisfaction measures (Rockbridge, 2013).

### 3.10.1 Trustworthiness

The researcher ensured the study's trustworthiness by maintaining transferability, credibility, dependability, and conformability. Trustworthiness is a demonstration that the evidence for the results reported is sound and the argument made based on such results is equally strong (LaBanca, 2010).

In ensuring credibility (confidence in the truth of the findings) the researcher used the following techniques as in line with Robert Wood Johnson Foundation (RWJF, 2008, 2012):

- Prolonged engagement with all participants. The researcher spent sufficient time in the settings, in order to learn and understand the culture or phenomenon of interest, thus the researcher was on the sites long enough to:
  - i. Become oriented to the prevailing circumstances so that the context is appreciated and understood.
  - ii. Be able to detect and account for any distortions that might emerge (the researcher blended in for the conformability of the respondents).
  - iii. The researcher rose above his own preconceptions.
  - iv. Built trust with the respondents.
- Persistent observation for provision of depth of the study.
- Triangulation to facilitate deeper understanding.
- Peer briefing which provided the researcher with catharsis and an opportunity to test and defend emergent hypotheses.
- Deviant case analysis, for refining an analysis until it can explain or account for a majority of cases, which concurs with Silverman (2006) who states that this involves different parts of data and making correlations between them.
- Referential adequacy, which involves identifying a portion of data to be archived.
- Member checking for establishing the validity of counts by providing respondents with the opportunity to assess adequacy of data and preliminary results.

In ensuring transferability (showing that the findings have applicability in other contexts) the researcher used “thick description”, which is an acceptable means of achieving external validity.

### **3.10.2 Validity**

The researcher concurs with Bapir (2012) who states that to achieve validity one needs to reduce the gap between reality and representation and the more data conclusions are correspondent, the more a piece of qualitative analysis is valid. Validity is, in essence, the degree to which a research study measures what it intends to measure. Hence, validity indicates how sound the research is. In ensuring construct validity was attained, the researcher resorted to open-ended questions in the questionnaires, which naturally required openness on the side of respondents. This in a sense accurately represented reality to respondents, so as to satisfy the research hypotheses, namely: “For students to understand the properties of circles through understanding on the use of chords, tangents, central, inscribed and related angles.”

In turn it helped to link with content validity that occurred when the experimental group provided adequate coverage of the subject being studied. The settings, methods and materials available required validity on the environmental perspective, where Bryman (2012, pp. 388-399) stated “this is a view and the opinion of the participant on a subject matter in a research project, and this is central to qualitative study.”

### **3.10.3 Transferability**

For the qualitative part of this study, the researcher used transferability to establish trustworthiness. Transferability is applying research results to other contexts and settings in order to get at generalizability (Robert Woods Johnson Foundation, 2012). The researcher provided a detailed description of the study's sites, participants, and procedures used to collect data for future researches.

### **3.10.4 Reliability**

Reliability is the degree to which an assessment tool produces stable and consistent results (Phelan and Wren, 2014). For the quantitative aspect, **reliability** is a method



used to establish trustworthiness. In ensuring that reliability is attained at all cost, the researcher visited the computer centres of schools in the study and checked the treatments in advanced, before the commencement of the study. The test-retest reliability was used, which is a measure of reliability obtained by administering the same test twice over a period of time to a group of individuals. The scores from Time 1 and Time 2 were then correlated to evaluate the test for stability over time. This was important since all the schools do not have technicians and the researcher did this to facilitate the smooth running of the study.

### **3.10.5 Dependability**

Dependability is a method qualitative researchers use to show consistency of findings (Robert Woods Johnson Foundation, 2012). The researcher described in detail the exact methods of collection, analysis and interpretation of data for the study to be auditable in describing the situation, and for any researcher to follow the study. The researcher used the following ways to show dependability:

- (1) There can be no validity without reliability, and hence no credibility without dependability. Maintaining validity as discussed fostered credibility in findings and hence more dependable results.
- (2) Overlap methods as a direct technique to exemplify a kind of triangulation.
- (3) Stepwise replication as a process of establishing reliability
- (4) Inquiry audit for the researcher auditor to examine the process of the study and determine its acceptability in terms of the dependability of the study.

### **3.10.6 Confirmability**

Confirmability is a degree of neutrality or the extent to which the findings of a study are shaped by the respondents and not researcher bias, motivation, or interest (RWJF, 2012). Confirmability includes an audit trail that include raw data, such as written field notes, documents, and records. Confirmability is a method used in qualitative part of this study to establish trustworthiness (RWJF, 2012). It helps for verification by another researcher when presented with the same data. The researcher during preliminary analysis requested participants to confirm the results before conducting the actual data analysis so as to confirm what they have written.

### **3.11 ETHICAL ISSUES**

McMillan and Schumacher (2010) stated that researchers are compelled to ensure that their research complies with ethical standards to protect the participants from unfair criticism that may arise from participating in the research, whilst Neumann (2006) writes that ethical dilemmas can be resolved through the protection of the participants' confidentiality and abstaining from deception or involvement with deviants. Permission from the Department of Basic Education was requested for the research (see Annexure 1) and a declaration form (Annexure 3) was submitted to the University of Zululand and research certificate was received (Annexure 4). The same letter was used to ask for permission to school managers. The informed consent of each participant was presented in writing (see Annexure 5).

The participants were given a workshop and asked to read and sign the consent form. The benefits, rights, risks and dangers involved as a consequence of their participation in the research were clearly explained. Participants were further asked that should any unforeseen circumstances occur, they are free to withdraw from participation. Participants voluntarily agreed to be part of the study. The information received during the research was treated with much confidentiality and all materials used were kept but to be destroyed immediately after the awarding of research. Even though both environments (school settings) enquire usage of English as medium of learning the researcher cautioned that translation to mother language (isiZulu), is very important for better understanding for all procedures involved. The researcher explained the purpose of the research to the principals and school management teams of the participating schools.

The researcher assured the participants that anything discussed during the study would be kept confidential and would not be used for purposes other than this study. The real names of the participants and the names of the schools would be and shall be kept anonymous in order to protect their identity from unnecessary criticism or ridicule. The description of the schools, the number of students was made in estimates in order to distort the precise location of the schools in the circuit or district.

The results of the study were communicated to the participants before the study was finalised in order to avoid possible misinterpretation and misuse. The researcher

allowed each participant to review the study before it was finalised to ensure that my transcriptions were in accordance with what the participants had written in open-ended questions. The researcher explained that there would be no rewards or payments due to them after they had participated in this particular research, however, the researcher committed to show them the results of the study when finalised.

The following principles guided this research as given by Terre Blanche, Durheim and Painter (2006) and Wassenaar and Mamotte (2012);

- The principle of Beneficence: research should make a positive contribution towards people's welfare. The results of the research could assist the schools of the UMkhanyakude District in the KwaZulu-Natal Province of the Republic of South Africa. A feedback of the results is expected to be given to the Department of Education and to the schools in the UMkhanyakude District.
- The principle of Non-Maleficence: research must not cause harm to the participants in particular. In this study there was no anticipated harm that could be caused to the participants.
- The principle of Autonomy: research must respect and protect the rights and dignity of other participants. All participants were consulted and made aware in this regard, in writing that they had the right not to participate.
- The principle of Justice: the benefits and risks of research must be fairly distributed. The research was conducted and planned in such a way that no risk anticipated.

The main purpose of the study was explained in the consent form (see Annexure 2). The participants were assured that the data would not be used for any purpose other than research and they were told that their names would be strictly kept confidential.

### **3.12 CONCLUSION**

The chapter presented the research methodology used in the study. The chapter discusses research paradigms, research design, target population and sample, the sample technique used in the study, data collection, data analysis, and measures used to enhance trustworthiness, and ethical issues covered in this study. The next chapter seeks to present results analysis of the study.

## **CHAPTER FOUR**

### **RESULTS ANALYSIS**

#### **4.1 INTRODUCTION**

The goal of this chapter was to evaluate the impact of using GeoGebra on academic achievement in terms of Euclidean geometry. In order to understand this impact, experimental and control groups were constituted with 56 students in each group. The achievement test as pre-test was administered in both groups. The students of experimental groups were acquainted at the beginning of implementation phase. The experimental groups were further taught with the materials, which were prepared by using GeoGebra. The control groups were further taught with traditional methods. At the end of the lessons, the post tests were applied to both groups, which was the same as the pre-test, and the opinions of experimental groups, about GeoGebra and its impact on Euclidean geometry learning, were taken.

The collected data were statistically analysed with MathPortal.org. The results of this evaluation were researched in reference to the below-mentioned questions:

1. Are the experimental and control groups at the same level during pre-test?
2. Is there a statistically significant difference between post-test results of both groups?
3. Is there a statistically significant difference between pre-test and post-test results of the experimental group?

This chapter describes the analysis of data collected and interprets the findings of the study in twofold; that is quantitative and qualitative study.

The purpose of quantitative correlational study was to determine the relationship between the use of GeoGebra software and Euclidean geometry academic achievement scores (Segori, 2006, p.73). Findings are presented in detail and describe the systematic application of the methodology.

The qualitative study analysis was written in order to convey human action, to show an emotional connection with the informants, and to conform to narrative traditions

(Creswell, 2003). The phenomenology described herein formed a significant part of the theoretical framework (see Chapter 2) of the overall study and also helped in developing a subsequent quantitative survey instrument; that is the questionnaire (see Annexure 7). Phenomenologies give some of the best tools for an in depth examination of the pre-reflective experience of being in particular environments (Van Manen, 2014).

This chapter presents the findings on how participants responded to the main questions:

1. How does application of GeoGebra in Euclidean geometry impact on learners' performance?
2. How does the use of GeoGebra improve learners' understanding of geometry theorems?
3. What are practical and theoretical implications of GeoGebra on;
  - 3.1 Teachers' confidence in teaching geometry?
  - 3.2 Learners' performance improvements?
  - 3.3 Justifying proofs and theorems of the circle geometry?

## **4.2 CONTEXT OF FINDINGS**

The study was conducted in five secondary schools in the Hlabisa circuit, in Empembeni and Ezibayeni wards. The students were all black Africans and taught by black African educators. The majority of students came from poor families, and learners either stayed in children headed families, with surrogate parents, with grandparents, or in single parent. Most educators who teach in the area commute to school. Learners travel to school over very long distances, some on foot, with parents' monthly rental panel vans and others even stay in the rented cottages close to school.

In the schools that were visited, learners generally expressed love for mathematics and even stated they intended pursuing mathematically related careers. However, they all indicated that they struggle in exercises in Euclidean geometry and firmly stated Euclidean geometry was a difficult topic in mathematics. The learners clearly sought plausible alternatives to do better in Euclidean geometry. There was a need for utilising resources already at their disposal, without any further costs. As

indicated these schools have computer laboratories, but they are used for subjects other than mathematics.

At this stage of research analysis, the steps, which are mentioned-below, were followed:

1. Specifying the experimental and control groups.
2. Making pre-post measurements with data collecting tools.
3. Analysing the Data and Used Statistical Techniques.

The researcher identified interval data as the main level of measurement. In interval data – data is continuous and has a logical order, data has standardised differences between values but exclude natural zero. Items were measured on a Likert scale of 1-4:

- 1- Strongly Disagree
- 2- Disagree
- 3- Agree
- 4- Strongly Agree

The procedures applied in this study were data tabulation, descriptive data, correlation, analysis of variance, and regression.

In data tabulation the combination of frequency distribution and percent distribution were used together. A frequency distribution as an organised tabulation was used to locate the number of individuals or scores in each category, while a percent distribution displayed the proportion of participants who were represented within each category.

Descriptive data was specifically used to determine mean – the numerical average of scores for a particular variable. Correlation as a statistical calculation described the nature of the relationship between two variables (i.e. strong and negative, weak and positive, statistically significant).

In this study five experimental and five control groups were used due to different settings. The equivalence of these groups were analysed from the point of the dependent variable. The dependent variable was described as academic

performance within Euclidean geometry section of mathematics. The reviews were thoroughly researched during the continuous assessment of data.

In order to understand whether the score of Euclidean geometry achievement tests displayed statistically significant difference or not, the independent t-test and paired samples t-test were applied. When the data of experimental and control groups were analysed, Mathportal.org, computer program was used.

### 4.3 FINDINGS AND ASSESSMENT

The findings of this research in the preceding sections are presented in accordance with the study questions:

#### 4.3.1 How Does the Use of GeoGebra Improve Learners' Understanding of Circle Geometry?

In order to find whether any significant differences existed between the pre-test mean score of the control and experimental groups, an independent sample *t*-test was performed as illustrated in Table 4.1

**Table 4.1: Results of the independent t-test on the pre-test of both groups**

Group	Mean	SD	T	Sig (2 tailed)
Experimental (56)	6.36	2.64	1.62	0.108
Control (56)	5.5	2.9		

T-value significant at  $p < 0.05$

Table 4.1 shows that the control group obtained a mean score of 5.5 while experimental group obtained a mean score of 6.36. The mean score difference between the groups was 0.86 with *t*-value of 1.619419. Nonetheless, the *p*-value was 0.108221 ( $p < 0.05$ ) indicating that the difference in the mean score was not significant. This result illustrated that both the students in the control and experimental group were similar in abilities before the treatment was administered.

**Table 4.2: Results of independent t-test on the post-test of both groups**

Group	Mean	SD	T	Sig (2 tailed)
Experimental (56)	9.45	1.92	4.38	0.000

Control (56)	7.52	2.64
--------------	------	------

T-value significant at  $p < 0.05$

To determine whether any significant differences exist between the post-test mean score of the control and experimental group, an independent sample  $t$ -test was carried out. Table 4.2 shows that the control group obtained a mean score 7.52, while the experimental group obtained a value of 9.45. The mean score difference between the groups was 1.93 with a  $t$ -value of 4.384833. However, the  $p$ -value was low at 0.000027 ( $p < 0.05$ ), indicating that the difference in the mean score of the groups was significant.

**Table 4.3 Results of the paired samples t-test**

Pair	Group	Mean	SD	T	Sig (2 tailed)
1	Post-test, Pre-test scores (Experimental)	3.09	2.72	13.7	0.000
2	Post-test, Pre-test scores (Control)	2.02	1.42	14.58	0.000

T-value significant at  $p < 0.05$

The findings presented in Tables 4.2 to 4.4 show that the students in the experimental group performed better using GeoGebra than students in the control group using the traditional learning method. The students in the experimental group performed better in the post-test compared to the control group. A paired samples  $t$ -test was conducted to compare the pre-test and post-test scores for the experimental and control groups. The result as illustrated in Table 4.3 shows that the mean score difference between the post-test and pre-test of the experimental group was 3.09 as compared to the control group with 2.02. For the experimental group, the  $t$ -value obtained was 13.7 and the  $p$ -value obtained was 0.000 which was low ( $p < .05$ ) indicating the difference between the pre and post-test score was significant. For the control group, the  $t$ -value obtained was 14.58 and the  $p$ -value obtained which was 0.000 that was low ( $p < .05$ ) indicating the difference between the pre-test and post-test score was significant.



**Table 4.4 The students' responses about learning Euclidean geometry with GeoGebra**

Questions	1	%	2	%	3	%	4	%
2. Were you excited about using GeoGebra?	0	0	3	5	20	36	33	59
3. Do you like studying circle geometry lessons with GeoGebra?	2	4	5	9	14	25	35	62
4. Did you feel confident using GeoGebra while learning?	2	4	6	14	23	41	25	45
5. Were you engaged in the learning process?	0	0	2	4	12	21	42	75
6. Did you benefit through teacher-student interaction?	3	5	2	4	23	41	28	50
7. Were you able to use visualisation skills during lessons?	3	5	3	5	32	57	18	33
8. Were you able to think creatively and critically in discussion?	3	5	2	1	25	45	27	48
9. Did you make logical assumptions while hypothesising?	3	5	6	11	20	36	27	48
10. Did you enjoy learning circle geometry using GeoGebra?	1	2	9	16	14	25	32	57
11. Did you make connections between new and previous lessons while using GeoGebra?	4	7	7	13	10	18	35	62
12. Will do well in Euclidean geometry in tests and examination?	5	9	7	13	3	5	41	73
13. Only brilliant learners can understand circle geometry without GeoGebra?	14	25	17	31	13	23	12	21
14. Can Euclidean geometry improve your mathematics results?	3	5	2	4	12	21	39	70
15. Can circle geometry develop good reasoning skills?	1	2	11	20	16	28	28	50

This indicated that there was a significant improvement in the scores of both the experimental and control groups. From these findings, it can be concluded that students gained from both approaches but the students in the experimental group appeared to have a higher mean difference or improvement in scores compared to the control group. In order to take student's opinions on learning Euclidean geometry with GeoGebra, the above questions were given.

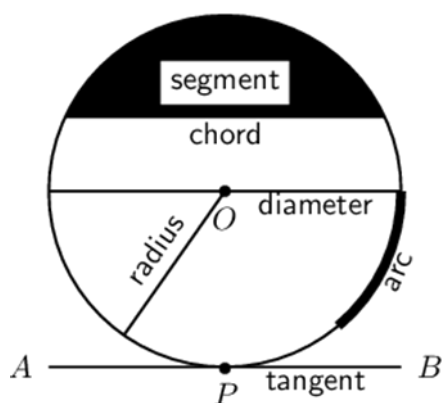
#### 4.3.2 How Does Application of GeoGebra Software in Euclidean Geometry Impact Learner Performance?

Application of GeoGebra software involves using set of one or more programs designed to carry out specific operations in calculus, algebra and geometry. This was one of the questions of the study; it intends to delve into the effect or influence of the GeoGebra software on learners' achievement in regards to Euclidean geometry. This is the topic that has been reintroduced in the mathematics curriculum after an absence of six years.

A learner labelled Euclidean geometry as the cause for them to struggle in mathematics. One learner Samkelo, further hinted at the bad background of Euclidean geometry:

*"I am repeating grade eleven. Mathematics was one of main cause especially these circle problems I don't know why it was returned." The learner continued and said: "At home when I was asking my older sister who did grade 12 eight years ago, to help me with homework last year, she said 'yooooohh that is the topic which changed my attitude towards mathematics and that is why I do not have it in my certificate'.....so how can we pass it then?"*

The comment by Samkelo clearly suggests that Euclidean geometry has a history of poor mathematics results. The learner hinted that circle geometry has even intimidated his sister, which inadvertently causes stigmas and impacts his view and confidence on the subject of mathematics. In this case Samkelo could only rely on school, therefore it is important for the school to utilise technological facilities first and foremost for development of learners, as opposed to surrounding communities. In addressing and bringing confidence to students like Samkelo, the researcher engaged learners in first drilling more on terminology, so as to acquaint them with the context of Euclidean geometry (see Figure 4.1).



**Figure 4.1 showing main terminology used in circle geometry**

The students used GeoGebra software to construct their manipulative:

A circle is the set of points equidistant from given point. A circle is named with a single letter in its centre, see Figure 4.1 and diagram 5 on Figure 4.2 below.

The radius of a circle is the segment with one endpoint at the centre of a circle and the other endpoint on the circle circumference, see Figure 4.1. All radii of a circle are congruent.

The diameter of a circle is the segment that contains the centre and whose endpoints are both on the circle. The length of the diameter is twice that of the radius, see Figure 4.1 and diagram 4 on Figure 4.2 below.

Circle segment is the part within a circle bounded by a chord of that circle and the minor arc whose endpoints are the same as those of the chord see Figure 4.1.

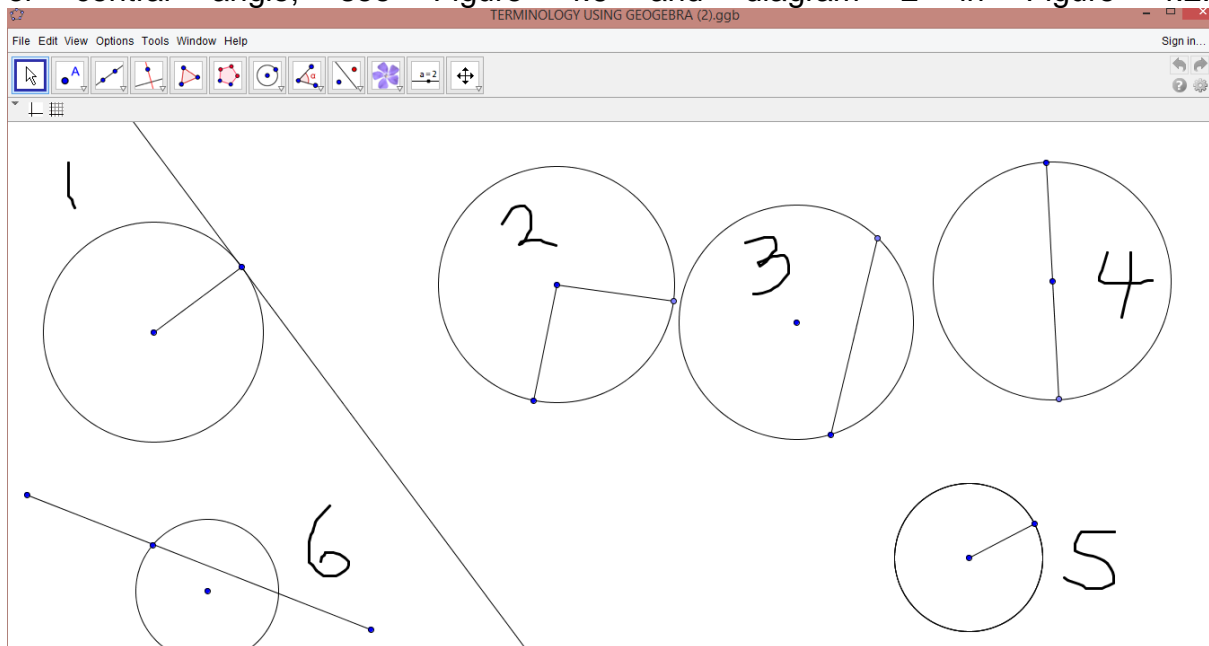
An arc is formed by two endpoints on a circle and all of the points on the circle between those two endpoints see Figure 4.1 and Figure 4.7.

A chord is a segment whose endpoints are on a circle, see Figure 4.1 and diagram 3 in Figure 4.2.

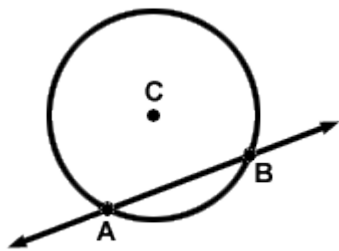
Secant is a line that intersects with a circle at two points, see diagram 6 in Figure 4.2 and Figure 4.3.

Tangent is a line that intersects with a circle at only one point (the point of tangency) (see diagram 1 in Figure 4.2 and Figure 4.4). The students again explored the notion

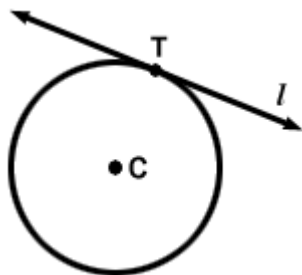
of central angle, see Figure 4.6 and diagram 2 in Figure 4.2.



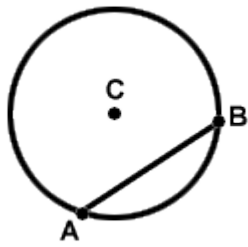
**Figure 4.2 Snapshot of GeoGebra screen on teaching terminology**



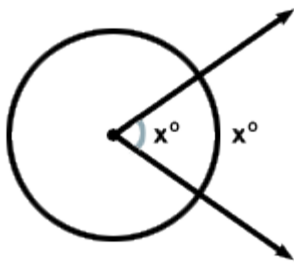
**Figure 4.3 Secant line**



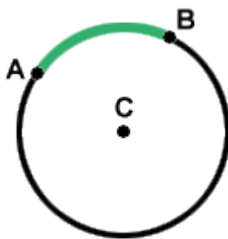
**Figure 4.4 Tangent line**



**Figure 4.5 Chord**

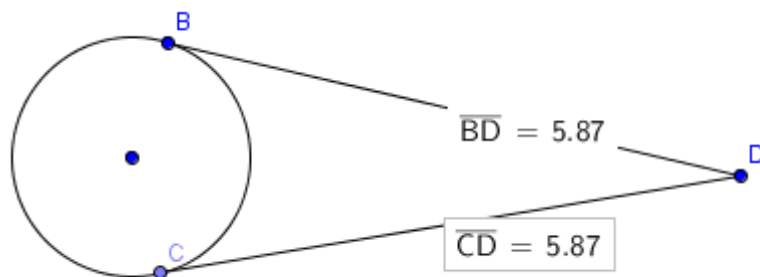


**Figure 4.6 Central angle**



**Figure 4.7 Arc**

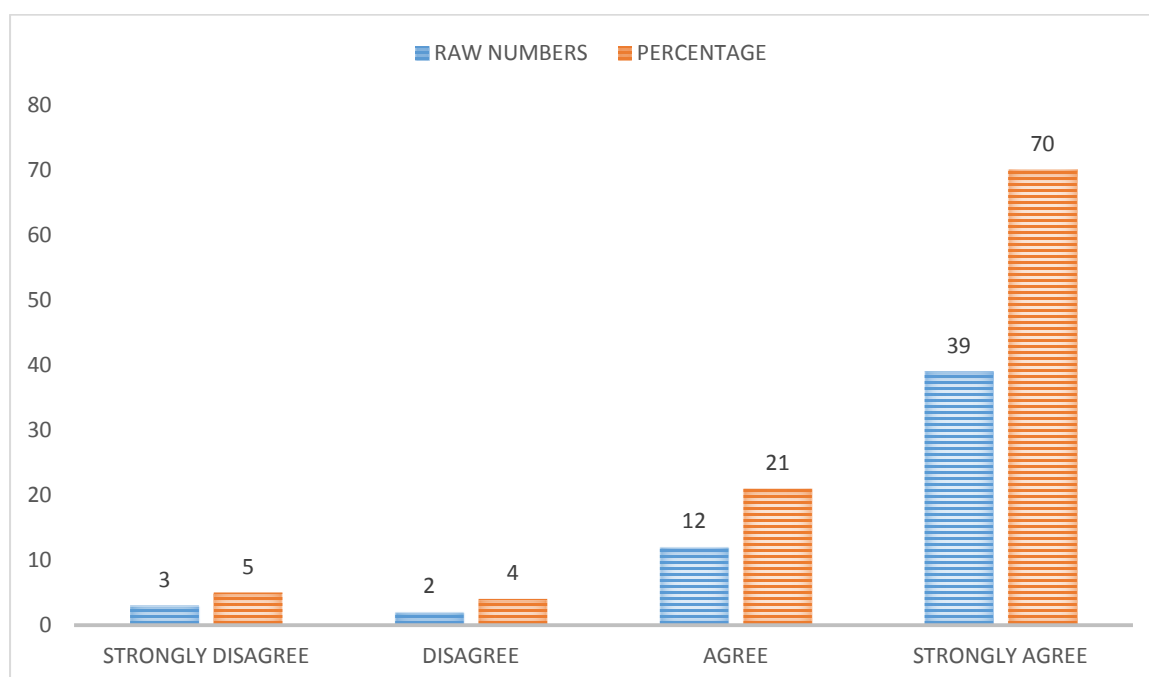
Furthermore, while using GeoGebra students learned some critical elements of theorems like “tangents meeting at one point of the same circle are equal in lengths” see figure 4.8 below.



**Figure 4.8 GeoGebra on two tangents meeting at one point.**

The students after multiple practices of using GeoGebra in understanding terminology of circle geometry, stated that they are more likely to perform better in circle geometry. As Antohe (2009) found that using GeoGebra students can see abstract concepts more clearly, students can make connections and discover mathematics on a higher level. Furthermore, the ability to assess students' solutions electronically may promote students' interests in mathematics while advancing their cognitive abilities.

The introduction of terminology through students construction made them more motivated in terms of what lies ahead of mastering mathematics and circle geometry, other than the traditional approach of being 'spoon fed' essentially. Students were eager to apply their problem solving skills on their own. As Bayazit & Aksoy (2010) found, GeoGebra promotes students' problem solving skills and helps students construct mathematical models of problems, through which students can conduct better analyses of situations and develop operational plans to effectively resolve the tasks at hand.

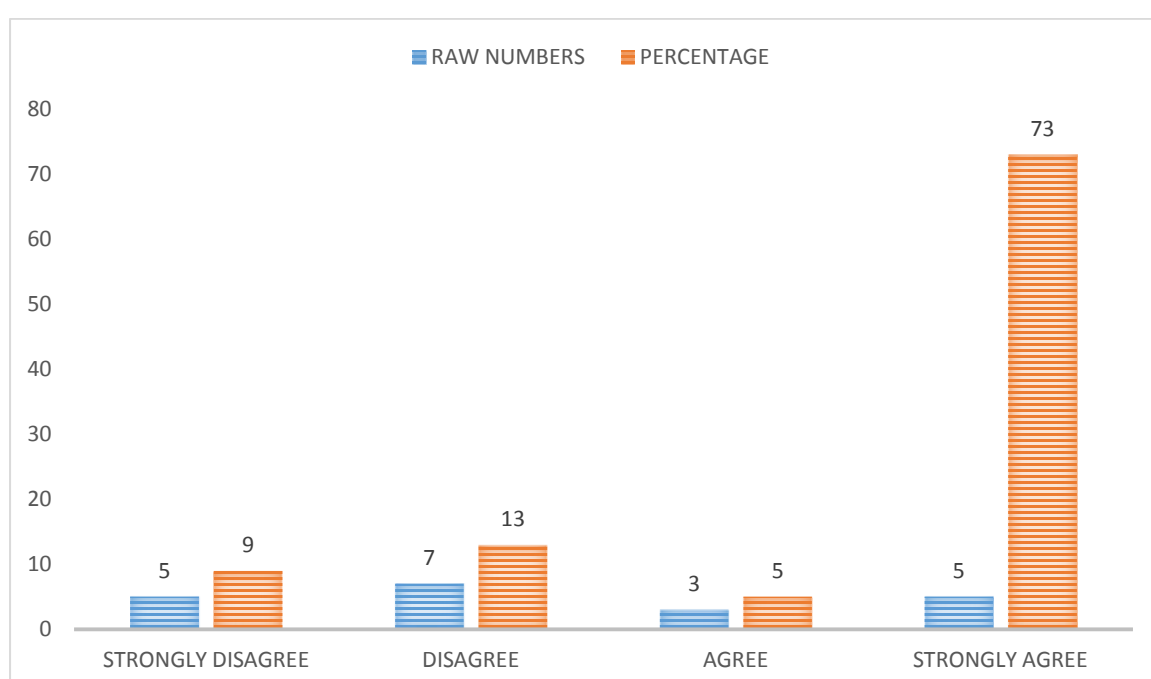


**Figure 4.9 Breakdown of results improvements**

The students according to statistical analysis believed that GeoGebra software can help in improving their mathematics results as illustrated in Table 4.4. Only 5% of students strongly disagreed, with 4% disagreeing. However, 12 students agreed and 39 students strongly agreed which collectively make 91% of students believing that

using GeoGebra software gave them confidence in improving mathematics results (see Figure 4.9).

Students had different views on their performance using GeoGebra while learning Euclidean geometry. Only 9% (which was 5 students) strongly disagreed, with 7 students disagreeing (13%). Nonetheless, 78% (46 students) collectively agreed or strongly agreed that using GeoGebra software in Euclidean geometry has helped them do well. This is further illustrated in the Figure 4.10 and Table 4.4.



**Figure 4.10 Breakdown of beliefs on performance in Euclidean geometry**

It is clear that GeoGebra had an impact on Euclidean geometry learning seeing that much more responses were positive toward doing better. The software had effectively changed the course of their mathematics views and prospective results for the better. This suggests that if they continue using GeoGebra their attitudes first toward Euclidean geometry will be more positive, thereafter yielding better mathematics interests and grades as a whole. This concurs with Reisa (2010) who found that the success of teaching with GeoGebra is higher compared with that of conventional teaching.

Furthermore, students used GeoGebra software in practising general geometry theorems in addressing the study question:

### 4.3.3 What Practical and Theoretical Assistance Does GeoGebra Offer to Learners to Justify Proofs and Theorems of Geometry?

One learner Qinisani in the same group who was repeating said:

*"I do not understand these theorems and I do not believe that the teacher knows this topic, because he cannot verify to us. He says 'it is just like that, ours is to study'. He even said he never learned Euclidean geometry at both high school and at tertiary. Thina senzenjani ke (what should we do then)? I do not know whether I will make it even this year. Kanti sibulawelani ngalezibalo engaziwa nango thisha (why are we being taught something even teachers are not sure of)?"*

Qinisani cited that learners do not understand theorems in geometry. The learning without understanding has been a traditional approach criterion, which was well suited to modern era students. The 21<sup>st</sup> century students seek understanding other than rote learning and they perform far better when they learn with understanding. The students were given multiple tasks of proving using GeoGebra. They had to prove and verify that an exterior angle of a triangle is equal to the sum of the two interior opposite angles see Figure 4.11 to view their theorem justification using GeoGebra.

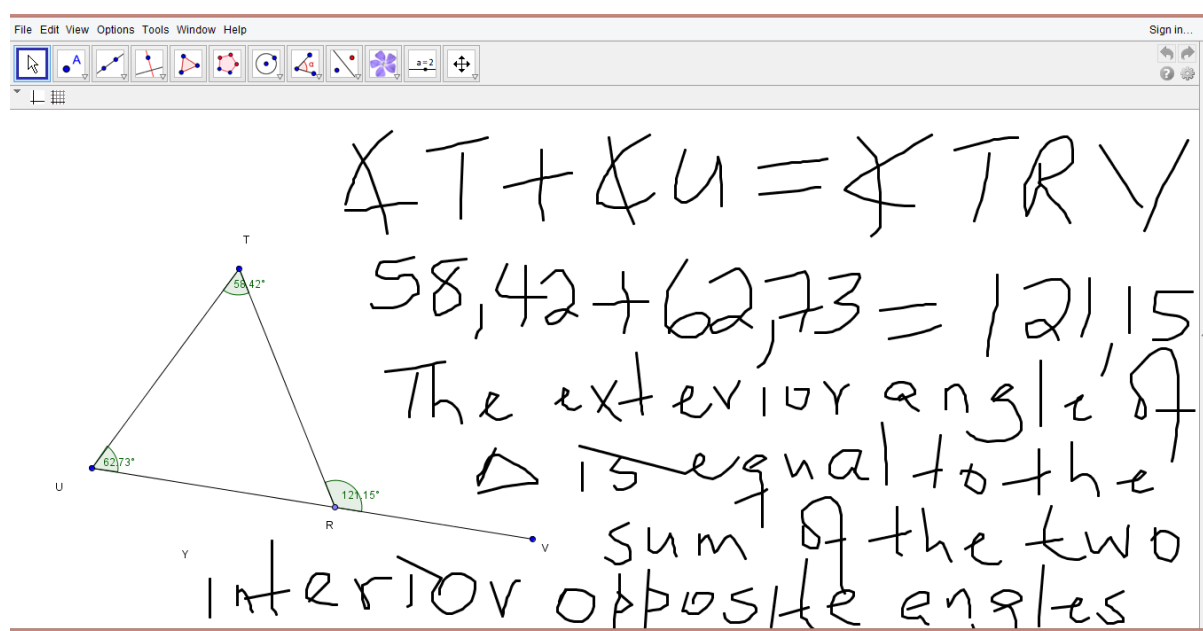


Figure 4.11 Snapshot showing exterior angle of a triangle theorem



Again Simphiwe had a similar problem. She stated that:

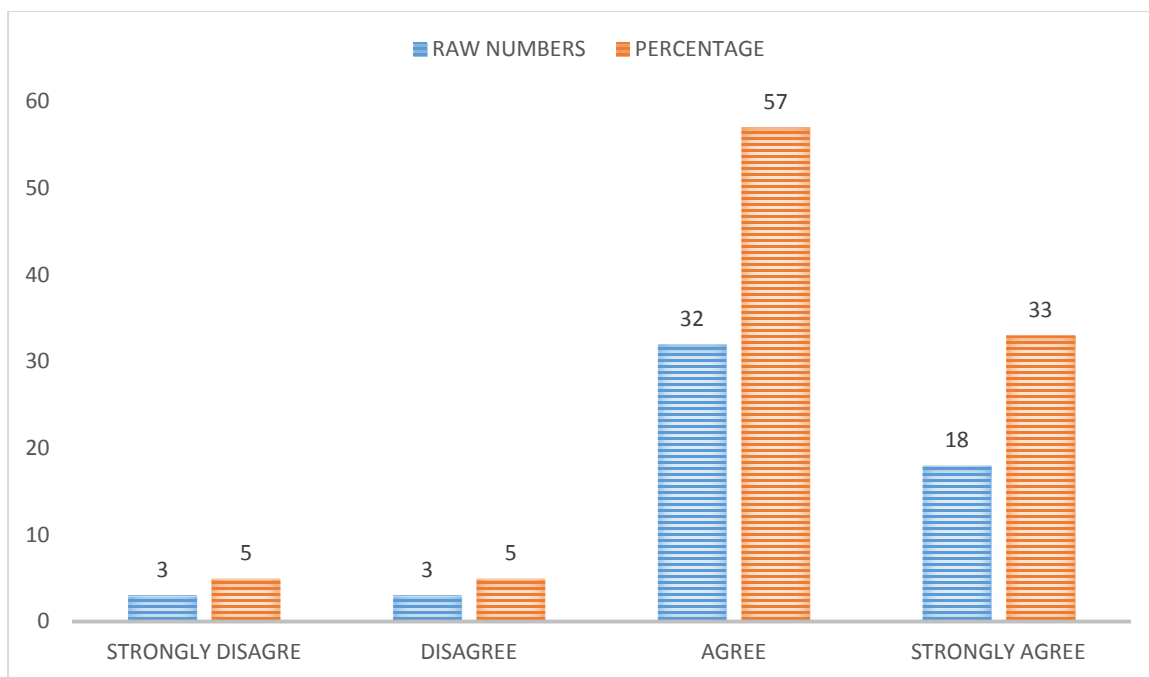
*"I have tried devoting my time on circle geometry hoping that this will improve my mathematics results but I see no change. I try by all means to follow the teacher during lessons' presentation but still I do not believe the teacher because when we used paper and pencil there are minimal accuracy, as a result I am left not sure. Sir  $45^\circ$  cannot be  $44.79^\circ$ . This totally distorts my visualisation"*

Simphiwe wanted to verify and justify when learning, and it is clear that to her accuracy is very important. So the researcher directed Simphiwe, and others in the group, into proving simple theorems with GeoGebra, like the one in Figure 4.11.

The students' responses in Figure 4.12 on visualisation showed that a total of only 10% distributed equally to both strongly disagree and disagree, while 57% agreed that GeoGebra helped improve their visualisation skills and 33% strongly agreed, meaning 90% of students believed that GeoGebra helped on visualisation.

The findings below clearly signify the importance of visualisation to students, more especially when learning mathematics. This confirms what Arcavi (2003, p.217) who defined visualisation, asserted:






*"Visualisation is the ability, the purpose of creation, interpretation, the use and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings."*

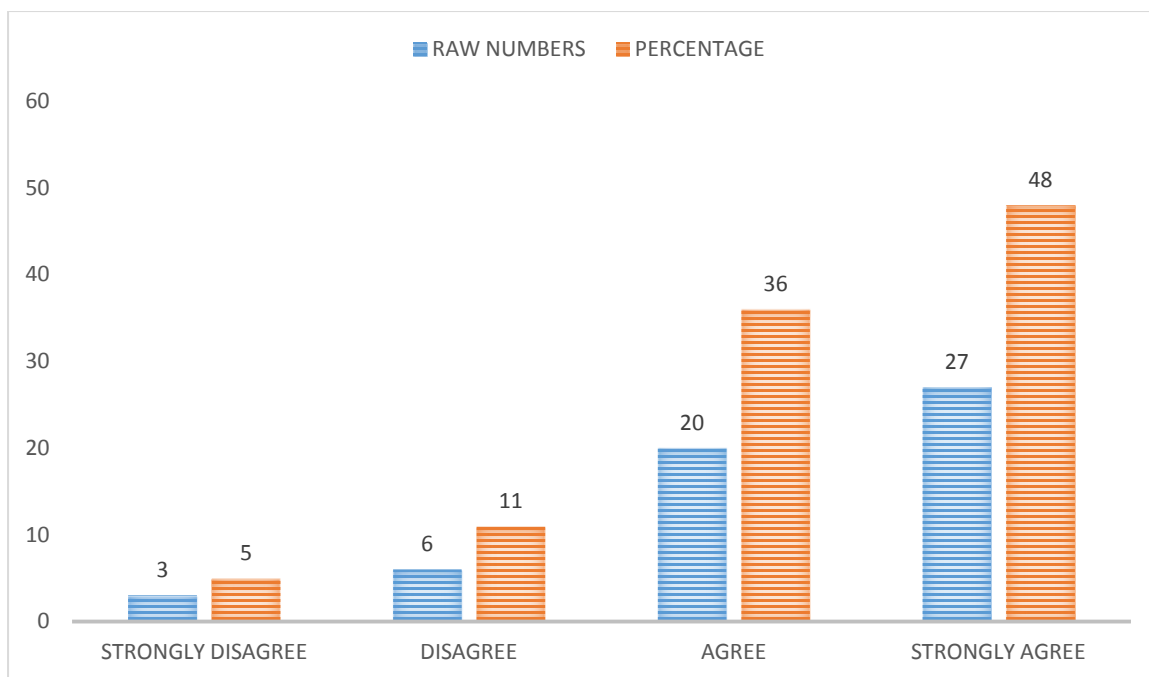


**Figure 4.12 Breakdown on visualisation through GeoGebra**

Visualisation in mathematics learning can be a powerful tool to explore mathematical problems and to give meaning to mathematical concepts and relationships between them. As Rösken and Rolka (2006) emphasised, visualisation allows for reducing complexity when dealing with a multitude of information. These findings show that GeoGebra had some help in attaining the critical aspects of mathematics, more especially geometry: The use of mental models in the lessons and hopefully in future careers too. Bayazit and Aksoy (2010) indicated that GeoGebra enhances students' visual ability and enables them to conduct visual strategies to resolve algebra problems.

In Figure 4.11 students used GeoGebra tools to verify that the exterior angle of a triangle is equal to the sum of the two interior opposite angles. The following tools

were utilised  for plotting four points R, T, U, and V,  for joining TU, TR, UR and RV,  for labelling,  for measuring  $\angle U = 62.73^\circ$ ,  $\angle T = 58.42^\circ$ , and  $\angle TRV = 121.15^\circ$ , the  for dotting down important information next to the figures so as make relations.



**Figure 4.13 Breakdown on justifying and making logical assumptions of theorems**

The Figure 4.13 above shows that 5% and 11% strongly disagreed and disagreed respectively. But 36% agreed that GeoGebra software help them justify proofs and circle geometry theorems, and make logical assumptions with a further 48% who strongly agreed. In total 84% were positive that GeoGebra helped in justification of proofs and theorems.

The above findings emphasise justification as a condition of proof. The importance of proof and formal reasoning helps in the development of mathematical understanding. This is particularly in line with what NCTM (2008) highlighted:

*“Students at all levels should, for instance, be able to communicate their mathematical thinking, analyse the thinking of others, use mathematical language to express ideas precisely, and develop and evaluate mathematical arguments and proofs.”*

To think mathematically, students must learn how to justify their results that is to explain why they think they are correct, and to convince their teacher and fellow students. As Ball and Bass (2003, p.29) stated:

*“Mathematical reasoning is as fundamental to knowing and using mathematics as comprehension of text is to reading. Readers who can only decode words can*

*hardly be said to know how to read.....Likewise, merely being able to operate mathematically does not assure being able to do and use mathematics in useful ways."*

Justification provides confidence in formally proving circle geometry theorems and this will in turn assist teachers in moving students toward advanced problems. As confirmed by Back, Mannila and Wallin (2010):








*"Justification is not only important to the students but also to the teacher, as the explanations (not the final answer) make it possible for the teacher to study the growth of mathematical understanding."*

How does the use of GeoGebra improve learners' understanding of Euclidean geometry theorems?

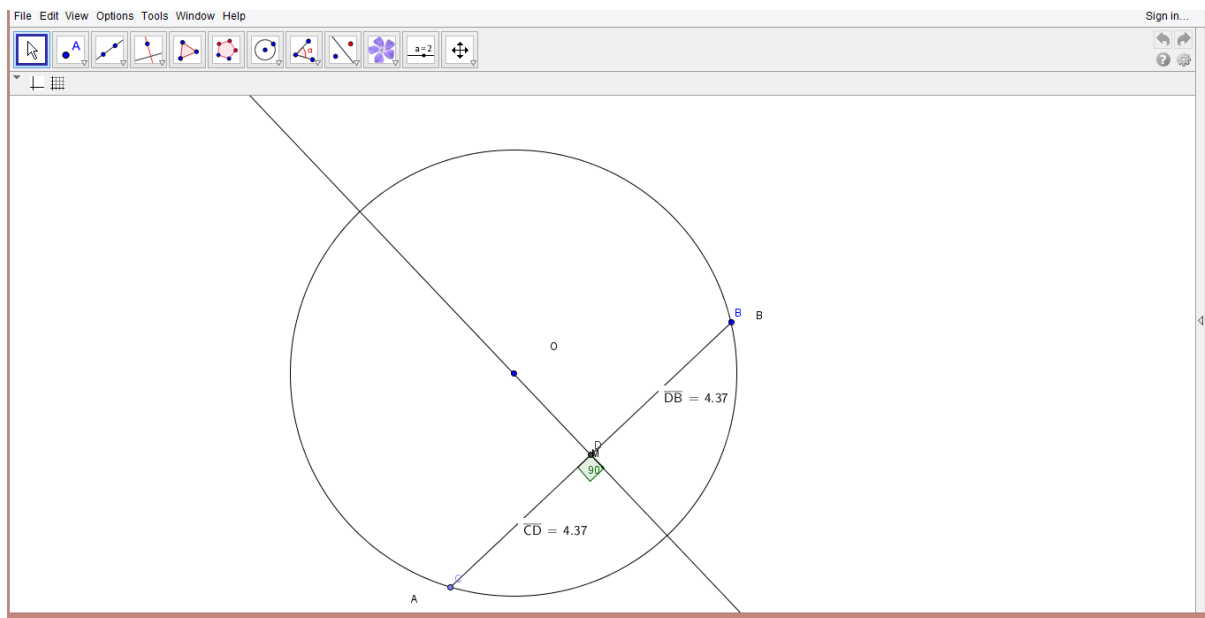
One student Vumisile stated:

*"I used to do well in mathematics but now I am getting in the region of 40% because of geometry. I am slowly losing hope little by little that I will ever get back where I used to be. This circle geometry needs reasons, reasons, reasons, and the teacher always emphasise about the need for precise reasoning. I do not understand how to construct these reasons as I do not follow theorems well. I definitely need assistance to this topic, because if it can be laid out to me clearly right from proving the theorems I can do eventually follow how do these required reasons come on board and improve my mathematics results."*

In addressing the problems like this by Vumisile, the researcher let the all students try and verify one theorem: The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord. The following GeoGebra tools

were used  for constructing the circle with given centre,  for plotting points B, D, and C,  for joining B and C,  for drawing OD which is perpendicular to BC,  for measuring or verifying that  $\angle ODC$  is  $90^\circ$ ,  for measuring distance  $BD = 4.37\text{cm}$  and  $DC = 4.37\text{cm}$ , and  for labelling. The emphasis was on verification, understanding and reasons writing.

From Vumisile's point of view, circle geometry is dragging his average down which means lowering his overall performance. He cited the lacking of core understanding. Moreover conceptual knowledge to Vumisile is more important than procedural knowledge. He is an honest and willing student who is desperate and capable of improving his understanding in Euclidean geometry. Vumisile shared the same sentiments as other students, who looked for the collective intervention to help them better understand Euclidean geometry, and in turn improve their mathematics grades. The researcher allowed students, in constructing theorems using GeoGebra, to instil more understanding during the process (Figure 4.14).

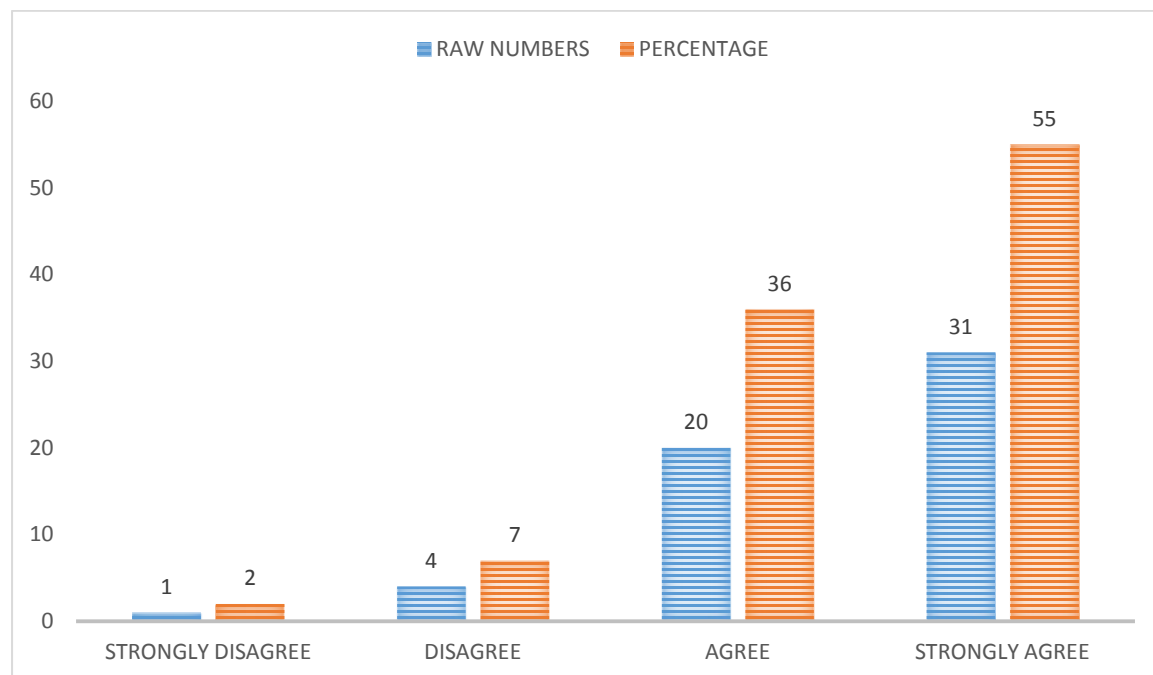


**Figure 4.14 Snapshot regarding understanding of circle geometry using GeoGebra**

The use of GeoGebra helped most learners in terms of circle geometry concepts, as illustrated in Figure 4.14. Almost 91% of students agreed and strongly agreed that GeoGebra gave them the right perspective when it came to understanding the basic concepts. This was in line with Velichová (2010) who contends that a simple drawing of mathematical objects and figures is not for the building of a comprehensive understanding of basic mathematical concepts, and Fahlberg-Stojanovska and Stojanovski (2009) found that using GeoGebra motivates and helps students learn at a higher level, while exploring and conjecturing as they draw and measure.

The responses from students using GeoGebra to circle geometry were very positive indeed. Students were satisfied due to the nature, features and benefits of the software, making it a motivating and attractive learning tool. According to the abovementioned statistics (see Figure 4.15), GeoGebra can be a suitable teaching aid. GeoGebra helps students understand geometric problems in intuitive and natural ways.

Dynamics geometry software like GeoGebra provide students with an interactive environment in which they can quickly and easily, create manipulative as well as measure and analyse digital representations of key concepts from geometry. GeoGebra allows students to drag points on geometric objects and to quickly be able to make and test conjectures and generalisations about properties.



**Figure 4.15 Breakdown on understanding circle geometry concepts**

This shows that with GeoGebra, students are more involved in the learning process and since more of the senses are engaged, higher success was achieved.

The purpose of administering the questionnaire to students was to confirm the data collected during the study. The open-ended questions, which have themes extracted from the main research questions and some items from the questionnaire, required students to have more freedom of expression rather than stereotype items. The students answered without the interference of the researcher. However, the

researcher was handy for translation and clarification purposes. The following themes were the main focus in the open-ended questions:

### **Theme 1: Understanding Euclidean geometry**

The responses from students revealed some differing impressions. Some students confessed that they thought circle geometry was a very difficult section in mathematics, and mathematics as a subject too.

Samkelo:

*"I experienced that circle geometry is not difficult it just needs someone to think creatively. I had lose hope in this topic and in mathematics generally. GeoGebra kept me toes right from making connections with terms that are used like chord, perpendicular, subtend, radii, etc. I am sure that as I understand circle geometry I will even concentrate when learning trigonometry as it is other section that constantly give problems in mathematics. I wished we also learned trigonometry with GeoGebra!"*

The student seemed to have gained a new perspective on Euclidean geometry. The student further mentioned trigonometry, just revealing confidence in the basic understanding here, and moreover wishing for the same in the other sections of concern.

Goodman stated in his response:

*"I learnt a lot about GeoGebra in learning geometry because now I have seen that circle geometry is an important section that has helped me to develop good reasoning skills. I've seen that Euclidean geometry can help me to improve my mathematics results so I enjoyed it!!!!!! I never understood when my mathematics teacher was teaching and I never took it seriously, but by using GeoGebra I got what was shot and I'm doing better in geometry now than before."*

The student cited that GeoGebra helped to bring special attention to the mathematics lessons, by bringing attention to circle geometry. The students even mentioned the development of reasoning skills, which geometry is well-known for. As Gunhan (2014) states:

*“Information about a student’s reasoning skills helps the teacher develop an opinion regarding the student’s thoughts, based on which he or she can review the procedures and techniques used in learning processes, if necessary.”*

Goodman maintained the above premise in terms of stating that he has already started doing well in geometry, therefore highlighting the effectiveness of GeoGebra in improving the understanding of Euclidean geometry.

## **Theme 2: Justification and verification of theorems**

How do you view GeoGebra in assisting you in the area of Euclidean geometry, more especially in justifying proofs and theorems?

Siziwe stated:

*“Without a doubt proving theorems was the main problems leading me to lose the concentration. With GeoGebra I have learned sequencing and making logical reasoning. This is so because GeoGebra have truly convinced me on all theorems and riders by easy verification. And too this has made some flows on making correct judgement by following each statement with correct and precise reason. Before using GeoGebra circle geometry was meaningless to me. I benefited a lot since in the classroom our teacher could not prove theorems for us to believe, but through this computer software theorems were easily and truly, believable and justifiable.”*

The student revealed that GeoGebra helped her follow all steps in proving theorems with precise reason next to every statement. Proofs have conventionally played an essential role in establishing the validity of a statement and in shedding light on the reasons or premises that support that statement (Hadas, Henke, & Regev, 2007). Proof construction is highly important as it goes beyond mathematics as affirmed by Hanna (1998, p. 5):

*“Further evidence of the importance accorded to proof in school geometry is the benefit which it is expected to bring beyond the borders of that subject. The consensus seems to be that the key goals of geometry instruction are the development of thinking abilities, of spatial intuition about the world, of knowledge*



*necessary to study more mathematics and of the ability to interpret mathematical arguments.”*

This concurs with Dynamic Geometry Environments (DGE) on theorem acquisition and justification:

*“Theorem acquisition and justification in DGE is a schematic cognitive-visual dual process potent with structured conjecture-forming activities, in which dynamic visual explorations through different dragging modalities are applied on geometrical entities. The activities stimulate argumentative/ transformational reasoning, which enables the process to converge towards integrated figural concepts that could bring about formal mathematical proofs, hence producing a cognitive unity in acquiring and proving geometrical theorems.”*

Siziwe further stated that GeoGebra helped convince her that the theorem is indeed true and can be proven. The student noted that the computer can be used dynamically as an excellent aid in the learning of mathematics, making the subject easier and more enjoyable.

### **Theme 3: Confidence**

What attributes of GeoGebra have given learners' greater confidence when engaging in circle geometry?

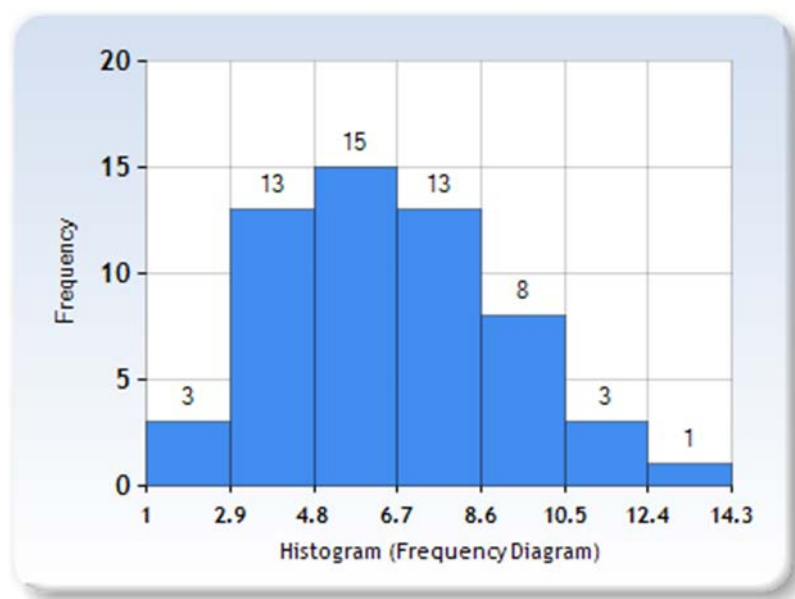
Snegugu was more convinced on circle geometry, and she stated:

*“I am oozing confidence towards circle geometry. I believe geometry more especially circle geometry is relative easy. Maybe it was because of GeoGebra. This topic is not as complicated as algebra. Circle geometry's options are straight forward. In fact anytime I will put more effort on geometry as I have realised it can quickly stay in my mind. Thank to computer software I am developing love for mathematics.”*

The student has gained immensely by using GeoGebra on circle geometry to such an extent that she even compared geometry to algebra stating that algebra is a bit complicated. This is in line with Caucasia's (2012) comment on GeoGebra who said users can acquire more confidence. Generally students had very positive comments on the use of GeoGebra in learning circle geometry.

#### 4.4 MEETING THE ASSUMPTIONS OF MRA USAGE

In analysing the results the researcher used MathPortal.org. This was done to check whether the underlying assumptions of multiple regression analysis hold or not. The dependent and independent variables were at the continuous level thus assumptions 1 and assumption 2 were met. Assumption 3 was met as the researcher evaluated linear relationships between the dependent and independent variables. Scatterplots were created to evaluate the linearity assumption, and positive correlation was observed (Figure 4.17). As observed in Figure 4.17 there are no significant outliers, clearly showing that homoscedasticity as well as all observations were independent.



**Figure 4.16 Results showing in histogram**

The researcher chose histogram (with a superimposed normal curve) of the residual to check on the validity of normality in terms of distribution, constant variance, and independence of variables.

This means the assumption of normality holds. The histogram represents the frequency table distribution of data and frequency table.

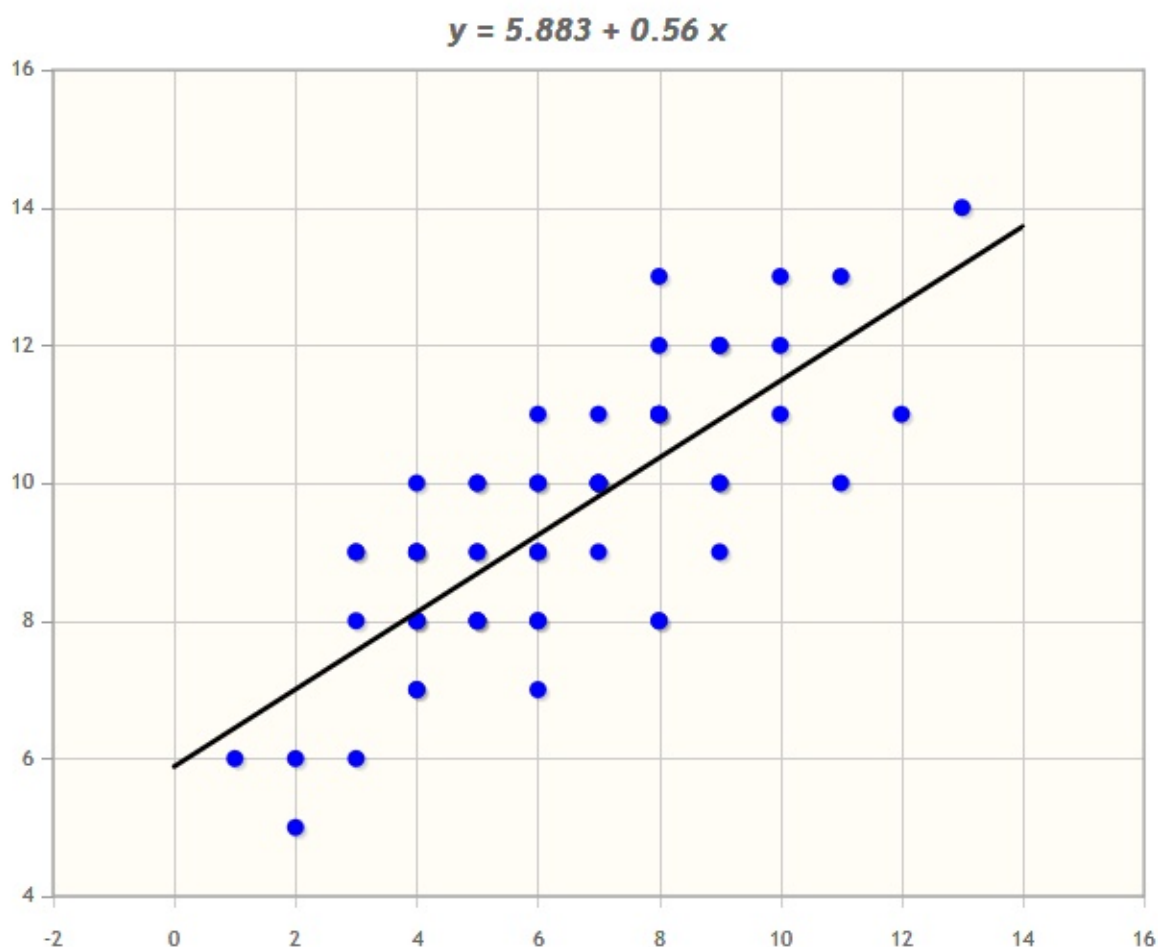
In analysing the results the regression analysis was performed by finding the sums first of the scores using MathPortal.org. The quantity  $r$ , called the linear correlation coefficient was calculated to measure the strength and direction of a linear relationship between two variables.

$R \approx 0.77613$  and  $r^2 \approx 0.60237776$

This depicts that there was a strong uphill relationship between variables, meaning that the use of GeoGebra yielded a positive increase in students' performance. The correlation is positive as illustrated by the diagram, which means X and Y have a strong positive linear correlation, thus r is close to +1.

$B \approx 0.56$   $a \approx 5.883$

The regression equation formula is  $y = 5.883 + 0.56x$



**Figure 4.17 Simple linear regression line**

In the simple linear regression equation 'a' is the y-intercept and 'b' is the slope of the line. The equation could be used to predict the academic achievement score of an individual. For instance, if the student obtained 5 out of 14 marks, the equation can predict the score marks after intervention of GeoGebra:

Predicted score marks =  $5.883 + .56 (5) = 8.683 \approx 9$  score marks

From the illustration below (Figure 4.17) it is clear that there was a strong linear relationship between GeoGebra (independent or predictor variable) and the academic achievement score (dependent variable or outcome) as points lie closer to the regression line.

#### **4.5 CONCLUSION**

In this chapter, data analysis methods, study results and a discussion of the findings have been presented. Data findings were described as correlations to the study variables and presented as tabulations. The researcher applied independent t-tests, to check whether both experimental and control groups were at the same level in the pre-test. Both groups were not statistically significant, meaning they were at the same level, see Table 4.1.

The statistical difference between post-test results of both groups again using independent variables showed that groups were statistically significant, but the experimental was more successful than control group, see Table 4.2. When paired, sample t-tests for post-tests and pre-tests, results were highly significant and both groups were successful at the end of the lessons as illustrated in Table 4.3. Further assessments were conducted based on questions covered in questionnaires, results in Tables and multiple figures. The researcher also elucidated on assumptions.

Findings from this study have been found to be consistent with the findings of several related studies on the use of GeoGebra in teaching and learning geometry. In consideration of these assessments, the teaching of Euclidean geometry with materials, which were prepared with GeoGebra, is more successful than traditional methods.

Accordingly, to integrate the educational technology into mathematics lessons fosters improved academic achievements by enhancing understanding and justification. This is owing largely to the practicality and appeal of the software in terms of engaging more mental structures or senses of students, such as stimulatory visual. Particularly, the visual component increased students' attention spans in mathematics lessons, meaning that the various abstract concepts associated with the subject became easier to focus on and grasp.

## **CHAPTER FIVE**

### **DISCUSSION OF FINDINGS**

#### **5.1 INTRODUCTION**

The first section of the chapter will discuss the findings in relation to the study research questions, literature and theoretical framework. A summary of the major results will be described. The second part of the chapter will discuss the researcher's deliberations. The third part will contemplate the ramifications of the study in connection with the scrutinised literature.

#### **5.2. DISCUSSION OF RESULTS**

##### **5.2.1 Section A**

###### **5.2.1.1 T-Tests**

The objective of this section is to set hypotheses regarding the sample to determine if there exist a significant difference between control and experimental groups within the sample. The two key variables (achievement test and GeoGebra software) that were used as the basis for the testing of hypotheses and questions. This procedure will help the researcher in getting more firm and concrete results. In hypothesis testing, two hypotheses are stated, that is, a null hypothesis- $H_0$  (the sample results are by chance), and an alternative hypothesis- $H_a$  (the sample results reflect what is actually happening in the population).

Based on the sample results a test statistic is calculated enabling either to accept the null hypothesis (that is, one concludes that the result is due to chance), or reject the null hypothesis (that is, one concludes that the result shows what is happening in the population). Also an exceedance probability or a p-value is calculated. The p-value is probability that result happens due to chance. If the p-value is small (less than 0.05) it implies that it is highly unlikely that the result is due to chance alone, that is, we reject the null hypothesis. If the p-value is large (larger than 0.05) we accept the null hypothesis.

$H_0$  – Mathematics students learning Euclidean geometry are performing the same.

$H_a$  – Mathematics students learning Euclidean geometry are not performing the same.

In Table 4.1, chapter 4, independent t-test is calculated for pre-test for both groups. Experimental group has a mean score of 6.36, standard deviation of 2.64 and control group has a mean score of 5.5, standard deviation of 2.9. The t-value is calculated as 0.1619419 and the p-value is 0.108221 ( $p > 0.05$ ). It means during the pre -test there was no significant difference, thus null hypothesis is accepted and alternative is rejected, mathematics students learning Euclidean geometry are performing the same. This was before treatment was applied.

In Table 4.2, experimental group in post has score mean of 9.45 with standard deviation of 1.92, and control group has 7.52 with standard deviation of 2.64. The t-value is calculated as 4.384833 and p-value of 0.000027 ( $p < 0.05$ ), that means mean scores of both groups are highly significant for unpaired post-test. And for the paired samples result, experimental group has mean score of 3.09, standard deviation of 2.72, t-value of 13.7 and control group has 2.02, standard deviation and t-value of 14.58. The resulting p-value is 0.00000 ( $p < 0.05$ ). This was after treatment has been administered to experimental group. Thus there is high significant difference.  $H_0$  is rejected and  $H_a$  is accepted.

#### **5.2.1.2 Discussion of the results of the t-tests**

If the students are taught in traditional method whether pencil and paper are used they perform the same but, though there seem to be improvement in the control group after continuous traditional approach is used, but it is noted that experimental group students performed much better than the control group after the application of GeoGebra in learning Euclidean geometry. This means GeoGebra have impact on learners' performance according to t-test and p-value calculation.

#### **5.2.2 Section B.**

In this section, relevant data about response frequencies of respondents of five groups of students in experimental group to the questionnaire is reported. The objective of Table 4.4, consisting of item 2 – 15 is to analyse those items which had

Strongly Disagree/Disagree/Agree/Strongly Agree as developed by the researcher to get the perceptions regarding their interaction with learning Euclidean geometry with GeoGebra

#### **5.2.2.1 The self-designed questionnaire**

A self-designed questionnaire was used to explore various issues factors influencing students' learning (Annexure 7). The questionnaire has the aim of measuring students overall insight into GeoGebra and Euclidean geometry. The questionnaire has the following items;

- Excitement / enjoyment
- Confidence
- Engagement
- Benefits of Interaction
- Visualisation
- Creativity and criticality
- Logic
- Connections
- Prediction
- Development and reasoning skills
- Future outcome
- Rating
- Improvement

#### **5.2.2.2 Discussion of the questionnaire results in Table 4.4**

**Logical assumptions:** About 84% (see also Figure 4.13) of students believe that application GeoGebra in learning Euclidean geometry has improved how work on step by step (logic) which is very important in proving theorems and breaking down information which might be embedded in circle geometry most probably from previous information, like knowledge of properties of parallel lines etc.

**Visualisation and reasoning skills:** Students agree and strongly agree that their visualisation has improved a lot after they have done lessons using GeoGebra (90%) (See also Figure 4.12). A further 78% agree and strongly agree that learning

Euclidean geometry using GeoGebra have improve their reasoning skills. This is very important as future mathematicians they will have to prune these important skills and in life generally.

**Confidence / excitement /enjoyment:** Learning must be fun as about 95% of students felt excited learning Euclidean geometry using GeoGebra, 82% enjoyed lessons and 86% felt confident.

**Interaction / creative and critical:** The results (91%) of students who like interaction between teacher-student interaction (See Figure 4.9), affirm the notion by Stols (2012) that using technology require an effective teacher who will be at all times ready to intervene to lift up the learning especially for clarifications. And 93% felt GeoGebra gave them to be critical in their view which is a higher skill required in mathematics.

**Outcome / improvement/ prediction:** There is a firm agreement from students (91%) in believing their Euclidean geometry results will improve as a result of using GeoGebra, and 78% of students in a sample felt they are going to do well in mathematics tests and examinations. And 96% of students felt that GeoGebra made fully engaged in their lessons.

**Connections / Rating:** The sample result shows that students can make connections with previous information. This is important as students will take each lessons seriously knowing its' impact in upcoming lessons. But a confliction notions as students believe only brilliant students can do well in Euclidean geometry without geometry while another group disagreed (44%: 56 %.)

### 5.2.3 Section C

#### 5.2.3.1 Sample result on regression analysis

In Figure 4.17 there is linear regression. The calculated value of B is 0.56 which is positive. And the correlation coefficient is 0.60. The value of A was calculated to be 5.33. These values were calculated based on the achievement test scores of experimental group for pre-test and post-test. In statistics, the correlation coefficient  $r$  measures the strength and direction of a linear relationship between two variables on a scatterplot. The value of  $r$  is always between +1 and -1.



### **5.2.3.2 Discussion in respect to regression analysis**

The test score from both pre-test and post-test shows an improvement to the performance by students, in that  $B = 0.56$  and  $r = 60$ . The values are positive meaning all variables increase together.

### **5.2.4 Section D**

#### **5.2.4.1 Findings from Question 1**

The question was meant to examine whether students by using GeoGebra will be able to perform better in Euclidean geometry and have high grades in mathematics.

The students asserted that geometry is difficult and with the re-introduction of Euclidean geometry in Grade 10, students tend to have found an excuse why they cannot perform sufficiently in mathematics. This was further highlighted in literature that students have difficulty in understanding Euclidean geometry concepts and their applications, as a result their overall mathematics performance suffers (Okazaki, 2008; Rollick, 2009). Fulton (2013) hinted that many students hit the geometry wall in high school and their mathematical journey comes to an abrupt and unfortunate stop.

The comment by the Samkelo (Section 4.3.2), illustrates that circle geometry is indeed a problem to students performing better in mathematics. It has a negative impact to such an extent that students blame it for repeating the entire grade; furthermore it seems to have an unwanted history of shattering students' dreams in the area of mathematics. The students traditionally also cannot get adequate support when completing homework and the main problem is that the teaching approach for Euclidean geometry in some materials mostly used by students (MINDSET LEARN GRADE 11 MATHEMATICS for CAPS, 2013) emphasises the need for rote learning. The theorems are only referred to and are expected to be memorised, so there is no core understanding being encouraged.

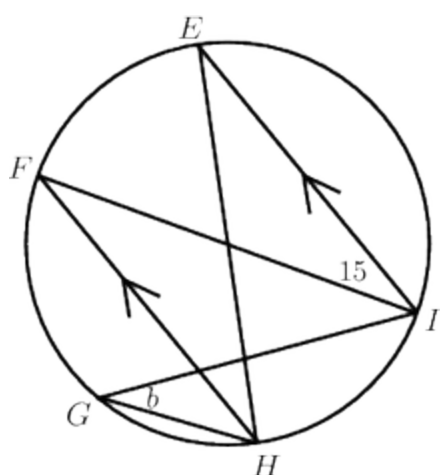
Statistics pointing to the poor performance of South African students in various levels of schooling are widely available. The performance of Grade 9 students for Grade 8 international testing in the *Trends in International Mathematics and Science Study*

(TIMMS) of 2011, revealed that South African students had the lowest scores as illustrated in Table 2.1, chapter 2 (Reddy, 2012).

The students encountered multiple challenges while doing exercises, such as not extracting the basic theorems from the figure, not realising vertically opposite angles of  $\angle C_1$  and  $\angle C_4$ , though these are from different circles, are equal, and can thus allow them to obtain the size of 'an' in exercise 4.2 (see Annexure 5). Approximately 83% of the students had challenges in inter circles, see Figure 4.2 (Annexure 5). The 83% applied the theorem that  $b$  is equal to  $51^\circ$  without realising that N is not on the circumference of circle centre P. They had to use the reasoning in the smaller circle first. This was detected during discussions though. The academic achievement test was a multiple choice test, meaning that if steps were marked these students would not get the actual marks. As a result this could severely jeopardise their mathematics results moving forward.

Students also wrote a number of true statements which were not required. This leads to answers becoming unnecessarily lengthy and confusing. The researcher also detected that some students liked to assume that RQ for instance in Figure 3.28 chapter 3, is a diameter and even went further assuming that the size of  $i = 51^\circ$  in Figure 3.28 chapter 3 again, stating that  $\angle RQO$  was  $90^\circ$ . Students did not label the angles correctly, for example  $\angle C = 32^\circ$  in Figure 3.25 chapter 3, instead of  $\angle ACB$ .

But after intervention using GeoGebra tools for learning concepts thoroughly and incorporating appropriate terminology with circle geometry theorems, there were vast changes noted in students' views on Euclidean geometry. Most of the students stated they are likely to perform better, as illustrated by Figure 5.1.



### Figure 5.1 Pointing to geometry terminology in theorems

Students while using GeoGebra learned that the key to performance is to learn terminology used in most geometry problems (see Figures 4.1 to 4.7 in chapter 4). Thus, in Figure 5.4 students firstly identify the parallel lines ( $EF \parallel FH$ ), thus resulting in  $\angle HFI = 15^\circ$ , as alternating angles are equal in size. But  $\angle HFI = \angle HGI$  (subtended by the same line segment, see diagram 3 of Figure 3.17 chapter 3 and Figure 3.18 chapter). Therefore  $b = 15^\circ$ .

Students having started off with a thorough drilling on terminology, easily answered the lengths of HG, JI and the size of  $d$  using Figure 3.27 in chapter 3 for task 5.

One student Gcwalisile demonstrated this by calculating that:

HG and HI are tangents at circle centre J, but they meet at one point H thus HG is also equal to 8cm (proving that tangents meeting at one point of the circle are equal in lengths, see also Figure 4.8 in chapter 4). The students having being helped by GeoGebra software, while proving theorems and their corollaries, easily recalled and knew when to apply information practised before (see also Figure 3.22 diagram 2). Thus they also used correct mathematical facts and identified the correct information in order to tackle problems. This is in line cognitive level 1. JI is perpendicular to HI at I and JG perpendicular to HG at G (student applying knowledge learned during intervention, see diagram 1 Figure 4.1 in chapter 4). And GJ is exactly equal to JI, equal to 5cm (see Figure 3.14 in chapter 3).

$\triangle HGJ$  and  $\triangle HIJ$  are congruent, reason being the right angle, hypotenuse and side. Therefore to compute the value of  $d$  the theorem of Pythagoras will be used.

Only  $\triangle HIJ$  will be used resulting in  $HJ^2 = JI^2 + HI^2 = 5^2 + 8^2 = 89$

$$d = 9.43$$

The students improved immensely in terms of layout when solving the circle geometry problems, right from mini class activities like the ones above in Figure 5.1 and Figure 5.2. Furthermore, the mean score of the pre-test was 6.36 for the experimental group (see Table 4.1 chapter 4) which went up to 9.45 after post-test results were taken (see Table 4.2 in chapter 4). This showed a true improvement of

performance by students. And this is further validated by upward movement of the regression line (see Figure 4.17 in chapter 4).

Figure 4.9 in chapter 4 shows that 91% of the students approved that use of GeoGebra in learning Euclidean geometry can enhance their performance. These results affirmed the notion by Van Wyk (2014) who indicated that technological tools like GeoGebra improve results, and thus have an impact on mathematics education. GeoGebra fundamentally helps to increase the probability of success. Indeed success equals improved teaching and learning too, and ultimately the improved educational outcomes we strive for (Ford, 2014).

And these concur with neuroscience theoretical framework, which according to Butterworth (2010), GeoGebra with its number-specific cognitive processes as compared to general cognitive processes support specific mathematics achievement. Students applied right cognitive level 2 for routine procedures as stipulated in the *Curriculum and Assessment Policy Statement* (CAPS, 2010) and *Mathematics Examination Guidelines Grade 12* (2014), when referring to proofs and deriving the formulae, with some further application of level 3 ( complex procedure) and level 4 (problem solving) as well.

The students showed some knowledge by clearly identifying correct formulae, using mathematical facts required to solve problems and using correct processes in calculations. Furthermore, students performed well-practised procedures in solving problems related to prescribed theorems. And again the students derived more sets of conjectures from the given information. The students timeously made significant connections between content and concepts learned in previous grades and previous lessons, where they used GeoGebra software in their presentations. They showed to reason at a relatively higher level. The students' analyses, syntheses and evaluation skills clearly showed the improvement in their level of performance in Euclidean geometry, which the study question intended to establish.

This was firmly confirmed by statistical results which found that though both experimental and control groups increased in their performance, the intervention using GeoGebra software yielded higher improvement levels in terms of academic achievement scores of the experimental group. This affirms that vast results can be

achieved in student performance through the proper application of GeoGebra software.

#### **5.2.4.2 Findings from Question 2**

The intention of this question was to find out to what extent interactions with GeoGebra tools help students in attaining knowledge surrounding circle geometry.

The student's comment (Vumisile, see Section 4.3.3), echoed what most of the students in the study felt. The students stated that their struggles were a result of not understanding reasons properly. They further showed some concerns over the precise reasoning when proving theorems. They cited the lack of understanding of circle geometry theorems as the main factor leading them to entirely lose interest in Euclidean geometry, and hence achieve poorly in mathematics.

Ndlovu (2014) indicates the basic understanding of dynamic geometry definitions is a challenge to teachers. This echoed some of the comments by students who did not have confidence in their teachers. In line with comments by Vumisile, research has shown that students have difficulty with the notion of proof (Senk, 1989; Usiskin, 2007; Bell, 1976), and this was further noted by Webber (2013) who indicates proofs are notoriously difficult mathematical concepts for students.

Indeed proof calls for developed reasoning skills. According to Department of Education in document curriculum 2005, reasoning is the most important skill needed to fall in line with the literature. Specific outcome 10, for mathematical literacy, mathematics and mathematical sciences, requires that learners "use various logical processes to formulate, test and justify conjectures." It is stated that:

*Reasoning is fundamental to mathematical activity. Active learners question, conjecture and experiment. Maths programmes should provide opportunities for learners to develop and employ their reasoning skills. Learners need varied experiences to construct arguments in problem settings and to evaluate the arguments of others.*

This entails the development of students in terms of mathematics programmes regarding modelling problems where their reasoning skills will be encouraged and maximised. Reasoning is a skill that is demonstrated during the advanced stages of

thought (Umay, 2003). In other words, during problem-solving processes, and which represents high-order mathematical thinking. Reasoning skills are an important component of education, and reasoning skills are necessary for understanding mathematics in particular since they present an important means of developing ideas (Gunhan, 2014).

Mathematical reasoning refers to the ability to formulate and represent a given mathematics problem, and to explain and justify the solution or argument. Mathematical reasoning can be gained at the elementary education level, where students make, refine, and test their own conjectures. Secondary students must be able to evaluate conjectures and assertions, in order to reason deductively and inductively by formulating mathematical assertions, and to develop and maintain their reasoning skills. Studies indicate that good mathematical reasoning skills are also imperative to improve writing performance. There is a direct relationship between reasoning skills and success in mathematics, where students who demonstrate better reasoning skills display good problem solving profiles with the interrelations they are able to identify, while also having better communication skills.

Poor reasoning involves unfounded and hasty reasoning processes resulting from insufficient understanding of the concept in question. Students continue to have difficulty with deduction and proof. Mukucha (2010) found in South Africa that most students lacked conceptual understanding and reasoning skills. These studies have suggested that different approaches and techniques are necessary for students to develop reasoning skills. Communication skills are important for the development of students' reasoning skills (Aineamani, 2011).

Goos, Stillman and Vale (2007) have indicated the processes of visualisation and reasoning to be part of mathematical thinking. TIMSS (2012) have indicated that a student with reasoning skills must be able to perform the following: Identify and use interrelations between variables in mathematical situations; dissociate geometric shapes in order to facilitate the resolution of a geometrical problem; draw the expansion of an object; visualise the information of three dimensional objects; and deduce valid results based on the provided information (analyse); think mathematically and describe anew the results obtained through problem solving and expand on these solutions (generalise); use mathematical operations in combination

and combine the results in order to obtain more advanced solutions (synthesise); use mathematical results or properties to provide evidence for the validity of an action or the truth of a mathematical expression (justify); and solve non-routine problems by applying his/her geometrical knowledge and appropriate mathematical processes (solve non-routine problems). These skills are grouped into meaningful themes and provide an overview of the skills required to successfully answer test items.

**Table 5.1 Levels of Mathematics Competency (Hungu, et al., 2010)**

Distribution of levels	Range on 500 point scale	Skills
<b>Level 1</b> <i>Pre-numeracy</i>	< 364	Recognises simple shapes. Matches pictures
<b>Level 2</b> <i>Emergent numeracy</i>	364 → 462	Recognises common two-dimensional shapes.
<b>Level 3</b> <i>Basic numeracy</i>	462 → 532	Interpretation of simple measurement
<b>Level 4</b> <i>Beginning numeracy</i>	532 → 587	Use of multiple operations
<b>Level 5</b> <i>Competent numeracy</i>	587 → 644	Converting basic measurement units
<b>Level 6</b> <i>Mathematically Skilled</i>	644 → 720	Make use of graphic representations
<b>Level 7</b> <i>Concrete problem Solving</i>	720 → 806	Extracts and converts information from tables, and visual presentations
<b>Level 8</b> <i>Abstract problem Solving</i>	> 806	Identification of unstated mathematical problem embedded on graphs

At the national averages only 40.2% of South African grade learners are functionally innumerate, and with a further 27.2% are functionally illiterate. From an educational perspective it is important to realise that a large number of students – particularly those from disadvantaged backgrounds – acquire learning deficits very early in their educational careers. Since education is a cumulative process, these deficits in numeracy and literacy are likely to stay with these students for the rest of their lives. As these students proceed to higher grades teachers and the curricula presume that students have acquired the necessary skills taught in previous grades.

However, in the South African education system, grade progression is not only determined by skills acquired but the need to maintain stable grade enrolments and normal pass rates. As a result, teachers do not exactly know at what level they should test their students, which automatically leads to poor or underperformance in exit examinations. This clearly reveals that the core skills to mathematics, which are concrete and abstract problem solving capabilities, have been eluding learners, right from primary level. In a span of eight years instead of learners improving in these skills, there seems to be a decline, where concrete problem solving abilities have dwindled. This shows that there is a huge challenge in terms of mathematics understanding, which if not addressed effectively, will keep South Africa's standard at being the lowest when measuring the mathematics levels by international bodies like TIMSS. The mathematics language is critical to understanding basic concepts of mathematics, Van de Walle (2004, pp.13), such as explaining, exploring, investigating etc. This allows students to understand that mathematics is about processes and products, see table 5.3.

**Table 5.2 Action verbs and products involved in mathematics**

<i>Processes</i>	<i>Products</i>
<i>Assuming</i>	Definition
<i>Computing</i>	Theorem
<i>Generalising</i>	Formula
<i>Solving</i>	Axiom
<i>Proving</i>	Corollary
<i>Testing</i>	Concepts

The knowledge of action verbs could let students focus appropriately when they are involved in any task, for example theorems need not be proved but computed. To prove means to establish the truth or genuineness of, as by evidence or argument, meaning only corollaries can be proved. Software like GeoGebra, computes the theorems.

Students were asked to draw and prove theorems such as exterior angle of the triangle is equal to the sum of the two interior opposite angles (see Figure 4.11). The students used point tool to plot points, text tool for labelling points, line segment tool and move tool to join the points, angle tool for measuring angles and pen tool for dotting down key concepts, as GeoGebra tools for computing the theorem.



For the circle geometry theorem: The line drawn from the centre of the circle perpendicular to the chord bisects the chord (see Figure 4.14). Students used the following GeoGebra software tools, move tool and circle with centre through point tool, point tool for plotting points and for labelling point text tool was used, line segment tool to join points, perpendicular bisector tool and angle tool to show that there is right angle formed and lastly distance tool to measure the line segments showing that lines are equal in length.

After lengthy cooperative and collaborative learning using GeoGebra software, students commented positively on understanding Euclidean geometry like Snegugu who said:

*"I am oozing confidence towards circle geometry. I believe geometry more especially circle geometry is relative easy. Maybe it was because of GeoGebra. This topic is not as complicated as algebra. Circle geometry's options are straight forward. In fact anytime I will put more effort on geometry as I have realised it can quickly stay in my mind. Thank to compare software I am developing love for mathematics. I follow all routine steps with more insight than before."*

Snegugu's comment echoed what the literature theory stated. Shellhorn (2011) stated that the free GeoGebra resource helps students understand constructive proofs in geometry in conjecturing as well as justification and thus verification when it comes to properties of geometric objects. The results clearly show that students approved the use of the GeoGebra to help them understand geometry theorems better, with 36% agreeing and 55% strongly agreeing.

GeoGebra provides multiple representations for a mathematical idea, which contributes to the complexity of the learning environment and cognitive performance when learning. Conceptual understanding is of paramount importance to young minds. This is provided for in GeoGebra software as Novak, Fahlberg-Stojanovska and Renzo (2010) reported that GeoGebra enables students' to have a deeper conceptual understanding of real-world scenarios and underlying mathematics, including conceptual understanding in experimentation and exploration in order to discover, generalise, conjecture and verify results (National Research Council, 2001) in the literature.

### 5.2.4.3 Findings from Question 3

The study question seeks an insight on GeoGebra improving students' poor performance by introducing quality learning and teaching of geometry. Furthermore, it seeks to ascertain whether GeoGebra adds real value in the level of students' mathematics achievements. And lastly the aid of GeoGebra in upholding the proofs and theorems of circle geometry is assessed.

Teachers' choice of technologies can be seen as related to their attitudes, conceptions and beliefs. Results show that teacher-students interactions were also the key to making GeoGebra appealing in circle geometry learning and teaching. About 91% indicated that through GeoGebra they tended to communicate better with the teacher, thus boosting both teacher and student confidence levels. This confirmed other studies that there has been an increasing awareness that interactions between humans and technological tools like GeoGebra software can facilitate effective teaching and learning.

Geometry theorem proving is one of the most challenging skills for students to learn in secondary school mathematics, as they were not introduced to justifications and informal proofs at elementary levels. Many students find it difficult to write down a formal proof because they do not understand the geometric properties in the proof (Wong, Yin, Yang, & Cheng, 2011). The students may have not smoothly gone through Van Heeled levels, like abstraction or relational, as geometry requires significant abstract thinking. As a result students cannot understand and formulate abstract definitions, distinguish between necessary and sufficient conditions for a geometric concept and cannot recognise shape differences effectively.

In the current study students echoed the same sentiments on struggling on geometry proofs and theorems, as Simphiwe stated:

*"I have tried devoting my time on circle geometry hoping that this will improve my mathematics results but I see no change. I try by all means to follow the teacher during lessons' presentations but still I do not believe the teacher because when we used paper and pencil there are minimal accuracy, as a result I am left not sure. Sir  $45^\circ$  cannot be  $44.79^\circ$ . This totally distorts my visualisation."*

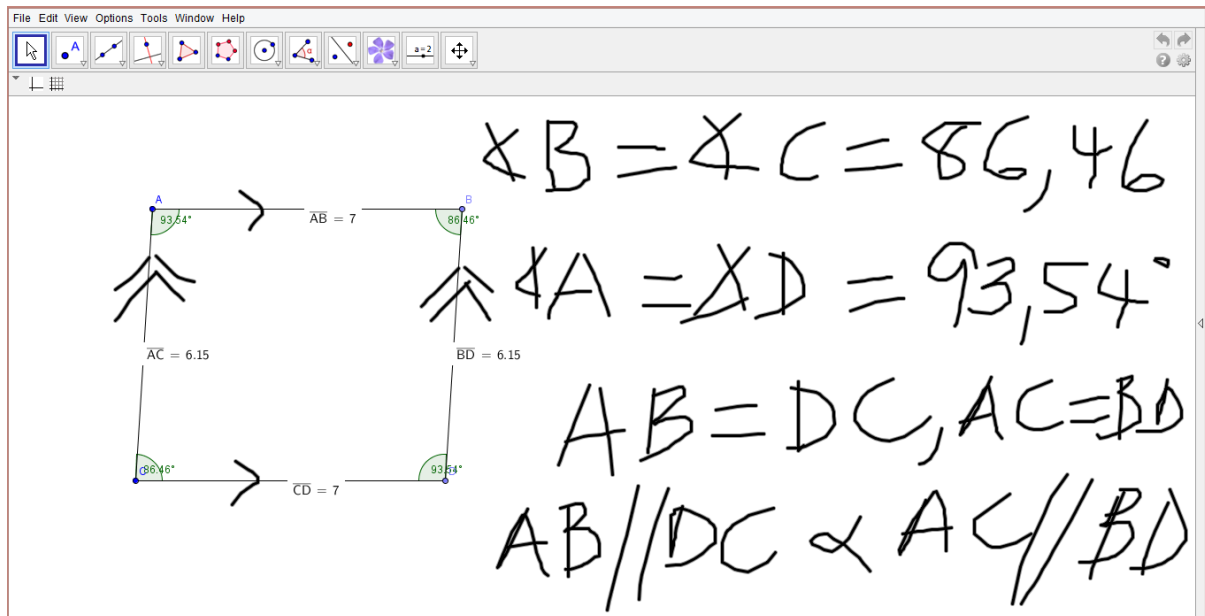
This stems from the fact that they did not believe if theorems were verified using pen and paper. And, this was in reference to inaccuracy in Figure 3.18 (chapter 3), where 11.15cm and 11.16cm tangents showed. Students argued these were not equal, contrary to the theorem. The visual image according to students ought to be acutely accurate. Visualisation and reasoning as cognitive processes are interconnected, promoting students' success in geometry. Arcavi (2003) indicated that visualisation is a skill that helps students recognise shapes, create new shapes or objects, and to reveal relationships between them. Studies reveal that visualisation can be improved by training and by GeoGebra software materials (Onyancha, Derov & Kinsey, 2009; Yildiz, 2009). In addition, visualisation and reasoning skills can be improved through the instruction methods (Arici, 2012; Goos et al., 2007). The instructional design is more important when teaching theorems to learners as this is central to understanding steps involved. Yang (2006) indicate students being forced into unfamiliar interaction styles and task structures may be distracted from the major task of exploring and understanding relationships between proofs, steps and geometry figures. This is an important factor to enhance the understanding of geometry proving.

The intervention was utilised to practically answer the research question by providing effective assistance through GeoGebra tools, helping justify the relevant geometry proofs and theorems. The practical task was for students to justify some of the properties of the parallelogram (Figure 5.2). Students used point tool for plotting four points, text tool to label points, line segment tool for joining labelled points, distance tool to measure line segment, angle tool to measure all interior angles and pen tool to indicate and dot the key properties of a parallelogram.

Figure 5.8 shows four points A, B, C, and D with lines AB, BD, DC, and CA, drawn by one student Bongephiwe. After drawing, she measured the line segments and got  $AB = 7\text{cm}$ ,  $BD = 6.15\text{cm}$ ,  $DC = 7\text{cm}$  and  $CA = 6.15\text{cm}$ . After long analysis she realised that  $AB = CD$  and  $BD = AC$ . She further measured angles,  $\hat{A} = 95.54^\circ$ ,  $\angle B = 86.46^\circ$ ,  $\hat{C} = 86.46^\circ$  and  $\angle D = 95.54^\circ$ . Line  $AB \parallel CD$  and  $AC \parallel BD$ . Bongephiwe made deductions by stating:

AB and CD are separated by two separate lines AC and BD which are equal in length ( $AC = BD = 6.15\text{cm}$ ), and AC and BD are separated by  $AB = CD = 7\text{cm}$ , this

means  $AB$  is parallel to  $CD$ , and  $AC$  parallel to  $BD$ . Furthermore  $\angle A + \angle B = 180^\circ$ .  $95.54^\circ + 86.46^\circ = 180^\circ$ . Angles  $A$  and  $B$  are co-interior angles, as these add up to  $180^\circ$  that means corresponding sides are parallel, in this case  $AC \parallel BD$ . So it also follows then, that  $AB \parallel CD$  from the same perspective.  $\angle A = \angle D$  and  $\angle B = \angle C$ , which is one of the key properties of a parallelogram.



**Figure 5.2 Snapshot of using GeoGebra tools in proof justification**

This showed that through GeoGebra tools improved visualisation and reasoning skills were evidenced, which helped substantially in justifying the proof using previous knowledge. GeoGebra activated Bongephiwe's existing schema that is when the distance between two lines remain the same it means those two lines will never meet, thus the lines are parallel.

The student, Bongephiwe seems to be applying theorems and elaborating definitions well. Bongephiwe made a proper distinction between features of the drawing and made use of auxiliary standard elements to complete the figure. For instance, extensive use of auxiliary objects proportionated by the software and correct capturing of the information given by the software allowed for the creation of the afore geometric object. Furthermore, the students seemed to have much more understanding of the logical structure of the problem as well as understanding of the ontological status of the objects. And more identification and construction of auxiliary elements took place, as more interactions with GeoGebra software in application of

said tools to more difficult problems, occurred. This was noted in terms of structural competence.

On visualisation competence the students fared well from visualisation of standard geometric properties of the figure (clear distinction between drawing and figure) to operative apprehension (extraction of similar figures, equivalent figures) to operative apprehension (introduction of auxiliary elements) right to dynamic visualisation (thinking about the variation of a point in a linked variation). As students got accustomed to GeoGebra, there seemed to be movement to recon figurative visualisation (transformation of two shapes in an equivalent shape that is deductive competence for visual proofs) to visualisation of algebraic-geometric elements, right to extraction of equivalent figures and mental visual transformations.

This affirms the proven results in Table 4.2, Table 4.3 and Table 4.4 all in chapter 4, and Figure 4.12 showing that 32 students agreed that GeoGebra helped them use visualisation skills more when learning Euclidean geometry, with 18 of them in the sample strongly agreeing- which meant a resounding 90% of students saw GeoGebra tools used during the study as extremely helpful in their learning. Visual tools can thus clearly facilitate the learning of mathematical concepts (Naidoo, 2012).

Dynamic geometry systems like GeoGebra are developed to assist users to create geometric constructions, explore geometry graphs, formulate conjectures, check facts and even build proofs (Geometry explorer; Kortenkamp & Fest, 2008; Narboux, 2007).

Altogether 84% of the students who used GeoGebra software feel that they justified the theorems correctly while they were proving right from general theorems. This was in line with other studies like Sanchez et al. (2010), who indicated that technology promotes and develops the functional language that is very important for the construction of intellectual proofs, and GeoGebra in constructing proofs provide an opportunity for exploration, discovery, conjecturing, refuting, reformulating and explaining.

GeoGebra promotes reasoning skills and geometric understanding. The relationships between visualisation and mathematical problem solving as well as between visualisation and mathematics achievement (Ünal, Jakubowski & Corey,

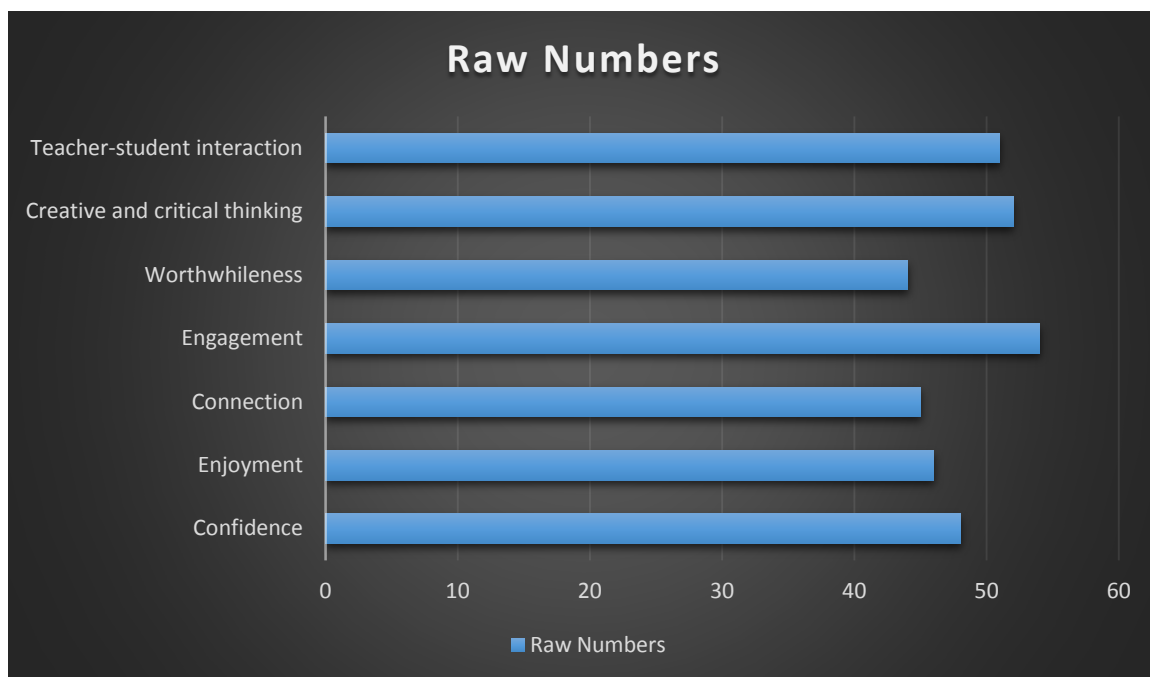
2009), have been shown in many studies. Moreover Kospentaris, Spyrou and Lappas (2011) proclaimed in their study that visualisation is an important factor that affects the choice of strategy when students undertake geometry tasks.

GeoGebra is more student-centred, thus engaging students more effectively. This entails greater stimulation of frontal lobes, which regulate functions such as attention, focus and decision making, which are of-course critical in learning Euclidean geometry and achieving ultimate career success. This concurs with Ansari (2013) who pointed that neuroscience supports the notion that young minds, when involved with active, and self-directed activities, engage their brains in deeper, faster and better learning than passive learning. It further confirms what the literature points to, as indicated by Sinclair and Jones (2009), that GeoGebra and Sketchpad illustrate how students can move from empirical and visual descriptions of spatial relations, to more theoretical, abstract ones.

### **5.3 STUDENTS FEEDBACK**

Figure 5.1 exhibits the results of subjective evaluation of GeoGebra software in learning Euclidean geometry. As shown, the confidence, teacher-student interaction, enjoyment, connections between lessons, logical assumptions, value of circle geometry, ability to be creative and critical and students' engagement in lessons all received good feedback from students. In addition, the students thought it was comfortable and enjoyable to use the GeoGebra tool.

While all students thought the tool was useful, they also offered some constructive suggestions. The students wished the GeoGebra software could provide them with different types of proofs, like the two-column proof, as they stated two-column proof is commonly used.



**Figure 5.3 Students' ratings of GeoGebra**

One student Sizwe openly said;

*“GeoGebra is so fascinating in all aspects of my learning circle geometry so far. But I wish this could lay out proofs in the form of two-column way, because that is format that you have been stressing all along to follow when formalising our learning.”*

The student seemed to be more dependent on the software, thus believing that the software could do more for them, similar to every statement being followed by a reason. As stated in Table 5.1, where the theorem needed to be computed by means of computer technology to complete the said tasks.

Using GeoGebra software can best help in the teaching and learning of mathematics, and especially geometry of the circle. This was confirmed by a significant increase in experimental students' conceptual grasping, grappling and understanding of circle and related subtopics as compared to those students in the control group. This finding is supported by Zengin, Furkan and Kutluca (2012), and Shadaan and Leong (2013) who conducted a study with two groups using an achievement test as pre-test and post-test to learn mathematics concepts.

These findings also confirm other studies previously done to determine the effects of a technology-rich environment on students learning (Dogan, 2010; Idris, 2006;

Roberts, 2012; White, 2012). Accordingly, it is important that the teacher as a custodian of the learning environment be equally enlightened regarding the advantages of a technologically-enabled mathematics classroom. There should be constant referral to studies done by professional mathematicians when reviewing the impact of new learning technologies (Shadaan & Leong, 2013).

The document, 'Technology in Teaching and Learning Mathematics', on technology stated:

*It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, communication. Effective teachers optimise the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students* (National Curriculum of Teachers of Mathematics, 2011, pp.1).

The findings also imply that technology is a great motivational tool as students' confidence increased when the GeoGebra was used to enhance the students' learning process. This was more beneficial to lower ability students. GeoGebra was the scaffold that enabled students to reach their zone of proximal development. This is in line with Dogan's (2012) study whereby it was noted that computer based activities encouraged higher order thinking skills, and had a positive effect in motivating students towards learning.








The findings in the current study support other findings from a number of studies, having shown that the strategic use of technological tools can support both the learning of mathematical procedures and skills as well as the development of advanced mathematical proficiencies, such as problem solving, reasoning, and justifying (e.g., Gadanidis & Geiger, 2010; Pierce & Stacey, 2010, pp. 41-55)

Where students were asked how the software affected them, they had many positives things to say, such as:

One of students, Khuphukile, said:



*“I experienced that using GeoGebra, one uses his or her mind quickly and with much caution. For instance calculating the values of  $a$ ,  $b$  and  $c$  in practice task 3 was relatively easy for me as I visualise theorems involved while solving that task. GeoGebra tools enlightened me a lot. My reasoning has grown sharply.”*

Again Khuphukile echoed what was stated by more 90 % of the students. Task 3 with Figure 3.17 (chapter 3) got students to apply knowledge they learned while using GeoGebra tools like circle with a given point , point tool  A, move tool , line segment tool , text tool  ABC, pen tool , and angle tool  while learning the circle geometry theorems; ‘angle subtended by a diameter in a circle is a right angle’ using figure 3.18, ‘opposite angles of a cyclic quadrilateral are supplementary’ with the use of Figure 3.19 a, and triangle theorem; ‘the sum of interior angles of a triangle is  $180^\circ$ ’ using Figures 2.1 and Figure 2.2 (chapter 2).






The student emphasised that GeoGebra’ helped in her development of good reasoning skills, experiences of visualising, and quickness in thinking and meticulousness in work.

Another student, Mpume, said:

*“I had never felt so confident in my mathematics in my entire learning life. I felt trusted by classmates as I was given an opportunity to show them how I had done my construction of theorem figures. The accuracy part held me in high esteem. The students listen to me and I did the same when other students were explaining their approaches. My classmates sometime were so critical thus enhancing me always creative. The teacher never disturbed us in our exploration other than when some computers jammed. We had a good relationship between ourselves and the teacher. I was more fascinated by dragging of the mouse. I was in real competition as I strived to be the first to get a correct proof.”*

Mpume highlighted again the teacher-students and students-students interactions as a core motivator to her striving higher. She even noted that accuracy of theorem proving made her feel more confident. This further showed that students were highly engaged in circle geometry with GeoGebra software. The students also learned to be more creative when using GeoGebra tools as they were proving theorems.

Deductive reasoning was witnessed as students made conclusion on theorems; for example, 'the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment' see figure 19 (chapter

3). They used circle with given point tool  for constructing the circle, point tool  for plotting the points, text tool  for labelling the points, line segment tool  for joining the points, and angle tool  for measuring angles.

This indicated that there was improvement of conceptual knowledge for students at all levels of competence, which suggests that the GeoGebra software affects such positive development among students. Furthermore, detailed evaluation of the experiment group students' responses clearly revealed that the students used GeoGebra construction properties for the tasks, and showed that they benefitted from these tools.

Competence and confidence was gained through practice, explorations and multiple reviews through GeoGebra tools. GeoGebra mediates the learning process and communicates indirectly the relevant mathematical concepts. As Antohe (2009) stated, students could visualise abstract concepts and make connections among these concepts in mathematics using GeoGebra.

## 5.4 CONCLUSION

The main results of the study are summarised as follows:

There is a statistically significant difference when learning Euclidean geometry using GeoGebra. It can be deduced that learners are highly engaged in their learning, think creatively and critically, thus have increased reasoning skills, when applying GeoGebra in this manner. The students are very confident and as a result are likely to improve their performance.

In this chapter, the description of the independent and dependent variables were described by means of frequencies and t-tests. In the next chapter, the researcher will be able to make recommendations and conclusions regarding GeoGebra software influencing the teaching and learning of Euclidean geometry.

## **CHAPTER SIX**

### **SUMMARY, CONCLUSIONS, LIMITATIONS AND RECOMMENDATIONS**

#### **6.1 INTRODUCTION**

The previous chapter presented in detail the findings of this study after an analysis of the data gathered. The last chapter of the research study addresses specific and appropriate recommendations and conclusions, based on the theoretical substructure (chapters 1 to 4), as well as the findings that came to light in the research results as discussed in chapter 5. The chapter includes noting implications from discussion, to make recommendations for present and future research based upon the findings. To facilitate research, the main aims of this study was subdivided into addressing three questions (see chapter 1), namely:

1. How application of GeoGebra in Euclidean geometry impact on learners' performance in Euclidean geometry?
2. How does the se of GeoGebra improve learners' understanding of geometry theorems?
3. What practical and theoretical implications GeoGebra have on justifying proofs and theorems of geometry?

#### **6.2 SUMMARY OF THE RESEARCH**

To ensure that this study is also an appraisal of a process and not only a dissertation based on a framework, the findings made in the proceeding chapters on the various objectives are summarised in this chapter in order to make recommendations and draw conclusion on the application GeoGbera software in Euclidean geometry on grade 11 rural high school learners.

The first purpose of these questions was to provide a conceptual analysis of GeoGebra that would lay the foundation to prepare the lessons for correct implementation. To achieve the abovementioned, it is necessary to understand the different views of researchers in respect of quality and usefulness of GeoGebra. Firstly in the research more authors have firm notion that technology, GeoGebra in particular, have proven to assist students in teaching and learning of geometry

(Bowie, 2010). Thus it was important to delve on the features of GeoGebra that can assist deep rural high school students so that institutions can have comprehensive decisions in bringing technology in their mathematics classrooms, to bring about quality in learning. From the above notion, it is clear that institutions can no longer afford to ignore in capacitating themselves.

The second purpose was to do a conceptual analysis of Euclidean geometry. The reason for this is that Euclidean geometry is arguably the most significant topic that have swept across institutions for years, but in South Africa, some part of it i.e. circle geometry has been optional for six years (from 2008 to 2013), but has come to the mainstream mathematics curriculum.

In the research literature there appears a notion that students struggle in Euclidean geometry (Section 2.1), and many well-known authors seem to have different perspectives on this issue, lamenting earlier for making Euclidean geometry optional, but more so after the news that it would be reinstated (Coetzee, 2014; Fulton, 2013; Kondratieva, 2011; Kutluca, 2013; Olivier, 2014). Institutions should adopt application GeoGebra for quality and quantity of results to ease Euclidean geometry as a demanding topic (Section 2.1, Table 2.5)

Most of researchers referred to, who have an opinion in respect of GeoGebra, regard it as top technological tool that is freely available and that can improve learners' geometry performance. Researchers who emphasise this notion are Bowie (2014), Howie, Wessels and Ndlovu (2013), Olivier (2014), Weinberg (2012), and Stols (2012). They found that GeoGebra should be directed towards (1) preparing students for their higher education and, in turn for their careers, (2) proving proof, (3) understanding algebra to attain more sophisticated levels of geometry, (4) improving students' spatial visualisation and reasoning skills, (5) transcending empirical arguments, thus engaging students in aspects of deductive argumentation, (6) engaging students in activities where they create mathematical definitions and discovering mathematical properties, (7) understanding constructive proofs, (8) offering increased potential for students to make choices about their learning space and sequence, (9) linking geometry and algebra, (10) removing complexity of mathematics, (11) understanding and creating intuitive.

Apart from these reasons, it was found that GeoGebra assisting independence of the students and be able to work anywhere apart from the school premises. From the conceptual of GeoGebra and Euclidean geometry it was found that students' service and satisfying the needs of all stakeholders are very important. Further it was important why GeoGebra fail and why Euclidean geometry has over the years being failed, which may provide insight into the importance of understanding what GeoGebra and Euclidean geometry entail. A thorough understanding was necessary of the barriers that can impede an effective, quality delivery of teaching and learning. Authors have identified a variety of reasons why GeoGebra and Euclidean geometry fail, and many researchers have been done on this subject.

Understanding the barriers that can hinder the success of GeoGebra and Euclidean geometry is essential for their sustainability. It was concluded that GeoGebra depends on the successful combined approach in avoiding these hindrances.

Therefore, it is recommended that, prior to implementing GeoGebra, institutions should first ensure that all involved in the activities of the institution are well aware of GeoGebra hindrances. In this way uncertainties in respect of the technical aspects will be eliminated. To ensure that activities do indeed address quality teaching and learning of Euclidean geometry, it is important that all involved are unanimous as far as GeoGebra is concerned.

Further explorations of main different types of geometry proofs in the form of two-column proof, flow chart proof and paragraph proof, as seen to hold key in Euclidean geometry. Subsequently GeoGebra models and simulations in building a link between students' empirical investigations and mathematics formalisations. Further explorations on the features of GeoGebra software, its uniqueness appealing to students and teachers in mathematics teaching and learning (Section 2.1.7). And consideration on mathematical understanding and model representations which are dominant themes in mathematics education reform.

The studies indicate that interactions between technological tools like GeoGebra facilitate effective teaching and learning, and technology is an essential tool for mathematics learning in the 21<sup>st</sup> century. Moreover technology promotes and develops the functional language that is so necessary for the construction of intellectual proofs and constructing proofs using GeoGebra provide opportunity for

exploration, discovery, conjecturing, refuting, reformulating and explaining (Section 1.2). GeoGebra allows students to move from empirical, visual description of spatial relations to a more theoretical abstract one (Section 1.2), and GeoGebra integrates multiple dynamic representations, various domains of mathematics, and a rich variety of computational utilities for modelling and simulations. Also GeoGebra through modelling, it helps students distinguish their mathematical conceptions, visualise the problem situations, and overcome algebraic barriers and thus focus on the geometrical reasoning behind learning tasks and enhances students' prior mathematical and cognitive background and too helps them make connections between real world situations and mathematical ideas. But barriers associated with the use of GeoGebra jeopardise smooth level of progression in knowledge advancement (Section 2.1.12).

In order to understand how students' cognitive systems associated with their mental processes while grappling and grasping key mathematical concepts, the information processing theoretical framework was used. This helps in full understanding and perception in the student cognitive development inclusive of linguistic skills, cognitions, conceptions, perception, recalling and thinking skills, problem solving skills (Section 2.2.1). In addition cognitive neuroscience is revisited to understand the neural bases of mental abilities like memory, attention, categorisations, and self-awareness, reasoning which encourages the development of social and creative thinking in student which are central to mathematics education (Section 2.2.2). Within this new environment where education is considered an invaluable commodity, GeoGebra software for its accessibility play a pivotal role and innovative teaching approach which can take mathematics teaching especially geometry to a new level.

The findings emanating from the literature review provided a conceptual and theoretical context and direction for the investigation of the present study. For the purposes of this study, an exploratory qualitative research theoretical framework was chosen because it was a means by which firstly a holistic understanding of the experiences and the students' perspectives of GeoGebra software as a tool and Euclidean geometry as totally different section after intervention (Table 4.4 and figure 5.9).

A quantitative research theoretical framework was chosen because it was a means to clearly and precisely specifying both the independent and the dependent variables under investigation, following firmly the original set of research goals, arriving at more objective conclusions and testing hypotheses, and achieving high levels of reliability of gathered data due to controlled observations.

The study presented was focused on the level of teaching and learning of Euclidean geometry with technological tool known as GeoGebra, mainly on students' academic achievements, geometry theorems' understanding and justifying of proofs. The central aim of the study presented in chapter 2 and chapter 3 was aimed at addressing the main research questions above.

The results in chapter 4 as further discussed in chapter 5 show sound agreement that students' academic achievements do improve by using GeoGebra in learning Euclidean geometry. Furthermore students do justify and verify geometry proofs and theorems. The students while using GeoGebra tools have also improved the cognitive abilities which resulted well in sound reasoning capabilities. The understanding of geometry theorems was attained as the students creatively and critically analysed, synthesised and evaluated the steps in geometry theorems by supporting with precise reasons.

The encompassing aim of the study presented in chapter 4 and chapter 5 was to identify benchmarks between Euclidean geometry academic achievements for students before and after intervention of GeoGebra software precisely for research question: "how does application GeoGebra in Euclidean geometry impact learners' performance?" The p-value which was 0.000027 as depicted by Table 4.2 chapter 4, in the post-test (academic achievement test) indicates that after intervention using GeoGebra there was improvement in learners' results.

It is argued that GeoGebra software encourages learner-centred approach of learning. As a result National Research Council suggests that GeoGebra as technological tool promotes (i) conceptual understanding, learning new ideas by connecting those ideas to students, (ii) procedural fluency in carrying out strategies efficiently and accurately, (iii) do proofs by using logical thought, explanation and justification, and (v) tendency to see mathematics as sensible and useful.

GeoGebra kept students engaged in lessons thus diminishing the level of difficulties in understanding Euclidean geometry concepts and their applications (Okazaki, 2008; Rollick, 2009)

In addressing the difficulties of proofs in geometry, the results show that students while using GeoGebra for understanding geometry theorems and justifying proofs, students were highly engaged. This is in line with Jones (2012) who indicated that students through GeoGebra can be highly engaged in activities where they create mathematical definitions and discover mathematical properties. The results affirm that GeoGebra helps in conjecturing, justification and verification about properties of geometric objects.

Students used their problem solving and evaluation skills using GeoGebra as the results confirmed. And this was further affirmed the notion of Curriculum 2005 which stated that 'students need varied experiences to construct arguments in problem solving and to evaluate the arguments.' The findings of the study alluded that integration of GeoGebra into teaching and learning of Euclidean geometry is more effective than the traditional method as GeoGebra stimulates students' mathematical thinking skills (Idris, 2009; Dimakos & Zaranis, 2010; Chew & Idris, 2012; Chew & Lim, 2013).

Academic achievement test was given to 112 participants; 56 in control group and another 56 in experimental group from five schools in rural area of Hlabisa circuit under UMkhanyakude district in KwaZulu-Natal, South Africa. The students ( $n = 56$ ) in the experimental group, were asked to complete a questionnaire about the research intensiveness of the learning with GeoGebra software (Van der Rijst et al., 2009).

It is also recommended that the views that various authors have on the meaning of quality GeoGebra software be analysed. Institutions also have to (1) understand the factors that can influence GeoGebra, (2) use existing successful manpower of education for implementation of GeoGebra, and (3) use internationally recognised technological tools for GeoGebra installation.

The preceding recommendations contain the first steps to be followed by institutions planning to establish the technology philosophy at their institutions. The conclusion



that can be drawn is that GeoGebra require specific principles and criteria for successful implementation. Therefore through the knowledge of the meaning of GeoGebra, should be optimised at institutions. A conceptual understanding of GeoGebra is one of the foundation tracks, required to move an institution from a traditional institution to a technological (GeoGebra) institution. The conceptual analysis of GeoGebra forms the initial track as a foundation whose purpose is to develop the groundwork for launching of a GeoGebra understanding.

### **6.3 CONCLUSIONS**

Conclusions for the present study are derived from three sources, that is, from an examination of the literature, theoretical framework and from empirical conclusions obtained from the findings.

#### **6.3.1 Conclusions from the Literature and Theoretical Framework**

An examination of the literature relating to students' poor performance in Euclidean geometry, improving understanding geometry theorems using GeoGebra, and assistances in justifying proofs and geometry theorems. From the evidence of this research study, it is possible to establish and reach conclusions, some of which confirms theorists' views, which are:

- There should be guidance from effective teacher to enhance students' mathematical reasoning, mathematical content and computational fluency.
- There is improvement in pedagogical competence when teaching and learning is conducted through GeoGebra software as compared to traditional approach.
- GeoGebra in Euclidean geometry teaching and learning helps in increasing the success rate as there is more collaboration resulting in high level of productive learning as students find solutions for themselves.
- The students can understand 2D and 3D objects and reverse their thinking, meaning they can start Euclidean geometry problems in any side having had more visualisation while using GeoGebra software.
- It is probable that students have high control of content, sequence and pace when learning Euclidean geometry using GeoGebra software.

- Modelling perspective is advanced using GeoGebra software as it allows the manipulation of information by assisting in problem solving.
- The students' geometric understanding levels of thinking are improved from lower level to higher level in accordance with the Van Hiele set levels which are pivotal in geometry thinking through the help of GeoGebra software.
- For automatic mental calculations, students keep the mathematics problem first in verbal working memory while computing the answer thus building long-term associations.
- The success of GeoGebra on Euclidean geometry critically depends upon the commitment of top management, who must be, and must be seen, to be involved.
- Top management must create and maintain the internal hospitable environment in which all stakeholders can become fully involved in achieving institutions' goals.
- A technological philosophy must be documented.
- Involvement, training and empowerment of educators, subject specialist and students are a must, as well as recognition that they are the primary source of a competitive advantage.
- Integrating the self-assessment procedure into the system's strategic planning process to ensure that it remains a core part of the institutions.
- Continuous improvement of technological tools used must be fitted for purpose and must be well understood by all stakeholders.
- Designing strategic plan that will contribute to directing all activities towards achieving high goals and objectives of students, institutions, districts and department's mission

## **6.4 RECOMMENDATIONS**

### **6.4.1 Suggestions for Further Research**

In analysing the present findings several areas for further are identified;

- The study suggests future work on the investigation of application of GeoGebra in basic geometry in relation to students with learning difficulties.

- Further research can be conducted to examine how GeoGebra supports primary school students' practices and explore the underlying theorems of mathematics, and how these representations enhance students' learning.

## **6.5 LIMITATIONS**

- The study only focused on the application of GeoGebra software in circle geometry for rural grade 11 high school learners in the district of UMkhanyakude, in KwaZulu Natal in the Republic of South Africa.
- For a more complete picture the opinions of the students and treatment could also have been obtained from other districts and provinces.

## **6.6 SUMMARY**

The objectives of the research have been identified and the findings of the data collected discussed. It has been concluded that using GeoGebra in the teaching and learning of Euclidean geometry (circle geometry) yields to improved students' performance, understanding and justifying of proofs and theorems of circle geometry and their application in the district of UMkhanyakude in KwaZulu Natal. The findings have revealed that technology problems remain a major problem due to lack of institutions' technicians.

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**ANNEXURE 1:**

University of Zululand

Department of Information Studies

Private Bag x1001

KwaDlangezwa

3886

30 June 2014

HEAD OFFICE KwaZulu-Natal Department of Education

247 Burger Street

Pietermaritzburg

3200

For attention: Dr Sishi

**Request for permission to conduct research in rural schools**

Dear Dr Sishi

My name is Mr M.Z. Mthethwa, and I am a Mathematics, Science and Technology Education student at University of Zululand in Kwa-Dlangezwa. The research I wish to conduct my Master's dissertation involves "Application GeoGebra in Euclidean Geometry Analysis in Rural High Schools grade 11 learners". This project will be conducted under the supervision of Prof Anass Bayaga (University of Zululand).

I am hereby seeking your consent to approach two rural high schools offering mathematics in the UMkhanyakude district to provide participants for this project.

I have provided you with a copy of my dissertation's proposal which includes copies of the measure and consent and assent forms to be used in the research process. I have as well sent you a copy of the approval letter from the University of Zululand Research Ethics Committee.

Upon completion of the study, I undertake to provide the Department of Education with a bound copy of the full research report. If you require any further information you can contact me 072 150 8983, email: mthembenimthethw3@gmail.com. Thanking you in advance for your time and consideration in this matter.

Thank you

Yours sincerely

Mr M. Z. Mthethwa

Department: Mathematics, Science and Technology Education.

Tell: 035 902 6169

Cell: 072 150 8983

Web: [www.lis.uzulu.ac.za](http://www.lis.uzulu.ac.za)

Declaration by Investigator

I Prof Anass Bayaga declare that:

I explained the information in this document to.....

..... I encouraged him to ask questions and take adequate time to answer them. I am satisfied that he adequately understands all aspects of the research, as discussed above.

I did not use an interpreter.

Signed at .....on .....20.....

.....

Signature of Investigator

**Annexure 2: Ethical declaration form for the research under the University of Zululand**

<b>Project Title</b>	Application of GeoGebra software in Euclidean in two Grade 11 Rural High Schools learners.						
<b>Principal Researcher(s)</b>	Mr M. Z. Mthethwa						
<b>Email Address</b>	Mthembenimthethwa3@gmail.com						
<b>Supervisor and Co-supervisor</b>	Prof. Anass Bayaga						
<b>Department</b>	Mathematics, Science and Technology Education						
<b>Nature of Project</b>	Honours/4 <sup>th</sup> Year		Master's	√	Doctoral		Departmental
<b>Research involves</b>	Human Health		Animals		Human Health and Animals		Desktop or Laboratory research only
Data collection from people	√	Children (Non-therapeutic research)		Children (Therapeutic research)		Other vulnerable persons	Special health and safety considerations
<b>Documents submitted for ethical clearance</b>	Project proposal			√	Participant Consent		√
	Questionnaire			√	Guardian Consent		
	Open-ended questions			√	Access to research participants		√
	Observation sheet				Research instrument permission		
	Other:						
<b>Documents still to be submitted</b>	Open-ended questions						

<b>Recommendation to the UZREC</b>			
<b>Faculty      REC Chairperson's Signature</b>		<b>UZREC Chairperson's Signature</b>	
<b>Date</b>		<b>Date</b>	

### **Annexure 3: Clearance certificate for research**

**UNIVERSITY OF ZULULAND  
RESEARCH ETHICS COMMITTEE**  
(Reg No: UZREC 171110-30- RA Level 01)



### **RESEARCH & INNOVATION**

Website: <http://www.unizulu.ac.za>  
Private Bag X1001  
KwaDlangezwa 3886  
Tel: 035 902 6887  
Fax: 035 902 6222  
Email: [MangeleSPunizulti.ac.za](mailto:MangeleSPunizulti.ac.za)

### **ETHICAL CLEARANCE CERTIFICATE**

Certificate Number	UZREC 171110-030-RA Level 01 PGM 2014/109		
Project Title	Application of GeoGebra software in Euclidean in two Grade 11 Rural High school learners		
Principal Researcher Investigator	MZ Mthethwa		
Supervisor and Co-supervisor	Prof A Bayaga		
Department	Mathematics, Science and Technology Education		
Nature of Project	Honours/4 <sup>th</sup> Year	Master's x	Doctoral Departmental

The University of Zululand's Research Ethics Committee (UZREC) hereby gives ethical approval in respect of the undertakings contained in the above-mentioned project proposal and the documents listed on page 2 of this Certificate.

**Special conditions:**

- (1) The Principal Researcher must report to the UZREC in the prescribed format, where applicable, annually and at the end of the project, in respect of ethical compliance.
- (2) Documents marked "To be submitted" (see page 2) must be presented for ethical clearance before any data collection can commence.

The Researcher may therefore commence with the research as from the date of this Certificate, using the reference number indicated above, but may not conduct any data collection using research instruments that are yet to be approved.

Please note that the UZREC must be informed immediately of

- Any material change in the conditions or undertakings mentioned in the documents that were presented to the UZREC
- Any material breaches of ethical undertakings or events that impact upon the ethical conduct of the research

**Classification:**

Data collection	Animals	Human Health	Children	Vulnerable pp.	Other
<b>X</b>					
<b>Low Risk</b>		<b>Medium Risk</b>		<b>High Risk</b>	
		<b>X</b>			

The table below indicates which documents the UZREC considered in granting this Certificate *and* which documents, if any, still require ethical clearance. (Please note that this is not a closed list and should new instruments be developed, these would require approval.)

Documents	Considered	To be submitted	Not required
Faculty Research Ethics Committee recommendation	<b>X</b>		
Animal Research Ethics Committee recommendation			<b>X</b>
Health Research Ethics Committee recommendation			<b>X</b>
Ethical clearance application form	<b>X</b>		
Project registration proposal	<b>X</b>		
Informed consent from participants	<b>X</b>		
Informed consent from parent/guardian			<b>X</b>
Permission for access to sites/information/participants	<b>X</b>		
Permission to use documents/copyright clearance			<b>X</b>
Data collection/survey instrument/questionnaire	<b>X</b>		
Data collection instrument in appropriate language		<b>Only if necessary</b>	
Other data collection instruments		<b>Only if used</b>	

The UZREC retains the right to

- **Withdraw or amend this Certificate if**
  - Any unethical principles or practices are revealed or suspected
  - Relevant information has been withheld or misrepresented
  - Regulatory changes of whatsoever nature so require
  - The conditions contained in this Certificate have not been adhered to
- **Request access to any information or data at any time during the course or after completion of the project**

The UZREC wishes the researcher well in conducting the research.

.....3

Professor Rob Midgley

Deputy Vice-Chancellor, Research and Innovation Chairperson: University Research Ethics Committee 20 August 2014



#### **Annexure 4: Participant Informed Consent-Research Study**

Title: Application GeoGebra in Euclidean geometry in UMkhanyakude District schools

1. I am aware that a research article will be published, and I agree that my name and identity be kept anonymous. I take note that Prof Anass Bayaga will have an access to my name and identity.
2. I agree that any arising queries concerning the research or/and my participation will be answered by Prof A. Bayaga.
3. I have been informed that feedback will be sent to me via email or postal on the findings obtained during the study. I have been informed that I will have access to full results of my own if I wish to view.
4. I have read all the above binding information with full understanding of my role in the research. I am satisfied by the answers to my queries and I am fully aware that I can ask to withdraw at any time if I feel uncomfortable with my participation without any harm.
5. I take this participation fully aware that there will be no incentives due to me.
6. The full ethics principles of the University of Zululand was provided to me in advance before the study commenced.
7. I am signing this informed consent exactly knowing that there are no legal obligations that will be ignored. A copy of this informed consent will be handed to me, and the original will be kept as proof of my participation.

I agree voluntarily to take part in the above- mentioned project.

Signed at .....on.....20.....

.....

Participant Signature

.....

Witness Signature

The researcher's Statement

I, Mr M. Z. MTHETHWA declare that I have explained the detailed information in this document to..... I asked him/her to ask questions that need clarification, and I conducted all our conversation in both ZULU and ENGLISH.

Signed at .....on.....20.....

.....

.....

Signature (Researcher)

Witness Signature

.....

.....

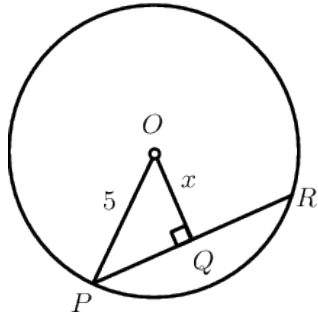
Signature (Researcher)

Witness Signature

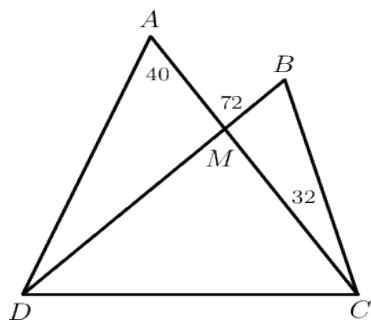


## **ANNEXURE 5:**

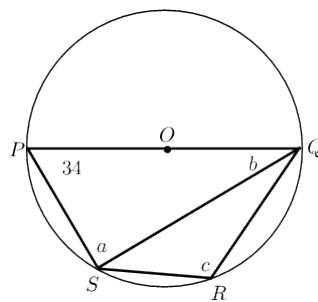
### **PRACTICAL EXERCISE**



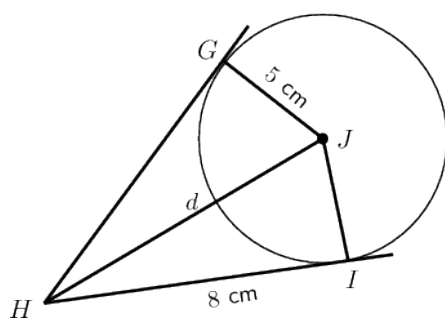
**Figure 3.18**



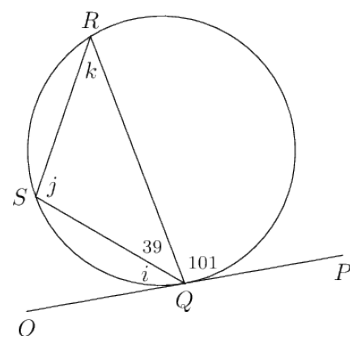
**Figure 3.19**



**Figure 3.20**



**Figure 3.21**



**Figure 3.22**

### Task 1

- 1) Complete the following statement: The line drawn from the centre of a circle to the midpoint of a chord is .....
- 2) Refer to the figure 3.15 above find the value of  $x$  given  $OP = 5$  units and  $PR = 8$  units.

### Task 2

- 1) Complete the following statement: Angles subtended by the same segment (arc) of a circle are .....
- 2) Refer to the figure 3.16 above to prove that ABCD form cyclic quadrilateral.

### Task 3

- 1) The opposite angles of a cyclic quadrilateral are .....
- 2) Refer to the figure 3.17 to calculate the values of
  - 3.4  $a$
  - 3.5  $b$
  - 3.6  $c$

### Task 4

- 1) Complete the following statement: If two tangents are drawn from the same point outside of a circle, then .....
- 2) Refer to the figure 3.18 to calculate the value of  $d$ .

### Task 5

- 1) Complete the following statement: The angle between a tangent to a circle and a chord drawn at the point of contact is .....
- 2) Refer to the figure 3.19 to find the values of
  - a.  $i$
  - b.  $j$
  - c.  $k$

**Annexure 6:**

**ACHIEVEMENT TEST**

**SCHOOL:** .....

- **Name:**
- **Date:**

**Objectives:** Stating theorems in words, calculations and applications of theorems in complex problems.

- 1. Please mark the correct answer from those given**

**The line drawn from the centre of the circle that bisects the chord is**

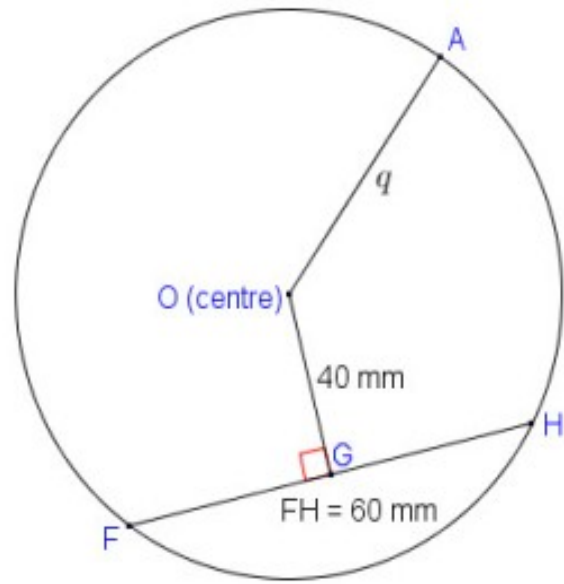
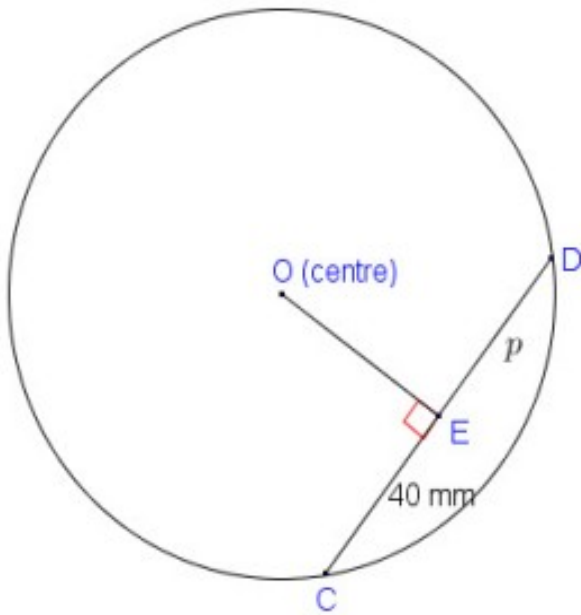
- (a) Parallel to the chord**
  - (b) Perpendicular to the chord**
  - (c) Vertical to the chord**
  - (d) Double to the chord**
- 2. Use the two following diagrams below to find the size of**

**2.1 p is**

- (a) 30 mm**
- (b) 80 mm**
- (c) 40 mm**
- (d) 50 mm**

**2.2 q is equal to**

- (a) 50 mm**
- (b) 40 mm**
- (c) 60 mm**
- (d) 30 mm**



3. Use the following diagrams below to answer letter a to f:

3.1 The size of a is

- (a)  $44^\circ$
- (b)  $49^\circ$
- (c)  $54^\circ$
- (d)  $98^\circ$

3.2 The size of b is

- (a)  $48^\circ$
- (b)  $24^\circ$
- (c)  $96^\circ$
- (d)  $44^\circ$

3.3 The size of c is

- (a)  $90^\circ$
- (b)  $40^\circ$
- (c)  $50^\circ$
- (d)  $45^\circ$

3.4 The size of d is

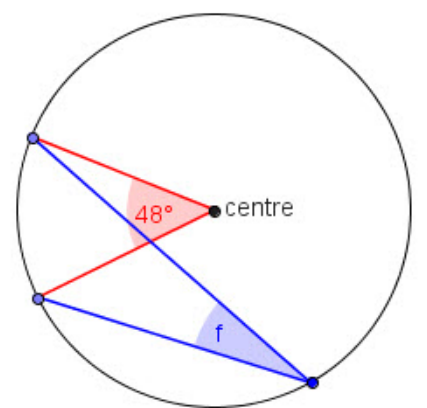
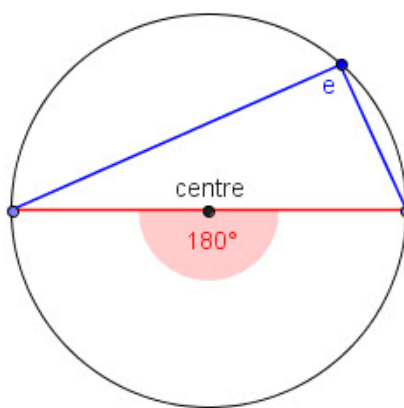
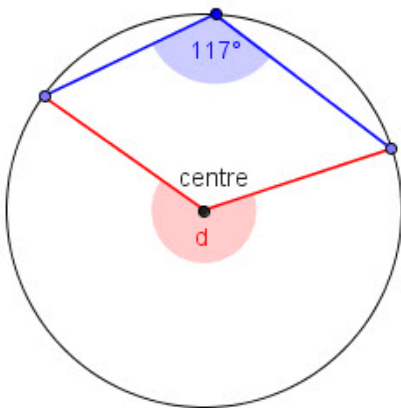
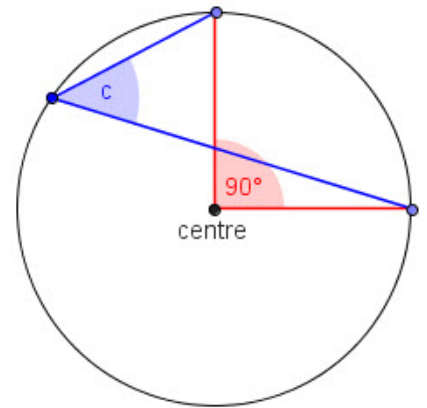
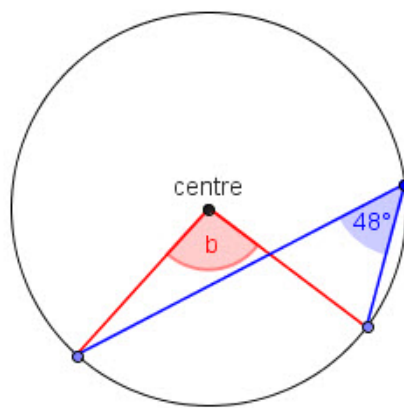
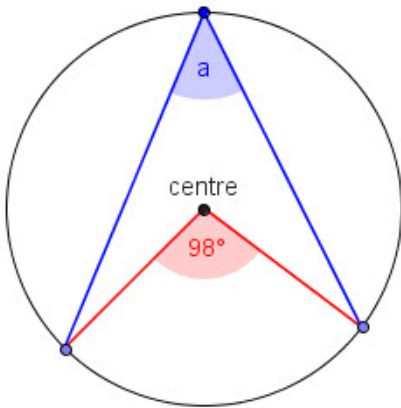
- (a)  $136^\circ$
- (b)  $58.5^\circ$
- (c)  $234^\circ$
- (d)  $117^\circ$

**3.5 The size of e is**

- (a)  $90^\circ$
- (b)  $100^\circ$
- (c)  $80^\circ$
- (d)  $180^\circ$

**3.6 The size of f is**

- (a)  $48^\circ$
- (b)  $96^\circ$
- (c)  $24^\circ$
- (d)  $44^\circ$



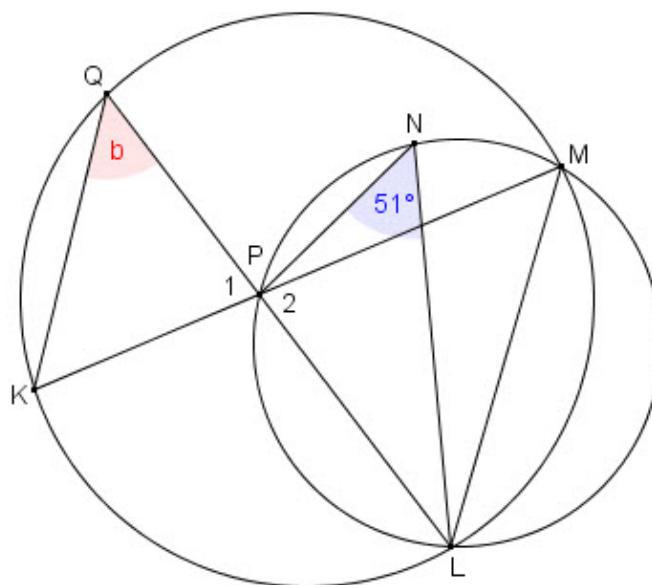
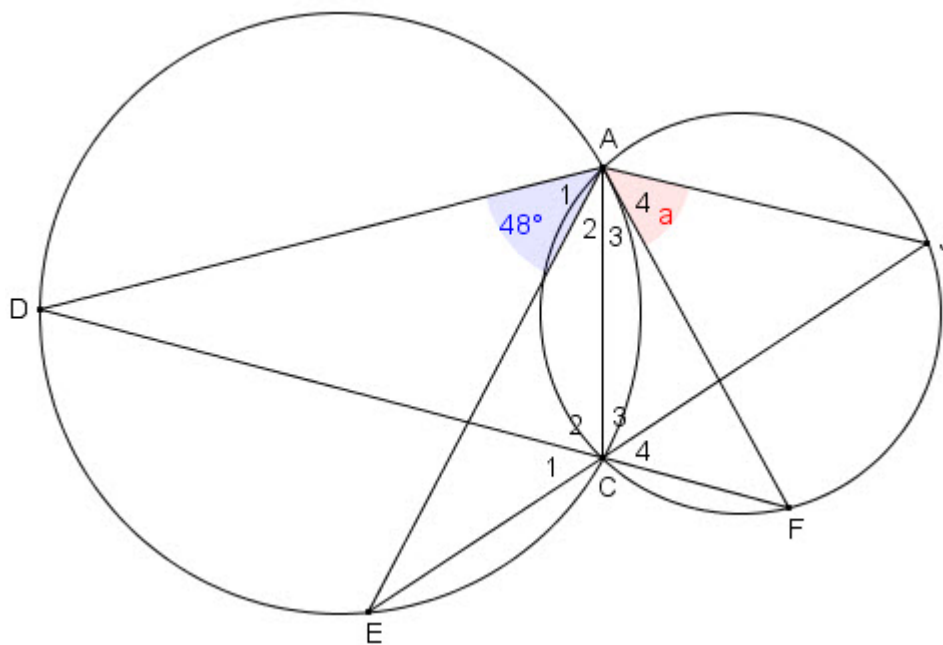
4. Use the following two figures to find the size of a and b

4.1 The size of a is

- (a)  $48^\circ$       (b)  $24^\circ$       (c)  $46^\circ$       (d)  $38^\circ$

4.2 The size of b is

- (a)  $71^\circ$       (b)  $51^\circ$       (c)  $102^\circ$       (d)  $61^\circ$



5. Complete the following statement

The opposite angles of a cyclic quadrilateral are

(a) Parallel (b) complimentary

(c)

supplementary (d)perpendicular

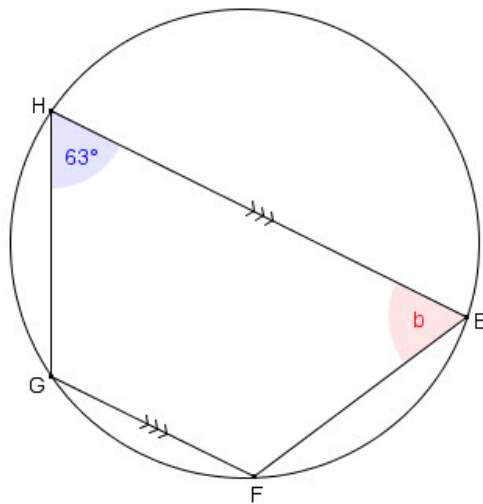
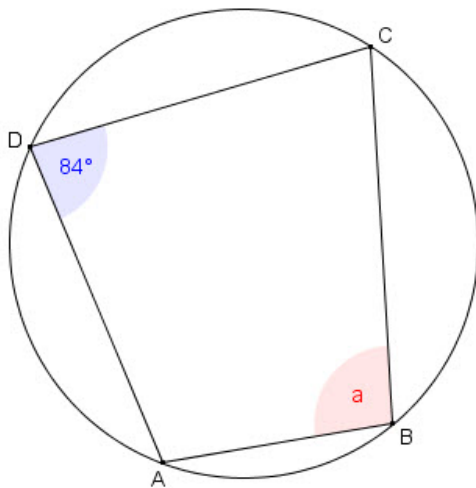
6. Refer to the following figures to find the size of a, b, c and d

6.1 The size of a is

(a)  $94^\circ$  (b)  $84^\circ$  (c)  $96^\circ$

6.2 The size of b is

(a)  $63^\circ$  (b)  $117^\circ$  (c)  $83^\circ$



## **ANNEXURE 7:**

### **RESEARCH QUESTIONNAIRE**

#### **1. GENERAL INFORMATION***(Tick where appropriate)*

Participant Name : .....

Date: .....

School A ☐ School B ☐ School C ☐ School D ☐ School E ☐

Male ☐ Female ☐  
16Yrs ☐ 17Yrs ☐ 18Yrs ☐ 19Yrs & Older ☐

#### **2. REFLECTIONS ON LEARNING EUCLIDEAN GEOMETRY USING GEOGEBRA SOFTWARE**

- Please only tick one answer from four possible given using the following numbers designed below:*

**1=strongly disagree, 2=Disagree, 3=Agree & 4=strongly agree**

Items	1	2	3	4
1. I was excited about using GeoGebra software.				
2. I like studying circle geometry lessons with GeoGebra software.				
3. I learnt a lot using GeoGebra more especially I understood the circle geometry concepts taught.				
4. I felt confident using the GeoGebra software during activities as there is accuracy.				
5. I was very engaged in the learning process.				



<b>6. I benefited a lot through teacher students' interaction.</b>				
<b>7. I was able to visualise and answer the questions after each activity.</b>				
<b>8. I was able to think creatively and critically in the discussions and during the question and answer session.</b>				
<b>9. I was able to make logical assumptions when attempting to hypothesise.</b>				
<b>10.I enjoyed learning circle geometry much using GeoGebra.</b>				
<b>11.I was able to form better connections between previous learning and new learning.</b>				
<b>12.I believe I will do well in Euclidean Geometry.</b>				
<b>13.Only brilliant learners can understand circle geometry without GeoGebra.</b>				
<b>14.Knowing Euclidean geometry will help me improve my mathematics results.</b>				
<b>15.Circle geometry is a worthwhile and important section that has helped me to develop good reasoning skills.</b>				

### 3. OPEN-ENDED QUESTIONS

- a) How has GeoGebra helped you to improve your understanding of Euclidean geometry?

.....

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- b) How exactly do you view GeoGebra in assisting you in Euclidean geometry, more especially in justifying proofs and theorems?

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- c) What attributes does GeoGebra have on your confidence when doing circle geometry, towards mathematics results improvement?

.....  
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.....  
.....  
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.....

THANK YOU VERY MUCH FOR YOUR PARTICIPATION!!!!

[preferences](#)

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# APPLICATION OF GEOGEBRA ON EUCLIDEAN GEOMETRY...

By Mthembeni Mthethwa

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Chapter One: Introduction and Overview 1.1 INTRODUCTION This study investigates the usage of geometry software called GeoGebra in five rural high schools. The essence was to assess the level of improvement in Euclidean geometry due to the application. The participants in this study were pupils who learn mathematics in Grade 11. The

**teaching and learning with the use of technology has many advantages such as providing greater learning opportunities for students, enhancing student engagement and encouraging discovery learning** 5

(Roberts, 2012, pp. 1 and White, 2012, pp. 1-2). In the

**teaching and learning of mathematics, and Euclidean geometry in particular, it is** 127

imperative that students take part in drawing, imagining, construction conjecturing, verifying, justifying shapes and making connections with the related facts of proofs and theorems. An opinion noted by Dugan (2010, pp. 1) suggested that a computer will assist students in imagining and understanding relevant constructs. It has also been highlighted that

**in a balanced mathematics program, the strategic use of technology strengthens mathematics learning and teaching (Dick & Hollebrands, 2011,** 164

pp. 1-3). There have been studies on a number of technology tools available, including calculators, Geometer Sketchpads and GeoGebra resources, such as those by Laborde, Kynigos, Hollebrands and Strasser who said: "Research on the use of technology in geometry not only offered a window on students' mathematical conceptions of notions such as angle, quadrilaterals, transformations, but also showed that technology contributes to the construction of other views of these concepts." (Laborde et al., 2006, pp. 275-304) Laborde et al., (2006) further argue that: "Research gave evidence of the research and the progress in student conceptualisation due to geometrical activities (such as construction activities or proof activities) making use of technology with the design of adequate tasks and pedagogical organisation. Technology revealed how much the tools shape the

1 1% match (student papers from 23-Feb-2015)  
Submitted to Albany State University

2 1% match (publications)  
Lingguo Bu, "Geogebra for Model-Centered Learning in Mathematics Education", Model-Centered Learning, 2011

3 1% match (publications)  
Reis, Zerrin Ayvaz, and Sebnem Ozdemir, "Using Geogebra as an information technology tool: parabola teaching", Procedia - Social and Behavioral Sciences, 2010,

4 1% match (student papers from 14-Jun-2012)  
Submitted to University of Zululand

5 1% match (student papers from 05-Jun-2013)  
Submitted to University of Malaya

6 1% match (publications)  
Kadam, Prashant, "Sample size calculation", International Journal of Ayurveda Research/09747788, 20100101

7 < 1% match (student papers from 26-Sep-2014)  
Submitted to Wawasan Open University


8 < 1% match (student papers from 07-Dec-2012)  
Submitted to Glasgow Caledonian University

9 < 1% match (student papers from 22-Mar-2015)  
Submitted to University System of Georgia

10 < 1% match (student papers from 18-Aug-2015)  
Submitted to National Institute of Technology Karnataka Surathkal

11 < 1% match (student papers from 10-Dec-2013)  
Submitted to University of Technology, Mauritius

## Annexure 9: Editing Report

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**Editor's Report**  
**RE: Mr Mthembeni Zebulon Mthethwa**  
**To: Whom It May Concern**



DISSERTATION	APPLICATION OF GEOGEBRA ON EUCLIDEAN GEOMETRY IN RURAL HIGH SCHOOLS - GRADE 11 LEARNERS
INSTITUTE	FACULTY OF EDUCATION UNIVERSITY OF ZULULAND
PURSUANT QUALIFICATION	MASTERS OF EDUCATION (MATHEMATICS EDUCATION)
SUPERVISOR	PROF. A. BAYAGA

Date	Page
20 November 15	1 of 1

Having edited the above document, I confirm that requisite changes to grammar, spelling, diction and punctuation as per English (SA) were attended to, in accordance with the necessary guidelines and writing standards adhered to at Genesis Articles. As a technical and research-based document, while content was also checked, no further changes were made to the context and substance underlying the written material and study. A large number of the changes mentioned seemed to stem from English being a second language of the writer.

Trusting the above is in order.

Sincerely,

  
**Editor:** Mark David Sing

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## Annexure 10: Editing certificate



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### Certificate of Completion

Date  
20 November 15

Page  
1 of 1

**This Certifies That The Document By:**

Mr Mthembeni Zeblon Mthethwa

**For The Purposes Outlined Below:**

DISSERTATION	APPLICATION OF GEOGEBRA ON EUCLIDEAN GEOMETRY IN RURAL HIGH SCHOOLS - GRADE 11 LEARNERS
INSTITUTE	FACULTY OF EDUCATION - UNIVERSITY OF ZULULAND
PURSUANT QUALIFICATION	MASTERS OF EDUCATION (MATHEMATICS EDUCATION)
SUPERVISOR	PROF. A. BAYAGA

**Has Been Edited To Completion**



**Editor:** Mark David Sing

**Proofreader:** Niru Sing



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